Stochastic efficiency analysis of alternative water conservation strategies

B Grové, F Nel & HH Maluleke[1]

Abstract

Stochastic efficiency with respect to an exponential utility function was used to determine utility-efficient water-conserving irrigation schedules for wheat and maize based on certainty equivalents. Total gross margin risk resulting from production risk of alternative deficit irrigation practices was quantified using an irrigation simulation model and stochastic budgeting procedures. Results showed increasing production variability with increasing levels of deficit irrigation, especially when rainfall has significant potential to contribute to the production process. Risk-averse decision makers are more willingly to adopt deficit irrigation schedules for maize due to increased effective rainfall. The conclusion is that the potential to use rainfall more effectively through deficit irrigation is a key variable determining adoption of deficit irrigation strategies by risk-averse decision makers. Localized weather forecasts may improve acceptance of deficit irrigation by risk-averse decision makers. The value of information for weather forecast might be low because of high risk premiums placed on full irrigation by risk-averse decision makers.

1. Introduction

Evidence of inefficient water usage can be found in all water usage sectors throughout the country, and the value of water seems largely unrecognised by most water users--particularly the privileged who, until recently, had access to water at inexpensive subsidised prices (DWAF, 1999). The realities of the new democratic South Africa demand improved management of our limited water resources. Currently, the government is reasoning that water savings in the agricultural sector will have a significant effect on the availability of water to other sectors and the protection of water resources (DWAF, 1999). One way irrigated agriculture can conserve water is through the use of deficit irrigation. Deficit irrigation is defined as the deliberate under irrigation of a crop with the aim to conserve water or to increase the profitability of the farming enterprise over the long term (Dent *et al*, 1988:19). Potential benefits include increased

---

[1] *Lecturer and Research assistants, Department of Agricultural Economics, University of the Free State, South Africa.*

irrigation efficiencies, cost savings, and gains from acknowledging the opportunity cost of water (English and Raja, 1996).

Much research has been done in South Africa to evaluate the profitability of deficit irrigation as a strategy to conserve water under limited water supply conditions. Early research showed that crops can be deficit irrigated in certain crop growth stages without reducing crop yield significantly (Virag, 1988; De Jager and Mottram, 1995). Later research focused on optimizing agricultural water use while taking water deficits in different crop growth stages into account (Mottram *et al*, 1995; Grové and Oosthuizen, 2002). The general conclusion of these researchers was that deficit irrigation is an economically viable option to follow under conditions of limited water supplies. However, water is a risk-reducing input, and decreasing water applications may increase production risk. Botes (1990) compared the risk associated with one deficit irrigation schedule with four alternative irrigation schedules for wheat, using stochastic dominance with respect to a function (SDRF). His results showed that the profitability of deficit irrigation can be improved by utilizing soils with large soil water holding capacities. Unfortunately, Botes (1990) did not quantify the effect of increasing levels of deficit irrigation. Currently, farmers are unsure whether they should reduce irrigation areas while maintaining optimal irrigation levels or to what extent they should practice deficit irrigation without reducing irrigation areas when irrigation water availability is reduced.

The main objective of this paper is to compare the risk efficiency of reduced area - full irrigation strategies for maize and wheat with alternative deficit irrigation strategies with the aim of conserving water.

2. Procedures

To conduct a thorough risk assessment one needs to know the probabilities of risky outcomes as well as the preferences held by decision makers for those outcomes (Hardaker and Lien, 2003). The first part of this section is devoted to quantifying the cumulative probability distributions of alternative deficit irrigation schedules and full irrigation strategies when water availability is reduced. The second part describes how the preferences of decision makers are taken into account to discriminate between mutually exclusive deficit irrigation schedules using certainty equivalence. The method is called stochastic efficiency with respect to a function (SERF) (Hardaker and Lien, 2003; Richardson, 2004).

2.1 Quantifying production risk of alternative irrigation schedules

Outputs from SAPWAT (Crosby *et al*, 2000) were used to quantify the production risk associated with alternative deficit irrigation schedules. The model is widely accepted as a tool to quantify crop water requirements under different conditions in South Africa. Currently, the model is used to determine lawful water use as compared to registered water use. SAPWAT uses readily available data on crop coefficients, soil water holding capacities, irrigation technology, and weather to model a daily cascading water budget taking all appropriate contributions and losses into account. The model does not calculate crop yield; however, evapo-transpiration deficits are calculated for each alternative water management scenario.

