The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

Skating on Thin Ice:
Rule Changes and Team Strategies in the NHL

Anurag N. BANERJEE

Johan F.M. SWINNEN

Alfons WEERSINK

Research Group on Food Policy, Transition, and Development

Katholieke Universiteit Leuven

Working Paper 2004/2

www.prgleuven.be

Skating on Thin Ice : Rule Changes and Team Strategies in the NHL. [1]

Anurag N. BANERJEE
Department of Economics, University of Southampton, Southampton, UK

Johan F.M. SWINNEN,
World Bank, Washington D.C, USA
and
Department of Economics, K. U. Leuven, Leuven, Belgium

Alfons WEERSINK,
Dept of Agricultural Economics and Business, University of Guelph,
Guelph, Ontario, CANADA

February, 2004

[1]The opinions expressed in this paper are those of the authors and do not represent those of the institutions they are associated with.

Abstract

In an effort to stimulate a more exciting and entertaining style of play, the National Hockey Association (NHL) changed the rewards associated with the results of overtime games. Under the new rules, teams tied at the end of regulation both receive a single point regardless of the outcome in overtime. A team scoring in the sudden-death 5-minute overtime period would earn an additional point. Prior to the rule change in the 1999-2000 season, the team losing in ovetime would receive no points while the winning team earned 2 points. This paper presents a theoretical model to explain the effect of the rule change on the strategy of play during both the overtime period and the regulation time game. The results suggest that under the new overtime format equally powerful teams will play more offensively in overtime resulting in more games decided by a sudden-death goal. The results also suggest that while increasing the likelihood of attacking in overtime, the rule change would have a perverse effect on the style of play during regulation by causing them to play conservatively for the tie. Empirical data confirm the theoretical results. The paper also shows that increasing the rewards to a win in regulation time would not prevent teams from playing defensively during regular time.

JEL Classification C72, L83

Keywords: Ice Hockey, Game Theory, NHL Overtime Rule.

1 Introduction

Economic studies on the influence of rules, institutions and incentive structures on behavior are complicated by the fact that incentives and observations on behavior are affected by a variety of factors outside the relationship of interest. For this reason, environments which eliminate some of these outside influences are of particular interest. Some recent studies have used observed data from sporting competitions to assess the effects of incentives. Rule changes in sports provide a unique natural experiment to test the consequences of theoretical payoff structures. Similar rule changes were recently introduced in professional ice hockey. For example, the performance of individual competitors has been shown to increase directly with the level of rewards in the sports ranging from bowling (Ehrenberg and Bognanno 1990) to auto racing (Maloney and Terkun, 2002). In terms of team sports, Banerjee and Swinnen (2003) present a game theoretic model along with empirical evidence that the introduction of a sudden death rule in soccer has led to more conservative play.

Despite the inherent attraction associated with the speed and action of ice hockey, the popularity in North America of the game's premier professional league, the National Hockey League (NHL), lags behind that of the other three major professional sports leagues, the National Football League (NFL), Major League Baseball (MLB), and the National Basketball Association (NBA). The NHL has attempted to increase the general appeal of the game by introducing several rule changes ranging from tougher fighting penalties to increasing the size of the area behind the nets. One of the most significant changes was the introduction in the 1983-84 season of a 5 minute, sudden-death overtime period to settle any regular season games that had ended in a tie. A team scoring in overtime would receive 2 points for the win while the losing team got 0 points for the loss. Each team received a single point if the game remained tied at the end of overtime. In an effort to combat the perceived conservative play in overtime, the NHL implemented a new point structure in the 1999-2000 season. Teams tied at the end of regulation would both receive a single point regardless of the outcome in overtime. A team scoring in the sudden-death overtime would earn an additional point. While the intent of the rule change was to increase the excitement of the game through a more attacking style of play, the change in the reward system could have a perverse effect on the style of play.

The purpose of this paper is to determine the effect of changes in overtime rules on the play by NHL teams during regulation time and in overtime. The paper begins with a game theoretic model that determines optimal team strategies under alternative overtime point systems. The theoretical results suggest that rule changes should result in more overtime games being decided within the extra-time period but that more games will end up tied after the normal 60 minute regulation time. Empirical results are then presented confirming the hypotheses in section 3. Implications of other rule changes are assessed in the conclusions.

2 Payoff Structures

The time points are defined such that teams can score a maximum of one goal during each time point.[1] We begin by breaking the game involving teams A and B into an arbitrary set of T discrete time points $N = \{1, \ldots, t, \ldots, T\}$. For example, t may represent a minute of play in a hockey game.

Consider the game from the point of view of team A. At every t, the state of the game for team A is described by the random state vector X_t, where

$$X_t = \begin{cases} 1, & \text{if team A scores a goal, at time } t, \\ 0, & \text{if neither team scores a goal, at time } t, \\ -1, & \text{if team B scores a goal, at time } t. \end{cases}$$

The probabilities of this random variable are given by

$$\Pr(X_t = 1) = p, \ \Pr(X_t = -1) = q \text{ and } \Pr(X_t = 0) = r = 1 - p - q.$$

The probability of team A scoring a goal is the same as the probability of team B conceding a goal. This implies that the state vector of team B is $-X_t$. At every instant t there is a zero sum game between A and B implying that both cannot score during the same time point. The probabilities p and q are functions of the actions of the teams.

Each team decides before the start of each time point on their playing strategy. We assume the strategies are defined in terms of two actions, defensive play, denoted by L, and offensive play, denoted by H. We define the actions sets of the two teams at a given moment t as S_t^A and S_t^B, where $S_t^i = \{H, L\}$, $i = A, B$.

The probability functions p and q are defined as

$$p : S_t^A \times S_t^B \rightarrow [0,1] \text{ and } q : S_t^A \times S_t^B \rightarrow [0,1].$$

We assume that the probability of scoring is higher with offensive play; formally:

$$p(H,.) > p(L,.) \text{ and } q(.,H) > q(.,L) \tag{2.1}$$

The ex-ante strategy sets of the two teams for the entire game are $\mathbf{S}^A$ and $\mathbf{S}^B$, where $\mathbf{S}^i = \prod_{t \in N} S_t^i$ ($i = A, B$), a set consisting of all possible $T-$tuple sequences of H and L. So an element of $\mathbf{S}^i$is the vector $\underline{s}^i = (s_1^i, \ldots, s_t^i, \ldots, s_T^i)$, where $s_t^i \in S_t^i$.

3 Optimal Overtime Strategies

The framework defined above is applied initially to the overtime period to determine how a changing reward system affects play in overtime. Overtime is the 5 minute additional period NHL teams play

[1] In hockey, there are three stop-time periods of 20 minutes in a regulation game and a 5 minute sudden-death overtime period if the game is tied after 60 minutes of regulation play.

at the end of a regular season game when the score is tied. The game may end before completion of the overtime period if a team scores a goal. The team that scores first in overtime wins the game. Since its introduction to the NHL in 1982, the length of overtime and its sudden-death format have remained unchanged. However, the rewards associated with overtime play were changed in the 1999-2000 season as described further below.

The sudden-death nature of overtime can be modeled as a stochastic process W_t defined in terms of a goal difference at time t. Initially, the goal difference is zero, $W_0 = 0$. If team A (B) scores during the next time interval, then $W_t = 1$ ($W_t = -1$) and team A (B) wins the game with no chance for the other team to come back and score. The team with a positive goal difference is the winner. Thus, W_t is similar to a random walk with absorbing barriers at 1 and -1. The transition probabilities associated with this stochastic process are given by;

$$\Pr\left(W_t = d' | W_{t-1} = d\right) = \begin{cases} 1 & \text{if } d' = d \neq 0, \\ p & \text{if } d' = +1 \text{ and } d = 0, \\ r & \text{if } d' = d = 0, \\ q & \text{if } d' = -1 \text{ and } d = 0, \\ 0 & \text{otherwise.} \end{cases}$$

with the initial condition of $\Pr\left(W_0 = 0\right) = 1$.

3.1 The old overtime rule

From the introduction of overtime to the NHL in 1982-1983 to the completion of the 1998-1999 season, the team scoring in overtime received two points for the win while the losing team received none. Both teams earned a single point if nobody scored and the game was still tied after the overtime period. We refer to this payoff system as the "old rule".

