A Re-examination of Excess Sensitivity Puzzle when Consumers Forecast the Income Process

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Abstract

In this paper, we address the issue of excess sensitivity/smoothness of consumption purely from an estimation and forecasting point of view. Suppose the agent estimates the income process using the past income data and obtains best linear unbiased forecasts of future income and then she computes permanent income using these forecasts. How far will she be o¤ if she misspeci…es the income process as a trend stationary process when truly it is di¤erence stationary? Using the recently developed sensitivity indices for forecasts, we argue that for a plausible characterization of the deterministic trend term in the income process, the mispeci…cation bias for estimate of marginal propensity to consume is of second order importance.

1 Introduction

A voluminous literature now exists addressing the issue of sensitivity of consumption to news about income. Using the simple version of the permanent income hypothesis (SPIH for short) with a constant interest rate, Flavin (1981) found that consumption is more sensitive to income innovations than one predicts from the SPIH. Mankiw and Shapiro (1987) argued that Flavins result is due to misspeci…cation. of the income process. Following the paper of Nelson and Plosser (1982), a growing number of researchers hold the view that the U.S. income can be represented by a time series process with a unit root. If income is truly a di¤erence stationary process and it is misspeci…ed as a trend stationary process as in Flavin (1982), the test of permanent income hypothesis will be biased against rejection. On the other hand, if income is posited as a process with a random walk, the true marginal propensity to consume is close to unity because an innovation to income raises the annuity value of income by the same magnitude. This has a strongly counterfactual implication that consumption is too volatile while measured US consumption is too smooth compared to income. This gives rise to an excess smoothness puzzle (also known as Deaton Paradox). Subsequently many papers tried to reconcile this excess sensitivity with excess smoothness puzzle. A far from exhaustive list includes explanation like liquidity constraint (Zeldes (1985)), ‡exible real interest rate (Christiano (1987)) precautionary savings, aggregation bias (Deaton (1987)).

While there is a lot of controversy about this issue of excess sensitivity versus excess smoothness , in the last decade there is also a growing time series literature which addresses the issue of near observational equivalence between trend stationary and di¤erence stationary process and low power of the tests for unit root. An excellent survey of this literature is available in Campbell and Perron (1991). In light of this literature it appears that purely from an estimation and forecasting point of view, the di¤erence in sensitivity of consumption to income should also be of a second order magnitude regardless of whether income is posited as a trend stationary or difference stationary process. This is precisely the issue we take up in this paper. We pose this question of excess sensitivity from a purely forecasting and estimation point of view. [1] Suppose the economic agent estimates the income process using the past income data and obtains the best linear unbiased forecasts of the future income. Then she computes the permanent income using these forecasts. The question is: how far will she be off if she misspecifies the income process as a trend stationary process when truly it is difference stationary? We explore this issue using some sensitivity criterion for the forecast itself. We first suggest some measures of sensitivity (call it a sensitivity index) of the forecast with respect to the true parameter that captures the persistence of the income process.

In the next step, we derive an expression for the estimator of the marginal propensity to consume (MPC) using the SPIH equation and examine using this sensitivity index how far the MPC is estimator is off if income is mispecified as trend stationary process while truly it is difference stationary. To address this issue we use a two component model of income where income is the sum of a trend and stochastic components. It turns out that sensitivity of the MPC estimate with respect to the persistence parameter critically depends on how the trend component is econometrically modelled. If one models trend as a constant, then the estimate of MPC will be different depending on whether income is estimated as trend stationary or difference stationary process. On the other hand if the trend component is modelled as a linear trend then the difference between MPC estimates is of a second order magnitude. Since the latter model of trend is more plausible than a constant trend, we conclude that there is near observational equivalence of the point estimates of MPC regardless of whether income is modelled as a trend stationary or difference

[1]Vinod and Basu (1995) examine the consumption smoothness in alternative models of interest rate forecasting. They do not however, address the issue of excess sensitivity when income trend is modeled differently.

stationary process.

The paper is organized as follows. In the next section using a two components model of income we develop two measures of sensitivity of forecast which we call first order and second order sensitivity indices. These sensitivity indices are designed to capture the sensitivity of income forecast when income is mispecified as a trend stationary process when it is truly difference stationary. In section 3, we apply these sensitivity indices to a SPIH model and examine how this sensitivity of income forecasts translates into the sensitivity of MPC estimator. This exercise addresses the issue whether from a purely forecasting point of view how smooth the consumption process is relative to income is an issue at all. For a plausible specification of the deterministic trend our results show that the misspecification bias of the MPC estimate is of second order importance. Section 4 ends with concluding comments.

