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On the Marginal Capital Costs of Peak and Off-Peak Transit Services**

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ABSTRACT

In this paper, the issue of the marginal capital costs of peak and off-peak services is revisited. A model is presented in which vehicle life is a function of usage (rather than elapsed time), and capital costs are determined using the capital recovery factor approach (in preference to the still-common "depreciation and interest" approach). These two features of the model interact to suggest explanations as to why the conditional marginal capital costs of off-peak services can be substantial, and why total capital costs may not be directly proportional to peak vehicle requirements.

INTRODUCTION

The allocation of capital costs between peak and off-peak services remains an unsettled issue in the transport economics literature. Some have argued that all capital costs should be allocated to peak services, because fleet size is determined by peak vehicle requirements. Others have argued that some capital costs should be allocated to off-peak services because if the peak was eliminated, positive capital costs would be incurred in the provision of off-peak services. The distinction here is between the conditional and the unconditional. For example, *if* vehicle life is determined by elapsed time and is independent of usage (as generally assumed in costing studies), then the marginal capital cost of off-peak services, *conditional* upon the (higher) peak service level being maintained, is indeed zero. In contrast, the *un*conditional marginal capital cost of providing service at any time of day is not zero. What is relevant for operational decision-makers, however, is the conditional marginal cost of each service.

This paper focuses on the conditional marginal costs of peak and off-peak services when vehicle life is determined by usage, rather than by elapsed time. While the analysis is confined to the case of transit services, the conclusions are relevant to any activity subject to peak demands and where capital life is at least partly a function of usage. Examples of these activities might include the generation of electricity and gas.

In this paper, it is argued that existing costing studies generally suffer from at least one of the following two drawbacks:

(1) Vehicle life is usually assumed to be solely determined by elapsed time. However, this does not seem to correspond with actual experience. For example, Pickrell (1987, p.16) has recently observed that:

> "In contrast to passenger cars, depreciation of transit vehicles appears to be almost exclusively the product of actual use rather than the passage of time. . .".

(2) In many studies, capital costs are determined as the sum of depreciation (generally calculated on a straight-line basis) and interest. However, the relevant variables for planners are cash flows and opportunity costs, rather than costs determined on the basis of arbitrary accounting allocations. Consequently, a theoretically superior means of determining capital costs is to use capital recovery factors to determine the annual outlays which would be equivalent, in terms of net present value, to the future cash outlays resulting from an investment decision. While capital recovery factors have long been used by some transport economists[1], the "depreciation and interest" approach is more frequently adopted.

Once account is taken of these factors[2], two conclusions follow:

(1) The conditional marginal capital cost of off-peak services is no longer zero. Off-peak services contribute to capital costs in two ways. Firstly, while they do not increase the fleet size required at a point in time, they do increase the number of vehicles purchased in the long-run. This is because running more vehicles in the off-peak increases average annual vehicle mileage, and therefore reduces average vehicle life in terms of years and increases vehicle replacement frequency. Secondly, the annualized capital cost attached to each vehicle in the fleet is increased as a result of the higher replacement frequency[3].

(2) Total capital costs are no longer directly proportional to peak vehicle requirements. This is because the capital costs of peak and off-peak services are now interdependent. As demonstrated below, total capital costs depend not only on the size of the fleet, but also on the relative number of vehicles run in the peak and off-peak periods.

A MODEL

In this section, equations are derived which specify the marginal capital costs of peak and off-peak services when vehicle life is determined solely by usage. As an example, the case of buses is discussed. For brevity, all exogenously-specified variables used in the derivations below, together with the values they take in the base case example, are given in Table (1); all variables given in this table are expressed in real terms.