A full irrigation schedule was developed for maize and wheat by refilling the soil profile to field capacity when a critical soil water depletion level was reached. The amounts and timing of the full irrigation schedules were recorded for each state of nature (normal weather, severe drought and wet) included in the SAPWAT weather database. Using the full irrigation schedules as a basis, deficit irrigation schedules were simulated by uniformly reducing the full irrigation water applications with a specified percentage. After each simulation, the evapo-transpiration deficits in each crop growth stage, which are needed to calculate crop yields, were calculated. Crop yields were calculated through the use of crop yield response factors (*ky*) which relate relative yield decrease (*1-Ya/Ym*) to relative evapo-transpiration deficit (*1-ETa/ETm*). More specifically, the Stewart multiplicative (De Jager, 1994) relative evapo-transpiration formula was used to calculate crop yield, taking the effect of water deficits in different crop growth stages into account:

$$Ya = Ym \ \times \ \prod_{g=1}^{4} \left(1 - ky_g \left(1 - \left(\frac{\sum_{t=1}^{m} ETa_{tg}}{\sum_{t=1}^{m} ETm_{tg}} \right) \right) \right) \tag{1}$$

with:

Ya	actual crop yield (ton/ha)
Ym	maximum potential crop yield (ton/ha)
ETa	actual evapo-transpiration (mm.ha)
ETm	maximum potential evapo-transpiration (mm.ha)
ky	crop yield response factor
g	growth stages
m	length of crop growth stage in days

The calculated crop yields for each state of nature were used as inputs in a triangular distribution to quantify production risk associated with each irrigation schedule. Stochastic budgeting (Hardaker *et al*, 1997) was used to link information on the irrigation schedules with economic parameters obtained from an agricultural cooperative to calculate the total gross margin for each crop and irrigation schedule based on the irrigation cost of a 50 ha center pivot. Output prices were kept constant at R900/ton for maize and R1,200/ton for wheat. SIMETAR (**Sim**ulation for **E**xcel **T**o **A**nalyze **R**isk) (Richardson *et al*, 2004) was used to conduct all the risk simulations.

2.2 Discriminating amongst distributions

SERF (Hardaker and Lien, 2003) utilizes certainty equivalence to determine the subset of utility-efficient alternatives given a range of risk aversion. Certainty equivalence is defined as the minimum amount of money a decision maker would require as a lump-sump payment to make him indifferent between the certainty equivalent (CE) and the future payment of a risky alternative (Richardson, 2004). The level of the CE is determined by the decision maker's expected utility function and the level of risk aversion. Due to the ordinal scale used for utility, it is not trivial to go from the shape of the utility function to a measure of the level of risk aversion (Hardaker *et al*, 1997). This difficulty is resolved by a measure that is constant for any positive linear transformation of utility, known as the coefficient of absolute risk aversion (RAC) (Hardaker *et al*, 1997:96-97). RAC is defined as (Pratt, 1964; Arrow, 1965):

$$RAC = -\frac{U''(W)}{U'(W)}$$

The property of constant absolute risk aversion means that the preferred option in a risky situation is unaffected by the addition or subtraction of a constant amount to all payoffs. The exponential utility function which exhibits the property of constant RAC was assumed for the SERF analyses. More specifically, SIMETAR uses the following equations to calculate the expected utility (E(U)) and the corresponding CE for a given RAC:

$$E(U) = \sum_i p_i\left(-e^{-RAC(X_i)}\right)$$

$$CE = \frac{\ln(E(U))}{RAC}$$

The relationship between CE and risk aversion for a specific alternative is calculated for 25 RAC intervals between any user-specified lower and upper RAC, where RAC <0 indicates increasing risk seeking preferences, RAC = 0 risk indifference, and RAC > 0 increasing risk aversion. When the relationship between CE and risk aversion is defined for all the alternatives, the utility-efficient set bounded by the upper and lower RAC consists of those alternatives which have the highest CE within the range of RACs. SERF allows for the simultaneous evaluation of alternatives and is therefore more discriminating than SDRF, which uses pair-wise comparisons (Hardaker and Lien, 2003).

