APPLICATION OF STOCHASTIC DYNAMIC PROGRAMMING (SDP) FOR THE OPTIMAL ALLOCATION OF IRRIGATION WATER UNDER CAPACITY SHARING ARRANGEMENTS

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Abstract

This study attempts to arrive at an optimal allocation of irrigation water using capacity sharing (CS) as an institutional arrangement, and stochastic dynamic programming (SDP) as an optimisation model. It determines the value of an additional unit of water under a crop enterprise mix of lucerne-maize-wheat (LMW). SDP is an improvement on linear programming (LP) under stochastic conditions. The SIM-DY-SIM Model was used to simulate optimal returns, decision and policy variables under varying conditions of capacity share. LP results show that wheat has the highest MVP of R0.39/m^3, with maize exhibiting the lowest value of R0.09/m^3. The MVPs generated with SDP range between R0.06/m^3 and R0.35/m^3 on the whole farm basis, with revenue to the farmer increasing with an increase in CS content and increased percentage water release. However, the MVP of water decreased with the increased supply of the resource – a phenomenon that follows the general rule of decreasing marginal utility of a resource as more of it is used.

1. INTRODUCTION

South Africa is predominantly an arid country with an average annual rainfall of 497 mm, which is far below the world average of 860 mm (DEAT, 1999). Furthermore, this scanty precipitation is unevenly distributed across the country. The development of South Africa's water economy is alleged to have reached a mature phase (Backeberg, 1997). Yet, water scarcity persists as demand increases against a dwindling supply and stiff competition between scores of users. The Department of Water Affairs and Forestry (DWAF) revealed that with the present population of about 42 million, only 1,200 kilolitres of fresh water is available per person, per annum in South Africa. This figure is far below the amount of water available in the relatively more

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arid countries such as Botswana and Namibia, with values of 9,300 and 10,200 kilolitres respectively (FAO, 2002). The scenario paints a bleak picture for the water-based economy of the country and places South Africa on the threshold of becoming a "water stressed" economy (DWAF, 1998).

The new South Africa poses additional challenges and demands on water, as new initiatives such as the Reconstruction and Development Programme (RDP) and Black Economic Empowerment (BEE) aspire to achieve racial, social, gender and economic equality, through the redistribution and reallocation of scarce resources, of which water is prominent. These initiatives call for immediate action on developing institutional and technological alternatives to pursue sustainable management and use of water resources. Water has also become an internationally traded commodity, in which international tradeoffs are observed and maintained through country commitments, as regards the provision of clean water, in line with the platform of the Millennium Development Goals (MDGs) and the global commitment to sustainable development. The government has acknowledged its commitment to responsible water management for South Africans and the global community, as outlined in the National Water Act of 1998. To cater for the country's diverse requirements, the Act stipulates that water should meet international obligations, the needs for basic human consumption, and that environmental or ecosystem protection must receive priority.

This creates a dilemma in the management and use of water for agricultural production to address issues of poverty, hunger, food insecurity and unemployment. To this end, the primary purpose of this paper is to investigate the most suitable ways in which to utilise the scarce water resource in South Africa, in order to maximise farm benefits and meet the national and global standards in using water sustainably. The central theme is the valuation of water with the aid of quantitative tools based on the efficient use of water available at reservoir and farm levels.

