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A Descriptive Supply Model for Demand-Responsive Transportation System Planning

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IN A CONVENTIONAL fixed route transit system, level of service characteristics such as wait time and ride time are generally not highly dependent on the demand for service. The estimation of these parameters is quite straightforward, given the specification of the system. For example, passenger wait time may be considered dependent only on the headway distribution, except, of course, in congested systems, while passenger ride time is dependent only on trip length, vehicle speed and dwell times. Level of service will influence demand, but routes and schedules can be considered the sole determinants of level of service.

In the case of demand-responsive transportation (DRT), however, in which door-to-door service is provided to each passenger, level of service is highly dependent on the demand for service. A passenger's ride time is dependent not only on the length of desired trip and the vehicle speed, but also on the number of other passengers served by the vehicle en route and the location of their origins and destinations. These factors, in turn, depend on the level of demand, its geographic distribution, the number of vehicles in service and the dispatching strategy, as well as other factors. Wait time and other service characteristics are at least as difficult to predict as ride time.

The complex nature of the supply side in DRT systems has made it unusually difficult to predict key inputs to DRT planning such as ridership, revenue, and net system cost. DRT system planners are generally forced to use simple rules of thumb in determining vehicle requirements, when vehicle fleet size decisions are not simply determined by economic constraints. The lack of a reliable tool for relating vehicle fleet size to level of service and ridership can result in demand-responsive systems with either a serious oversupply or undersupply of vehicles. The much discussed Santa Clara County system provides a classic example of the consequences of mismatching the number of vehicles and the population served.

There have been a number of models of DRT supply developed in recent years, but each of the models has shortcomings. The most widely used technique for modelling DRT service has been computer simulation, an appropriate tool because of the complex and stochastic nature of the system. A simulation model developed at MIT (1) (2) has been validated with data from the UMTA sponsored Dial-a-Ride demonstration project in Haddonfield, New Jersey. This model, which was developed both to predict DRT system performance and to assist in the development of computer algorithms for DRT dispatching, offers considerable flexibility in analyzing different systems and options. However, it requires a fairly large computer facility and is relatively expensive and time-consuming to use; thus, its effectiveness as a planning tool is severely limited.

There have also been attempts at developing a simple DRT supply model. Among the better known models are those developed by Wilson (1), Arrillaga and Medville (3) and Lerman and Wilson (4). The Wilson model consists of a simple equation based on simulation results that relates vehicle fleet size to demand density and level of service. This model was applied by Bechtel (5) to the Santa Clara system and indicated that there was an insufficient supply of vehicles to meet the expected demand. However, the model is weak in terms of range of validity, the limitation of only one service output measure, output bounds, and the number of input parameters, and is probably oversimplistic for most planning efforts. The Arrillaga and Medville model is a linear regression model calibrated with data from nine very different operating DRT systems. This model has a number of major problems, and in fact gives clearly erroneous service time estimates for some feasible input sets because of the simplistic linear form.

Both of the above models are descriptive models in the sense that they are not based on any underlying theory, and hence, have a limited range of validity. The Lerman and Wilson model is based on queuing theory, but because of many

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simplifying assumptions, has also been shown to be valid only over a very limited range of inputs.

In light of the inadequacies of existing DRT supply models, there is a clear need for a simple model that can be used to predict DRT service levels, and enable a planner to easily investigate a wide range of demand-responsive transportation system design options. This paper presents a descriptive supply model that can be used in this manner.

A DESCRIPTIVE SUPPLY MODEL FOR DRT

The model described in this paper was developed as part of a combined supply/demand/equilibrium model of many-to-many DRT systems.[1] (Many-to-many service, in which point-to-point service is provided as demand anywhere within the service area, is at once the most common form of DRT service and the most complex; and as such, is the most important form of DRT to model.) The supply model can be used on its own in parametric analyses, in which demand and other exogenous factors are varied over wide ranges. This can serve as an effective method of investigating system performance and determining vehicle requirements for a range of system configurations and demand levels.