We can define the incentive scheme $Uold_T^A$ for team A as,

$$Uold_T^A = I\left\{W_T > 0\right\} - I\left\{W_T < 0\right\} \tag{3.2}$$

where $I\{.\}$ is the indicator function. Team B's end of play payoff will be $Uold_T^B = -Uold_T^A$. If team A at the end of the game has more goals than team B, then team A wins (receives 1) and otherwise team B wins (team A gets -1). The teams get nothing when the goal difference is zero. Since the strategies will only depend on the differences in payoff, this normalisation does not matter. The expected payoff

$$E\left(Uold_T^A\right) = \Pr\left(W_T > 0\right) - \Pr\left(W_T < 0\right),$$

under the old incentive scheme are derived for each team in Lemma 1 as[2]:

Lemma 1 *Under the old rule, the expected payoff of team A, $Vold_T^A\left(\underline{s}^A, \underline{s}^B\right) = E\left(Uold_T^A\right)$, is:*

$$\sum_{t=1}^{T} R\left(\underline{s}^A, \underline{s}^B \middle| \Omega_{t-1}\right)\left(p\left(s_t^A, s_t^B\right) - q\left(s_t^A, s_t^B\right)\right),$$

[2]Formal proofs of lemmas and theorems are in the appendix.

and the expected payoff for team B is

$$Vold_T^B\left(\underline{s}^A,\underline{s}^B\right)=-Vold_T^A\left(\underline{s}^A,\underline{s}^B\right)$$

where $R\left(\underline{s}^A,\underline{s}^B\middle|\,\Omega_{t-1}\right)=\prod_{t\in\Omega_{t-1}}r\left(s_t^A,s_t^B\right)$ *and* $\Omega_{t-1}=\{1,\ldots,t-1\}$.

With the payoff structures associated with the overtime point systems established above, we can now define the strategies of play that will maximize a team's expected payoff (or minimize in the case of team B). To derive the equilibrium strategy sequences, we assume that the teams involved are equally likely to score under similar situations and similar styles of play. This is a reasonable assumption given the fact that a game is more likely to go to extra times if the teams are of similar qualities (a similar assumption is made by Palomino et al. (1999) and Banerjee and Swinnen (2003)). Formally: teams A and B are defined to be equally powerful if and only if

$$p\left(s^A,s^B\right)=q\left(s^B,s^A\right) \tag{3.3}$$

for all $\left(s^A,s^B\right)\in S^A\times S^B$.

Under the assumption of equality, using lemma (**??**) we have

$$Vold_T^A\left(\underline{s}^A,\underline{s}^B\right)=-Vold_T^A\left(\underline{s}^A,\underline{s}^B\right).$$

This implies if team A maximises (team B minimises) its expected payoffs a) the value of the game is 0 (See Owen 1995, page 29) and b) at any equilibrium the teams will play similar sequences of actions $\underline{s}^A=\underline{s}^B=\underline{s}$.

To see this, assume for a moment that there is only one period in a game. (This assumption is purely for illustration and will be generalized later.) Let team A chose strategy, $\underline{s}^A$ and team B $\underline{s}^B$. Then with the assumptions of equality of teams the expected payoff matrix (of Team A) under the old rule is:

$s^B\diagdown s^A$	H	L
H	0	$-\left(p(H,L)-p\left(L,H\right)\right)$
L	$p(H,L)-p\left(L,H\right)$	0

(3.4)

where the columns (rows) represent the possible strategy of team A (B).

From the above matrix if $p(H,L)>p\left(L,H\right)$, then team A will not be better off deviating from H if team B plays H. Therefore $[H,H]$ is the equilibrium. If $p(H,L)<p\left(L,H\right)$, then the last row and the last column dominates and $[L,L]$ is the equilibrium.

Denote $p(H,L)-p\left(L,H\right)=\alpha$. The equilibrium therefore depends on the sign of α. We refer to the value of α as the comparative advantage of team A. Team A has a comparative advantage in playing offensive hockey if and only if

$$p\left(H,L\right)>p\left(L,H\right) \tag{3.5}$$

A team has a comparative advantage in playing offensive hockey if the team is more likely to score playing an offensive strategy against a defending team of equal quality compared to when it plays defensively against an equal quality team playing offensively. The previous analysis implies that if team A has a comparative advantage in playing offensive ($\alpha > 0$), then so would team B because both are equal. Thus, playing offensive is the optimal strategy for both teams in all time periods. This also holds for general $T - period$ strategies of the overtime game:

Theorem 1 *Under the old rule and assuming the teams are of equal quality, if team A maximises (team B minimises) its expected payoff, and:*

a) if teams have a comparative advantage in playing offensive hockey ($\alpha > 0$) then $(\underline{H}, \underline{H})$ is the only equilibrium;

b) if $\alpha < 0$, then $(\underline{L}, \underline{L})$ is the only equilibrium,

c) and the value of the game is zero for all T,

where $\underline{H}$ is a $T-$vector of $H's$ and $\underline{L}$ is a T-vector of $L's$.

Theorem 1 implies that in equilibrium the optimal strategy sequences chosen by the teams depend on whether the teams have a "comparative advantage" in playing offensively or defensively. This is the generalisation of the result of the one period game. In some sense it is even stronger as it holds for any given T. This implies that even if the teams are given the freedom to change their style of play as many times as they like, they will always chose the style which is consistent with their comparative advantage.

3.2 The new overtime rule

In the 1999-2000 season, the NHL introduced a new reward system for overtime games. Under the new rule (also called Rule 89), both teams receive one point if they draw in regulation regardless of the overtime result. A team that wins the game in overtime gets an additional point, hence two in total, as in the old rule. However, the team that loses in overtime still keeps its one point- unlike the old overtime rule.

The new incentive scheme is no longer a zero sum game. If team A wins in overtime, team B receives the same reward as if the game was tied. In terms of the notation defined above, team A receives 1 point and team B earns 0 points which is what both teams would get if the game remained tied after overtime. Again, since the strategies will only depend on the differences in payoff, this normalisation does not matter.

Define the end of play payoff $Unew_T^A$ of team A under the new rule as,

$$Unew_T^A = I\{W_T > 0\} \tag{3.6}$$

where $I\{.\}$ is the indicator function.

Team B's end of play payoff will be

$$Unew_T^B = I\{W_T < 0\} \tag{3.7}$$

The payoff structure with the new rule for overtime scoring is given in Lemma 2.

Lemma 2 *Under the new rule the expected payoff of team A,* $Vnew_T^A\left(\underline{s}^A, \underline{s}^B\right) = E\left(Unew_T^A\right)$ *is:*

$$\sum_{t=1}^{T} R\left(\underline{s}^A, \underline{s}^B \middle| \Omega_{t-1}\right) p\left(s_t^A, s_t^B\right),$$

and the expected payoff of team B, $Vnew_T^B\left(\underline{s}^A, \underline{s}^B\right) = E\left(Unew_T^B\right)$*, is:*

$$\sum_{t=1}^{T} R\left(\underline{s}^A, \underline{s}^B \middle| \Omega_{t-1}\right) q\left(s_t^A, s_t^B\right),$$

where $R\left(\underline{s}^A, \underline{s}^B \middle| \Omega_{t-1}\right) = \prod_{t\in\Omega_{t-1}} r\left(s_t^A, s_t^B\right)$ *and* $\Omega_{t-1} = \{1, \ldots, t-1\}$.

The intended effect of the change in the overtime incentive scheme was to encourage offensive play. To determine the effects, we evaluate it from the point of view of Team A. Since Team B's expected utility function is symmetric, it will behave the same way as team A.

With the payoff structures associated with the overtime point systems established above, we can now define the strategies of play that will maximize a team's expected payoff. To derive the equilibrium strategy sequences, we assume as in the old overtime game, that the teams involved are equally likely to score under similar situations and similar styles of play (3.3). Note that this is not a zero sum game anymore, for both teams the payoffs are always positive, which we shall denote in a payoff matrix as $\left(Vnew_T^A\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}^B\right), Vnew_T^B\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}^B\right)\right)$. But since the teams are similar, the teams will play similar sequences of actions $\underline{\mathrm{s}}^A = \underline{\mathrm{s}}^B = \underline{\mathrm{s}}$.

To see this, assume for a moment that there is only one period in a game. (Again this assumption is purely for illustration and will be generalized later.). Let team A chose strategy, s^A and team B s^B. Then with the assumption of equality of teams, the expected payoff matrix (of Team A and Team B) under the new rule is:

$s^B \diagdown s^A$	H	L
H	$(p(H,H), p(H,H))$	$(p(L,H), p(H,L))$
L	$(p(H,L), p(L,H))$	$(p(L,L), p(L,L))$

(3.8)

where the columns (rows) represent the possible strategy of team A (B). From the above matrix since $p(H,H) > p(L,H)$ and $p(H,H) > p(H,L)$ by (2.1), then team A will not be better off deviating from H if team B plays H. Therefore $[H,H]$ is the equilibrium. This result is true for any $T-period$ strategy:

Theorem 2 *Under the new rule and assuming the teams are of equal quality, if team A maximises (team B minimises) its expected payoff and:*

a) if teams have a comparative advantage in playing offensive hockey ($\alpha > 0$) then ($\underline{H}, \underline{H}$) is the equilibrium;

b) and $\lim_{T\to\infty} Vnew_T^i\left(\underline{H}, \underline{H}\right) = \frac{1}{2}$*, i=A,B.*

where $\underline{H}$ *is a* $T-$*vector of* H'*s.*

To illustrate the implication of Theorem 2, consider the case that team A has a comparative advantage in playing defensive hockey ($\alpha < 0$). Note that in (3.8) even if, $p(H, L) < p(L, H)$, $[L, L]$ is not a equilibrium. Therefore even though the team has a comparative advantage in playing defensively, it will not do so since playing offensively increases the expected payoff to the team. This implies that even teams with a comparative advantage of playing defensive hockey ($\alpha < 0$) will not play defensively throughout the game. Thus, our model shows that the rule change will lead to an increase in offensive play in overtime.