For simplicity, only two service levels are assumed: peak and off-peak. Peak services are

TABLE 1

List of Exogenous Variables & Their Values in the Base Case

Variable	Definition	Value taken in example
BP	Number of peak buses run.	100
BO	Number of off-peak buses run.	50
P	Vehicle purchase price.	\$150,000
S	Vehicle scrap value after L miles. (assumed equal to 10 percent of initial purchase price).	\$15,000
A	Average annual outlays of a non-operating nature (e.g., registration and insurance). Annual outlays are likely to decline with vehicle age; A can be thought of as the annualized equivalent of these outlays.	\$5,000
r	Real interest rate.	4%
s	Average vehicle speed (assumed constant over time periods).	12 m.p.h.
L	Bus life in miles.	400,000 miles
WD	Number of weekdays per annum.	250
WE	Number of non-weekdays per annum.	115
HP	Number of peak hours per weekday.	4
	+	
HO	Number of off-peak hours per weekday.	14
(HP + HO)	Number of off-peak hours per non-weekday.	18

assumed to be provided for a total of HP hours per weekday; at all other times when the system is operating the off-peak service level is assumed to be provided. Using the capital recovery factor (CRF) approach, the capital cost per annum, AKC, for a particular vehicle can be expressed as:

$$AKC = A + (P - S.(1+r)^{-n}).CRF \quad (1)$$

where: $CRF = \dfrac{r}{(1-(1+r)^{-n})}$

However n, the average vehicle life in years, is now endogenous and depends on M, the average number of miles run per bus per annum, where M is given by:

$$M = s.(WD.(BP.HP+BO.HO) + WE.BO.(HP+HO))/BP \quad (2)$$

Some rearrangement yields:

$$M = s.(b + a.BO/BP) \quad (3)$$

where: $a = 365.HO + WE.HP$

and $b = WD.HP$

Note that a and b are, respectively, the numbers of off-peak and peak hours for which the system is operated each year. Thus, average vehicle life in years is given by:

$$n = L/M = \frac{L}{s.(b + a.BO/BP)} \quad (4)$$

It is assumed for simplicity that no reserve buses are held, so the total fleet size is therefore equal to the number of buses run at peak times, BP. Thus, total capital cost, TKC, is given by the product of AKC and BP, that is:

$$TKC = BP.\left(A + \left(P - \frac{S}{(1+r)^{n}}\right).\frac{r}{1-(1+r)^{-n}}\right) \quad (5)$$

By differentiating equation (5) while recognizing the dependence of n on both BO and BP, the marginal capital costs per vehicle of running peak and off-peak buses, respectively, can be determined as follows:

$$\frac{\delta TKC}{\delta BP} = AKC - \frac{n^{2}.a.s.(P-S)}{L}.\frac{r.\ln(1+r).(1+r)^{n}}{(1-(1+r)^{n})^{2}}.\frac{BO}{BP} \quad (6)$$

$$\frac{\delta TKC}{\delta BO} = \frac{n^{2}.a.s.(P-S)}{L}.\frac{r.\ln(1+r).(1+r)^{n}}{(1-(1+r)^{n})^{2}} \quad (7)$$

Some simple manipulation yields:

$$\frac{\delta TKC}{\delta BP} = AKC - \frac{BO}{BP}.\frac{\delta TKC}{\delta BO} \quad (8)$$

These marginal costs reflect the annual equivalent capital costs of permanently running one extra bus for the relevant time period. In order to determine the marginal costs per vehicle-mile, we need to divide by the resulting increases in total bus miles run per annum in the respective time periods, which are given by s.b for peak services and by s.a for off-peak services.

Thus, the marginal capital costs per vehicle mile are as follows:

Service type
Marginal capital cost per vehicle-mile

Peak

$$\frac{\delta TKC}{\delta BP}\cdot\frac{1}{s.b} = \frac{AKC}{s.b} - \frac{n^2.a}{b}\cdot\frac{(P-S)}{L}\cdot\frac{r.\ln(1+r).(1+r)^n}{(1-(1+r)^n)^2}\cdot\frac{BO}{BP} \quad (9)$$

Off-peak

$$\frac{\delta TKC}{\delta BO}\cdot\frac{1}{s.a} = \frac{n^2.(P-S)}{L}\cdot\frac{r.\ln(1+r).(1+r)^n}{(1-(1+r)^n)^2} \quad (10)$$

where n is given by equation (4).

Note from equations (8)–(10) that the marginal cost of each service depends on the level of service provided in the other period. In this sense the marginal costs are interdependent, and therefore conditional. Equation (9) indicates that the marginal capital cost of peak service is always less than the average capital cost per vehicle-mile (AKC/s.b), because additional peak-only service tends to lengthen vehicle replacement cycles. Conversely, equation (10) implies that the marginal capital cost of off-peak service is positive because additional off-peak service shortens vehicle replacement cycles.