Since RAC's are dependant on the scale or range of the data, it is not possible to compare the results across different studies for the same RAC's. Ferrer (1999) adopted a procedure proposed by Nieuwoudt and Hoag (1993) to standardise the RAC's to give a unit-less expression of the absolute risk aversion function. The relationship between the scaled RAC and the original RAC is $RAC_{scaled} = RAC_{original} (X_{max}\text{-}X_{min})$, where X_{min} and X_{max} are respectively the minimum and maximum values of the distribution of risky outcomes. In this research, the RAC's were scaled to enable comparisons between different studies.

3. Results

Results are presented for four alternative water conservation strategies (C8%, C16%, C24% and C32%) conserving between 8% to 32% water while utilizing a full area deficit irrigation strategy (D) or a reduced area full irrigation strategy (F).

3.1 Gross margin risk of alternative deficit and full irrigation schedules

Figure 1 show the gross margin risk for irrigated maize in Vaalharts associated with alternative deficit irrigation schedules and corresponding reduced area full irrigation schedules with the aim of conserving water.

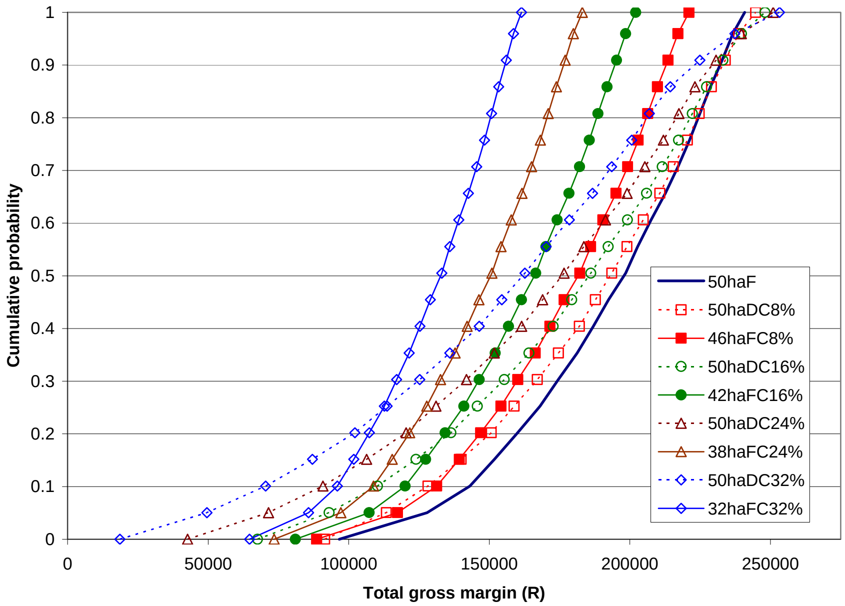

Figure 1: Total gross margin of alternative maize deficit (D) and full (F) irrigation schedules conserving (C) specified percentage water, 2004

As expected, a reduction in irrigated area to conserve water does not impact significantly on total variability of gross margins. Rather, there is an almost parallel shift in the cumulative distribution functions (CDF) to the left as the irrigated areas are reduced, which results in reduced total gross margins.

Of more interest is the risk-increasing effects of deficit irrigation. All the deficit irrigation strategies evaluated have a higher maximum gross margin compared to a full irrigation strategy when water is not limiting (50haF). The higher maximum gross margins with the deficit irrigation strategies are attributed to reduced irrigation cost and the more efficient use of rainfall and applied irrigation water. However, as the level of deficit irrigation increases, the risk also increases. Deficit irrigation leaves extra capacity in the soil profile for rainfall. Due to reduced irrigation, cost-deficit irrigation strategies will result in higher total gross margins than the full irrigation strategies in seasons with high rainfall. However, when you practise deficit irrigation in seasons with low rainfall, your downside risk will increase significantly as the level of deficit irrigation increases. For instance, the area under the CDF of strategy 50haD8% to the left of the corresponding full irrigation strategy (46haF8%) is much less than the corresponding area when you have to conserve 32% irrigation water.

Figure 2 show the gross margin risk for irrigated wheat in Vaalharts associated with alternative deficit irrigation schedules and corresponding reduced area full irrigation schedules.