Before describing the model, it is important to understand the context in which the model is best applied. A descriptive model, by definition, implies a simple representation of a system, and is valid only over the range of calibration. Although an attempt has been made to incorporate as many parameters as possible in the model, many simplifying assumptions were made. The model cannot be expected to offer the same degree of flexibility as a simulation model. It is intended less as a method for accurately predicting service levels than as a screening tool, enabling the planner to identify feasible options and determine the impact of modifying vehicle fleets and service levels.

MODEL OUTPUTS

The basic desired output of the supply model was level of service (LOS), defined as the ratio of total travel time to the direct auto travel time. To obtain this measure, travel time was broken into its two components, wait time (WT) and ride time (RT). These represent the most commonly used service measures, and experience has indicated that they vary differently with respect to factors such as demand density. Measures such as wait and ride time uncertainty may also influence the demand for DRT service, but at this point there has been no attempt to model uncertainty directly.

MODEL INPUTS

Experience with both the simulation model and with actual DRT systems suggested that the following factors influence passenger wait time and ride time, and thus should serve as model inputs:

- Demand density (D)
- Service area size (A)
- Load and unload times (l and u)
- Street network characteristics (f_a, ratio of street distance to airline distance to airline distance between two points)[2]
- Mean direct trip length between origin and destination (L)
- Vehicle speed characteristics (V)
- Dispatching system

Two difficulties were encountered when consideration was given to incorporating dispatching within the model formulation. The first was the inherent difficulty of attempting to parameterize the many possible dispatching schemes, while the second was the fact that the simulation model could provide a consistent predictor of computerized dispatching only. It was recognized, however, that dispatching clearly impacts wait and ride time. The decision was reached to develop the model based on a dispatching system that effectively minimizes total travel time, and then consider the impact of different dispatching systems externally. The impact of dispatching will be discussed in a later section.

Another factor that could potentially influence level of service is vehicle capacity. Experience and research have both indicated, however, that the level of service is sensitive to vehicle capacity only over a small range of vehicle sizes (6). The reason for this is that for vehicles with capacities of perhaps seven or more, the desired service quality more actively constrains the number of passengers that can be picked up than does physical capacity. It was decided that a more effective approach than incorporating vehicle capacity explicitly within the model would be to develop separate models for the operating scenarios most closely related to vehicle capacity. One model would represent traditional DRT service which uses vans or small buses that seat 10 passengers or more, while the second model would represent a shared-ride taxi system, with a vehicle capacity of 4 or 5.

MODEL FORMULATION

The approach taken in formulating the model was to develop bounds for both wait time and ride time and formulate models which were bounded correctly and demonstrated the observed relationship with respect to the input parameters discussed above. Calibration of the models was based on a series of simulation experiments; bear in mind that the MIT simulation model had previously been validated. A primary objective of the model development was accuracy within $\pm$ 10% of mean system performance as measured by the simulation model, since fluctuation between systems caused by factors not modelled would be at least at this level.

System bounds are developed below. To simplify the following discussion, define:

Vehicle Productivity $\lambda = \frac{D x A}{N} =$ demands /vehicle/hour

Effective Vehicle Speed $V_{eff} = \frac{(60-\lambda\ (1+u)\ )V}{60}$ (where, effective vehicle speed is the net speed, incorporating all delays, to pick up and drop off passengers.)

1. As vehicle productivity approaches zero:

a. the ride time approaches the direct ride time:

$$RT = \frac{f_a x L}{V_{eff}}$$

b. wait time becomes the expected travel time between a point and the closest of N randomly distributed points in an area size A: (7)

$$WT = \frac{f_a}{2V_{eff}} \sqrt{\frac{A}{N}}$$

(where V_{eff} approaches V as λ approaches 0)

