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Two Moments Estimation of the Delay On A Partially Double-Track Rail Line With Scheduled Traffic

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INTRODUCTION

The purpose of a rail line delay model is to estimate the mean and variance of the running time for each train type under various track configurations and operating policies. Such a model is used in many different contexts: the analysis of scheduling policies, analysis of capital investment decisions, etc. In addition, these models are often used as a component of strategic planning models for rail operations.

The papers by Petersen (1974, 1975) are among the first analytical line delay models for single and partially double-track railroads. English (1977) extended this work to include multiple meets and overtaking of trains in the case where traffic volumes are low; Frank (1966) considered a similar situation for heavy traffic conditions. Petersen and Taylor (1982) provide a simulation framework for the analysis of more general situations. In all of these papers, it is assumed that the trains depart from their originating yard in a Poisson manner; i.e., the train departure times are assumed to be uniformly distributed over the planning period.

While the uniform distribution assumption may be mathematically convenient, it is oftentimes unrealistic. Many trains have a scheduled departure time; in North American railroads, the current trend is to give almost every train a schedule and in Europe, scheduled operations have been the norm for quite some time. In such a situation, trains may depart with some error about the stated departure time, but they will not depart purely at random. In addition, the use of the delay models in analyzing schedule policies (Jovanovic and Harker 1990) or in the real-time control of train operations (Harker 1990) makes it imperative that schedules be an integral part of such a model. Chen and Harker (1990) have recently provided an extension of the Petersen single-track model in order to estimate the mean and variance (and hence, probability of on-time arrival) of the delay for each train by relaxing Petersen's assumption that the trains are uniformly distributed over time (i.e., Poisson departures).

The model which is presented in this paper is significant in two major respects. First, the model explicitly calculates both the mean *and variance* of the delays for every train on the line. Most previous models simply compute the mean delay (or travel time). However, the reliability and hence, the quality of freight rail service, must include the variance term. For example, which train service is better: a train that takes 10 hours with a standard deviation of 5 hours, or a train that takes 12 hours with only a 30 minute standard deviation? Recent marketing surveys of railroad customers tend to support the choice of the latter train service due to its predictability. Thus, the variance (or standard deviation) of travel time is crucial if the model is to be a useful planning or forecasting tool.

The second significant feature of the proposed model was briefly discussed above; namely, that trains are not assumed to arrive randomly but rather, enter a rail line at or near some scheduled departure time. As railroads move toward greater use of scheduled operations in response to the market demand for more reliable service, the assumption of random departures becomes less and less realistic. The proposed model is the only analytical approach for dealing directly with the scheduled train issue.

The purpose of this paper is to complete the work in Chen and Harker (1990) by considering the case in which the rail line consists of a series of single and double-tracked sections. Section 2 provides the mathematical formulation of the partially double-track line delay model along with a brief discussion of the numerical methods for the solution of this model. Section 3 will illustrate the properties of this model through the use of an example from a major Class I railroad, and conclusions are drawn in Section 4.

MODEL OF A PARTIALLY DOUBLE-TRACK RAILROAD

This section will develop a model of delay for a partially double-track railroad between two yards which consists of single-track sections with sidings and double-track sections. The following assumptions are used to derive this rail delay model (note that most of these assumptions can easily be relaxed at the expense of more complex notation):

- Each single-track section is of equal length; each double-track section is also of equal length, although this length will be different from the single-track distance in general.
- Sidings are uniformly distributed within the each single-track section.
- Only a faster train can overtake a slower train.
- Meetings and overtakings occur only between two trains at a time.
- The meeting of two trains may occur at a siding or at any point along the double-track section.
- When a meet occurs at a siding or at the end of a double-track section, the train which arrives first must stop and thus, will incur a delay while the switch is set (the switching delay).
- The overtaking of two trains occurs at a siding or at the end of a double-track section.
- When an overtake occurs, only the overtaken train will incur a switching delay.
- The train schedules are cyclic for the track section considered; i.e., the schedule will repeat itself after some fixed period τ. The actual departure time for each train is a random variable centered about the scheduled departure time.
- The probability that train i will be delayed by train j, P_{ij}, is a function not only of train i and train j's physical characteristics; such as their horsepower per trailing ton ratio, the commodity being transported, etc.; but also of the relative tightness of their schedules; i.e., a late train may receive priority over a higher priority train if the latter is running ahead of schedule.
- The free running (unobstructed) travel time of any train is greater than or equal to twice the switching delay. In other words, the track section is of sufficient length so that the delays caused by the interference from other trains are not more than half of the free running time.