SOME RESULTS

For the values taken by the variables given in the base case example in Table (1), the marginal capital costs per vehicle-mile for peak and off-peak services are 105.83 and 34.08 cents respectively[4], giving a ratio of around 3.1:1. Thus, the marginal capital costs of off-peak services are not insignificant. When the capital costs are combined with marginal operating costs per vehicle-mile[5] of around 410 and 325 cents in the peak and off-peak respectively, this yields a ratio of the overall marginal costs of peak and off-peak services of approximately 1.4:1.

Figures (1)–(4) illustrate how total capital costs, as well as the marginal capital costs of peak and off-peak services, vary with the inverse of the peak-to-base vehicle ratio. Because marginal capital costs depend on the ratio BO/BP, but not on the absolute values of BO and BP (see equations (9) and (10)), transit operators of different sizes will have the same capital costs per bus and the same marginal capital costs per time period if they have the same BO/BP ratio[6]. Figure (1) demonstrates that, for a given fleet size, total capital costs vary greatly, depending on the degree of fleet utilization in the off-peak. For example, for a fleet size of 100 buses, total capital costs almost triple (from $1.3M to over $3.5M) as the number of buses run in the off-peak rises from 0 to 99. Similarly, for a given number of buses run in the off-peak (BO = 50), a doubling of the fleet size (from BP = 100 to BP = 200) will increase total capital costs by only 53%.

In summary, we have shown that total capital costs are not necessarily proportional to fleet size.