2. As vehicle productivity becomes very large: both wait time and ride time approach infinity.

3. As area becomes very large, wait time approaches infinity.

4. As number of vehicles approaches zero: wait time and ride time approach infinity.

A number of functional forms were developed, with parametric relationships that were based on observations of actual DRT systems and experience with the DRT simulation model. Both wait time and ride time appear to be exponentially related to productivity (with wait time more sensitive to productivity than is ride time) which is not surprising, given the complex queuing process represented by DRT service. The functional forms that provided the best results are:

$$WT = \frac{f_a}{2xV_{eff}} \sqrt{\frac{A}{N}} \exp\left(k_1 \ x \sqrt{\frac{A+4}{N+12}}^{\,k_2} \lambda\right)$$

$$RT = \frac{f_a x L}{V_{eff}} \exp\left(k_3 \left(\frac{A x \lambda}{N}\right)^{k_4}\right)$$

These models were calibrated via log linear regression over the following range of inputs.

A (area) = 4 mi^2 - 24 mi^2

N (vehicle fleet size) = 4 - 34

f_a (street adjustment factor) = 1.2 - 1.4

V (vehicle speed) = .20 mi/min - .30 mi/min

l, u (load and unload times) = .375 min - 1.25 min

D (demands per sq. mi. per hour) = 1 - 45

λ (demands per vehicle per hour) = 4 - 12.7

The resulting constant values were:

k_1 = .22 for a bus system and .20 for a shared ride taxi system

k_2 = .9 for a bus system and 1.0 for a shared ride taxi system

k_3 = .084 for both bus and shared ride taxi systems

k_4 = .7 for both bus and shared ride taxi systems

The percent RMS error, a measure of the mean percent difference between the simulation model results and the descriptive model results was approximately 10% for the wait time model (6% for the shared ride taxi version) and just under 5% for the ride time model. Chi-square and t-tests indicated that the output distributions of the simulation and descriptive models were not significantly different.

REFINEMENTS TO THE MODEL: SOME TRIAL APPLICATIONS

Trial applications to the model combined with further simulation experiments led to insights into the impact of the dispatching system on service levels, which in turn led to suggested refinements to the basic model form. Two trial applications will be presented; the first is a test of the model against data from the Haddonfield, New Jersey DRT system. The Haddonfield system provided the only experience with a fully operational computer dispatch system.[8] The Haddonfield experiment utilized the dispatching algorithm that is incorporated in the simulation model used for calibration; thus, this application represents a true test of the descriptive model's predictive capability.

Before presenting the results of the test, the concept of "effective vehicle fleet size" must be introduced. This concept was developed during the validation of the simulation model (2). It was discovered that when vehicles leave and then reenter service, for example, for driver reliefs, the system operates as if it had fewer vehicles in operation than it actually has. Since passengers waiting for service cannot be assigned to vehicles scheduled to leave service, the mean system wait time is greater than it would be if there were a constant vehicle fleet size. As far as system wait time is concerned, the "effective" vehicle fleet size is smaller than the actual fleet size. Ride time, however, does not appear to be similarly affected. Although a backlog of passengers will appear before a vehicle reenters service, resulting in a longer than average ride for these passengers, those passengers onboard a vehicle scheduled to leave service will be dropped off directly, and thus receive a shorter than average ride. These two factors appear to offset each other.

In testing the descriptive model against real world results, an estimate of effective vehicle fleet size must be made.[4] Effective vehicle fleet size appears to be a function of the number of vehicles in service, the number of times vehicles leave and enter service, and the point where driver reliefs are made in relation to the service area. To date there has been no attempt to model effective vehicle fleet size. Simulation experiments have suggested that the effective fleet size may be as much as 20-25% smaller than the actual vehicle fleet size in cases where vehicles leave service frequently (8).

In obtaining the results for the Haddonfield trial, shown in Table 1, an effective vehicle fleet size of 8 was assumed for wait time prediction. This figure, obtained by simulation experiments during the validation of the simulation model, is 18% lower than the actual mean fleet size of 9.2 used for the prediction of ride time.