4 Impact on Regular Time Game Strategies

Changing the points awarded in overtime will not only affect the style of play in overtime but may also have an effect on the play during the 60 minute regular time game. To show this, we define a random variable Z_t, as the goal difference between teams A and B from the perspective of team A at time t. This stochastic process is a Markov chain (more precisely a random walk on integers) with $Z_t = Z_{t-1} + X_t, t = 1, \ldots, T$ and $Z_0 = 0$. The transition probabilities of this stochastic process are

$$\Pr(Z_t = d' | Z_{t-1} = d) = \begin{cases} p & \text{if } d' = d + 1 \\ r & \text{if } d' = d \\ q & \text{if } d' = d - 1 \\ 0 & \text{otherwise} \end{cases} . \tag{4.9}$$

with the initial condition of $\Pr(Z_0 = 0) = 1$.

A winner is determined after the regular time period of T, if there is a positive goal difference. If there is no goal difference, the game goes to overtime with the payoff structures defined in the previous section. Therefore, we define the end of play payoff $Ureg^A$ of team A as,

$$Ureg^A = I\{Z_T > 0\} - I\{Z_T < 0\} + \lambda I\{Z_T = 0\}$$

where $I\{.\}$ is the indicator function and λ is the expected outcome from the extra time game. Since the overtime game under the old rule was zero sum with a skew symmetric payoff matrix, the value of the game is zero (Theorem 1 part c)). Hence, under the old rule, $\lambda = 0$. Under the new rule, the game is no longer a zero sum game, and the expected value of overtime is always positive, but the exact value depends on the number of periods,-T the strategy has been considered. But as a limiting case, i.e when T is large enough, $\lambda = \frac{1}{2}$(Lemma 2 part b)).

Team B's end of play payoff will be

$$Ureg^B = I\{Z_T < 0\} - I\{Z_T > 0\} + \lambda I\{Z_T = 0\}$$

Recall that we assumed that a maximum of one goal can be scored in each period t and that $N = \{1, \ldots, t, \ldots, T, \}$. Define then T_1 as the periods in which team A scores a goal, T_2 as the periods in which Team B scores a goal and T_3 when neither scores. Therefore the probability that team A scores at times $t \in T_1$ and team B scores (team A concedes a goal) at times $t \in T_2$, is

$$\Pr(X_{t \in T_1}, X_{t \in T_2}, X_{t \in T_3}).$$

Then $|T_1|$ is the number of goals scored by team A and $|T_2|$ is the number of goals scored by team B. So if $|T_1| - |T_2| = d$, team A wins by d goals. Notice that T_1, T_2 and T_3, is an arbitrary partition of N, so team A can win by d goals with any such partition as long as $|T_1| - |T_2| = d$, therefore summing over all such partitions will give us the probability of winning by d goals as,

$$\Pr(Z_T = d) = \sum_{T_3, T_1, T_2 : |T_1| - |T_2| = d} \Pr(X_{t \in T_1}, X_{t \in T_2}, X_{t \in T_3}).$$

We can then derive the expected payoff of Team A,

$$E\left(Ureg^A\right) = \sum_{d=1}^{T} \left[\Pr(Z_T = d) - \Pr(Z_T = -d)\right] + \lambda \Pr(Z_T = 0)$$

and Team B,

$$E\left(Ureg^B\right) = \sum_{d=1}^{T} \left[\Pr(Z_T = d) - \Pr(Z_T = -d)\right] + \lambda \Pr(Z_T = 0)$$

as follows,

Lemma 3 *In the regular game, the expected payoff of team A,* $V_T^A reg\left(\underline{s}^A, \underline{s}^B\right) = E\left(Ureg^A\right)$ *is:,*

$$\begin{aligned} &\sum_{d=1}^{T} \sum_{T_1, T_2 : |T_1| - |T_2| = d} R\left(\underline{s}^A, \underline{s}^B \middle| T_3\right) D\left(\underline{s}^A, \underline{s}^B \middle| T_1, T_2\right) \\ &+ \lambda \sum_{T_1, T_2 : |T_1| = |T_2|} R\left(\underline{s}^A, \underline{s}^B \middle| T_3\right) P\left(\underline{s}^A, \underline{s}^B \middle| T_1\right) Q\left(\underline{s}^A, \underline{s}^B \middle| T_2\right), \end{aligned}$$

and the expected payoff of team B, $V_T^B reg\left(\underline{s}^A, \underline{s}^B\right)$ *is:*

$$\begin{aligned} &-\sum_{d=1}^{T} \sum_{T_1, T_2 : |T_1| - |T_2| = d} R\left(\underline{s}^A, \underline{s}^B \middle| T_3\right) D\left(\underline{s}^A, \underline{s}^B \middle| T_1, T_2\right) \\ &+ \lambda \sum_{T_1, T_2 : |T_1| = |T_2|} R\left(\underline{s}^A, \underline{s}^B \middle| T_3\right) P\left(\underline{s}^A, \underline{s}^B \middle| T_1\right) Q\left(\underline{s}^A, \underline{s}^B \middle| T_2\right), \end{aligned}$$

where T_1, T_2, T_3 *is a partition of* N *and*

$$D\left(\underline{s}^A, \underline{s}^B \middle| T_1, T_2\right) = P\left(\underline{s}^A, \underline{s}^B \middle| T_1\right) Q\left(\underline{s}^A, \underline{s}^B \middle| T_2\right) - Q\left(\underline{s}^A, \underline{s}^B \middle| T_1\right) P\left(\underline{s}^A, \underline{s}^B \middle| T_2\right),$$

s.t $P\left(\underline{s}^A, \underline{s}^B \middle| \Omega\right) = \prod_{t \in \Omega} p\left(s_t^A, s_t^B\right)$, $Q\left(\underline{s}^A, \underline{s}^B \middle| \Omega\right) = \prod_{t \in \Omega} q\left(s_t^A, s_t^B\right)$ *and* $R\left(\underline{s}^A, \underline{s}^B \middle| \Omega\right) = \prod_{t \in \Omega} r\left(s_t^A, s_t^B\right)$.

Note that when the overtime game is under the old rule (i.e , $\lambda = 0$), the expected payoff for team B is

$$V_T^B reg\left(\underline{s}^A, \underline{s}^B\right) = -V_T^A reg\left(\underline{s}^A, \underline{s}^B\right).$$

An alternative way to compute $\Pr(Z_T = d)$ is to use the Markov equations in (4.9), by defining,

$$\Pi_{t,d}\left(\underline{s}^A, \underline{s}^B\right) = \Pr(Z_t = d).$$

Then $\Pr(Z_T = d)$, is the solution for the recursive system

$$\begin{aligned} \Pi_{t,d}\left(\underline{s}^A, \underline{s}^B\right) &= \begin{array}{l} p\left(s_t^A, s_t^B\right)\Pi_{t-1,d+1}\left(\underline{s}^A, \underline{s}^B\right) + q\left(s_t^A, s_t^B\right)\Pi_{t-1,d-1}\left(\underline{s}^A, \underline{s}^B\right) \\ +r\left(s_t^A, s_t^B\right)\Pi_{t-1,d}\left(\underline{s}^A, \underline{s}^B\right) \end{array} \\ \Pi_{0,0}\left(\underline{s}^A, \underline{s}^B\right) &= 1 \text{ and } \Pi_{0,d}\left(\underline{s}^A, \underline{s}^B\right) = 0 \text{ for } d \neq 0. \end{aligned} \tag{4.10}$$

As before we assume that the teams which are playing are equally likely to score under similar situations and similar styles of play (see equation (3)).

4.1 Under the old overtime rule

The extratime game under the old rule was zero sum with a skew symmetric payoff matrix. The value of the game is zero (Theorem 1 part c)). Hence under the old rule $\lambda = 0$. Therefore under the assumption of equality, using lemma (3) we have

$$Vreg_T^A\left(\underline{s}^A, \underline{s}^B\right) = -Vreg_T^A\left(\underline{s}^B, \underline{s}^A\right).$$

The payoff matrix is skew-symmetric, which implies that under the old overtime rule if team A maximises (team B minimises) their expected payoffs a) the value of the game is 0 (See Owen 1995, page 29) and b) at any equilibrium the teams will play similar sequences of actions $\underline{s}^A = \underline{s}^B = \underline{s}$.