2. METHODOLOGY

2.1 Theoretical framework

Several models have been applied in the optimal allocation of scarce resources, mainly from a static equilibrium consideration. Simulation and optimisation models are the two basic categories of models applied in water resource allocation. In their work, Reca *et al* (2001) argue that these two models differ significantly. With simulation, the optimal allocation of water is determined independently for each time interval. On the other hand, optimisation models carry out multi-interval analysis on optimal solutions. Thus, most water management and planning models fail to fully integrate the management of water supply and demand into a total system within the stochastic environment (Dudley and Hearn, 1993). The authors stress that where both water supply and demand are stochastic, there is a hierarchy of short-intermediate and long-term decisions to be optimised, in order to maximise returns from irrigation water. According to Dudley (1988), failure to take an integrated approach to water management could result in reduced sectoral or regional income, especially in cases of water scarcity resulting from increasing demand. Dudley and others in their works developed a methodology that enhances efficiency in water use by utilising capacity sharing as an institutional arrangement, and by optimally integrating the stochastic water demand with the stochastic water supply. Various research publications describe the development path of Dudley's work, which was mainly executed in Australia. The following articles, published in Agrekon, provide some insight into Dudley's approach and the value of his expertise in the South African context: Dudley, 1999a; Viljoen, Dudley & Gakpo, 2000; Gakpo, Du Plessis & Viljoen, 2001; Gakpo & Du Plessis, 2001).

Dudley, Reklis and Burt (1976) combined simulation and linear programming (LP) models to optimise water use. The simulation model simulates the water flow through the reservoir for each of the years of historical data flow needed to determine state variable transition probabilities for each season, or sub-stages. It adjusts the benefit function for the LP outputs to cater for water shortages occurring within a season. The dynamic programming (DP) model employs these transition probabilities and benefits to determine the optimal amount of water to allocate to irrigation during each season (sub-stage) throughout the planning horizon, based on the conditions that prevail at the start of each season.

2.2 The Dynamic Programming model

The DP model is a vital tool in decision-making, regarding the modelling of optimal utilisation of natural resources through time. According to Dudley (1999b), the separation of time into stages or decision intervals (period of time over which a particular level of control is held constant) results in multi-stage decision processes. The use of DP requires that at each decision point and stage, all factors that influence the response of the system to different decision processes must be considered in describing the "state of the system". The state of the system is defined by a specific combination of discrete values of state variables. DP uses the recurrence relation recursively to calculate the optimal remaining returns for each state at stage n, given the optimal remaining returns for each state of stage $n-1$.

A typical DP model of the form used in the study is shown in Equation 1 below:

$$f_n(i) = \text{optimum}\left[V_n(i,k) + B_n \sum_{j=1}^{J} P_n(i,j,k)\, f_{n-1}(j)\right] \tag{1}$$

$f_n(i)$ = present value of optimal expected returns over the remaining n stages in the planning horizon given that the current stage is i and optimal decisions are followed in each remaining stage.

n = number of stages left in the planning horizon, where $n = 1,2, \ldots, N$.

i = discrete state variable combination for the start of stage when n stages remain where $i = 1, 2, \ldots, I$.

j = discrete state variable combination at the end of stage when n stages remain and $j_n = i_{n-1}$, $j = 1, 2, \ldots, J$.

$V_n(i,k)$ = i x k matrix of expected immediate returns (returns in the immediate or just-beginning stage) when n stages remain, state i exists and decision k is implemented.

$P_n(i,j,k)$ = probability of the system state changing from discrete level i to j over the stage when n stages remain and decision k is followed.

$f_{n-1}(j)$ = present value of optimal expected return over the remaining $n-1$ stages when the state at the end of the immediate stage is j and optimal decisions are followed in each remaining stage.

B_n = the discount factor for the current stage.

Source: Gakpo, 2002.

The DP model is used as a tool for generating values of state variables within vital sub-units of a general system of computer-aided algorithms – the SIM-DY-SIM Model. This model is a combination of double simulation and stochastic dynamic programming (SDP). It has the capacity to integrate water demand and supply, taking hydrological factors into consideration to help water users and managers make short-, medium- and long-term water management decisions easily. The model is based on a recursive DP optimisation algorithm, which incorporates the discounting factor for the current stage and the probability of a change in the system at the various stages. It is programmed to combine two-computer simulations with dynamic programming to generate stream flow, water losses, water releases and farm revenue. The working mechanism of the SIM-DY-SIM model is illustrated in Figure 1.