The prediction of total travel time is within another 7.4% of the actual travel time, which suggests that the model is highly reliable. The predictions of wait and ride time were less accurate (wait time within 9.5% and ride time within 24%); however, given the stochastic nature of the system and some of the assumptions used in setting up the trial runs, these results are considered very reasonable.[5]

Note that wait time was predicted to be lower than ride time. With the dispatching algorithm used during calibration, the simulation model will consistently predict lower wait than ride time. Yet, observations of manually dispatched DRT systems have indicated that the opposite occurs. For example, data from the La Habra, California system indicates mean system wait and ride times of 21 and 15 respectively for a three day period in June, 1975 (9). This dichotomy can be directly traced to the type of dispatching system being used.

First of all, it has been widely assumed that computer dispatching would have a significant impact on service levels. The Haddonfield experiment is the only test of this hypothesis; the advent of computer dispatching in Haddonfield resulted in a 20% decrease in wait time, a decrease in wait time uncertainty, and no change in ride time. Although this represents only a single data point these results help explain part of the reason for the difference between simulated and real world results.

The remaining difference can be explained by the way in which the dispatching system treats wait and ride time. In any dispatching system, either manual or computer, passenger assignments are in some way based on providing the best overall service. In the

COMPARISON OF DESCRIPTIVE MODEL WITH DATA FROM HADDONFIELD*

	# of Veh. in Service	Mean System Wait Time	Mean System Ride Time	Mean System Travel Time
Haddonfield Data	9-11 (mean = 9.2)	9.5 min.	9.5 min.	19.0 min.
Model Results	8 (WT) 9.2 (RT)	8.6 min.	11.8 min.	20.4 min.

*Data covers a six hour period of operation during which 262 demands were served. A = 11.25 mi2, $f_a = 1.4$, and L = 1.47. Estimates of V, l, and u were .25, .375 and .375 respectively (2).

TABLE 1

algorithm used in Haddonfield and in the simulation model, passenger wait and ride time were treated equally within the assignment process. This approach has been found to yield the minimum total travel time, although different "weightings" of wait and ride time should not significantly impact total travel time (10). On the other hand, observations of manual systems, including Rochester, have shown that dispatchers consider ride time more onerous than wait time, and will seek to minimize ride time at the expense of wait time. In addition, some drivers will at times drop off a passenger already on board even when they have been assigned a prior pick up. As a result of these actions, wait time will generally be higher than ride time.

As a vivid illustration of the way dispatching can impact wait and ride time, consider a second trial application of the descriptive model, a test against data from the Rochester, N.Y. system while it was under computer control. In response to suggestions of the dispatcher, the computer assignment algorithm in Rochester initially weighted ride time 50% more heavily than wait time. (This has since changed and wait and ride time are now being weighted equally). A comparison of the model predictions with the actual data is shown in Table 2.

The model, calibrated on the basis of an algorithm that weighted wait and ride time equally, underpredicts wait time by 36% and overpredicts ride time by 47%. Yet the total travel time estimate, the desired output of the model, is within 7.5% of the actual value.

The conclusion to be reached is that the descriptive model as it has been presented can accurately predict total travel time for a computer dispatched DRT system and wait time and ride time for systems whose dispatching is predicated on an equal weighting of these two variables, and where the driver plays no role in stop sequencing. To make the model more generally applicable would require the addition of parameters that can account for changes in dispatching.

It is hoped that additional experience with computer dispatching in DRT systems will provide sufficient data to calibrate such a model. In the interim, however, the following refinements to the model are suggested in order to make the model more general.

$$WT_a = (1 + \alpha + \beta)WT^1$$
$$RT_a = RT - \beta WT$$

Where:

WT_a = Wait time adjusted for dispatching system

RT_a = Ride time adjusted for dispatching system

α and β are parameters which reflect different dispatching algorithms

β = a measure of weighting of wait and travel time[6]

α = an indication of whether the system is computer dispatched.

Suggested ranges for these parameters are:

$\alpha = 0$ for computerized dispatching

$\alpha = .1\text{-}.3$ for manual dispatch, depending on the demand level. In Haddonfield, α was approximately .2.

$\beta = -.6$ to $.6$ (negative if wait time is weighted higher than ride time, positive if weighted lower).

A word of caution in using this form: it is possible for WT or RT to go below their minimum bound in certain situations. This constraint places an upper and lower bound on β.