2 Sensitivity Indices for Income Forecasts

We consider the vector of time series income observations $y = (y_1, \ldots, y_t, \ldots, y_n)'$, and we write y_t as a sum of deterministic trend x_t and a stochastic term u_t,

$$y_t = x_t + u_t. \tag{2.1}$$

The trend x_t is known up to a finite dimensional parameter vector β. The stochastic component u_t follows an AR(1) process, with normal innovations,

$$\begin{aligned} \delta u_1 &= \varepsilon_1 \\ u_t &= \theta u_{t-1} + \varepsilon_t, \quad t = 2, \ldots, n \end{aligned} \tag{2.2}$$

with $\varepsilon_1, \ldots, \varepsilon_n \sim$ iid $N(0, \sigma^2)$, where $\sigma^2 > 0$ and δ is a constant ($\neq 0$). The parameter θ characterizes the persistence of the income process. If $0 < \theta < 1$, income is a strictly trend stationary process. If θ is close to unity, income process has a unit root or difference stationary

If $\delta = 1$; we have the covariance matrix of Berenblut and Webb (1973). In the time series forecasting literature, similar assumptions have been made in different contexts by several authors (see Dufour and King (1991), Magnus and Rothenberg (1988), Magnus and Pesaran (1991) and Fuller and Hasza (1980)).

The variance-covariance matrix of $u = (u_1, \ldots, u_n)$, is $\sigma^2\Omega(\theta)$, where

$$\Omega(\theta)_{ij} = \theta^{|i-j|}\left(\sum_{k=0}^{\min(i,j)-2}\theta^{2k} + \frac{1}{\delta^2}\theta^{2(\min(i,j)-1)}\right). \tag{2.3}$$

When θ is known, and the deterministic term is specified and known, the Minimum Mean Square Error $l-$ period ahead forecast of the y_t process given by (2.1) and (2.2) is

$$\hat{y}_{n+l}(\theta) = x_{n+l} + \theta^l(y_n - x_n) \tag{2.4}$$

(Harvey (1990, 1993)).

Since $x_t = x_t(\beta)$ and the parameter vector β is unknown, we use the Maximum Likelihood Estimate (MLE) of β, $\hat{\beta}(\theta)$, based on the past values of y_t, in (2.4).

We define the s^{th} order sensitivity of the forecast $\hat{y}_{n+l}(\theta)$ at $\theta = 1$ as

$$\lambda_l^{(s)} = \lim_{\theta\to 1}\frac{d^s\hat{y}_{n+l}(\theta)}{d\theta^s},$$

whenever the derivatives and the limit exist. If s=1, the derivative may be called the first order sensitivity index for income forecast. Accordingly if s=2, it is called the second order sensitivity index. Justification for our proposed definition of sensitivity can be motivated by developing $\hat{y}_{n+l}(\theta)$ as a Taylor series expansion in the neighborhood of $\theta = 1$;

$$\hat{y}_{n+l}(\theta) = \hat{y}_{n+l} + (1-\theta)\lambda_l^{(1)} + \frac{1}{2}(1-\theta)^2\lambda_l^{(2)} + \cdots. \tag{2.5}$$

We would consider $\hat{y}_{n+l}(\theta)$ and $\hat{y}_{n+l}$ to be almost equal if

$$\lambda_l^{(s)} \approx 0, \quad (s = 1, 2) \tag{2.6}$$

If the statistic $\lambda_l^{(s)}$; is near to zero we need not discriminate between stationary and non stationary (unit root) process, in sofar as forecasting is concerned. If $\lambda_l^{(1)}$ is zero or "near to zero" but $\lambda_l^{(2)}$ is not, then the order of error is $(1-\theta)^2$:

Given that the sensitivity measures exist, and since we are interested in knowing how close are $\hat{y}_{n+l}(\theta)$ and $\hat{y}_{n+l}$, we need to study how close the first order and second order sensitivity indices, $\lambda_l^{(s)}$ $(s = 1; 2)$ are to zero. A detailed analysis of the sensitivity of forecasts is done in Banerjee (1998)[2]. In this paper we shall only be concerned with 1st order sensitivity of income forecasts and its relation to sensitivity of MPC.