The simplest way of dividing the regular time game is to think of it in terms of three equal periods. This makes intuitive sense as the regular time play is divided into three 20-minute (stop-time) periods. The teams may chose different strategies in each of the three periods. If team A chooses strategy, $\underline{s}^A$ and team B $\underline{s}^B$,the expected payoff matrix in regular time for Team A (when T = 3) is:

$s^B \diagdown s^A$	H,H,H	H,H,L	H,L,H	L,H,H	H,L,L	L,H,L	L,L,H	L,L,L
H,H,H	0	αA	αA	αA	αB	αB	αB	αC
H,H,L	$-\alpha A$	0	0	0	αD	αD	αE	αF
H,L,H	$-\alpha A$	0	0	0	αD	αE	αD	αF
L,H,H	$-\alpha A$	0	0	0	αE	αD	αD	αF
H,L,L	$-\alpha B$	$-\alpha D$	$-\alpha D$	$-\alpha E$	0	0	0	αG
L,H,L	$-\alpha B$	$-\alpha D$	$-\alpha E$	$-\alpha D$	0	0	0	αG
L,L,H	$-\alpha B$	$-\alpha E$	$-\alpha D$	$-\alpha D$	0	0	0	αG
L,L,L	$-\alpha C$	$-\alpha F$	$-\alpha F$	$-\alpha F$	$-\alpha G$	$-\alpha G$	$-\alpha G$	0

(4.11)

where

$$
\begin{aligned}
A &= (1 - 2(1 - p(H,H))p(H,H)), \\
B &= (1 + r(H,L)\, r(H,H)), \\
C &= (3r(H,L) + \alpha^2 + 6p(H,L)\, p(L,H))), \\
D &= (1 - p(H,H)\, r(L,L) - p(L,L)), \\
E &= (r(H,L) + \alpha^2 + 2p(H,L)\, p(L,H))), \\
F &= (1 + r(H,L)\, r(L,L)), \\
G &= (1 - 2(1 - p(L,L))p(L,L))
\end{aligned}
$$

Note that A, B, C, D, E, F and G are positive numbers. Assuming there is no comparative advantage to offensive play by the teams ($\alpha > 0$), team A will not be better off deviating from (H, H, H) if team B plays (H, H, H) during regulation time. Therefore, $[(H, H, H), (H, H, H)]$ is the equilibrium when the old overtime ($\lambda = 0$) incentive rule prevailed. If $\alpha < 0$, then the last row and the last column dominates and $[(L, L, L), (L, L, L)]$ is the equilibrium. This also holds for a general $T - period$ strategy and the result is summarized in Theorem 3.

Theorem 3 *Under the old rule and assuming the teams are of equal quality, if team A maximises (team B minimises) its expected payoff in the regular time game and:*

a) if teams have a comparative advantage in playing offensive hockey ($\alpha > 0$) then ($\underline{H}$, $\underline{H}$) is the only equilibrium;

b) if ($\alpha < 0$), then ($\underline{L}$, $\underline{L}$) is the only equilibrium,

c) and the value of the game is zero for all T,

where $\underline{H}$ is a $n-$vector of $H's$ and $\underline{L}$ is a T-vector of $L's$.

Theorem 3 (proof: see appendix) implies that under the old overtime rule in equilibrium the optimal regular time strategy sequences chosen by the teams depends on whether the teams have a "comparative advantage" in playing offensively or defensively. This is the generalisation of the result for 3 periods as discussed above. This implies that even if the teams are given the freedom to change their style of play as many times as they like, they will always chose the style which is consistent with their comparative advantage, under the old rule.

4.2 Under the new overtime rule

The new overtime rule may have a perverse effect of the style of play during the regular time game. Note that under the new rule, the game is no longer zero sum and the expected value of the overtime is always positive. The exact value depends on the number of periods (T) but the strategy as a limiting case, i.e when T is large enough, $\lambda = \frac{1}{2}$(Lemma 2 part b)).

An important implication is that teams may no longer play offensive hockey even if they have a comparative advantage in doing so. Formally, if the teams are equal (3.3) and have a comparative advantage in playing offensive hockey (3.5) $(\underline{H},\underline{H})$ may not be an equilibrium for some teams unlike under the old incentive regime. Consider the following example for the three period game ($T = 3$) of two equal teams with a comparative advantage in offensive play.

Example 4 *Let* $(p(H,H), p(H,L), p(L,H), p(L,L)) = (\frac{1}{4}, \frac{1}{12}, \frac{1}{16}, \frac{1}{20})$. *Considering the infinitely divisible overtime game under the new rule we have* $\lambda = \frac{1}{2}$. *Note that the comparative advantage of playing offensive hockey is* $\alpha = p(H,L) - p(L,H) = \frac{1}{48} > 0$. *Using lemma (3) or the Markov equations (4.10) we obtain the expected payoff matrix of Team A and B as:*

$s^B \diagdown s^A$	H,H,H	H,H,L	H,L,H	L,H,H	H,L,L	L,H,L	L,L,H	L,L,L
H,H,H	(0.16,0.16)	(0.19,0.17)	(0.19,0.17)	(0.19,0.17)	(0.25,0.19)	(0.25,0.19)	(0.25,0.19)	(0.38,0.27)
H,H,L	(0.17,0.19)	(0.18,0.18)	(0.22,0.22)	(0.22,0.22)	(0.24,0.21)	(0.24,0.21)	(0.34,0.31)	(0.38,0.3)
H,L,H	(0.17,0.19)	(0.22,0.22)	(0.18,0.18)	(0.22,0.22)	(0.24,0.21)	(0.34,0.31)	(0.24,0.21)	(0.38,0.3)
L,H,H	(0.17,0.19)	(0.22,0.22)	(0.22,0.22)	(0.18,0.18)	(0.34,0.31)	(0.24,0.21)	(0.24,0.21)	(0.38,0.3)
H,L,L	(0.19,0.25)	(0.21,0.24)	(0.21,0.24)	(0.31,0.34)	(0.23,0.23)	(0.34,0.34)	(0.34,0.34)	(0.37,0.34)
L,H,L	(0.19,0.25)	(0.21,0.24)	(0.31,0.34)	(0.21,0.24)	(0.34,0.34)	(0.23,0.23)	(0.34,0.34)	(0.37,0.34)
L,L,H	(0.19,0.25)	(0.31,0.34)	(0.21,0.24)	(0.21,0.24)	(0.34,0.34)	(0.34,0.34)	(0.23,0.23)	(0.37,0.34)
L,L,L	(0.27,0.38)	(0.3,0.38)	(0.3,0.38)	(0.3,0.38)	(0.34,0.37)	(0.34,0.37)	(0.34,0.37)	(0.37,0.37)

Clearly Team A (or B) will not play (H,H,H) *as deviating to* (L,L,L) *yields a better payoff. In fact, in this example* $[(L,L,L),(L,L,L)]$ *is the dominant strategy equlibrium.*

This example shows that under the new rule some teams will not play attacking hockey even if their comparative advantage lies in doing so. Thus, the change in incentive to create more attractive play in overtime has the perverse effect of resulting in more conservative, defensive play in regulation time.

This is rational behavior for players and coaches, because why risk losing a point by aggressively pursuing a win in regulation if the safer route is to get a regulation tie and go for the extra point in overtime? In short, with the current rules, you can lose a game but still get a point in the standings.

5 Empirical evidence

Data on the results of individual games for the last eight NHL seasons were collected from the website http://www.hockeynut.com/archive.html. The number of regulation and overtime games played and the results in the overtime games are summarized in Table 1 for the whole NHL and by conference within the NHL in Table 2. The first four seasons represent the time when the losing team in overtime game received no points while the last four seasons are when each team tied at the end of regulation received a single point regardless of the outcome in overtime.

The results in Table 1 confirm the first theorem that teams take a more offensive approach in overtime if both teams are guaranteed a single point going into the overtime. If a team could lose a point by giving up a goal in the 5 minute overtime, teams would play a conservative strategy to avoid losing what they had gained over the 60 minutes of regulation play. Under such a rule for the

1995-1996 season through the 1998-1999 season, over 70% of the games tied in regulation remained tied after the 5 minute overtime (see Table 1). In contrast, overtime goals were scored in close to 50% of the games tied after regulation for the last four seasons under the rule change.

Table 1

It is important to note that the modification to the NHL overtime rule at the beginning of the 1999-2000 season not only changed the point system but also the number of skaters on the ice. In order to further increase the possibility of scoring, the NHL went from the 5-on-5 play used in regulation time to 4-on-4. Since both the point system and number of players for overtime were changed at the same time, the effect of each on the significant increase in overtime wins cannot be determined. However, Abrevaya (2002) gathered overtime results for the American Hockey League (AHL) which is a minor league affiliate of the NHL. The two overtime rule changes were implemented separately by the AHL. The data indicate that the awarding a single point in overtime was primarily responsible for the increased number of overtime wins. Prior to the rule change, 68% of overtime games remained tied and this dropped to 59% with the change in point system. The percentage dropped slightly further to 55% with the introduction of 4-on-4 play. Thus, the scoring system change has increased the likelihood of offensive strategy in overtime as desired.