To illustrate the way in which this format can be applied, let us return to the Rochester example. Set $\alpha=0$ and, since ride time was weighted 50% higher than wait time, we might set $\beta=.5$. The resulting values of WT and RT are 29.1 and 13.9 respectively, extremely close to the actual values of 30.5 and 16.0.

COMPARISON OF DESCRIPTIVE MODEL WITH DATA FROM ROCHESTER*

	# of Veh. in Service	Mean System Wait Time	Mean System Ride Time	Mean System Travel Time
Rochester Data	2-5 (mean = 3.4)	30.5 min.	16.0 min.	46.5 min.
Model Results	2.9(WT); 3.4(RT)	19.4 min.	23.6 min.	43.0 min.

*Data is for one evening of service; to-date the computer system has been successfully used for evening service only. Data represents a 4.6 hour period during which 62 demands were served. Five instances of "no shows" were treated as half-demands. Effective vehicle speed, accounting for street adjustment factors was measured at 11.3 miles per hour. L-2.2, A = 17.5. The effective vehicle fleet size was assumed to be 15% smaller than the actual mean fleet size.

TABLE 2

USE OF THE MODEL TO DETERMINE PARAMETRIC RELATIONSHIPS

Given the ability of the descriptive model to predict total travel time (for a computer dispatched system), there is an opportunity provided to explore the behavior of a many-to-many system in a wide range of situations, and determine how various system characteristics effect level of service.

Figure 1 tests the hypothesis that there are significant economies of scale in DRT systems, i.e., as demand density increases, productivity can be increased without decreasing service quality. It is clear from this figure that such economies of scale do exist at all service levels, for some range of demand density. This behavior is due to the increasing probability with increasing demand densities that different passengers will be travelling along similar paths at similar times, making the formation of more efficient tours. As service quality is relaxed, the time window to provide service by a common vehicle is also lengthened, resulting in higher productivities at a given demand density. The higher the quality of service provided, however, the more quickly these economies of scale are exhausted. For example, if the mean LOS is about 2, economies effectively disappear beyond a demand density of about 10-15 passengers per square mile per hour. For a LOS of 4, however, economies exist even at demand densities of greater than 20.

In developing this graph, demand density was varied from 1 to 20 demands per square mile per hour. Most existing many-to-many systems operate in the range of 2-5 demands per square mile per hour. In that range the model predicts productivities of 2.5-4.0 for a high quality service (LOS-1.7), and 6.0-8.5 for a relatively poor quality service (LOS=4.2). The productivities of existing systems have typically been well within these ranges, and although these productivities may be low when compared with projections made before any systems were implemented (11), they should not be surprising given the low demand densities.

PRODUCTIVITY vs. DEMAND DENSITY

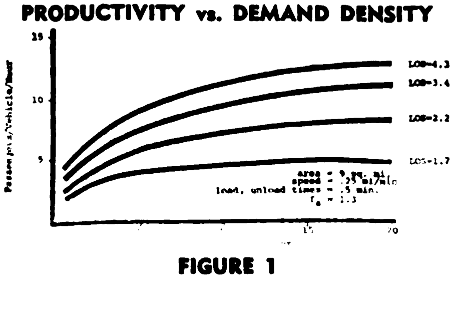

FIGURE 1

LEVEL OF SERVICE vs. SERVICE AREA SIZE

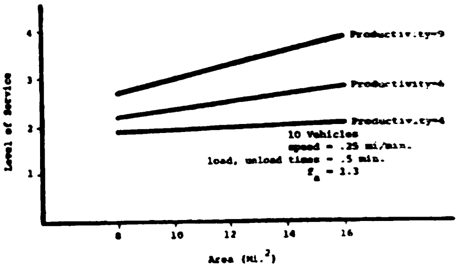

FIGURE 2

It has been noted that area size plays an important role in a DRT system. As area size increases, and the number of demands remain the same, the distance between stops on a vehicle tour would be expected to increase. Thus level of service would be expected to increase with increasing area size. Figure 2 supports this hypothesis. Level of service is shown as a function of area size, for constant demand levels (not demand densities) and vehicle fleet sizes. Notice that at fairly low productivity levels, service levels are less sensitive to area size. Interestingly, level of service appears to be linearly related to service area size. As productivity increases, the level of service becomes more sensitive to area size; this, of course, is a reflection of the non-linear relationship between level of service and productivity.

