Compte Rendu de la Conférence Internationale sur la Recherche en Matière de Transport

Proceedings of the International Conference on Transportation Research

PREMIÈRE CONFÉRENCE

FIRST CONFERENCE

Bruges, Belgium
Juin, 1973

Bruges, Belgium
June, 1973

COLLEGE D'EUROPE

TRANSPORTATION RESEARCH FORUM

Recent Development and Applications of Lowry Type Land Use Models

by

*B. G. Hutchinson**
*O. Stradal***

DURING THE PAST 20 years comprehensive transport planning studies have been conducted in many urban areas throughout the world. Most of these studies have been directed towards dimensioning the capacity requirements of alternative transport networks geared to serving trend type development prognostications. In many jurisdictions throughout the world, transport plans produced by the traditional process have been rejected by decision makers. The sentiment has been expressed by many (1, 2,3) that the urban transport problem cannot be viewed simply as a problem in transport capacitv determination. It has been suggested that a major improvement in strategic planning would be realized if the problem was viewed as a question of urban spatial organization.

In a number of countries throughout the world, studies have been initiated with the aim of exploring in a quantitative way, the transport and other infra-structure requirements of a number of alternative strategic development concepts. In Switzerland, current strategic planning studies range in scope from the national scale to analyses of Bern and Zürich. In India, one concern is with the implications of alternative development plans for the Delhi region.

In the Province of Ontario, the Government has been attempting to articulate comprehensive development plans for each of the economic regions of the Province. These regions range in size from the Toronto Centred Region of about 4 million persons, to smaller regions of 1 - 200,000 persons.

It is the purpose of this paper to describe the manner in which several versions of the Lowry family of land use models are being applied to strategic planning problems in the countries mentioned previously.

**Professor of Civil Engineering, University of Waterloo, Waterloo, Ontario, Canada.*

***Professor of Planning and Management in Building, Swiss Federal Institute of Technology, Zurich, Switzerland.*

THE LOWRY MODEL

The Lowry model (4) conceives of the major spatial features of an urban area in terms of three broad sectors of activity which are the basic employment sector, the population serving employment sector and the household sector. The fundamental structure of the Lowry model is illustrated in Figure 1.

In the Lowry model basic employment is usually defined as employment in those industries whose outputs are sold principally in markets external to the region under study. However, this definition cannot be interpreted literally and the important criterion is that the location of basic employment within a region is essentially independent of the population distribution within the region. Examples of basic industries are the various primary sectors, manufacturing, national financial institutions, university and other tertiary education employment, and so on.

In contrast to basic employment, population serving employment location is highly dependent upon the population distribution. Retail trade, personal serv-

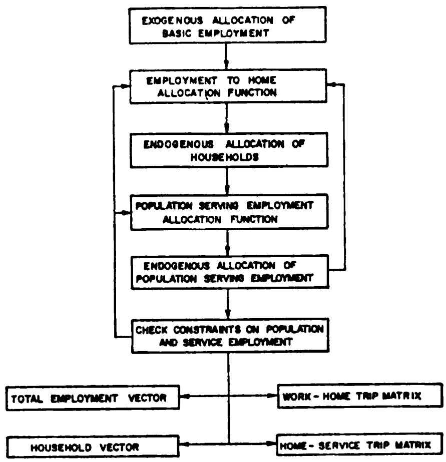

FIG. 1 - FUNDAMENTAL STRUCTURE OF LOWRY LAND USE MODEL

ices, elementary and high school employment are examples of population serving employment.

In the Lowry model, the spatial distribution of basic employment is allocated exogenously to the model, and the spatial distributions of households and population serving employment are calculated by the model. The zonal allocation rules for both households and population serving employment are specified within the model structure. In addition, the constraints on the maximum number of households for each zone and the minimum population serving employment thresholds for any zone are specified.

The Lowry model may be regarded as a static equilibrium type model based on the concept that inter-activity accessibilities are the major determinants of the equilibrium distribution of activities. Future states forecast by the Lowry model must be regarded only as conditions of quasi-equilibrium in that they do not reflect the development history of the urban system under study.