The effects of the rule change on the probability of an overtime result were also assessed through the use of a Logit model with the dependent variable defined in terms of whether a team scored in overtime (1=yes, 0=no (game remained tied)). The factors hypothesized to influence the chance of scoring in overtime include the introduction of the rule change in the 1999-2000 which is proxied by a dummy variable equal to 1 for seasons with the new point system and 0 for years prior to the 1999-2000 season. Team payrolls were used to proxy the effect of absolute and relative offensive abilities by teams. Player salary information was obtained from http://users.pullman.com/rodfort/SportsBusiness/Bi for the majority of the seasons and http://www.lcshockey.com/extra/1997/salary.asp for the 1997-98 years and http://www.lcshockey.com/extra/1998/salary.asp for the 1998-99 season. The sum of the team payrolls is assumed to increase the likelihood of a goal scored in overtime since player salary is generally directly related to offensive skills. The absolute value of the difference is also assumed to increase the chance of the game being settled in overtime since there is a relative difference in the comparative ability of the teams to play offensive hockey.

A dummy variable was included to capture the effects of games played between teams within the same or different conferences. The 30-team NHL is divided into two conferences of 15 teams (East and West). The top 8 teams within each conference make the playoffs at the end of the 82 game regular season. A team's overall profitability is determined in large part by the revenue earned from ticket sales of home playoff games. Thus, making the playoffs generates income but so does finishing further up the league standings as the higher placed team earns home-ice advantage in any playoff series. Thus, incentives and correspondingly team strategy in overtime will vary depending upon whether the opponent is from the same conference. Since relative positions determine playoff ranking and potential home-ice advantage, teams within the same conference wish to avoid giving an additional point in overtime to their opponents. While teams from within the same conference could gain from an additional point in overtime, the consequences of giving up a goal are significant. In contrast, both teams have nothing to lose by attempting to score in overtime if they are in different conferences. Summary statistics presented in Table 2 confirm the hypothesis that an overtime goal is more likely to be scored by if the teams are from different conferences but the percentage differences are not statistically significant.

Results of the Logit regression on the likelihood of a goal being scored in overtime are listed in Table 3. The new overtime rule has significantly increased the likelihood of a goal being scored in overtime as predicted by the model and supported by the summary statistics of Table 1. The only other statistically significant variable was the sum of the salaries for the two teams involved in an overtime game. As expected, high payroll teams are more likely to be involved in an overtime game that results in a win for one team. The difference in salary between two team and the conference effect had the expected signs but were not statistically significant.

The theoretical results suggested the overtime rule changes would not only affect strategy within the overtime but also play during regulation. The fifth row of Table 2 indicates that slightly more games of the total games played have gone into overtime with the change in point system. The percentage of games ending in a tie after the 60 minute regulation time increased from 20.2% to 22.6% with the change in the payoff structure. The increase is statistically significant.

Table 2

Table 3

6 Increasing rewards for a win

Another contemplated change to the incentives facing NHL teams is to give a winning team three points for a regulation win as opposed to two. Such a change in the point structure has been implemented in most professional soccer leagues. One version of the reward system would give a team three points for a win in regulation, two for a win in overtime, one for a tie or an overtime loss, and zero for a loss in regulation. The intent of such a change is to create a more attacking style of game since teams would be rewarded for doing better and trying to win in regulation rather than play conservatively and settle for a guaranteed one point arising from a tie in regulation (Mullin, J (2003)).

To analyze how such a proposed rule change would affect team strategies, we will use the model developed above. We define the end of play payoff $Ureg^A$ of team A as,

$$Ureg^A = 2(I\{Z_T > 0\} - I\{Z_T < 0\}) + \frac{1}{2}I\{Z_T = 0\}$$

where $I\{.\}$ is the indicator function and the expected payoff from tieing a regulation time game is $\frac{1}{2}$.

Team B's end of play payoff will be

$$Ureg^{B}=2(I\left\{Z_{T}<0\right\}-I\left\{Z_{T}>0\right\})+\frac{1}{2}I\left\{Z_{T}=0\right\}$$

We can then derive the expected payoff of Team A as

$$E\left(Ureg^{A}\right)=2\sum_{d=1}^{T}\left[\Pr\left(Z_{T}=d\right)-\Pr\left(Z_{T}=-d\right)\right]+\frac{1}{2}\Pr\left(Z_{T}=0\right)$$

and for Team B,

$$E\left(Ureg^{B}\right)=2\sum_{d=1}^{T}\left[\Pr\left(Z_{T}=d\right)-\Pr\left(Z_{T}=-d\right)\right]+\frac{1}{2}\Pr\left(Z_{T}=0\right)$$

where the probabilities are calculated as in (4.10).

Let us look at the example 4 in the previous section and see how the same teams will fare under the proposed rules.

Example 5 *Let* $(p\left(H,H\right),p(H,L),p(L,H),p(L,L))=(\frac{1}{4},\frac{1}{12},\frac{1}{16},\frac{1}{20})$. *Considering the infinitely divisible overtime game under the new rule we have* $\lambda=\frac{1}{2}$.*Note that the comparative advantage of playing offensive hockey is* $\alpha=p(H,L)-p(L,H)=\frac{1}{48}>0$. *Using lemma (3) or the Markov equations (4.10) we obtain the expected payoff matrix of Team A and B as:*

$s^B \diagdown s^A$	H,H,H	H,H,L	H,L,H	L,H,H	H,L,L	L,H,L	L,L,H	L,L,L
H,H,H	(0.16,0.16)	(0.2,0.15)	(0.2,0.15)	(0.2,0.15)	(0.28,0.16)	(0.28,0.16)	(0.28,0.16)	(0.43,0.22)
H,H,L	(0.15,0.2)	(0.18,0.18)	(0.22,0.22)	(0.22,0.22)	(0.25,0.19)	(0.25,0.19)	(0.36,0.29)	(0.41,0.27)
H,L,H	(0.15,0.2)	(0.22,0.22)	(0.18,0.18)	(0.22,0.22)	(0.25,0.19)	(0.36,0.29)	(0.25,0.19)	(0.41,0.27)
L,H,H	(0.15,0.2)	(0.22,0.22)	(0.22,0.22)	(0.18,0.18)	(0.36,0.29)	(0.25,0.19)	(0.25,0.19)	(0.41,0.27)
H,L,L	(0.16,0.28)	(0.19,0.25)	(0.19,0.25)	(0.29,0.36)	(0.23,0.23)	(0.34,0.34)	(0.34,0.34)	(0.39,0.32)
L,H,L	(0.16,0.28)	(0.19,0.25)	(0.29,0.36)	(0.19,0.25)	(0.34,0.34)	(0.23,0.23)	(0.34,0.34)	(0.39,0.32)
L,L,H	(0.16,0.28)	(0.29,0.36)	(0.19,0.25)	(0.19,0.25)	(0.34,0.34)	(0.34,0.34)	(0.23,0.23)	(0.39,0.32)
L,L,L	(0.22,0.43)	(0.27,0.41)	(0.27,0.41)	(0.27,0.41)	(0.32,0.39)	(0.32,0.39)	(0.32,0.39)	(0.37,0.37)

Clearly Team A (or B) will not play (H,H,H) as deviating to (L,L,L) yields a better payoff. This example shows that the change in the rewards to a regulation time win will not be sufficient to stimulate more offensive play for some teams. In other words, while increasing the rewards for a win in regulation time may stimulate more offensive play for some teams, it will not fully offset the negative incentive effects caused by the introduction of the new overtime rule. For at least some of the teams, the negative incentive effects of the overtime rules changes in 2000 are stranger than any positive effects from increasing the rewards for regulation time victories from 2 to 3 points. The incentives for some of the teams will still be to play defensively in regulation time to capture the point from a tie after 60 minutes of play and hope to get an extra point in overtime.

7 Concluding comments

Institutional reforms affect behavior, but not always in the way intended. Recent studies analyzing the effect of rule changes on strategic behavior in sporting competitions yield interesting insights. They are particularly interesting because due to the very nature of the sporting competitions many external factors which typically influence behavior are controlled for, or excluded.

In this paper we have analyzed the impact of the recent introduction of a rule change in the National Hockey League on the strategies of hockey teams. The NHL decreed that as of the 1999-2000 season, in case of a tie after the regulation time, both teams would get one point each; and that the winner in overtime would get an additional point. This change rewarded the teams which tied in regulation time but lost in overtime with a point which they did not get under the old rules. The rule change was intended to enhance the general appeal of the game by stimulating more offensive play in regulation time. Our analysis shows that this was a correct assumption. We demonstrate with our theoretical model that teams are more likely to play offensively in overtime under the new rule. The empirical evidence we present confirms these conclusions.

These findings appears to provide strong support for the NHL's decision to introduce the 1999 rule change. However, such conclusion is not justified, since it is based on an incomplete analysis. The rule change also has another, unintended, impact. Our theoretical analysis shows how the rule change has a perverse effect on team strategies in regulation time, causing more defensive play during the main part of the game. The empirical evidence provides support for this conclusion. Hence the conclusions based on the full effects of the rule change should be less positive, and more nuanced.