FIGURE 1

TOTAL CAPITAL COST & BO/BP

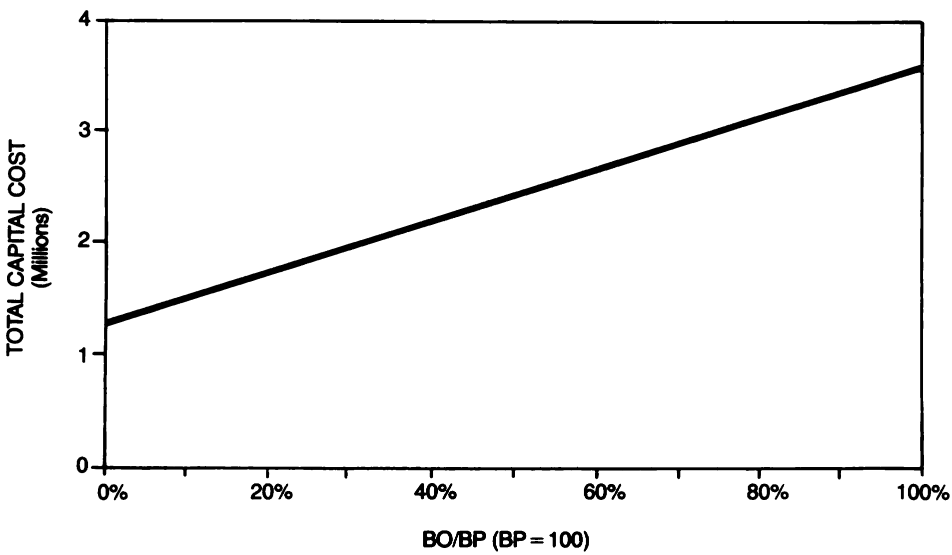


FIGURE 2

MARGINAL CAPITAL COST IN THE PEAK

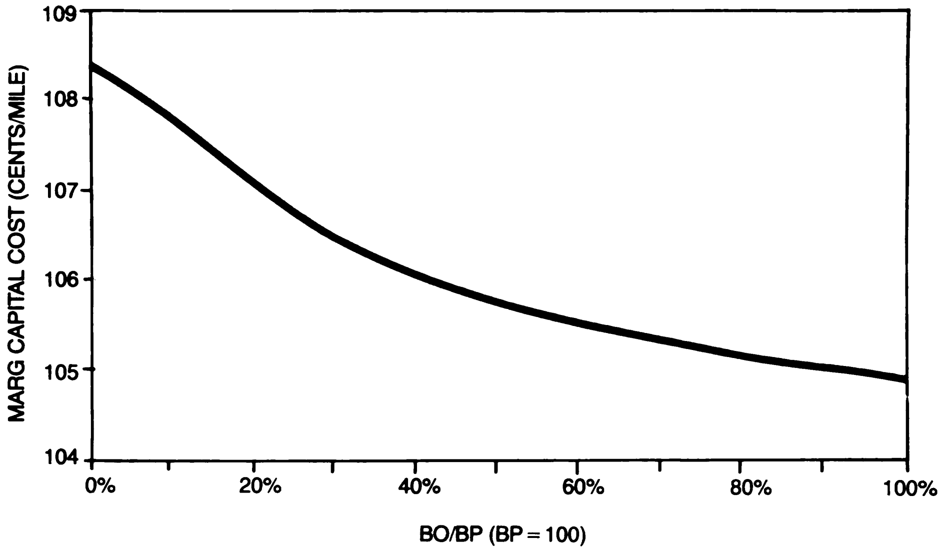
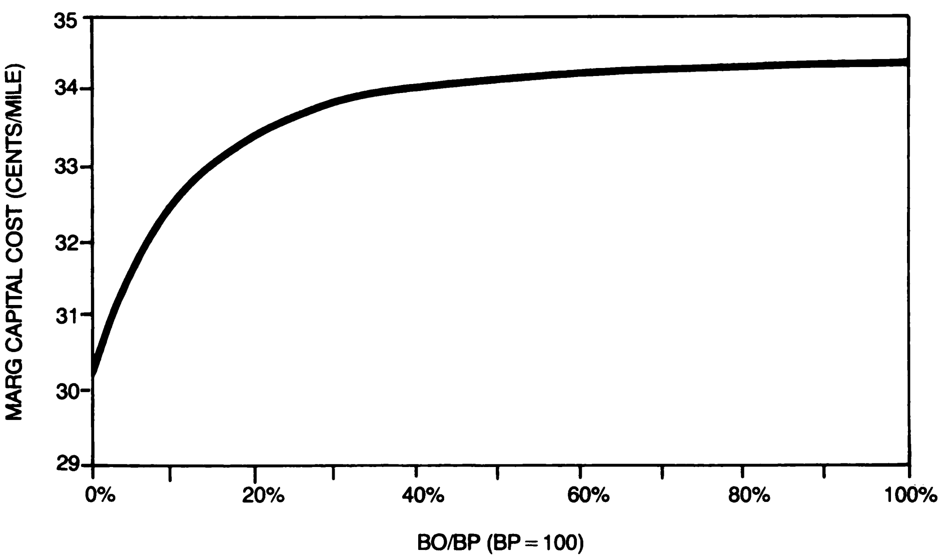


FIGURE 3

MARGINAL CAPITAL COST IN THE OFF-PEAK



This is because total capital costs also depend significantly on fleet utilization during the off-peak. Consequently, total capital costs associated with a given peak vehicle requirement can vary greatly depending on the number of vehicles run during the off-peak. Total capital costs will only be proportional to fleet size if the ratio BO/BP is held constant.

Figures (2)–(4) demonstrate how the marginal capital costs of peak and off-peak services vary with the degree of off-peak fleet utilization. Figure (2) shows that the marginal capital cost of peak services *declines* as the ratio BO/BP rises. The reason for this is as follows. Remembering that TKC = BP.AKC, peak services have two offsetting effects on TKC:

(a) the (positive) direct effect on TKC, due to the increase in BP;

(b) the (negative) indirect effect, due to the effect on AKC. A rise in BP, given BO, will increase n, and therefore reduce AKC. Now, it is a property of discounted cash flow analysis that a given absolute change in n will have a greater effect on the present value of a cash flow stream the smaller is the initial value of n. But the initial value of n is smaller the higher is BO/BP. Thus, effect (b) becomes stronger the higher is BO/BP, and this results in a lower marginal cost.

As an example, suppose that BO = 50. Then, using the base case parameter values, a one unit increase in BP from 51 to 52 would increase n from 5.16 to 5.24, while a one unit increase in BP from 100 to 101 would raise n from 8.81 to 8.87. Although the absolute change in the value of n is similar, the effect on AKC is greater for the smaller initial value of n.