Lowry Model Equation System

The Lowry model structure shown in Figure 1 may be expressed in terms of the following system of equations:

$$p = eA \quad (1)$$
$$e^s = pB \quad (2)$$
$$e = e^b + e^s \quad (3)$$

in which p = a row vector of the population (or households) within each of the n zones; e = a row vector of the total employment in each zone; e^s = a row vector of the population serving employment in each zone; A = an nxn matrix of the workplace to household accessibilities; B = an nxn matrix of the household to service centre accessibilities.

The A matrix may be developed as follows:

$$A = [a'_{ij}]\,[a_j] \quad (4)$$

in which $[a'_{ij}]$ = an nxn matrix of the probabilities of an employee working in i and living in j; $[a_j]$ = an nxn diagonal matrix of the inverses of the labour participation rates, expressed either as population per employee, or households per employee.

The B matrix may be developed as follows:

$$B = [b'_{ij}]\,[b_i] \quad (5)$$

in which $[b'_{ij}]$ = an nxn matrix of the probabilities that the population in j will be serviced by the population serving employment in i; $[b_i]$ = an nxn diagonal matrix of the population serving employment to population ratios.

Empirical Estimation of the Accessibility Matrices

The a'_{ij} elements may be estimated empirically in the following way:

$$a'_{ij} = h_j f_{ij}^w(d_{ij}) / \sum_j h_j f_{ij}^w(d_{ij}) \quad (6)$$

where h_j = a measure of the attractivity of zone j for household location; d_{ij} = the travel time between i and j; the function f_{ij}^w of travel time is determined through calibration of the model to empirical data.

The alternative functional forms of f_{ij}^w that have been used in practical applications of the model include:

f_{ij}^w = an area wide polynomial function of travel time f^w which represents an average for the study area,

$f_{ij}^w = d_{ij}^{-n}$, where the index n also represents an average value for the study area,

$f_{ij}^w = \exp(-\alpha_{ij}^w d_{ij})$, where the parameter α_{ij} has been used as an area wide average α, as a zone specific parameter α_i and sometimes as an i-j zone specific magnitude α_{ij}.

These various functional forms of d_{ij} incorporated in equation (6) reflect the influence that travel time has on workplace—household location decisions.

The b'_{ij} elements may be estimated empirically in a manner similar to that described above for a'_{ij} where the expression analagous to equation (6) is:

$$b_{ij}'^r = s_i^r f_{ij}^s(d_{ij}) / \sum_i s_i^r f_{ij}^s(d_{ij}) \quad (7)$$

where s_i^r = a measure of the attractivity of zone i for satisfying the service type r needs of people; f_{ij}^s = service travel time factor function.

The Zonal Constraints

The distribution of activities produced by the above set of equations should be such that the following constraint equations are satisfied:

$$p \leqslant p^c \quad (8)$$
$$e^{sr} \geqslant e^{srmin} \quad (9)$$

where p^c = a row vector of the population holding capacities of each of the n zones; e^{srmin} = a lxr matrix of the population serving employment thresholds for type r service employment that are considered to be viable for any zone.

If equations (8) and (9) are violated, then new accessibility matrices must be developed and the equation set solved again using the new matrices. One approach to this problem of developing a distribution of activities that satisfies the constraints is outlined in the following:

$$a_{ij}^{*} = h_j^{*} f_{ij}^{w}(d_{ij}) / \sum_j h_j^{*} f_{ij}^{w}(d_{ij}) \qquad (10)$$

$$h_j^{*} = h_j(h_j/p_j) \qquad (11)$$

$$b_{ij}^{*r} = s_i^{*r} f_{ij}^{s}(d_{ij}) / \sum_i s_i^{*r} f_{ij}^{s}(d_{ij}) \qquad (12)$$

$s_i^{*r} = s_i^{r}$ for zones in which

$$e_i^{sr} \geqslant e_i^{srmin} \qquad (13)$$

$= 0$ for zones in which

$$e_i^{s} < e_i^{srmin} \qquad (14)$$

in which a_{ij}^{*} and b_{ij}^{*r} are the new accessibility elements to be used in the next interaction of the model.