A series of additional rule changes to offset the perverse regulation time effect are being considered. The most prominent proposal is to raise the reward for the winning team to 3 points and keep the rest the same. In the last section of the paper we show that this proposal, if implemented, may mitigate the perverse effect in regulation time, but will not eliminate it. In fact, there will still be more defensive play in regulation time under the proposed rule changes than if the 1999 rule change would be eliminated and one would reverse to the pre 1999 rule system.

References

Abrevaya, J. (2002). Fit to Be Tied: The Incentive Effects of Overtime Rules in Professional Hockey. Working Paper, Dept of Economics, Purdue University.

Banerjee, A.N. and J. F.M. Swinnen (2003). Does A Sudden Death Liven Up The Game ? Rules, Incentives, And Strategy In Football, Economic Theory, (Forthcoming).

Ehrenberg, R.G. and M.L. Bognanno (1998). Do Tournaments Have Incentive Effects? Journal of Political Economy, 98, 1307-1324.

Fyffe, I, (2002), Is it Fit to be Tied? Puckerings, Hockeyzoneplus.com (http://www.hockeyzoneplus.com/puck_e.htm)

Hobson, R (1999), NHL Overtime Rule a Big Hit, Everyrule.com (http://www.everyrule.com/ frame-

sets /newsroomframes.htm#nhl).
Maloney, M.T. and R.E. McCormick (2000). The Response of Workers to Wages in Tournaments: Evidence from Foot Races. Journal of Sports Economics, 1, 99-123.
Mullin, J (2003), NHL Rule Changes! Yay!, In the Crease: Articles, Redwingsnetwork.com (http://www.re
Maloney, M.T. and K. Terkun (2002). Road Warrior Booty: Prize Structures in Motorcycle Racing. Contributions to Economic Analysis & Policy, 1(1), Article 3.
Owen, G (1995), *Game Theory (Third Edition)* Academic Press, San Diego.
Palomino, F., L. Rigotti and A. Rustichini, (1999), Skill, Strategy and Passion: an Empirical analysis of hockey , Center Discussion Paper series No.98125, Tilburg University (http: //grey-www.kub.nl:2080 / greyfiles/ center/ 1998/ doc/ 129.pdf).

Appendix:

Proof of Lemmas:

Proof of Lemma 1 : By definition (3.2),

$$E\left(Uold_T^A\right) = \Pr\left(W_T = 1\right) - \Pr\left(W_T = -1\right)$$

Notice that,

$$\begin{aligned}
\Pr\left(W_T = 1\right) &= \sum_{t=1}^{T} \Pr\left(X_t = 1,\ X_{t'} = 0\ \forall\ t' < t\right) \\
&= \sum_{t=1}^{T} \Pr\left(X_t = 1\right) \prod_{t' \in \Omega_{t-1}} \Pr\left(\ X_{t'} = 0\right) \\
&= \sum_{t=1}^{T} p\left(s_t^A, s_t^B\right) R\left(\underline{s}^A, \underline{s}^B \middle| \Omega_{t-1}\right) \qquad (7.12)
\end{aligned}$$

Similarly

$$\Pr\left(W_T = -1\right) = \sum_{t=1}^{T} q\left(s_t^A, s_t^B\right) R\left(\underline{s}^A, \underline{s}^B \middle| \Omega_{t-1}\right). \qquad (7.13)$$

Combining (7.12) and (7.13) we get

$$E\left(Uold_T^A\right) = \sum_{t=1}^{T} \left(p\left(s_t^A, s_t^B\right) - q\left(s_t^A, s_t^B\right)\right) R\left(\underline{s}^A, \underline{s}^B \middle| \Omega_{t-1}\right).$$

Proof of Lemma 2: Follows from (7.12) of Lemma 1.
Proof of Lemma 3: By definition it follows that

$$\begin{aligned}
E\left(Ureg_T^A\right) &= \Pr\left(Z_T > 0\right) - \Pr\left(Z_T < 0\right) + \lambda \Pr\left(Z_T = 0\right) \\
&= \sum_{d=1}^{T} \Pr\left(Z_T = d\right) - \Pr\left(Z_T = -d\right) + \lambda \Pr\left(Z_T = 0\right)
\end{aligned}$$

Define a partition of $N=\{1,\ldots,T\}$ as $T_1=\{t:X_t=1\}\,,T_2=\{t:X_t=-1\}$ and $T_3=\{t:X_t=0\}\,.$ Then $|T_1|$ is the number of goals scored by team A and $|T_2|$ is the number of goals scored by team B. So if $|T_1|-|T_2|=d$, team A wins by d goals. Notice that T_1,T_2 and T_3, is an arbitrary partition of N, so team A can win by d goals with any such partition as long as $|T_1|-|T_2|=d$.

Using the notation defined in (3) the probability that team A scores at times $t\in T_1$ and team B scores (team A concedes a goal) at times $t\in T_2$, as

$$\Pr\left(X_{t\in T_1},X_{t\in T_2},X_{t\in T_3}\right)=P\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)R\left(\underline{s}^A,\underline{s}^B\middle|\,T_3\right) \tag{7.14}$$

Hence the game ending with d goals can be written as the sum of all such terms in (7.14) as long as the $|T_1|-|T_2|=d$. Therefore

$$\begin{aligned}\Pr\left(Z_T=d\right)&=\sum_{k=0}^{T-d}\sum_{T_3:T-|T_1|+|T_2|}\sum_{T_1:|T_1|=k+d}\sum_{T_2:|T_2|=k}P\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)R\left(\underline{s}^A,\underline{s}^B\middle|\,T_3\right)\\&=\sum_{T_3,T_1,T_2:|T_1|-|T_2|=d}P\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)R\left(\underline{s}^A,\underline{s}^B\middle|\,T_3\right).\end{aligned}$$

where T_1,T_2 and T_3 is a partition of N. Similarly (notice that team A now loses by d goals)

$$\Pr\left(Z_T=-d\right)=\sum_{T_3,T_1,T_2:|T_1|-|T_2|=d}R\left(\underline{s}^A,\underline{s}^B\middle|\,T_3\right)P\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)$$

and

$$\Pr\left(Z_T=0\right)=\sum_{T_3,T_1,T_2:|T_1|=|T_2|}R\left(\underline{s}^A,\underline{s}^B\middle|\,T_3\right)P\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)$$

Hence,

$$\begin{aligned}&\sum_{d=1}^{T}\left[\Pr\left(Z_T=d\right)-\Pr\left(Z_T=-d\right)\right]-\lambda\Pr\left(Z_T=0\right)\\&=\sum_{d=1}^{T}\sum_{T_3,T_1,T_2:|T_1|-|T_2|=d}R\left(\underline{s}^A,\underline{s}^B\middle|\,T_3\right)D\left(\underline{s}^A,\underline{s}^B\middle|\,T_1,T_2\right)\\&-\lambda\sum_{T_3,T_1,T_2:|T_1|=|T_2|}R\left(\underline{s}^A,\underline{s}^B\middle|\,T_3\right)P\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)\end{aligned}$$

Proof of Theorems:

Proof of Theorem 1:

a) Given $p\left(H,L\right)-p\left(L,H\right)=\alpha>0$ and $\underline{H}=(H,\ldots,H)$ .We prove the theorem in two parts

1) $(\underline{H},\underline{H})$ is an equilibrium and

2) Any other pair of strategies $(\underline{s},\underline{s})$ are not equilibrium strategies.

Let $\underline{s}^A$ be the strategy vector with $s_{t_0}^A=L$ and $s_t^A=H\ \forall\ t\neq t_0$ Team A will then deviate at time t_0, if

$$Vold_T^A\left(\underline{s}^A,\underline{H}\right)-Vold_T^A\left(\underline{H},\underline{H}\right)>0$$

>From (1) we know that $V_w\left(\underline{\mathrm{H}}, \underline{\mathrm{H}}\right)=0$, for any $\underline{\mathrm{s}}$. So team A deviates if

$$Vold_T^A\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\right)>0$$

Notice that

$$\begin{aligned}
Vold_T^A\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\right) &= \sum_{t=1}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle|\, \Omega_{t-1}\right)\left(p\left(s_t^A, H\right)-q\left(s_t^A, H\right)\right) \\
&= \sum_{t=1, t\neq t_0}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle|\, \Omega_{t-1}\right)\left(p\left(H,H\right)-q\left(H,H\right)\right) \\
&+ R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle|\, \Omega_{t_0-1}\right)\left(p\left(L,H\right)-q\left(L,H\right)\right) \\
&= R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle|\, \Omega_{t_0-1}\right) p\left(L,H\right)-q\left(L,H\right) \;\; \text{(by (3.3))} \\
&= R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle|\, \Omega_{t_0-1}\right) p\left(L,H\right)-p\left(H,L\right) \;\; \text{(by (3.3))} \\
&= -\alpha R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle|\, \Omega_{t_0-1}\right)
\end{aligned}$$

So there is a negative payoff for deviation and therefore team A does not deviate. The same holds for team B. If the teams want to deviate at multiple time points $\{t_1, \ldots, t_k\}$,there is also a negative payoff, the proof of which is similar as above.