2.3 Pre-Dynamic Programming Simulation (SIM 1)

SIM 1 calculates expected gross margins ($V_n(i,k)$) or the objective function matrix and state variable transition probabilities (P_n) for each combination of state and decision variable levels under consideration. To do this, the model uses three factors as inputs, namely values of farm gross margins, calculated from the LP application to the farm data; hydrology data for the farm area, as well as the discount factor or the cost of capital.

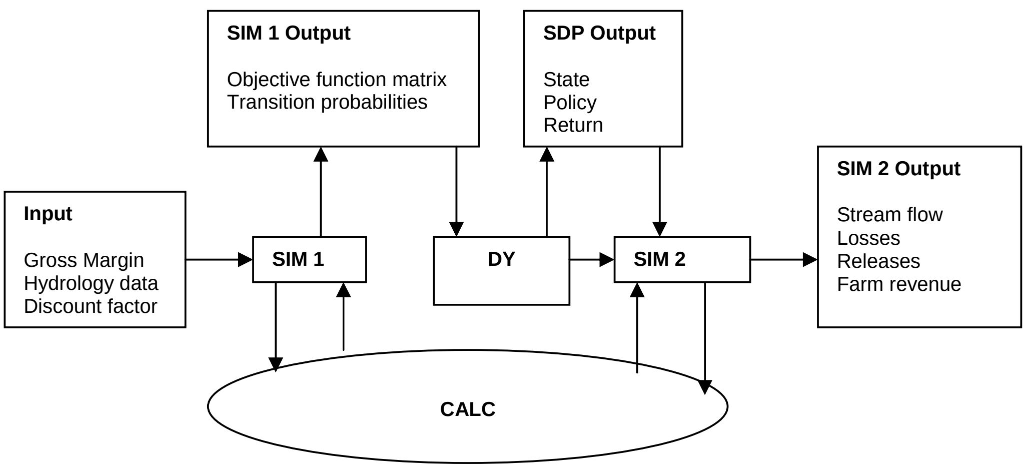

Figure 1: Illustration of SIM-DY-SIM model

Source: Gakpo, 2002.

2.4 Stochastic Dynamic Programming (SDP)

The SDP selects the best water management strategies as the optimal values of decision *k* for each state *i* in each decision interval, to satisfy the objective over the entire planning horizon. The main objective is to maximise expected gross margins over the planning horizon using the optimal quantity of water. The SDP uses the SIM 1 input to generate the following outputs:

- the state (quantity of water in the reservoir);
- policy decision variable (how much water to release); and
- expected returns (gains to the water user over the planning horizon, if a specific policy is pursued) for each stage in the planning horizon.

The marginal value product (MVP) of water is then derived directly from the expected returns (calculated return on investment in irrigation water) as the amount of money a farmer is willing to pay for an additional unit of water.

2.5 Post-Dynamic Programming (SIM 2)

This sub-unit of the model simulates the effects of using optimal decisions on optimal discrete releases from the farmer's capacity share, as derived by the SDP. SIM 2 calculates changes in the farmer's capacity levels across decision intervals by adding net flows (inflows less spills), reservoir losses and releases, as required to satisfy optimal water use at the farm level. At the end of the season and year, the programme outputs seasonal and annual revenues respectively. At the start of a new season, SIM 2 reads the inflow for the season and checks to ensure that the reservoir level is sufficient to justify any current releases for the season. If the reservoir level is low in a season, due to the stochastic inflows, water releases are reduced proportionately. The optimal decision on water release from the DP, in the immediate decision interval, represents maximum rather than mere release. Therefore, SIM 2 will not permit a release that would exceed the quantity, which is recommended by the DP. It will actually release a lesser amount in the event of insufficient water levels.