The home based work trip matrix for any time period (say the peak hour) may be derived from:

$$T^{w} = WA' \qquad (15)$$

where T^{w} = an nxn matrix of home based work trips where the elements t_{ij}^{w} equal the number of work trips between zones i and j; W = an nxn diagonal matrix of the work trip generation rate per employee during the time period under consideration; A' = the workplace to home accessibility matrix.

The home based service trip matrices for any time period may be calculated in an analagous way:

$$T^{sr} = RB^{r\prime} \qquad (16)$$

where T^{sr} = an nxn matrix of the home based service trips to type r service opportunities; R = an nxn diagonal matrix of the type r service trip generation rate per person (or household) during the time period under consideration; $B^{r\prime}$ = the home to type r service accessibility matrix.

Figure 2 illustrates a flow diagram of the sequence of activities required to obtain a stable co-distribution of population and employment using equations (1), (2), (3), (6), (7), (10) - (14), and which satisfies the constraints (8) and (9). Equations (1) and (2) are linked together by equation (3) and the stable distributions of population and employment are achieved through an iterative approach.

Garin (5) has shown that the iterative process illustrated in Figure 2 converges to the following expression:

$$p = e^{b}(I-AB)^{-1}A \qquad (17)$$

where I = an nxn identity matrix; and the other terms have been defined previously.

The validity of equation (17) is subject to a certain condition on the product matrix AB which is that the sum of the elements in a row is less than unity. This means that $(AB)^{x}$ must tend to zero as x increases. Garin argues that if this were not the case then an infinite amount of population serving em-

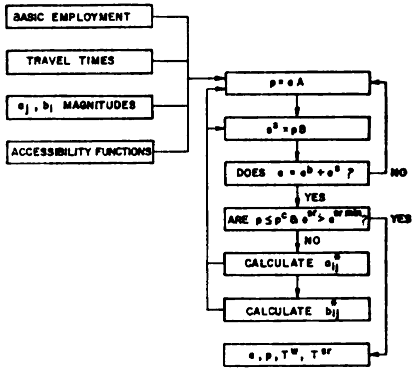

FIG. 2 - LOWRY MODEL PROCESS FOR STABLE CO-DISTRIBUTION OF EMPLOYMENT AND POPULATION

ployment would be generated by a finite amount of basic employment, which is not observed empirically.

Both Wilson (6) and Batty (7,8) have provided interesting extensions to the original Lowry model framework. The principal contribution of Wilson has been in the development of a consistent procedure for generating alternative forms of the Lowry model for different sets of input and output conditions. Batty has developed systematic techniques for both calibrating the model and using it to simulate future conditions. Other applications of the Lowry model are described in references (9, 10, 11, 12).

Disaggregation of the Equation System

The socio-economic characteristics of individuals within urban areas have a significant impact on the spatial location and modal transport choice decisions of the various groups living in urban areas. In certain types of strategic planning studies activity distributions and the associated transport flows are required by socio-economic group, rather than as aggregated quantities.

If k socio-economic groups exist in an urban area, then the equations described previously may be re-written as follows:

$$p^{k} = e^{k}A^{k} \qquad (18)$$

$$e^{sr} = pB^{r} \qquad (19)$$

$$e^{k} = e^{bk} + e^{sk} \qquad (20)$$

and the other equations described above may be disaggregated in a similar manner.

Wilson (6) and others have proposed disaggregated forms of the Lowry model but the data requirements of these proposals are excessive. Any disaggregation of the Lowry model must be compatible with the types of variables that are expressed normally in planning proposals and which are forecastable.

The basic employment vector disaggregated by person type k and contained in equation (20) may be derived in the following way:

$$[e^{bk}] = [e^{kc}][e^{bc}] \quad (21)$$

where $[e^{bk}]$ = a row vector of basic employment partitioned into k person types; $[e^{kc}]$ = a kxc matrix of the probabilities that person type k will be employed in industry category c; $[e^{bc}]$ = a row vector of basic employment partitioned into industry categories c.