  This assumption is made in order to employ the Case I and IV results in Peterson (1975).

Other than the assumptions dealing with the double-track section, the above assumptions are identical to those employed in Chen and Harker (1990).

In order to state the line delay model, let us define the following notation:

L_S = the total length of the single-track sections,

L_D = the total length of the double-track sections,

L = the total length of the track section where $L = L_S + L_D$,

n_S = the number of single-track sections,

n_D = the number of double-track sections,

n = the total number of sections $n = n_S + n_D$,

m = the total number of sidings,

K = the set of all the trains running in one direction (eastbound) within a period τ,

$\bar{K}$ = the set of trains running in the other direction (westbound),

Sw_i = the switching time of train i when passing or meeting other trains,

H_i = the minimum headway which train $i \in K \cup \bar{K}$ must maintain while following another train; this headway is a function of the type of signal blocks which exist,

D_i, A_i = scheduled departure and arrival time of train i,

P_{ij} = the probability that train i is delayed when it interferes with train j, $P_{ij} = 1 - P_{ji}$, for all i, j.

d_i = the actual departure time of train i, which is a random variable centered about D_i,

FR_i^S = the total free running time of train i when it travels through all the single-track sections without any interference,

FR_i^D = the total free running time of train i when it travels through all the double-track sections without any interference,

FR_i = the total free running time of train i when it travels through the entire section without any interference; $FR_i = FR_i^S + FR_i^D$,

d^S_{ij} = the interference delay of train i caused by meeting or overtaking train j on a single-track section,

d^D_{ij} = the interference delay of train i caused by meeting or overtaking train j on a double-track section,

d_{ij} = the total interference delay of train i caused by meeting or overtaking train j where $d_{ij} = d^S_{ij} = d^D_{ij}$,

q^{SM}_{ij} = the probability that train i will meet train j in a single-track section, where $i \epsilon$ K, $j \epsilon \bar{K}$,

q^{SO}_{ij} = the probability that train i and train j will overtake each other in a single-track section, where $i, j \epsilon$ K or $i, j \epsilon \bar{K}$,

q^{DM}_{ij} = the probability that train i will meet train j in a double-track section, where $i \epsilon$ K, $j \epsilon \bar{K}$,

q^{DO}_{ij} = the probability that train i and train j will overtake each other in a double-track section, where $i, j \epsilon$ K or $i, j \epsilon \bar{K}$,

$$q_{ij} = q^{SM}_{ij} + q^{SO}_{ij} + q^{DM}_{ij} + q^{DO}_{ij}.$$

$$t_{ij} = \begin{cases} d_{ij} & \text{where train } i \text{ and train } j \text{ interfere with each other,} \\ 0 & \text{otherwise,} \end{cases}$$

$t_i = FR_i + \Sigma_{j \epsilon K \cup \bar{K}} t_{ij}$ = total travel time (delay) of train i when passing through the entire track section.

Note that, by assumption, the track need not consist of a *single* piece of single-track followed by a *single* piece of double-track. Rather, the sections must be of equal length in each category (single or double-track), but can be located anywhere within the total portion of railroad under consideration.

The Delay Probabilities

The first important piece needed to build a description of how a rail line operates is a model of the dispatcher's choice process. Chen and Harker (1990) have provided a formulation for the probability that a train i is delayed when it meets or overtakes train j which is based upon a behavioral model of the dispatcher's operations of the rail line. This model assumes that the dispatchers trade off the lateness of a train vis-a-vis its schedule and the overall priority of the train γ_i which depends on its composition (intermodal, priority freight, grain, etc.) in deciding which train to delay when two trains meet or overtake. This model assumes that each train has its own priority number ρ_i which is given by

$$\rho_i = \alpha\gamma_i - (1 - \alpha)\frac{A_i - D_i - FR_i}{A_i - D_i}$$

where

α = the relative weight of the train type and the tightness of schedule $0 \leq \alpha \leq 1$.

γ_i = a constant related to the train type.

That is, the dispatcher or management is assumed to define the priority of a train (ρ_i) as a weighted combination of the priority γ_i assigned to the train type (e.g., intermodal, boxcar, unit coal, etc.) and the percentage of "slack" or excess time built into the schedule. The parameter α is used to represent the weight assigned to each of these priority components.