Hence ($\underline{\mathrm{H}}$,$\underline{\mathrm{H}}$) is an equilibrium.

Secondly, we need to prove the uniqueness of the equilibrium. As the payoff matrix is skew symmetric the there is no asymmetric equilibrium. So any equilibrium must be of the form $\left(\underline{\mathrm{s}}, \underline{\mathrm{s}}\right)$. We shall show that if $\underline{\mathrm{s}} \neq \underline{\mathrm{H}}$, there is a positive payoff for team A to deviate.

If $\underline{\mathrm{s}} \neq \underline{\mathrm{H}}$, then $\exists\ t_0\ s.t\ s_{t_0}=L$.

Let team A deviate at t_0, and plays H. Let $\underline{\mathrm{s}}^A$ be the strategy vector with $s_{t_0}^A=L$ and $s_t^A=s_t \forall$ $t\neq t_0$. Then

$$\begin{aligned}
Vold_T^A\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}\right) &= \sum_{t=1}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}\middle|\, \Omega_{t-1}\right)\left(p\left(s_t^A, s_t\right)-q\left(s_t^A, s_t\right)\right) \\
&= \sum_{t=1, t\neq t_0}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}\middle|\, \Omega_{t-1}\right)\left(p\left(s_t, s_t\right)-q\left(s_t, s_t\right)\right) \\
&+ R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}\middle|\, \Omega_{t_0-1}\right)\left(p\left(H,L\right)-q\left(H,L\right)\right) \\
&= 0+R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}}\middle|\, \Omega_{t_0-1}\right)\left(p\left(H,L\right)-p\left(L,H\right)\right) \\
&= \alpha R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}}\middle|\, \Omega_{t_0-1}\right)>0
\end{aligned}$$

Hence team A will deviate. The same holds for team B. Therefore $\left(\underline{\mathrm{s}}, \underline{\mathrm{s}}\right)$ is not an equilibrium strategy pair. Hence ($\underline{\mathrm{H}}$,$\underline{\mathrm{H}}$) is an unique equilibrium.

b) If $\alpha<0$, then $\left(\underline{\mathrm{L}}, \underline{\mathrm{L}}\right)$ will be the only equilibrium. Proof is similar as a)

c) since the payoff matrix is skew symmetric the value of the game is zero.

Proof of Theorem 2. Let $\underline{\mathrm{s}}^A$ be the strategy vector with $s_{t_0}^A = L$ and $s_t^A = H \;\forall\; t \neq t_0$ Team A will not deviate at time t_0, if $Vnew_T^A(\underline{\mathrm{H}}, \underline{\mathrm{H}}) > Vnew_T^A\left(s_{t_0}^A, \underline{\mathrm{H}}\right)$. Consider

$$Vnew_T^A(\underline{\mathrm{H}}, \underline{\mathrm{H}}) = \sum_{t=1}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle|\, \Omega_{t-1}\right) p(H, H)$$

$$Vnew_T^A\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\right) = \begin{array}{l} \sum_{t=1}^{t_0} R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}}\middle|\, \Omega_{t-1}\right) p(H, H) + p(L, H)\, R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}}\middle|\, \Omega_{t_0-1}\right) \\ + \sum_{t=t_0+1}^{T} R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}}\middle|\, \Omega_{t-1}\backslash(t_0)\right) p(H, H)\, r(L, H) \end{array}$$

therefore subtracting,

$$\begin{aligned}
& Vnew_T^A(\underline{\mathrm{H}}, \underline{\mathrm{H}}) - Vnew_T^A\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\right) \\
= \; & (p(H,H) - p(L,H))\, R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}}\middle|\, \Omega_{t_0-1}\right) \\
& + (r(H,H) - r(L,H))\, p(H,H) \sum_{t=t_0+1}^{T} R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}}\middle|\, \Omega_{t-1}\backslash(t_0)\right) \\
= \; & \left(((p(H,H) - p(L,H))) + (r(H,H) - r(L,H))\, p(H,H) \frac{1 - r(H,H)^{T-t_0+1}}{1 - r(H,H)}\right) r(H,H)^{t_0-1}
\end{aligned}$$

since $r(H,H)^{t_0-1}$ is positive we need to consider only

$$\begin{aligned}
& p(H,H)(r(H,H) - r(L,H)) \sum_{t=1}^{T-t_0} r(H,H) + (p(H,H) - p(L,H)) \\
& = p(H,H)(r(H,H) - r(L,H)) \frac{1 - r(H,H)^{T-t_0+1}}{1 - r(H,H)} + (p(H,H) - p(L,H)) \\
& = (r(H,H) - r(L,H)) \frac{1}{2}\left(1 - r(H,H)^{T-t_0+1}\right) + (p(H,H) - p(L,H)) \\
& = (p(H,H) - p(L,H)) - \left(1 - r(H,H)^{T-t_0+1}\right) \frac{1}{2}\left(2p(H,H) - p(H,L) - p(L,H)\right) \\
& = (p(H,H) - p(L,H)) - \left(1 - r(H,H)^{T-t_0+1}\right)\left(p(H,H) - \frac{p(H,L) - p(L,H)}{2}\right) \\
& = (p(H,H) - p(L,H)) - \left(1 - r(H,H)^{T-t_0+1}\right)\left(p(H,H) - \frac{p(H,L) - p(L,H)}{2}\right) \\
& = \left(\frac{p(H,L) - p(L,H)}{2}\right) + r(H,H)^{T-t_0+1}\left(p(H,H) - \frac{p(H,L) + p(L,H)}{2}\right)
\end{aligned}$$

The above expression is positive since $\alpha > 0$, and (2.1). Since Team B is symmetric to Team A the same will hold for them, hence, $s^A = \underline{\mathrm{H}}$ and team B $s^B = \underline{\mathrm{H}}$ is the equilibrium.

b) If team A has a comparative advantage of playing in offensively ($\alpha > 0$), we know (from part a)) that under the new rules that ($\underline{\mathrm{H}}$,$\underline{\mathrm{H}}$) is an unique equilibrium for all T. Therefore, the expected utility of Team A at T is

$$Vnew_T^A(\underline{\mathrm{H}}, \underline{\mathrm{H}}) = p(H,H) \frac{1 - r(H,H)^T}{1 - r(H,H)}.$$

Hence $\lim_{T\to\infty} Vnew_T^A(\underline{\mathrm{H}}, \underline{\mathrm{H}}) = \frac{p(H,H)}{1 - r(H,H)} = \frac{1}{2}$. The proof is same for Team B.

Proof of Theorem 3:

a) Given $p(H,L) - p(L,H) = \alpha > 0,\ \lambda = 0$ and $\underline{\mathrm{H}}$=$(H,\ldots,H)$. We prove the theorem in two parts that

1. $(\underline{\mathrm{H}},\underline{\mathrm{H}})$ is an equilibrium and

2. Any other pair of strategies $(\underline{\mathrm{s}},\underline{\mathrm{s}})$ are not equilibrium strategies.

Let $\underline{\mathrm{s}}^A$ be the strategy vector with $s_{t_0}^A = L$ and $s_t^A = H\ \forall\ t \neq t_0$ Team A will then deviate at time t_0, if

$$Vreg_T^A\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\right) - Vreg_T^A\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\right) > 0$$

>From (3) we know that $Vreg_T^A\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\right) = 0$, for any $\underline{\mathrm{s}}$. So Team A deviates if

$$Vreg_T^A\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\right) > 0 \tag{7.15}$$

Notice that for any given partition of $N : T_1, T_2$ and T_3,

$$\begin{aligned}
P\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_1\right) &= P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\backslash\{t_0\}\right)p(L,H)\ \text{ if } t_0 \in T_1 &(7.16)\\
&= P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\right)\ \text{ if } t_0 \notin T_1\\
Q\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_2\right) &= P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_2\backslash\{t_0\}\right)q_{L,H}\text{if } t_0 \in T_2 &(7.17)\\
&= P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_2\right)\ \text{ if } t_0 \notin T_2\\
R\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_3\right) &= R\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_3\right)\ \text{ if } t_0 \in T_1 \cup T_2 &(7.18)\\
&= R\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_3\backslash\{t_0\}\right)r(L,H)\ \text{ if } t_0 \in T_3
\end{aligned}$$

Let us look at the contributions of individual terms on the summation of $Vreg_T^A\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\right)$

$$\sum_{d=1}^{T}\sum_{T_3,T_1,T_2:|T_1|-|T_2|=d} R\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_3\right)D\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_1,T_2\right)$$

If $t_0 \in T_3$, then $R\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_3\right)D\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_1,T_2\right) = 0$, since

$$\begin{aligned}
D\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1,T_2\right) &= P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\right)Q\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_2\right) - Q\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\right)P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_2\right)\\
&= 0
\end{aligned}$$

by (3.3).