A disaggregated service employment vector may be derived in an analogous manner;

$$[e^{sk}] = [e^{kr}][e^{sr}] \quad (22)$$

where $[e^{sk}]$ = a row vector of the service employment partitioned into k person types; $[e^{kr}]$ = a kxr matrix of the probabilities that person type k will be employed in service type r; $[e^{sr}]$ = a row vector of service employment partitioned into service employment types r.

The residential locations of any socioeconomic group k may be estimated by formulating group specific accessibility matrices $a_{ij}'^k$ such that:

$$\sum_j a_{ij}'^k = 1.0 \quad \text{for } i = 1, 2, \ldots, n \quad (23)$$

$$a_{ij}'^k = h_j^k f^{wk}(d_{ij}) / \sum_j h_j^k f^{wk}(d_{ij}) \quad (24)$$

where h_j^k = a measure of the attractivity of housing opportunities in zone j to type k persons; $f^{wk}(d_{ij})$ = a function of the i-j travel time which reflects the influence that travel time has on the residential location decisions of group k persons.

In certain applications of the Lowry model, the measure of residential attractivity h_j^k is set equal to the number of housing opportunities o^k available to group k people in zone j. The housing opportunities by group k may be estimated from:

$$[o^k] = [h^{kd}][g^d] \quad (25)$$

where $[o^k]$ = a row vector of housing opportunities partitioned into k groups; $[h^{kd}]$ = a kxd matrix of the probabilities that group k persons will live in housing density class d; $[g^d]$ = a row vector of the number of housing opportunities g partitioned into d housing density classes.

With this approach to disaggregation, residential density has been used as a proxy for housing type and price. Residential density is one of the few variables that is normally expressed in planning proposals for future time horizons. In certain situations the residential density variable may not be an adequate reflection of the housing market and the density may have to be disaggregated into housing quality groups.

Many studies of modal transport choice have demonstrated that trip makers must be thought of as two separate groups which are usually referred to as captive transit riders and choice transit riders. Captive transit riders are defined as those trip makers who do not have access to a car for the particular trip being studies. Choice transit riders are defined as those trip makers who may choose freely between public transport and private car for a particular journey. Mode specific transport flows may be estimated from the following expressions:

$$t_{ij}^{wkm} = t_{ij}^{wk} pr_{ij}^{wkm} \quad (26)$$

$$t_{ij}^{wm} = \sum_k t_{ij}^{wkm} \quad (27)$$

where t_{ij}^{wkm} = the number of home based work trips between zones i and j of group k persons by mode m; t_{ij}^{wk} = the number of trips between zones i and j of group k persons; pr_{ij}^{wkm} = the probability that group k persons on a work trip between zones i and j will use mode m.

Certain k groups may be classified immediately as transit captives and the modal patronage probabilities in equation (26) may be specified immediately. For those k groups which are identified as choice transit riders, the modal patronage probabilities may be estimated from logistic functions of the following form:

$$pr_{ij}^{wt} = \frac{e^{z_{ij}}}{1+e^{z_{ij}}} \quad (28)$$

$$pr_{ij}^{wc} = \frac{1}{1+e^{z_{ij}}} \quad (29)$$

where pr_{ij}^{wt} and pr_{ij}^{wc} = the probabilities of transit and car usage respectively; z_{ij} = the difference in the generalized costs of travelling by transit and car between zones i and j for the home based work trip.

The generalized cost of travel is derived from the notion that travel is considered to have a number of characteristics which create costs and the generalized cost of a trip is calculated from:

$$z_{ij}^m = a_1 x_{1ij}^m + a_2 x_{2ij}^m + \ldots + a_n x_{nij}^m \quad (30)$$

where x_{nij}^m = the nth characteristic of mode m between zones i and j which creates costs of travel; a_n = the unit contribution (in cents) that the nth characteristic of mode m makes to the generalized costs of travel.

Dynamic Extensions to the Model

Dynamic and quasi-dynamic extensions to the basic Lowry model structure have been proposed by Crecine (12), Rogers (13) and Echenique (14). There is some difficulty in applying the model structures advanced in the above references because of the absence of suitable data sets. A simple quasi-dynamic version of the Lowry model is presented below which is compatible with data that are normally available in planning studies.