Even though we know the priorities ρ_i, we have no guarantee that the dispatchers will choose which train to delay on this basis alone. That is, other unknown attributes of the trains may enter into the dispatcher's decision process. Thus, we can only state the probability that a dispatcher will delay a given train in a meet or overtake. Of course, the priorities should influence these probabilities. In order to capture the behavior of the dispatchers, the delay probabilities are estimated by a discrete choice (logit) model (Ben Akiva and Lerman 1985). Letting P_{ij} denote the probability that train i will be delayed by train j, this model can be stated as:

$$P_{ij} = \frac{e^{-\chi\rho_i}}{e^{-\chi\rho_i} + e^{-\chi\rho_j}}$$

where χ is a scalar parameter estimated from historical dispatching behavior.

To illustrate this model, consider the case of an eastbound intermodal train (i = 1) with priority γ_1 = 1.0 and a westbound priority freight train (i = 2) with priority γ_2 = 0.5. The intermodal train departed Yard A at 12:00 a.m. and is scheduled to arrive at Yard B at 6:00 a.m.; the free running time for this train is 5 hours. The priority freight train departed late from Yard B at 2:00 a.m. (it was scheduled to depart at 12:00 a.m. as well) and is scheduled to arrive at Yard A at 7:00 a.m.; its free running time is 6 hours. A meet now occurs between these two trains; which will take the siding? By the above formulas, the priorities of each train are:

$$\rho_1 = 1\alpha - (1-\alpha)\frac{1}{6} = \frac{7}{6}\alpha - \frac{1}{6},$$

$$\rho_2 = \frac{1}{2}\alpha - (1-\alpha)\frac{-1}{5} = \frac{3}{10}\alpha + \frac{1}{5}.$$

Thus, when $\alpha = 1$, $\rho_1 = 1.0$ and $\rho_2 = 0.5$; i.e., no weight is placed on the fact that the freight train is late. When $\alpha = 0$, all the weight is placed on the lateness of the freight train ($\rho_1 = -1/6$, $\rho_2 = 1/5$). When equal weight ($\alpha = 0.5$) is placed on both criteria, the intermodal train is given slight priority ($\rho_1 = 5/12$, $\rho_2 = 7/20$).

Now consider the use of these priorities in modelling the behavior of a dispatcher (assume $\alpha = 0.5$ in what follows). If there were no attributes used by the dispatcher other than the priorities, the probability that Train 1 (intermodal) would be delayed by Train 2 (freight) would be zero since ($\rho_1 = 5/12, > \rho_2 = 7/20$. This case corresponds to χ being equal to infinity in the logit model. As other unknown attributes enter into the picture, we cannot say for sure that Train 1 will not be delayed. In this case, χ takes on some finite value. For example, $\chi = 25$ implies that $P_{12} = 15.9\%$; i.e., there is a 15.9% chance that Train 1 will be delayed by Train 2 and of course, a 1-15.9% or 84.1% chance that Train 2 will be delayed. When $\chi = 0$, the probability that either train is delayed will be 50%. Thus, the parameter is estimated from historical information and represents the importance which a dispatcher places on the computed/assigned priority of a train.

Interference Delays

Given the above model of dispatching behavior, the next important components in the description of total line delay are the estimates of the mean $E(d_{ij})$ and variance $Var(d_{ij})$ of the interference delays. To compute these values, we shall modify the results found in Chen and Harker (1990) and Peterson (1975). Defining the average velocities and differences in average velocities over each track section by:

$$\lambda^S_{ij} = \frac{FR^S_i + FR^S_j}{(m+1)n_S}$$

$$\lambda^D_{ij} = \frac{FR^D_i + FR^D_j}{n_D}$$

$$\mu^S_{ij} = \frac{FR^S_i - FR^S_j}{(m+1)n_S}$$

$$\mu^D_{ij} = \frac{FR^D_i - FR^D_j}{n_D},$$

the following results can be derived:

Case 1: Meet delays on a single-track section.

$$E(d^S_{ij}) = Sw_i + \frac{1}{2}P^3_{ij}\lambda^S_{ij} \quad \forall i \in K, j \in \bar{K} \tag{1}$$

$$Var(d^S_{ij}) = (\mu^S_{ij})^2 P^3_{ij}\left(\frac{1}{3} - \frac{1}{4}P_{ij}\right) \quad \forall i \in K, j \in \bar{K} \tag{2}$$

Case 2: Overtake delays on a single-track section.