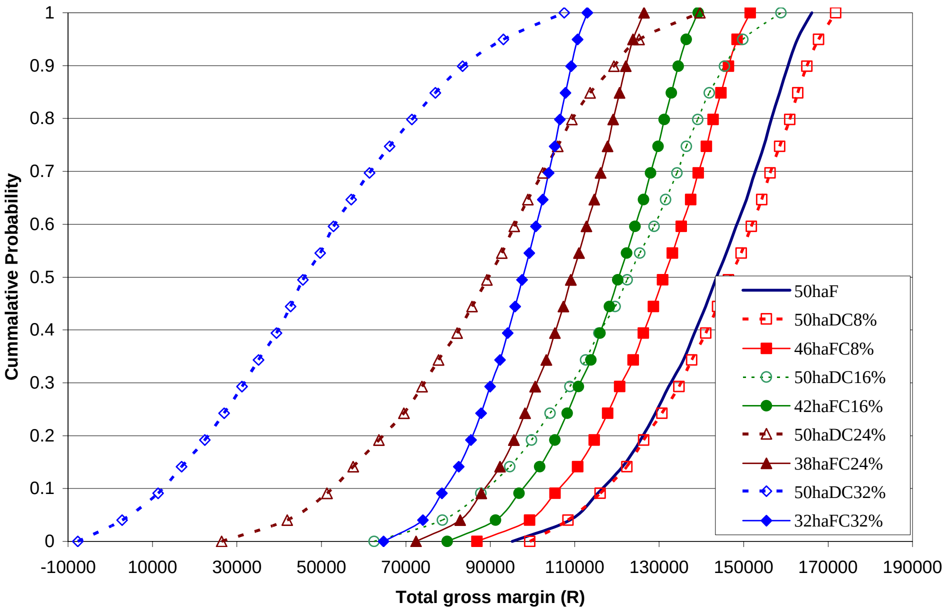

Figure 2: Total gross margin of alternative wheat deficit (D) and full (F) irrigation schedules conserving (C) specified percentage water, 2004

Due to the low rainfall in winter, increasing deficit irrigation results in rather parallel shifts of the CDF's to the left as the level of deficit irrigation increases. However, the magnitude of the shifts increases as the level of deficit irrigation increases from 8% to 32%. Because of the parallel shifts, the area under the CDF by which the full irrigation strategies dominates the deficit irrigation schedules at low probabilities increases rapidly as the level of deficit irrigation increases. When concerning a deficit of 32%, the full irrigation strategy dominates the deficit irrigation schedule by first-degree stochastic dominance. Interesting is the fact that the Strategy 50haD8% to a large extent dominates strategy 50haF, indicating that strategy 50haF may not be the optimal full irrigation schedule when water is not limiting.

3.2 Preference for deficit irrigation by decision with varying levels of risk aversion

Figure 3 shows the SIMETAR-calculated CE for RAC's that ranges between -14 and 36 for alternative maize irrigation schedules, while the results for wheat are given in Figure 4.

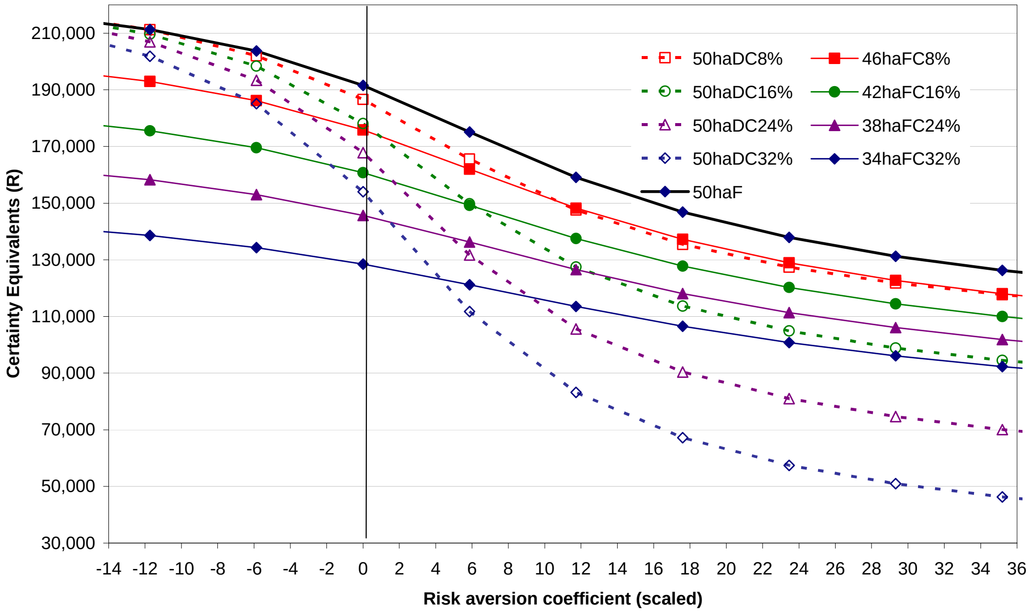

Figure 3: Certainty equivalents of alternative maize deficit (D) and full (F) irrigation schedules conserving (C) specified percentage water using the exponential utility function, 2004