The population distribution at some future time period is given by the following expression:

$$p^k(t+\Delta t) = p^{kf}(t) + p^k(\Delta t)$$
$$= p^k(t)L(k)+e^k(\Delta t)A^k(t+\Delta t) \quad (31)$$

where $p^k(t+\Delta t)$ = a row vector of population at time $t+\Delta t$ partitioned into k socio-economic groups; $p^{kf}(t)$ = a row vector of population at time t which does not relocate in the time interval Δt partitioned into k socio-economic groups; $L(k)$ = an nxn diagonal matrix of the probabilities that a person type k household will not relocate during the period Δt; $A^k(t+\Delta t)$ = the workplace to home accessibility matrix for time $t+\Delta t$; $e^k(\Delta t)$ = a row vector of the change over Δt of total employment.

The change in total employment over Δt consists of the following components;

(i) the jobs freed by households which relocate during the period Δt,

(ii) the change in basic employment over Δt, and

(iii) the change in service employment over Δt due to the change in basic employment over Δt.

The new workplace to home accessibility matrix reflects both the changes in transport system properties and household opportunities over Δt. The probabilities $L(k)$ may be established readily for most urban areas from real estate records.

The population serving employment distribution for time $t+\Delta t$ is given by:

$$e^{sr}(t+\Delta t) = p(t+\Delta t)B^r(t+\Delta t) \quad (32)$$

Equation (32) implies that service employment relocation follows closely in time any relocations in population throughout a time period. The P matrices in equation (32) will reflect any transport system property changes as well as any changes in service centre location policies.

Equations (31) and (32) are simply examples of the way in which the equation set introduced early in this paper may be extended to yield a quasi-dynamic version of the Lowry model. Space does not permit the full development of a dynamic version of the Lowry model in this paper.

APPLICATIONS OF THE LOWRY MODEL

A series of applications of the Lowry model to a range of strategic planning problems have been undertaken at the Institut für Orts-, Regional-, und Landesplaning of the Swiss Federal Institute of Technology, Zurich. A computer model, called ORL-MOD-1, has been developed and described by Stradal and Sorgo (15). Exponential type accessibility functions of the following type are used in this model:

$$a_{ij}' = h_j\exp(-\alpha_{ij}d_{ij}) / \sum_j h_j\exp(-\alpha_{ij}d_{ij}) \quad (33)$$

$$b_{ij}' = h_j\exp(-\beta_{ij}{}^r d_{ij}) / \sum_j h_j\exp(-\beta_{ij}{}^r d_{ij}) \quad (34)$$

ORL-MOD-1 has been used to examine the implications of alternative spatial development alternatives for the Zürich region. The accessibility function of equation (33) was calibrated for three work place zones located in three different parts of the region. The results of these calibrations are shown in Figure 3. This diagram illustrates the influence of using h_j factors equal to the zonal populations. This diagram illustrates that the observed trip length frequency distributions of commuters to jobs in the region are simulated satisfactorily by the above accessibility function. The criterion used to establish the best estimate of α_{ij} was the minimum absolute difference in the actual and calculated a_{ij}' accessibility magnitudes.

The population serving employment accessibility function of equation (33) was calibrated for three types of service employment in the Zürich region and these were r = food shops, r = large central Zürich department store, and r = regionally located supermarket. The results of these calibrations are shown in Figure 4. It should be noted from equation (34) that the weighting factor used is the population of the zones served by the particular service zones. It should also be noted from Figure 4 that an unweighted version of equation (34) was used for the accessibilities of food shops.