$$E(d^S_{ij}) = \begin{cases} \frac{1}{2}P^3_{ij}\mu^S_{ji} & \forall i,j \in K \text{ or } i,j \in \bar{K} \text{ and } FR^S_j \geq FR^S_i \\ \frac{1}{2}P^3_{ij}\mu^S_{ij} + 2H_j + Sw_i & \forall i,j \in K \text{ or } i,j \in \bar{K} \text{ and } FR^S_i \geq FR^S_j \end{cases} \tag{3}$$

$$Var(d^S_{ij}) = \begin{cases} (\mu^S_{ji})^2 P^3_{ij}(\frac{1}{3} - \frac{1}{4}P_{ij}) & \forall i,j \in K \text{ or } i,j \in \bar{K} \text{ and } FR^S_j \geq FR^S_i \\ (\mu^S_{ij})^2 P^3_{ij}(\frac{1}{3} - \frac{1}{4}P_{ij}) & \forall i,j \in K \text{ or } i,j \in \bar{K} \text{ and } FR^S_i \geq FR^S_j \end{cases} \tag{4}$$

Case 3: Meet delays on a double-track section.

$$E(d^D_{ij}) = \begin{cases} \frac{1}{6}(4Sw^2_i + Sw^2_j)/\lambda^D_{ij} & \forall i \in K, j \in \bar{K} \text{ or } i \in \bar{K}, j \in K \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$Var(d_{ij}) = \begin{cases} \frac{1}{24}(12Sw^2_i + Sw^2_j)/\lambda^D_{ij} - E(d^D_{ij})^2 & \forall i \in K, j \in \bar{K} \text{ or } i \in \bar{K}, j \in K \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Case 4: Overtake delays on a double-track section.

$$E(d_{ij}) = \begin{cases} 2H_j + Sw_i - \frac{1}{2}\mu^D_{ij} & \forall i,j \in K \text{ or } i,j \in \bar{K} \text{ and } FR^D_i \geq FR^D_j \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$Var(d_{ij}) = \begin{cases} \frac{1}{12}(\mu^D_{ij})^2 & \forall i,j \in K \text{ or } i,j \in \bar{K} \text{ and } FR^D_i \geq FR^D_j \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Note that any form of the meet and pass interference delays can be used in the nonlinear equation system defined below; the above formulas are presented only as an illustration.

Estimation of the Mean and Variance of Travel Time

Having defined the interference along with the probabilities of meeting and overtaking given in Chen and Harker (1990), the mean and variance of the total running time is given by:

$$E(t_i) = FR_i + \sum_{j \in K \cup \bar{K}} q_{ij}E(d_{ij}) \tag{9}$$

$$Var(t_i) = \sum_{j \in K \cup \bar{K}} \{q_{ij}Var(d_{ij}) + q_{ij}(1-q_{ij})E^2(d_{ij})\} + \sum_{h,k \in K \cup \bar{K}, h \neq k} Cov(t_{ih}, t_{ik}), \tag{10}$$

where *Cov (. , .)* represents the covariance of the delays. That is, the expected travel time of train *i*, $E(t_i)$, is equal to the free running time of this train plus the expected delays caused by interference with other trains. This latter term is equal to the sum of the probabilities of being delayed by a train times the expected delay from such an interference. The variance term consists of the direct variance caused by a meet or overtake plus the indirect variance represented in the covariance of travel time.

Two cases were analyzed in Chen and Harker (1990): the situation where the only uncertainty is the probability of interference with other trains (the departure time is fixed), and the case where there exists a probability that the trains will not depart at the scheduled departure time. To specialize the above equations for these cases, define for all $i \in K$, $j \in K$

$$D_i(j) = \begin{cases} D_j - D_i & \text{if } |D_j - D_i| \le \frac{\tau}{2} \\ D_j - D_i - \tau & \text{if } D_j - D_i > \frac{\tau}{2} \\ D_j - D_i + \tau & \text{if } D_j - D_i < -\frac{\tau}{2}. \end{cases}$$

The value $D_i(j)$ can be interpreted as the difference between the departure times of train *i* and train *j* after considering the characteristics of the cyclic schedule. Note that $D_i(j) = -D_j(i)$ by definition and $D_i(j) < \frac{1}{2}\tau$ if all the D_i's are taken from one cycle.