Recall that the utility-efficient set for a specific range of RACs is determined by the highest CE. For RACs less than -12.31, the efficiency set consists of all the deficit irrigation strategies, and for RACs greater than -12.31, the efficiency set consists of strategy 50haF for maize. Thus, a decision maker will only engage in deficit irrigation practices willingly if such a person is risk seeking. When comparing deficit irrigation and reduced area full irrigation strategies with each other, it is interesting to note that the deficit irrigation strategies can be preferred by risk-averse decision makers when they have to conserve water. The breakeven RACs where the reduced area strategies (F) are preferred compared to deficit irrigation strategies (D) by more risk averse decision makers are 6.09, 4.77, and 4.15 respectively when you have to conserve 16%, 24%, and 32% of water.

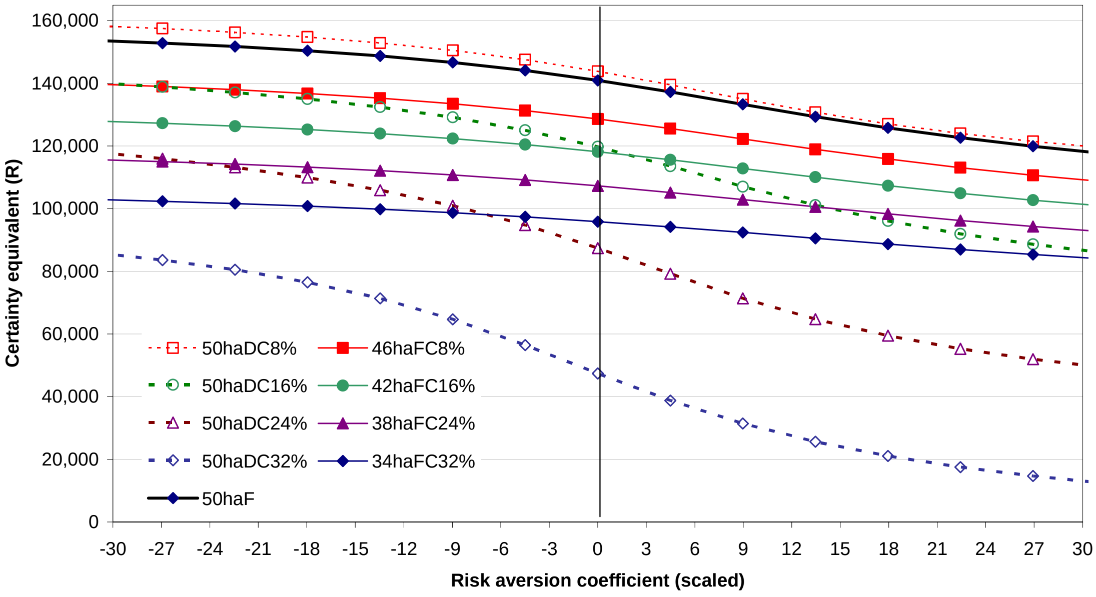

Figure 4: Certainty equivalents of alternative wheat deficit (D) and full (F) irrigation schedules conserving (C) specified percentage water using the exponential utility function, 2004

From Figure 4, it is clear that when strategy 50haD8% for wheat is ignored, no decision maker within the range -30 and 30 will willingly engage in deficit irrigating wheat since only strategy 50haF for wheat is included in the efficiency set. The breakeven RACs for a decision maker to prefer reduced area full irrigation strategies above deficit irrigation are 2.00 to conserve 16% of water and -24.72 to conserve 24% of water. Reduced area full irrigation always dominates deficit irrigation by first degree stochastic dominance when you have to conserve 32% of water while irrigating wheat.