The calibrated model has been used to test a number of alternative development concepts for Canton Zürich in the

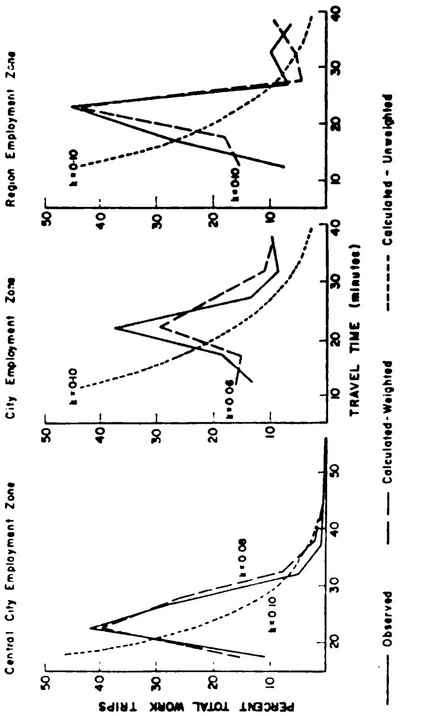

FIG. 3 - ZURICH REGION WORKPLACE - HOME ACCESSIBILITY FUNCTION CALIBRATIONS

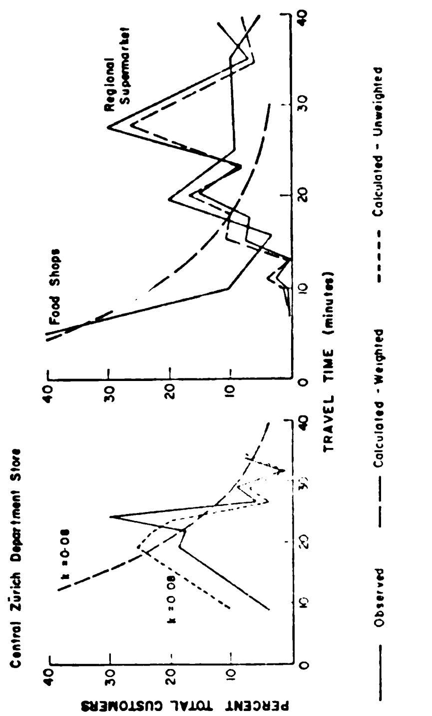

FIG. 4 - ZÜRICH REGION HOME - SERVICE ACCESSIBILITY FUNCTION CALIBRATIONS

year 2000 and these alternatives included:

(i) Trend Development: existing development trends extrapolated to 2000 with the proposed transport network.

(ii) Trend Development With Extended Transport Network: same trend development as above but with proposed transport network for the year 2030.

(iii) Two Band Cities: two centres exist in Zürich, the main centre and a secondary centre in the industrial section of the city; this concept would emphasize growth of the secondary centre and would promote growth along the two valleys of Zürich Lake and Greifen Lake.

(iv) New Regional Centres: new regional centres away from the City of Zürich would be emphasized through the subsidization of hospitals, cultural facilities and schools in these growth centres.

Distributions of population and employment were estimated along with the transport flows created by the activity allocations. These estimates were obtained by using the properties of the future transport system, a forecast of employment growth and a detailed estimate of the parameters α_j through the use of estimated house rents within the region.

ORL-MOD-1 has also been used to estimate the peak period work trip flows in Bern for 2000. In the traditional transport planning approach population and employment estimates are prepared for all zones and the flows derived using the traditional trip generation and trip distribution models. Hutchinson (16) has pointed out some of the difficulties with the traditional approach. ORL-MOD-1 was used to derive the zonal population allocations and the inter-zonal transport flows simultaneously, given a 2000 employment allocation and the estimated 2000 transport network properties.

ORL-MOD-1 is being applied currently to problems of national planning in Switzerland and to regional planning in Hamburg, Germany. Three alternative development strategies are being considered for the whole of Switzerland and the servicing and transport implications of these alternatives are being explored with the aid of ORL-MOD-1. In Hamburg, a major new airport north of the City is planned. The transport facilities which are to be used to connect the airport to the city are also to be used to stimulate urban development in a northerly direction. Three alternative rapid transit routes to the airport are to be evaluated with respect to their probable impact on urbanization.