In contrast, Figure (3) shows that the marginal capital cost of off-peak services *increases* with the ratio BO/BP. This is because a change in BO has only an indirect effect on TKC, through the effect on n. As BO rises, for given BP, n falls, causing AKC to increase. But this indirect effect is stronger the lower is the initial value of n, and hence the higher is BO/BP.

Combining the results of Figures (2) and (3), we see in Figure (4) that the relative marginal capital cost of off-peak services rises as the degree of off-peak fleet utilization rises. However, although the marginal capital costs of both peak and off-peak services do vary with the peak-to-base vehicle ratio, Figures (2)–(4) and Table (2) indicate that the degree of variation is limited. These marginal costs exhibit little change even when the ratio of peak to off-peak buses is increased from 2:1 to 4:1, nor do they change greatly when this ratio approaches 1:1, as long as the ratio strictly exceeds unity. This is because, while a change in the peak-to-base ratio clearly has a direct effect in equations (9) and (10) by altering the ratio BO/BP, this effect tends to be offset by the associated indirect effect on n. For example, if only peak buses are run, average bus life is 33.33 years, but as the ratio BO/BP approaches unity, average bus life approaches 5.12 years.

The sensitivity of the marginal capital costs to changes in the values of certain key parameter values is demonstrated in Table (2) by examining what happens when one variable at a time is changed from its assumed value in the base case. It is evident that, while the marginal capital cost of off-peak services is independent of the annual outlays for registration and insurance, A, and is rela-

FIGURE 4

RATIO OF MARGINAL CAPITAL COSTS

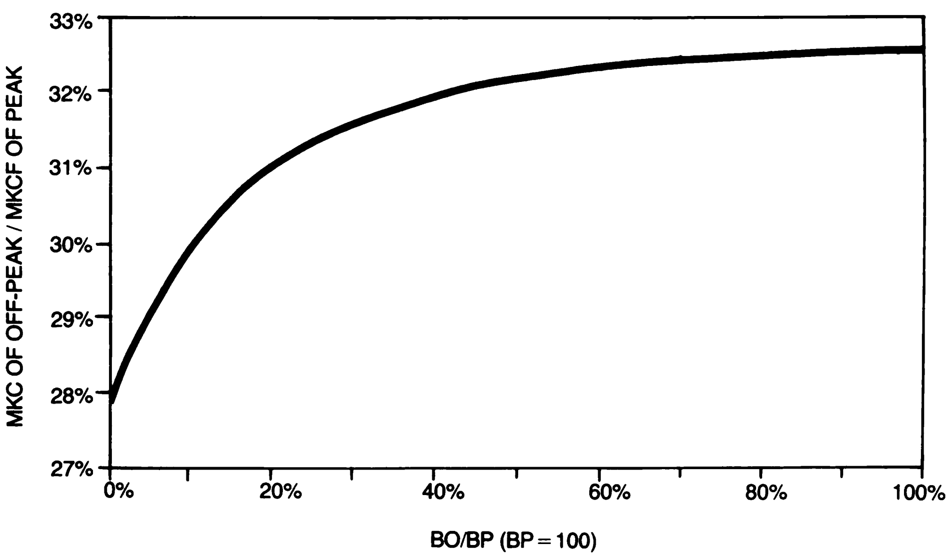


TABLE 2

SENSITIVITY OF RESULTS TO CERTAIN KEY VARIABLES

Variation	Marginal capital cost (cents per vehicle mile) Peak	Off-Peak	Average Annual Capital Cost ($ per bus)	Average Bus Life (years)
Base case	105.83	34.08	24,089	8.81
BP = 200	106.80	33.58	18,427	13.93
BO = 99	104.98	34.31	35,297	5.12
BO = 0	108.36	29.91	13,003	33.33
S = $0	107.40	37.87	25,543	8.81
S = $75,000 (= 50% of P)	99.53	18.93	18,272	8.81
S = $150,000 (= 100% of P)	91.67	0.00	11,000	8.81
A = $0	64.16	34.08	19,089	8.81
A = $10,000	147.49	34.08	29,089	8.81
r = 2%	90.07	34.00	22,171	8.81
r = 6%	122.64	34.00	26,080	8.81