For the case of no uncertainty on the departure times of the trains, the equations can be written as:

$$E(t_i) = FR_i + \sum_{j:D_i(j)<0} P(t_j \ge -D_i(j))E(d_{ij}) + \sum_{j:D_i(j)\ge 0} P(t_j \ge D_i(j))E(d_{ij}) \quad (11)$$

$$\begin{aligned} Var(t_i) = & \sum_{j:D_i(j)<0} P(t_j \ge -D_i(j))Var(d_{ij} + (1 - P(t_j \ge -D_i(j)))E^2(d_{ij}) + \\ & \sum_{j:D_j(j)\ge 0} P(t_j \ge D_i(j))Var(d_{ij} + (1 - P(t_j \ge D_i(j)))E^2(d_{ij}) + \\ & \sum_{D_i(h)>D_i(k)\ge 0} P(t_i > D_i(h))(1 - P(t_i > D_i(k)))E(d_{ih})E(d_{ik}) + \\ & \sum_{D_i(k)>D_i(h)\ge 0} P(t_i > D_i(k))(1 - P(t_i > D_i(h)))E(d_{ih})E(d_{ik}) \end{aligned} \quad (12)$$

where

$$\begin{aligned} P(t_i \ge D_i(j)) &= \int_{D_i(j)}^{\infty} \frac{1}{\sqrt{2\pi V_i}} e^{-\frac{(t-T_i)^2}{2V_i}} dt \\ &= \frac{1}{\sqrt{2\pi}} \int_{\frac{D_i(j)-T_i}{\sqrt{V_i}}}^{\infty} e^{-\frac{t^2}{2}} dt. \end{aligned}$$

In the case of uncertain departure times, let $[D_i^-, D_i^+]$ be the interval in which the actual departure time d_i of train *i* is known to occur. Assume that this interval of uncertainty is the same for all trains, and define the uncertainty interval by $\Delta D = |D_i^- - D_i^+|$. Using this notation, the system of equations for the uncertainty case becomes:

$$\begin{aligned} E(t_{ij}) = FR_i + & \sum_{j:D_i(j)<-FR_i} P(t_j \ge -d_i(j))E(d_{ij}) + \sum_{j:D_i(j)\ge FR_i} P(t_j \ge d_i(j))E(d_{ij}) + \\ & \sum_{0<D_i(j)<FR_i} E(d_{ij}) + \sum_{0>D_i(j)>-FR_i} E(d_{ij}) \end{aligned} \quad (13)$$

$$\begin{aligned} Var(t_i) = & \sum_{j:D_i(j)<-FR_i} P(t_j \ge -d_i(j))Var(d_{ij}) + (1 - P(t_j \ge -d_i(j)))E^2(d_{ij}) + \\ & \sum_{j:D_j(j)\ge FR_i} P(t_j \ge d_i(j))Var(d_{ij}) + (1 - P(t_j \ge d_i(j)))E^2(d_{ij}) + \\ & \sum_{0<D_i(j)<FR_i} Var(d_{ij})E^2(d_{ij}) + \sum_{0>D_i(j)>-FR_i} Var(d_{ij})E^2(d_{ij}) + \\ & \sum_{D_i(h)>D_i(k)\ge 0} P(t_i > D_i(h))(1 - P(t_i > D_i(k)))E(d_{ih})E(d_{ik}) + \\ & \sum_{D_i(k)>D_i(h)\ge 0} P(t_i > D_i(k))(1 - P(t_i > D_i(h)))E(d_{ih})E(d_{ik}) \end{aligned} \quad (14)$$

where

$$P(t_i \ge d_i(j)) = \int_{D_i(j)-\Delta D}^{D_i(j)+\Delta D} g_{j-i}(t)P(t_i \ge t)dt$$

and $g_{j,i}(t)$ is the distribution of the random variable $D_i(j)$ which is defined in Chen and Harker (1990).

Reliability of Train Schedule

Since shippers rate the reliability of average transit time as one of the most important variables influencing freight transportation (Allen, Mahmoud and McNeil 1985), it is important to evaluate the performance of a given set of train schedules based on their reliability. The reliability of a schedule is defined as the probability that a train arrives on time. Chen and Harker (1990) provide the following equations to measure the reliability of a given schedule:

Case 1: No Uncertainty on Departure Time

$$\begin{aligned} P(t_i < A_i - D_i) &= \int_{-\infty}^{A_i-D_i} \frac{1}{\sqrt{2\pi V_i^2}} e^{-\frac{(t-T_i^2)^2}{2V_i^2}} dt \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{A_i-D_i-T_i^2}{\sqrt{V_i^2}}} e^{-\frac{t^2}{2}} dt \end{aligned}$$