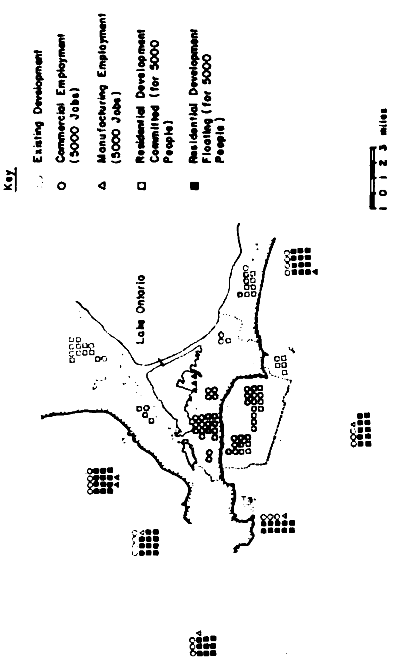

FIG. 5 - ONE DEVELOPMENT CONCEPT FOR HAMILTON, ONTARIO

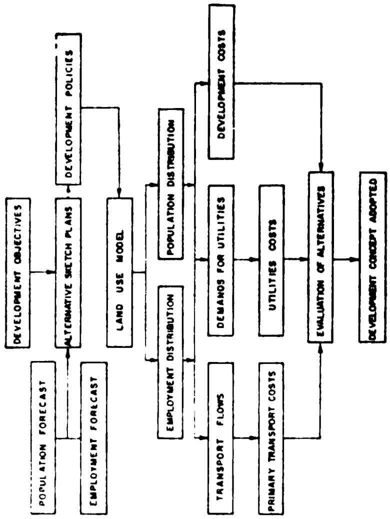

FIG 6 - ACTIVITIES IN STRATEGIC LAND USE - TRANSPORT - UTILITIES PLANNING

The applications of the Lowry model to planning problems in Ontario are directed towards the estimation of the infra-structure requirements of alternative development strategies for urban regions. Figure 5 shows one of the eleven development alternatives being considered for Hamilton, Ontario. Figure 6 outlines the flow of activities which may be used in planning problems of this type. A Lowry model disaggregated by socio-economic group may be used to estimate the spatial distributions of population, employment and transport demands by mode for various development policy alternatives. The population and employment distributions may then be converted into water supply, sewage disposal and storm water drainage demands. Computer techniques have been developed to design the trunk water, sewerage and storm drainage systems and to estimate the costs of these systems (17). Other techniques are available for estimating the development costs within zones as well as the transport network investment costs (18). With this type of information for each development policy alternative the planning team is then free to explore the non-quantifiable impacts of the alternatives.

In the applications of the Lowry model in Ontario accessibility functions which are area wide polynomial functions of travel time have been found to be the most suitable.

Urban populations in India have been growing rapidly. For example, the Delhi region is expected to increase in population from 3.2 million in 1971 to 5.5 million in 1981. Master plans were prepared for most Indian cities in the early 1960's but formal analyses of the activity allocations and their interactions which were implied by these plans were not conducted. At the present time in Delhi, several major development alternatives are being considered and these include construction of a rapid transit line, densification of certain parts of the city and redevelopment of older sections of the city.

A disaggregated form of the Lowry model is being used at the University of Waterloo to explore the implications of alternative development policies of the above type. The importance of using a model disaggregated by economic group in Delhi is illustrated by the information presented in Table 1. This table shows the distribution of households by income range for the Delhi urban area as well as the influence of income on modal transport choice. The equation system introduced earlier in this paper for disaggregated economic groups forms the basis of the computer model being used in this study.

Other applications of the Lowry family of land use models are described in reference (19).

CONCLUSIONS

Applications of the Lowry model to a number of urban areas have demonstrated that it is capable of explaining the major spatial features of urban areas, and the activity interactions, in

TABLE 1
DISTRIBUTION OF HOUSEHOLDS BY INCOME
DELHI, 1969

Income Range (Rupees/Month)*	Percent of Households	Modal Patronage: Car	Motorcycle	Bicycle	Mass Transit	Other
<100	4.1	1.4	0.1	47.8	44.6	6.1
100-149	9.4	0.9	0.5	51.5	40.4	6.7
150-249	26.7	0.8	2.0	54.6	37.0	5.6
250-499	32.3	1.8	8.7	32.6	47.9	12.0
500-749	12.9	8.8	30.0	12.1	36.6	15.0
750-999	4.4	16.2	41.5	5.8	24.6	11.9
1000-1499	4.9	36.4	30.6	2.9	15.5	14.6
1500-1999	2.2	78.1	9.1	0.6	3.7	9.0
2000-2500	3.0	78.8	6.2	1.4	3.0	10.6
>2500	0.1	86.3	2.4	0	2.1	9.2