tively insensitive to the discount rate, r, the marginal capital cost of peak services is sensitive to both of these variables. Finally, note the sensitivity of the marginal capital cost of off-peak services to the scrap value of vehicles, S. In fact, when scrap value is equal to the initial purchase price, this marginal capital cost is zero since, although off-peak services still tend to increase vehicle replacement frequencies, replacement costs nothing as there has effectively been no depreciation. However, the marginal capital cost of peak services is not zero in this case, because although the initial purchase price will eventually be recouped, there is an opportunity cost in the interim; furthermore, the other annual outlays, A, are still attributable to the peak.

FURTHER CONSIDERATIONS

It is useful to distinguish between the marginal private and marginal social costs of transit provision. For example, consider the annual insurance costs, which are presently incorporated in the term "A" in the model, and are therefore allocated entirely to the peak. From the private viewpoint of the transit operator, it may be appropriate to treat this as a capital cost determined solely by peak vehicle requirements. However, even here it is possible that if the operator agreed to reduce off-peak service provision, an insurance company may be willing to reduce the insurance charge per bus in the fleet. Furthermore, from a societal viewpoint, accident costs (which the insurance charges reflect) are more of an operating cost; insurance charges are merely lump-sum charges designed to cover accident risks over all operating periods. Although the accident risk per vehicle-mile may be higher in the peak, more miles are generally run in the off-peak. To the extent that some (probably most) of the insured risk is associated with off-peak service, the results presented in this paper tend to *under*state the relative marginal cost of off-peak services.

At the expense of a little more complexity, the model can also be extended to incorporate even more realistic assumptions, such as allowances for reserve buses, differential vehicle speeds in the peak and off-peak, more than two service levels and vehicle life being a *joint* function of usage and elapsed time. Most of these modifications would tend to raise the relative marginal capital costs of peak period services. Nevertheless, the marginal capital cost of off-peak services generally would still be substantial enough to warrant serious consideration by transit decision-makers.

CONCLUSION

In this paper, a methodology for the determination of the marginal capital costs of peak and off-peak services has been presented. It is suggested that this methodology is based on more realistic assumptions and stronger theoretical foundations than those of most existing costing models. The analysis indicates that the marginal capital costs of off-peak services can be substantial, and warrant consideration by transit planners. Planners need to be mindful that changes in the peak-to-base vehicle ratio can have sizable implications for total capital costs, even if fleet size is fixed.

Finally, for operational planning purposes, the above costing model can be easily set up using a standard spreadsheet package, with the user only being required to input the basic information described in Table (1).

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ENDNOTE

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1. For example, Meyer, Kain and Wohl (1965) and Mohring (1979, 1983).
2. I know of only one other paper, Mohring (1979), that accounts for *both* of these factors, by expressing vehicle life in terms of mileage and using CRF's. However, Mohring confined his calculation to an appendix to the main paper, and did not attempt to draw out the implications of this approach. Indeed, the analysis contained in the body of that paper treated the peak and off-peak independently, ignoring the interrelatedness between time periods that is generated by this approach. It is also interesting to contrast Mohring's 1979 paper with his earlier papers, such as Mohring (1972), in which the "depreciation and interest" approach is used, and vehicle life is expressed on an elapsed time basis, resulting in the marginal capital cost in the off-peak being zero.
3. Note that, although raising off-peak service levels results in higher capital costs per bus in the fleet, it reduces the overall capital cost per vehicle-mile.
4. Note that the marginal capital cost of running an extra off-peak bus *per annum* is considerably *higher* than that of running an extra peak bus (by a factor of approximately 1.8, using the base case data). However, because the ratio of off-peak to peak hours per annum is around 5.6:1, the marginal capital cost *per hour* in the off-peak is less than that in the peak.
5. These marginal operating costs are based on the operating costs for full-size buses quoted in Mohring (1983, p.300), after excluding capital costs and factoring up to maintain relativity with the bus purchase price.
6. In practice, large operators may gain economies, because a large fleet size may enable them to operate at a lower reserve bus to fleet size ratio.