Finally, Figure 3 illustrates the "trade-off" between demand density and area size for constant level of service and constant vehicle fleet size. The same number of vehicles that are needed to maintain a particular level of service in a small area with a particular demand density would be needed to maintain the

DEMAND DENSITY vs. AREA SIZE
Constant Vehicle Fleet Size

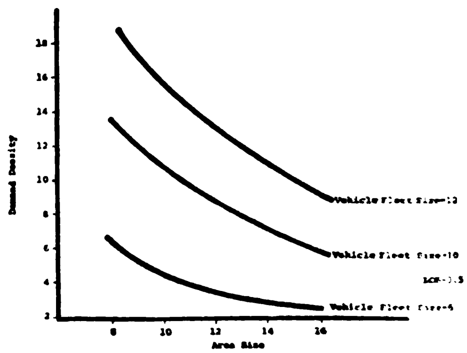

FIGURE 3

same level of service in a larger area with a lower demand density.

PERSPECTIVE

The descriptive supply model presented in this paper is intended to be used primarily as a screening tool. In this context, the model would typically be used to predict system performance for a range of possible service area sizes and vehicle fleet sizes. This would enable the planner to determine which options are feasible and how many vehicles would be needed to maintain a certain service level for a given system. The service time outputs of the model could be used as input to a demand model; alternatively, in the absence of an actual demand model the supply model could be used to predict performance over a range of feasible demands. Clearly, assumptions will have to be made about such input parameters as speed, load/unload times, and perhaps effective vehicle fleet size. However, the model can be easily used to perform sensitivity analyses on each of these parameters. One word of caution; since the model is a descriptive model its range of reliability extends only to the range of calibration.

The model has been able to accurately predict total system travel time for both the Haddonfield, N.J. and Rochester, N.Y. DRT systems, and it is this measure that the model can most reliably predict. An approach has been suggested that would make the model responsive to differences in the dispatching system used. With this approach, it should be possible to predict reasonable total travel times for manually dispatched systems, and to predict reasonable wait and ride time values in general. However, the impact of dispatching on level of service is clearly an area which requires additional research.

An objective of the research which led to this model was to develop an inexpensive, simple to use model that could predict level of service as reliably as the MIT simulation model, and avoid the problems of earlier descriptive models by being properly bounded, being sensitive to a wide range of input parameters, being reliable over a fairly wide range of inputs, and displaying the proper relationships between system parameters. The model described in this paper has achieved this objective, and should provide a valuable tool for the planning of many-to-many DRT systems.

FOOTNOTES

1 Contract #DOT-TSC-977.

2 A perfect grid system would have an "adjustment factor" of 1.273. Measurements of actual distances for the Haddonfield, N.J. and Rochester, N.Y. DRT service areas resulted in adjustment factors of 1.4 and 1.2 respectively.

3 The Rochester, N.Y. DRT system is implementing a computer dispatch system and, although it is not as yet fully operational, some data are available, as will be discussed later.

4 When using the model as a screening tool it does not seem unreasonable to assume an idealized system in which vehicles never leave service.

5 The discrepancies may be in part traced to the estimates for V,1,u and effective vehicle fleet size. A number of simplifying assumptions used in calibrating the model should be pointed out, however. The model was calibrated with the assumptions of uniform demand density and square service areas. Simulation experiments have indicated that, except in extreme cases, neither of these assumptions will significantly affect the reliability of the results. However, this may account for small differences between predicted and actual system performances.

6 One of the reasons that this approach is feasible is that total travel time has been found not to vary significantly with different weightings of wait and ride time. Thus β represents the relative weight of wait and ride time within total travel time.

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