3 SPIH model and the Sensitivity of MPC to Specification of Income Process

We consider a standard permanent income model with constant interest rate (r) where the consumption date n (call it C_n) equals the permanent income y_n^p. We take the permanent income hypothesis in period n equal to contemporaneous ex ante permanent income, y_n^p, which is the annuity value of expected wealth

$$C_n = y_n^p \equiv (1-R)\sum_{l=0}^{\infty} R^l E_n y_{n+l} \tag{3.1}$$

$$\text{where } R = \frac{1}{1+r}$$

[2] A similar study had been done by Banerjee and Mganus (1998), on the sensitivity of the Ordinary Least Squares (OLS) estimates. Their paper Banerjee and Mangnus (1998) also shows the relation between the DW test statistic (and other related statistic like the Dicky-Fuller statistic and Kings alternative DW statistic) and sensitivity of OLS variance estimates of linear models. The proofs to Lemma A and B in the following section use some relevant results on the sensitivity of income forecasts from banerjee and Magnus (1998).

with the given income process specification.[3]

So given θ; we estimate $E_n y_{n+l}$; as $\hat{y}_{n+l}(\theta)$; and we have the estimated consumption function as

$$\hat{c}_n(\theta) = y_n^p \equiv (1 - R) \sum_{l=0} R^l \hat{y}_{n+l}(\theta)$$

therefore the MPC of the estimated income process is

$$S(\theta) \equiv \frac{\partial \hat{c}_n(\theta)}{\partial y_n} = (1 - R) \left(1 + \sum_{l=1} R^l \frac{\partial \hat{y}_{n+l}(\theta)}{\partial y_n} \right) \tag{3.2}$$

In this paper we will analyse the sensitivity of the MPC. Since $x_t = x_t(\beta)$; and the parameter vector β is unknown, we use the Maximum Likelihood Estimate (MLE) of β; $\hat{\beta}(\theta)$; based on the past values of y_t. Therefore the estimated MPC, using (2.4) takes the form, for all $0 \leq \theta \leq 1$;

$$S(\theta) = \frac{(1 - R)}{(1 - R\theta)} + (1 - R) \left(\sum_{l=0} R^l \frac{\partial x_{n+l}\left(\hat{\beta}(\theta)\right)}{\partial y_n} - \frac{1}{(1 - R\theta)} \frac{\partial x_n\left(\hat{\beta}(\theta)\right)}{\partial y_n} \right) \tag{3.3}$$

Hence the bias introduced by estimating the income process by the consumer is

$$(1 - R) \left(\sum_{l=0} R^l \frac{\partial x_{n+l}\left(\hat{\beta}(\theta)\right)}{\partial y_n} - \frac{1}{(1 - R\theta)} \frac{\partial x_n\left(\hat{\beta}(\theta)\right)}{\partial y_n} \right). \tag{3.4}$$

Furthermore if the deterministic component is a linear function of β, that is $x_t(\beta) = x_t'\beta$ where x_t is a vector regressors, the corresponding MLE of β is the standard GLS estimate of β. Using the linear model we have

If $x_t(\beta) = x_t'\beta$; then for all $0 \leq \theta \leq 1$;

$$S(\theta) = \frac{(1 - R)}{(1 - R\theta)} + (1 - R) \left(\sum_{l=0} R^l x_{n+l}' - \frac{x_n'}{1 - R\theta} \right) \frac{\partial \hat{\beta}(\theta)}{\partial y_n}. \tag{3.5}$$

It readily follows from (3.5), when the process is a nonstationary unit root process the MPC takes the form,

$$S(1) = 1 + (1 - R) \left(\sum_{l=0} R^l x_{n+l}' - \frac{x_n'}{1 - R} \right) \frac{\partial \hat{\beta}(1)}{\partial y_n}.$$

[3] Such a permanent income model can be derived from a utility maximizing problem where the utlity function is quadratic see, Blanchard and Fischer , 1989

Since the controversy about the excess sensitivity puzzle lies in the estimation of MPC at and around the unit root process we examine the sensitivity of the estimated MPC at $\theta = 1$.

To motivate our definition of sensitivity, we consider a Taylor series expansion of $S(\theta)$ around $\theta = 1$,

$$S(\theta) = S(1) + (1 - \theta) \left.\frac{\partial S(\theta)}{\partial \theta}\right|_{\theta=1} + \cdots \tag{3.6}$$

We shall consider

$$S(\theta) \approx S(1)$$

if

$$\left.\frac{\partial S(\theta)}{\partial \theta}\right|_{\theta=1} \approx 0.$$

We define the sensitivity of our estimated MPC at $\theta = 1$ as

$$s \equiv \left.\frac{\partial S(\theta)}{\partial \theta}\right|_{\theta=1}. \tag{3.7}$$

Our primary interest here is to understand the bias introduced by the estimation of the income process as trend stationary when truly it is difference stationary. It is more clearly seen in rewriting (3.6) as

$$S(\theta) - S(1) = (1 - \theta)s + \cdots$$

$S(\theta) - S(1)$ is the bias introduced by the mispecification of the process. Therefore $(1 - \theta)s$ is the misspecification bias and it is of the order $(1 - \theta)$. Hence if $s \approx 0$, this will imply that the misspeciation bias is of the order $(1 - \theta)^2$; typically the second and the subsequent terms of the Taylor series expansion (3.6). We now establish the relationship between the sensitivity of income forecasts $\left(\lambda_l^{(1)}\right)$'s and the sensitivity of MPC (s).