Case 2: Limited Uncertainty on Departure Time

$$\begin{aligned} P(t_i < A_i - d_i) &= \int_{D_i^-}^{D_i^+} \frac{1}{\Delta D} P(t_i < A_i - x)\, dx \\ &= \frac{1}{\Delta D} \int_{D_i^-}^{D_i^+} \int_{-\infty}^{A_i-x} \frac{1}{\sqrt{2\pi V_i^2}} e^{-\frac{(t-T_i^2)^2}{2V_i^2}} dt. \end{aligned}$$

NUMERICAL EXAMPLES

In order to illustrate the behavior of the proposed model, data from a portion of a major U.S. Class I railroad will be analyzed in this section. We assume that each train runs at a constant speed over the track section and that the switching time and the headway of each train are same (5 minutes each). Twelve eastbound and ten westbound trains are considered within a $\tau = 24$ hour planning period. Table 1 lists the data used in this model where τ_i denotes the train priority. It is assumed that a train runs at a constant speed over all sections. For this example, 60% is single-track and 40% is double-track.

The nonlinear equations system (9) - (10) was solved by using the HYBRID subroutine from the MINPACK library at Argonne National Laboratories; this nonlinear equation solver is based on Powell's (1970) hybrid method. The program was implemented in FORTRAN on Apollo DN 3000 workstation. In all cases considered, the program converged to a solution within 50 iterations given a fairly stringent convergence tolerance; the approximate CPU time is in the order of 30 seconds.

Table 2 shows the comparison of a single-track section with three and five sidings and no uncertainty on the departure times. In this case, $\alpha = 0.5$ and $\chi = 0.5$. The quantity STD refers to the standard deviation of the travel times and "Reliab." denotes the probability that the train will arrive on time. As one can see, the increase in the number of sidings has a dramatic effect on the reliability of some trains but for other trains which are "hopelessly" behind schedule, the effect is minimal.

In order to study the impact on changing the behavior of the dispatchers, the value of the parameter α altered; Table 3 lists the results. As one can see, increasing the importance of the overall train priority relative to the schedule of the trains (increasing the value of α) tends to increase the reliability of the high priority trains and can decrease the reliability of the others. For example, the reliability of Train 3 increases at the expense of Train 12.

Next consider altering the track to now be 60% double-track and 40% single-track; Table 4 lists the results of this addition of track capacity. As expected, the reliability of the trains increases.

Finally, consider the impact of increasing the uncertainty of the departure time on the performance of the rail line; Table 5 lists the results of this example. As one can see, reliability generally decreases with the level of uncertainty as expected, although this result varies by train type and the number of meets and passes in which this train is involved.

CONCLUSION

This paper has presented an extension of the model by Chen and Harker (1990) to estimate the mean and the variance of the total travel time of train over a partially double-track section with scheduled rail traffic. The model is computationally efficient and as the results of Section 3 illustrate, tend to conform to our intuitive understanding of how such a rail line would perform under changes to the operating policy or the track topology.

The practical significance of the model presented herein lies in the fact that one should not plan operations, either tactically or in a real-time environment, solely on the basis of mean (average) travel times and delays. For example, Train 1 in the numerical examples presented above seems as if it would perform quite well since its expected travel time of 2:22 (in the case of five sidings) would permit it to arrive at 6:37, three minutes ahead of the 6:40 scheduled arrival time. However, the reliability of this train is below 50%. Focusing solely on means would have never enabled the analyst to uncover such poor expected quality of service.

Future research will be directed toward the empirical verification of this model using data from a major railroad along with its incorporation into the real-time planning models described in Harker (1990).