4 Conclusions

Generally speaking, deficit irrigation is thought of as a risk-increasing strategy which will not be adopted by decision makers who are risk averse. However, the results from deficit irrigating maize showed that decision makers who are slightly risk averse will adopt deficit irrigation. In contrast, decision makers need to be rather risk seeking to adopt deficit irrigation practices when irrigating wheat. The conclusion is made that the potential to use rainfall more efficiently has a significant impact on the adoption of deficit irrigation strategies by risk-averse decision makers. Any information that will increase the potential to use rainfall more efficiently, such as improved localised weather forecasts, will improve the adoption of deficit irrigation strategies. However, the improvement in adoption might be low because of the high risk premiums placed on reduced area full irrigation by risk-averse decision makers. In areas where rainfall is low, risk-averse decision makers will not be willing to adopt deficit irrigation strategies.

Acknowledgements

Financial assistance provided by the Water Research Commission (WRC) is gratefully acknowledged. The views of the authors do not necessarily reflect those of the WRC.

References

Arrow KJ (1965). *Aspects of the theory of risk-bearing.* Academic Bookstore, Helsinki.

Botes JHF (1990). *An economic evaluation of wheat irrigation scheduling strategies using stochastic dominance.* (Afrikaans). MScAgric Dissertation. Department of Agricultural Economics, University of the Orange Free State, Bloemfontein.

Crosby CT, De Lange M, Stimie CM & Van der Stoep I (2000). *A review of planning and design procedures applicable to small scale farmer irrigation projects.* Water Research Commission, Report No 578/2/00, Pretoria.

De Jager JM (1994). Accuracy of vegetation evaporation formulae for estimating final wheat yield. *Water SA* 20(4):307-314.

De Jager JM & Mottram R (1995). *Current research on improving water management and water use efficiency on multi-farm irrigation projects.* Proceedings of the Southern African Irrigation Symposium, 4-6 June 1991, Durban. WRC Report No TT 71/95, The Water Research Commission, Pretoria.

Dent MC, Schulze RE & Angus GR (1988). *Cropwater requirements, deficits and water yield for irrigation planning in Southern Africa.* Report to the Water Research Commission on the project: A detailed regional soil moisture deficit analysis for irrigation planning in Southern Africa. WRC Report No 118/1/88, ACRU Report 28, The Water Research Commission, Pretoria.

Department of Water Affairs and Forestry (DWAF) (1999). *Draft: Water conservation and demand management national strategy framework*. Department of Water Affairs and Forestry, Pretoria.

English M & Raja SN (1996). Review: Perspectives on deficit irrigation. *Agricultural Water Management* 32(1):1-14.

Ferrer SRD (1999). *Risk preferences and soil conservation decisions of South African commercial sugarcane farmers.* Unpublished PhD Thesis. University of Natal, Pietermaritzburg.

Grové B & Oosthuizen LK (2002). An economic analysis of alternative water use strategies at catchment level taking into account an instream flow requirement. *Journal of the American Water Resources Association* 38(2):385-395.

Hardaker JB, Huirne RBM & Anderson JR (1997). *Coping with risk in Agriculture*. Wallingford, Oxon, UK.

Hardaker JB & Lien G (2003). *Stochastic efficiency analysis with risk aversion bounds: A simplified approach.* Working Paper No 2003-1. University of New England, Department of Agricultural and Resource Economics.

Mottram R, De Jager JM, Jackson BJ & Gordijn RJ (1995). *Irrigation water distribution management using linear programming*. Proceedings of the Southern African Irrigation Symposium, 4-6 June 1991, Durban. WRC Report No TT 71/95, The Water Research Commission, Pretoria.

Nieuwoudt WL & Hoag DL (1993). *Standardizing Arrow-Pratt absolute risk aversion to the range and scale of the data*. Unpublished paper, University of Natal, Pietermaritzburg.

Pratt JW (1964). Risk aversion in the small and in the large. *Econometrica*, 32:122-36.

Richardson JW (2004). *Simulation for applied risk management with an introduction to the Excel simulation add-in: Simetar*. Department of Agricultural Economics, Texas A & M University.

Richardson JW, Schumann K & Feldman P (2004). *Simetar: Simulation for Excel to analyse risk*. Department of Agricultural Economics, Texas A & M University.

Virag T (1988). *Input-output relationships and expected economic returns in crop production with variable water supply.* MScAgric Dissertation. Department of Agricultural Economics, University of Pretoria, Pretoria.