*$C1 = Rs. 7

terms of a few variables. The Lowry family of land use models has a simple causal structure and modest data requirements. These features make the model particularly useful for regional planning problems where extensive data bases are usually not available. The causal structure of the model allows the principal development policy variables available to most municipalities to be tested.

This paper has shown that the model may be disaggregated readily and may be expressed in a quasi-dynamic form. The disaggregation and the dynamic extension proposed in this paper require information that is normally available in strategic planning studies. Applications of the model have demonstrated that it may be used for a variety of planning problems of different scales and levels of detail.

REFERENCES

1. Roberts, P. O., **"Model Systems for Urban Transportation Planning: Where Do We Go From Here?,"** Record No. 309, Highway Research Board, Washington, D.C., 1970.
2. Hutchinson, B. G., **"Some Urban Transportation Investment Myths,"** Proceedings, Roads and Transportation Association of Canada, 1972.
3. Organisation for Economic Co-operation and Development, **"The Urban Transportation Planning Process,"** Paris, 1971.
4. Lowry, I. S., **"A Model of Metropolis,"** RM-4035-RC. The RAND Corporation, Santa Monica, California, 1964.
5. Garin, R. A., **"Matrix Formulation of the Lowry Model for Intra-Metropolitan Activity Location,"** Journal, American Institute of Planners, December, 1966.
6. Wilson, A. G., **"Entropy in Urban and Regional Modelling,"** Pion Press, 1970.
7. Batty, M. J., **"Some Problems of Calibrating the Lowry Model,"** Environment and Planning, Vol. 2, 1970.
8. Batty, M. J., **"Design and Construction of a Sub-Regional Land Use Model,"** Socio-Economic Planning Sciences, Vol. 5, 1971.
9. Cordey-Hayes, M., and Massey, D. B., **"An Operational Urban Development Model of Cheshire,"** Working Paper 64, Centre for Environmental Studies, London, 1970.
10. Cripps, E., and Foot, D. H. S., **"A Land Use Model for Sub-Regional Planning,"** Regional Studies, Vol. 3, 1969.
11. Goldner, W., **"Projective Land Use Model,"** Bay Area Transportation Study Commission, Berkeley, California, September, 1968.
12. Crecine, J. P., **"Time Oriented Metropolitan Model,"** CRP Technical Bulletin No. 5, Pittsburgh City Planning Department, January, 1964.
13. Rogers, A., **"A Note on the Garin-Lowry Model,"** Journal, American Institute of Planners, December, 1966.
14. Echenique, M., **"Urban Systems: Towards an Explorative Model,"** CES UWP 2, Centre for Environmental Studies, London, 1969.
15. Stradal, O., and Sorgo, H., **"ORL-MOD-1: A Model for Regional Allocation of Activities,"** Working Paper No. 25.1, Institute for National, Regional and Local Planning, Swiss Federal Institute of Technology, Zurich, 1970.
16. Hutchinson, B. G., **"Transport Analysis of Regional Development Plans,"** Journal Urban Planning and Development Division, A.S.C.E., Vol. 99, UP1, March, 1973.
17. Kropf, R, and Hutchinson, B. G., **"Trunk Services Design and Costing Sub-Routines,"** Working Paper, Transport Group, Department of Civil Engineering, University of Waterloo, 1973.
18. D'Andrea, N., and Hutchinson, B. G., **"Development Cost Sub-Routines for Garin-Lowry Model,"** Working Paper, Transport Group, Department of Civil Engineering, University of Waterloo, 1972.
19. Stradal, O., and Hutchinson, B. G., **"Notes for a Short Course on Practical Applications of Regional Development Models,"** Transport Group, Department of Civil Engineering, University of Waterloo, 1971.