The next step we consider the terms where $t_0 \in T_2$, then

$$\begin{aligned}
D\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_1,T_2\right) &= P\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_1\right)Q\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_2\right) - Q\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_1\right)P\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_2\right)\\
&= P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\right)Q\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_2\right) - Q\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\right)P\left(\underline{\mathrm{s}}^A,\underline{\mathrm{H}}\middle| T_2\right)\\
&= \begin{array}{c} P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\right)Q\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_2\backslash\{t_0\}\right)q(L,H)\\ -Q\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1\right)P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_2\backslash\{t_0\}\right)p(L,H)\end{array}\\
&= P\left(\underline{\mathrm{H}},\underline{\mathrm{H}}\middle| T_1 \cup T_2 - \{t_0\}\right)\left(q(L,H) - p(L,H)\right)
\end{aligned}$$

Notice that since $|T_1| = |T_2| + d$, then $|T_1|$ must be at least $d = 1, \ldots, T$. So let $t_1 \in T_1$. Construct $T_1' = T_1 \cup \{t_0\} \setminus \{t_1\}$ and $T_2' = T_2 \cup \{t_1\} \setminus \{t_0\}$. Notice that T_1', T_2' and T_3' is a partition such that $|T_1'| = |T_2'| + d$ and $T_3' = T_3$ therefore

$$R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_3\right) = R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_3'\right)$$

and

$$\begin{aligned}
D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1', T_2'\right) &= P\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1'\right) Q\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_2'\right) - Q\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1'\right) P\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_2'\right) \\
&= \begin{array}{c} P\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1 \cup \{t_0\} \setminus \{t_1\}\right) Q\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_2 \cup \{t_1\} \setminus \{t_0\}\right) \\ -Q\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1 \cup \{t_0\} \setminus \{t_1\}\right) P\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_2 \cup \{t_1\} \setminus \{t_0\}\right) \end{array} \\
&= \begin{array}{c} P\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_1 \setminus \{t_1\}\right) p(L, H)\, P\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_2 \cup \{t_1\} \setminus \{t_0\}\right) \\ -P\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_1 \setminus \{t_1\}\right) q(L, H)\, P\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_2 \cup \{t_1\} \setminus \{t_0\}\right) \end{array} \\
&= P\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_1 \cup T_2 \setminus \{t_0\}\right) \left(p(L, H) - q(L, H)\right) \\
&= -D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1, T_2\right)
\end{aligned}$$

Hence,

$$R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_3'\right) D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1', T_2'\right) + R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_3\right) D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1, T_2\right) = 0,$$

the terms will cancel each other. There will be no positive contribution to the payoff due to change of strategy by team A.

The next case is when $t_0 \in T_1$. We will also show this in two parts.

Part 1: when for a given d, $|T_1| > d$, then $|T_2| > 0$, so that as in case of the proof above (case of $t_0 \in T_2$) we have $T_1' = T_1 \cup \{t_0\} \setminus \{t_1\}$ and $T_2' = T_2 \cup \{t_1\} \setminus \{t_0\}$ where $t_1 \in T_2$, and show that $R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_3\right) D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1, T_2\right) = -R\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_3'\right) D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1', T_2'\right)$. Therefore, the terms cancel again. There is no positive contributions by these terms to the payoff of team A.

Part 2: when given $|T_1| = d$ (notice $d > 1$), then $T_2 = \emptyset$

$$\begin{aligned}
D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1, \emptyset\right) &= P\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1\right) - Q\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}} \middle| T_1\right) \\
&= P\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_1 \setminus \{t_0\}\right) \left(p_{L,H} - q_{L,H}\right) \\
&= -\alpha P\left(\underline{\mathrm{H}}, \underline{\mathrm{H}} \middle| T_1 \setminus \{t_0\}\right)
\end{aligned}$$

so this term contributes a negative amount if $\alpha > 0$. Adding up all the terms we have,

$$Vreg_T^A\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\right) < 0.$$

So there is a negative payoff for deviation and therefore team A does not deviate. The same holds for team B. If the teams want to deviate at multiple time points $\{t_1, \ldots, t_k\}$,there is also a negative payoff, the proof of which is similar as above.

Hence ($\underline{\mathrm{H}}$,$\underline{\mathrm{H}}$) is an equilibrium.

Now we show the second part of the proof : any other pair of strategies $(\underline{\mathrm{s}}, \underline{\mathrm{s}})$ are not equilibrium strategies.

Let $\underline{s}^A$ be the strategy vector with $s^A_{t_0} = L$ and $s^A_t = H \; \forall \; t \neq t_0$ Team A will then deviate at time t_0, if

$$Vreg^A_T\left(\underline{s}^A, \underline{s}\right) - Vreg^A_T\left(\underline{s}, \underline{s}\right) > 0$$

>From (3) we know that $Vreg^A_T\left(\underline{s}, \underline{s}\right) = 0$, for any $\underline{s}$. So team A deviates if $Vreg^A_T\left(\underline{s}^A, \underline{s}\right) > 0$ and then any other pair of strategies $\left(\underline{s}, \underline{s}\right)$ are not equilibrium strategies.

By using similar logic as the proof in part a) we look at the contributions of each individual terms on the summation of $Vreg^A_T\left(\underline{s}^A, \underline{s}\right)$

$$\sum_{d=1}^{T} \sum_{T_1, T_2 : |T_1| - |T_2| = d} R\left(\underline{s}^A, \underline{s} \middle| T_3\right) D\left(\underline{s}^A, \underline{s} \middle| T_1, T_2\right)$$

If $t_0 \in T_3$, then $R\left(\underline{s}^A, \underline{s} \middle| T_3\right) D\left(\underline{s}^A, \underline{s} \middle| T_1, T_2\right) = 0$, since $D\left(\underline{s}^A, \underline{s} \middle| T_1, T_2\right) = 0$.

Similar to the proof in part a) if $t_0 \in T_2$,there will be zero contribution to the deviation due to change of strategy by team A. Also if $t_0 \in T_1$ and $|T_1| > d$, there will be a zero contribution. The only case where there is a non-zero contribution is the case where $t_0 \in T_1$ and $|T_1| = d$. Then, for each d (Case 2 in part 1),

$$\begin{aligned} D\left(\underline{s}^A, \underline{H} \middle| T_1, \emptyset\right) &= P\left(\underline{s}^A, \underline{H} \middle| T_1\right) - Q\left(\underline{s}^A, \underline{H} \middle| T_1\right) \\ &= P\left(\underline{H}, \underline{H} \middle| T_1 \backslash \{t_0\}\right)\left(p_{H,L} - q_{H,L}\right) \\ &= \alpha P\left(\underline{H}, \underline{H} \middle| T_1 \backslash \{t_0\}\right) > 0 \end{aligned}$$

if $\alpha > 0$. Therefore, again adding up all the terms we have,

$$Vreg^A_T\left(\underline{s}^A, \underline{H}\right) > 0.$$

and team A deviates from L to H at time t_0. The same holds for team B. So $\left(\underline{s}, \underline{s}\right)$ is not an equilibrium strategy pair. Hence $(\underline{H}, \underline{H})$ is a unique equilibrium.

b) If $\alpha < 0$, then $(\underline{L}, \underline{L})$ will be the only equilibrium. The proof is similar as a)

8 Tables:

Season	95-96	96-97	97-98	98-99	99-00**	00-01	01-02	02-03
# of Teams	26	26	26	27	28	30	30	30
# of Games Played	1066	1066	1066	1107	1148	1230	1230	1230
# of Overtimes Played	201	214	219	222	260	261	263	307
% of All Games Going into Overtime	19.9	20.1	20.5	20.1	22.6	21.2	21.4	25.0
# of Overtime Wins	64	70	54	60	114	122	119	154
% of Overtime Wins to Overtime Played	31.8	32.7	24.7	27.0	43.8	46.7	45.2	50.2
**Year of overtime rule change.								

Table 1: Number and Results of Overtime Games in NHL, 1995-2003.

Season	Total	Same Conference	Different Conference	Z Test Statistic
1995-1996	31.8	_		
1996-1997	32.7	30.9	35.5	-0.67
1997-1998	24.7	29.9	14.9	2.41
1998-1999	27.0	24.3	28.3	-0.64
1999-2000	43.8	42.5	43.5	-0.15
2000-2001	46.7	45.4	50.0	-0.70
2001-2002	45.2	44.0	50.0	-0.92
2002-2003	50.2	49.1	52.9	-0.63

Table 2: Percentage of Overtime Games Ending in a Win by Conference, 1995-2003,

Dependent variable: Win in overtime (1=yes, 0=no)		
Variables	Coefficient	Standard Error
New Overtime Rule	0.713*	0.118
Conference	-0.001	0.103
Salary Sum	0.0035	0.002
Salary Difference	-0.0031*	0.005
RLR Chi-Sq = 71.45		
* Statistically significant at 5%		

Table 3: Logit Regression Results of Likelihood of Goal Being Scored in an Overtime Game