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TABLE 1

Train Data For Examples

Train i	Direction	γ_i	D_i	A_i	Sw_i	H_i	FR_i	FR_i^S	FR_i^D
1	east	0.8	4:15	6:40	0:05	0:05	2:18	1:22	0:56
2	east	0.6	4:45	6:15	0:05	0:05	2:18	1:22	0:56
3	east	1.0	5:30	7:25	0:05	0:05	1:46	1:03	0:43
4	east	0.6	8:15	10:40	0:05	0:05	2:18	1:22	0:56
5	east	0.2	9:00	12:00	0:05	0:05	2:22	1:25	0:57
6	east	0.6	13:55	16:35	0:05	0:05	2:18	1:22	0:56
7	east	0.4	16:00	19:30	0:05	0:05	2:18	1:22	0:56
8	east	0.8	16:25	19:05	0:05	0:05	2:18	1:22	0:56
9	east	0.4	17:55	21:05	0:05	0:05	2:18	1:22	0:56
10	east	0.2	19:45	24:15	0:05	0:05	2:22	1:25	0:57
11	east	0.8	20:50	23:30	0:05	0:05	2:18	1:22	0:56
12	east	0.6	22:00	24:30	0:05	0:05	2:18	1:22	0:56
13	west	0.8	3:45	5:10	0:05	0:05	2:00	1:12	0:48
14	west	0.6	4:35	6:00	0:05	0:05	2:00	1:12	0:48
15	west	0.8	14:30	16:25	0:05	0:05	2:18	1:12	0:48
16	west	0.8	16:35	20:10	0:05	0:05	2:00	1:12	0:48
17	west	0.4	18:25	21:55	0:05	0:05	2:39	1:35	1:04
18	west	0.6	20:10	23:45	0:05	0:05	2:00	1:12	0:48
19	west	1.0	21:25	23:10	0:05	0:05	1:45	1:03	0:42
20	west	0.4	21:30	26:00	0:05	0:05	2:39	1:35	1:04
21	west	0.4	23:05	26:05	0:05	0:05	2:39	1:35	1:04
22	west	0.8	23:35	27:10	0:05	0:05	2:00	1:12	0:48

TABLE 2

Effect of the Number of Sidings on Reliability

	3 Sidings			5 Sidings		
Train i	$E(t_i)$	$STD(t_i)$	Reliab.	$E(t_i)$	$STD(t_i)$	Reliab.
1	2:26	0:09	0.4499	2:22	0:09	0.4621
2	2:26	0:09	0.0000	2:22	0:09	0.0000
3	1:54	0:09	0.5415	1:50	0:06	0.5537
4	2:18	0:00	1.0000	2:18	0:00	1.0000
5	2:22	0:00	1.0000	2:22	0:00	1.0000
6	2:22	0:06	0.9967	2:20	0:06	0.9972
7	2:28	0:11	1.0000	2:23	0:10	1.0000
8	2:29	0:11	0.8102	2:24	0:11	0.8228
9	2:29	0:11	1.0000	2:24	0:11	1.0000
10	2:38	0:14	1.0000	2:31	0:13	1.0000
11	2:39	0:16	0.5161	2:28	0:15	0.5364
12	2:38	0:14	0.2882	2:29	0:14	0.3030
13	2:12	0:12	0.0000	2:06	0:12	0.0000
14	2:12	0:11	0.0000	2:06	0:07	0.0000
15	2:11	0:11	0.0000	2:06	0:07	0.0000
16	2:12	0:11	1.0000	2:06	0:07	1.0000
17	2:57	0:14	0.9865	2:48	0:09	0.9893
18	2:15	0:13	1.0000	2:08	0:09	1.0000
19	1:56	0:11	0.1434	1:52	0:07	0.1484
20	2:51	0:11	1.0000	2:45	0:07	1.0000
21	2:54	0:12	0.9218	2:53	0:11	0.9298
22	2:05	0:07	1.0000	2:02	0:04	1.0000

TABLE 8

Effect of Priorities on Reliability

		$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.8$
Train i	γ_i	Reliability	Reliability	Reliability
1	0.8	0.4583	0.4599	0.4621
2	0.6	0.0000	0.0000	0.0000
3	1.0	0.5484	0.5506	0.5537
4	0.6	1.0000	1.0000	1.0000
5	0.2	1.0000	1.0000	1.0000
6	0.6	0.9972	0.9972	0.9972
7	0.4	1.0000	1.0000	1.0000
8	0.8	0.8225	0.8226	0.8228
9	0.4	1.0000	1.0000	1.0000
10	0.2	1.0000	1.0000	1.0000
11	0.8	0.5353	0.5357	0.5364
12	0.6	0.3049	0.3041	0.3030
13	0.8	0.0000	0.0000	0.0000
14	0.6	0.0000	0.0000	0.0000
15	0.8	0.0000	0.0000	0.0000
16	0.8	1.0000	1.0000	1.0000
17	0.4	0.9894	0.9894	0.9893
18	0.6	1.0000	1.0000	1.0000
19	1.0	0.1477	0.1480	0.1484
20	0.4	1.0000	1.0000	1.0000
21	0.4	0.9307	0.9304	0.9298
22	0.8	1.0000	1.0000	1.0000