The SIM-DY-SIM therefore uses both the CS and inflow share (IS) to simulate different states of water availability. Dudley and Bryant (1995) define capacity share as an institutional arrangement and property rights structure for allocating water among multiple users of water resource systems, which includes storage reservoirs. CS provides each user or group of users' reservoir water with perpetual or long-term rights to a percentage of the reservoir inflows as well as a percentage of total reservoir capacity or space, in which to store those inflows, and from which to control releases. Inflow share represents the actual amount of entitlement in quantity or percentage terms to the water flowing into the storage space, which a farmer possesses in the CS system. Different levels of returns and MVPs can be simulated under different scenarios of water availability, by changing the CS and IS for a given farmer under different enterprise combinations and water management regimes.

The model was applied to data on 75-ha farms among 17 farmers using a centre-pivot irrigation system at the Ramah Canal, in the Vanderkloof Dam area in the Northern Cape Province. The enterprises on the farm include the three crops namely lucerne, maize and wheat, which are perennial, summer and winter crops respectively.

3. RESULTS AND DISCUSSION

3.1 LP output

The marginal value products generated from the LP for the 75-ha farms, which, as stated previously, produce maize, lucerne, and wheat, are presented in Table 1. This crop combination is typical in the area. The common practice in a 75-ha farm is for farmers to plant a maximum of 15 ha of lucerne and the remaining 60 ha will be planted with maize in the summer and wheat in the winter.

Table 1: MVP of water for summer and winter crops for a 75-ha farm on Ramah Canal, Vanderkloof Dam, 2000

Crop mix	Seasonal MVPs for the crops		
	Summer		Winter
1	Lucerne (R0,18/m^3)	Maize (R0,09/m^3)	Wheat (R0,39/m^3)

The results show that the value of irrigation water in the study area, Ramah Canal at Vanderkloof Dam is highest in the production of wheat, and lowest in maize production. The table shows that a farmer is prepared to pay R0,18, R0,09 and R0,39 for an additional cubic metre of water to produce lucerne, maize and wheat respectively.

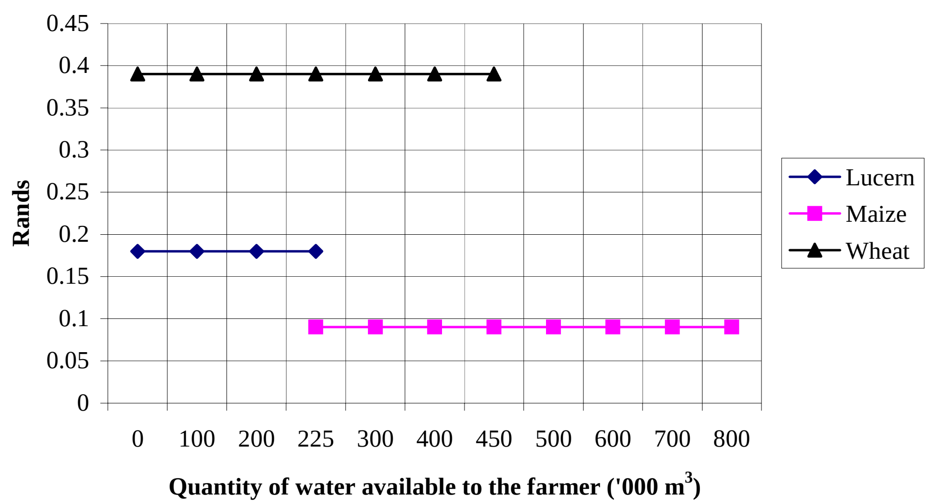

Figure 2: Water MVPs (R/m^3) resulting from LP simulation for a 75ha farm producing lucerne, wheat and maize on Ramah Canal, Vanderkloof Dam, 2000

Under a strict water scarcity condition, the sequence of demand and utilisation of water, based on its scarcity value, is such that for a lucerne-maize-wheat (LMW) combination and an 800,000 m^3 capacity share, the farmer would allocate the first 233,000 m^3 of water to the production of lucerne and wheat. Beyond this level of availability, water can then be allocated to maize production (see Figure 2).