Theorem 1 We have,

$$s = (1 - R) \sum_{l=1}^{\infty} R^l \frac{d\lambda_l^{(1)}}{dy_n} \tag{3.8}$$

Proof of Theorem 1 The proof follows by differentiating (3.2) with respect to θ at $\theta = 1$ and interchanging the limits, which is possible since the derivative is continuous. //

We shall now study the sensitivity of two special deterministic components of interest, the constant ($x_t(\beta) = \beta_1$) and the linear trend ($x_t(\beta) = \beta_1 + t\beta_2$):

Lemma A: We have, when $x_t(\beta) = \beta_1$

a) $\hat{y}_{n+l} = y_n$;

b) $\lambda_l^{(1)} = l(y_n - y_1)$;

where if $u \sim N(0; \sigma^2\Omega(\theta))$;

Proof: Appendix

Theorem 2 We have, when $x_t(\beta) = \beta_1$

$$s = \frac{1}{r} \tag{3.9}$$

Proof of Theorem 2 The proof follows from the expression of λ_l in following lemma A and substituting in the expression for s (3.8).//

Lemma B: We have, for $x_t(\beta) = \beta_1 + t\beta_2$

a) $\hat{y}_{n+l} = y_n + l\frac{(y_n - y_1)}{n-1}$;

b) $\lambda_l^{(1)} = 0$:

Proof: Appendix

Theorem 3 We have, when $x_t(\beta) = \beta_1 + t\beta_2$,

$$s = 0 \tag{3.10}$$

Proof of Theorem 3 The proof follows from the expression of λ_l in lemma B and substituting in the expression for s in (3.8).//

This implies that the error in estimating the MPC due to misspecification of the income process is of the order $(1 - \theta)^2$: This is true when the income process has a deterministic linear trend term. This means if we ignore the bias of order $(1 - \mu)^2$, there is an observational equivalence between, a income process which has been subjected to either transient or permanent shocks (unit root in income process). That is in infinite horizon the bias to the MPC introduced by the consumer mistakenly identifying a transient income shock as a permanent shock smooths out very quickly.

On the other hand, this is not the case where the consumer faces a stream of future income which is constant but subjected to random shocks that may be transient or permanent. In this case, the if the random shocks are of transient variety but the consumer thinks its permanent variety, the bias of MPC is $(1 - \mu)/r$; which can be large is r is small.

4 Concluding Remarks

In this paper, we reexamine the excess sensitivity puzzle from a purely forecasting and estimation point of view. It is well known in the literature that the sensitivity of consumption to income innovation crucially depends on how one models the income process. If income is modeled as a trend stationary process, consumption is less sensitive to change in income than when income is modeled as a random walk. The question still remains whether from a purely estimation point of view the misspecification of the income process is of first order importance. To address this issue formally, we apply some recently developed forecasting sensitivity index to characterize the true sensitivity of the MPC estimator with respect to the misspecification of the income process. If income is modeled as a trend stationary process with a deterministic linear trend term while truly the income is difference stationary, the error of estimation of the marginal propensity to consume is of second order importance which is revealed by our sensitivity index. On the other hand, if the trend in income is a constant the bias in esimation of the MPC is of first order importance.

This tells us that how one models the deterministic trend component in income may have an effect on the observed sensitivity or smoothness of consumption to news about income.

Since modeling income as a trend stationary process with a deterministic linear trend is quite common in the literature, our theoretical result suggests that from an estimation and forecasting point of view the sensitivity of consumption to news about income may be immune to the misspecification of the income process. Since marginal propensity to consume is good measure of the degree of sensitivity of consumption to income innovation, our results suggest that the observed consumption sensitivity to income innovation is relatively robust with respect to the modelling of income trend. This result resembles the near observational equivalence between a trend stationary and difference stationary processes. The point is that a similar equivalence also holds for the marginal propensity to consume estimator with a plausible characterization of the linear trend term in the income process. Of course, the question still remains whether the same equivalence also holds for an income process with nonlinear deterministic trend . This may be a subject matter of further research.