TABLE 4

Change in Reliability Due To Change of Double Track

	40% Double Track			60% Double Track		
Train i	$E(t_i)$	$STD(t_i)$	Reliab.	$E(t_i)$	$STD(t_i)$	Reliab.
1	2:25	0:09	0.4621	2:23	0:10	0.5578
2	2:25	0:09	0.0000	2:23	0:10	0.0000
3	1:53	0:09	0.5537	1:51	0:09	0.6408
4	2:18	0:00	1.0000	2:18	0:00	1.0000
5	2:22	0:00	1.0000	2:22	0:00	1.0000
6	2:21	0:06	0.9972	2:20	0:07	0.9965
7	2:28	0:10	1.0000	2:24	0:11	1.0000
8	2:29	0:11	0.8228	2:25	0:12	0.8764
9	2:29	0:11	1.0000	2:25	0:12	1.0000
10	2:37	0:13	1.0000	2:33	0:14	1.0000
11	2:38	0:15	0.5364	2:31	0:16	0.6882
12	2:37	0:14	0.3030	2:31	0:15	0.4586
13	2:11	0:12	0.0000	2:08	0:13	0.0000
14	2:11	0:11	0.0000	2:08	0:12	0.0000
15	2:11	0:11	0.0000	2:07	0:12	0.0000
16	2:11	0:11	1.0000	2:08	0:12	1.0000
17	2:57	0:14	0.9893	2:51	0:15	0.9945
18	2:15	0:13	1.0000	2:10	0:13	1.0000
19	1:56	0:11	0.1484	1:53	0:12	0.2506
20	2:50	0:11	1.0000	2:47	0:12	1.0000
21	2:54	0:11	0.9298	2:49	0:12	0.9495
22	2:05	0:07	1.0000	2:03	0:07	1.0000

TABLE 5

Effect of Departure Time Uncertainty

	$\Delta D = 0:10$			$\Delta D = 0:15$			$\Delta D = 0:20$		
Train i	$E(t_i)$	$STD(t_i)$	Reliab.	$E(t_i)$	$STD(t_i)$	Reliab.	$E(t_i)$	$STD(t_i)$	Reliab.
1	2:25	0:09	0.4638	2:25	0:09	0.4659	2:25	0:09	0.4684
2	2:25	0:09	0.0000	2:25	0:09	0.0000	2:25	0:09	0.0000
3	1:53	0:09	0.5487	1:53	0:09	0.5460	1:53	0:09	0.5426
4	2:18	0:00	1.0000	2:18	0:00	1.0000	2:18	0:00	1.0000
5	2:22	0:00	1.0000	2:22	0:00	1.0000	2:22	0:00	1.0000
6	2:21	0:06	0.9944	2:21	0:06	0.9896	2:21	0:06	0.9804
7	2:28	0:10	1.0000	2:28	0:10	1.0000	2:28	0:10	1.0000
8	2:29	0:11	0.8059	2:29	0:11	0.7972	2:29	0:11	0.7856
9	2:29	0:11	0.8059	2:29	0:11	0.9995	2:29	0:11	0.9992
10	2:37	0:13	1.0000	2:37	0:13	1.0000	2:37	0:13	1.0000
11	2:38	0:16	0.5264	2:38	0:16	0.5254	2:38	0:16	0.5240
12	2:37	0:14	0.3042	2:37	0:14	0.3083	2:37	0:14	0.3136
13	2:11	0:12	0.0000	2:11	0:12	0.0000	2:11	0:12	0.0000
14	2:11	0:11	0.0000	2:11	0:11	0.0000	2:11	0:11	0.0000
15	2:11	0:11	0.0000	2:11	0:11	0.0000	2:11	0:11	0.0000
16	2:11	0:11	1.0000	2:11	0:11	1.0000	2:11	0:11	1.0000
17	2:57	0:14	0.9877	2:57	0:14	0.9861	2:57	0:14	0.9836
18	2:15	0:13	1.0000	2:15	0:13	1.0000	2:15	0:13	1.0000
19	1:56	0:11	0.1564	1:56	0:11	0.1659	1:56	0:11	0.1785
20	2:50	0:11	1.0000	2:50	0:11	1.0000	2:50	0:11	1.0000
21	2:54	0:12	0.9128	2:54	0:12	0.9005	2:54	0:12	0.8841
22	2:05	0:07	1.0000	2:05	0:07	1.0000	2:05	0:07	1.0000

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ENDNOTE

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