3.2 SIM-DY-SIM model

The SIM-DY-SIM model consists of two main variables, namely a state and decision variable. State refers to the quantity of water in the reservoir. This variable takes a minimum value of 1 (for an empty reservoir), increasing at two per cent points per state to 51 (for a reservoir filled to capacity). A decision variable refers to the quantity of water released. It assumes a minimum value of 1 (for no release) and increments by two units up to 51 (for 100 per cent release). Results for SIM-DY-SIM model are discussed below.

3.3 SIM 1 outputs

A segment of the outputs of the pair of summer and winter objective function matrixes, and corresponding gross margins (GMs) for different states (farmer's reservoir contents) and decisions (amount of water releases) are presented in Table 2.

Table 2: Gross Margins (R'000) for 75-ha farm generated for different CS contents and decisions on Ramah Canal, Vanderkloof Dam, 2000

State	Decision					
	1	11	21	31	41	51
1	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	23.73	23.73	23.73	23.73	23.73
21	0.00	23.73	44.13	44.13	44.13	44.13
31	0.00	23.73	44.13	55.99	55.99	55.99
41	0.00	23.73	44.13	55.99	67.85	67.85
51	0.00	23.73	44.13	55.99	67.85	79.71

Source: Adapted from Table A7 of Gakpo, 2002.

The results show that the GM increases with an increase in CS and the amount of water releases. For example, in the Table, a State 11, in which CS content is 20 per cent full under Decision 41, where 80 per cent of the water is released, will yield a gross margin of R23,730 for the 75-ha farm. This value is less than the sum of R67,850 realisable at State 51 (CS content is 100 per cent full) under the same Decision 41 (80 per cent of water to be released). Note that State 1, Decision 1 represents conditions in which the CS content is zero, therefore no release is made. Furthermore, at the end of the farmer's planning horizon, any water left in storage is assumed to have zero value to the farmer.

3.4 SDP output

The SDP model selects the optimal quantity of water and the best water management strategies that would maximise expected returns (gross margin) over the planning horizon. Table 3 presents SDP output for stage 13 (thirteenth season of the farmer's planning horizon). The results indicate an increasing return on investment from R810,633 (State 1 Policy 1) to R890,112 (State 51 Policy 44) with the MVP for water ranging between R0.16 to R0.08/m^3 for the 75-ha farm with lucerne, maize and wheat crop combination. Similarly, if the farmer is at State 41, which corresponds to 80 per cent of CS content, the Policy is 34, implying that 66 per cent of the water is released. When the MVP for water is R0.08/m^3 the returns increase to R879,054. These results further show the thrift and dynamics of the model, whereby water is reserved for future use, as long as returns are increasing – thus influencing the decision to irrigate. Expected returns for each stage of the entire planning horizon are important parameters from which MVPs of water can be determined. The results further show that at the shorter planning horizon, MVPs do not change significantly, but in the long-run, the MVPs tend to decline and assume a fair spread (Table 3).

Table 3: MVPs derived from present value of expected optimal remaining returns in Stage 13 for base case lucerne, maize and wheat for 75 ha farm on Ramah Canal, Vanderkloof Dam, 2000

State	Policy	Percentage water release	Optimal return (R'000)	MVP (R/m^3)
1	1	0	810.633	0.16
11	11	20	834.238	0.16
21	18	40	854.834	0.12
31	24	60	867.229	0.08
41	34	80	879.055	0.08
51	44	100	890.112	0.08

3.5 SIM 2 outputs

Results simulated for the 75-ha farm with a lucerne-maize-wheat crop combination operating at maximum CS content for 19 years of two seasons (winter and summer) per year are presented in Table 4.