Appendix

<u>Proof of Lemma A:</u>

We consider the model with a constant but unknown trend $x_t = \beta_1$: From equation (2.4) the $l -$ period ahead forecast is

$$\hat{y}_{n+l}(\theta) = \hat{b}_1(\theta) + \theta^l \left(y_n - \hat{b}_1(\theta) \right); \tag{A.1}$$

where

$$\hat{b}_1(\theta) = \frac{(1-\theta)^2 \sum_{t=2}^{n-1} y_t + (1-\theta)(y_n - \theta y_1) + \delta^2 y_1}{(1-\theta)^2 (n-1) + \delta^2} \tag{A.2}$$

are the Generalised Least Squares (GLS) estimates of β_1.[4]

a) We have from (A.1) and (A.2) at $\theta = 1$, $\hat{y}_{n+l} = y_n$.

b) From equation (A.1), differentiating (A.2) at $\theta = 1$, we have
$\lambda_l^{(1)} = l\left(y_n - \hat{\beta}_1(1)\right)$ and $\hat{\beta}_1(1) = y_1$.

<u>Proof of Lemma B :</u>

We consider the model with a linear trend $x_t = \beta_1 + t\beta_2$, with unknown coefficients β_1 and β_2. From equation (2.4) the $l -$ period ahead forecast is

$$\begin{aligned} \hat{y}_{n+l}(\theta) &= \hat{\beta}_1(\theta) + (n+l)\hat{\beta}_2(\theta) \\ &\quad + \theta^l\left(y_n - \hat{\beta}_1(\theta) - n\hat{\beta}_2(\theta)\right), \end{aligned} \tag{A.3}$$

where $\hat{\beta}_1(\theta)$ and $\hat{\beta}_2(\theta)$ are the GLS estimates of β_1 and β_2. We can write $\hat{\beta}_1(\theta)$ and $\hat{\beta}_2(\theta)$ as,

$$\hat{\beta}_1(\theta) = \frac{(\tau'\Omega^{-1}\tau)(i'\Omega^{-1}y) - (i'\Omega^{-1}\tau)(\tau'\Omega^{-1}y)}{(\tau'\Omega^{-1}\tau)(i'\Omega^{-1}i) - (i'\Omega^{-1}\tau)^2} \tag{A.4}$$

$$\hat{\beta}_2(\theta) = \frac{(i'\Omega^{-1}i)(\tau'\Omega^{-1}y) - (i'\Omega^{-1}\tau)(\tau'\Omega^{-1}y)}{(\tau'\Omega^{-1}\tau)(i'\Omega^{-1}i) - (i'\Omega^{-1}\tau)^2}, \tag{A.5}$$

where i is a $n \times 1$ vector of ones, and $\tau = (1, \ldots, n)'$, and $\Omega^{-1} = \Omega(\theta)^{-1}$. Note that $\Omega(\theta)^{-1}$ can be written as, (see Dufour and King (1991))

$$\Omega(\theta)^{-1} = (1-\theta)^2 I_n + \theta D'D + \left(\theta + \delta^2 - 1\right) e_1 e_1' + \theta(1-\theta) e_n e_n' \tag{A.6}$$

where I_n is the $n \times n$ identity matrix, e_1 and e_n are $n \times 1$ vectors $e_1 = (1, 0, \ldots, 0, 0)'$,

[4] It has been shown that $\hat{y}_{n+l}(\theta)$ is the BLUP (Best Linear Unbiased Predictor) when $0 < \theta < 1$; see Goldberger (1962)

$e_n = (0; 0; \ldots; 0; 1)'$, and D is defined as the $n \times n$ matrix of first differences,

$$D = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{pmatrix}. \qquad \text{(A.7)}$$

a) Using (A.6) we have $\lim_{\theta \to 1} \hat{\beta}_1(\theta) = \hat{\beta}_1$, $\lim_{\theta \to 1} \hat{\beta}_2(\theta) = \hat{\beta}_2$; and $\hat{y}_{n+l} = y_n + l\hat{\beta}_2$.

b) Differentiating (A.3) and taking limits as $\theta \to 1$; we get $\lambda_l^{(1)} = l\left(d\hat{\beta}_2(\theta) + y_n - \hat{\beta}_1 - n\hat{\beta}_2\right)$. We also have from (A.4), (A.5), $d\hat{\beta}_2 = \lim_{\theta \to 1} \frac{d\hat{\beta}_2(\theta)}{d\theta} = 0$; and $y_n = \hat{\beta}_1 + n\hat{\beta}_2$; using (A.6).

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