Table 4: Simulated results for lucerne-maize-wheat crop combination operating at maximum CS content for 19 years of summer and winter seasons on 75-ha farm Ramah Canal, Vanderkloof Dam, 2000

Season	S3 %	SFLOW 10^3m^3	EVAP 10^3m^3	Release 10^3m^3	DI %	FMREV (R'000)	RESCB 10^3m^3	RESCF 10^3m^3	RECET 10^3m^3
Winter	100.0	443.9	16.06	407.2	56.0	138.5	727.2	747.8	357.6
Summer	100.0	1521.1	77.59	625.4	86.0	71.4	727.2	1545.4	562.9
Winter	39.94	240.8	7.97	276.3	38.0	96.3	283.2	239.7	248.7
Summer	38.36	296.3	30.50	261.8	36.0	41.8	279.0	283.1	235.6

Where:

S3 = beginning season farm CS content as a percentage CS capacity.

SFLOW = inflow into farm CS in $10^3m^{3.}$

EVAP = seasonal evaporation from farm CS in $10^3m^{3.}$

RELEAS = seasonal farm release from farm CS in $10^3m^{3.}$

D1 = optimal decision from SDP as a percentage of farm CS capacity.

FRMREV = gross margin from releasing D1 (R'000).

RESCB = farm CS contents at the start of season in $10^3m^{3.}$

RESCF = farm CS content at end of season plus season's spills in $10^3m^{3.}$

RECET = water received at farm (release less approximately half CS surface evaporation losses and dam farm transmission losses) in $10^3m^{3.}$

The results in Table 4 show that at 100 per cent CS capacity, there is more evaporation in summer and therefore more release. However, optimal farm revenue is less in summer than in winter at R138,500 and R71,400 respectively, probably because of the higher valued wheat, which is produced in winter. The pattern remains the same when the CS content is about 38 per cent with revenue again more in winter than in summer at R96,300 and R41,800 respectively.

The results further indicate that there is variation in water demand, which can be due to seasonal disparities in water availability, as well as variations in the crop enterprise mix. The CS content (RESCF) is higher in summer than in winter (Table 4). This could be due to the tendency for farmers to save and accumulate water in summer, as a result of higher potential for water losses in summer than in winter. The CS helps to control potential water fluctuations. Surplus water accumulates in the CS of the reservoir and it is released based on demand, which is mainly guided by the profitability of the crops.

3.6 MVPs from SDP for inter season decisions

The MVPs generated by the SDP model for inter season decisions are illustrated in Figure 3. These MVPs represent values for a marginal unit of water to be used at any time in the future, under a non-steady water flow regime. In the case of the LP, the MVPs reflect what the farmer can expect to pay for a marginal unit of water that is delivered to the farm for immediate use. The figure depicts synergy between the behaviour of the model and normal economic principles of decreasing marginal utility as the supply of a scarce resource increases.

The figure demonstrates that the MVP for the summer season, at the start of the last summer in the planning horizon (1//1) is almost the same as when 13 seasons remain in the farmers planning horizon (1//13). Furthermore, the figure illustrates that at a low CS content with a low release of water, the marginal productivity of this resource and its corresponding MVP are high and remain constant at R0.16/m^3 from the point of zero CS content and release, to the point of 30 per cent CS content and release. At the State level 13, beyond the 30 per cent CS content and release, the MVP drops to a low of R0.08/m^3.

The winter season is represented by the following two curves, 1//2 and 1//14. Figure 3 illustrates that during this season, from zero to half the CS and release content, the MVP per cubic metre of water is R0.35, irrespective of the seasons remaining in a farmer's planning horizon. Beyond this point (from 50 per cent CS and above) the MVP declines sharply to R0.12 and thereafter fluctuates between R0.12 and R0.06/m^3.

Comparable results on the MVPs of water obtained from dynamic irrigation modelling studies are not available in South Africa. However, research executed by Louw (2001) and Conradie (2002), as summarised by Nieuwoudt *et al*, (2004) provides a number of comparable and plausible results. Conradie (2002) constructed linear programming models for 16 model types of farm

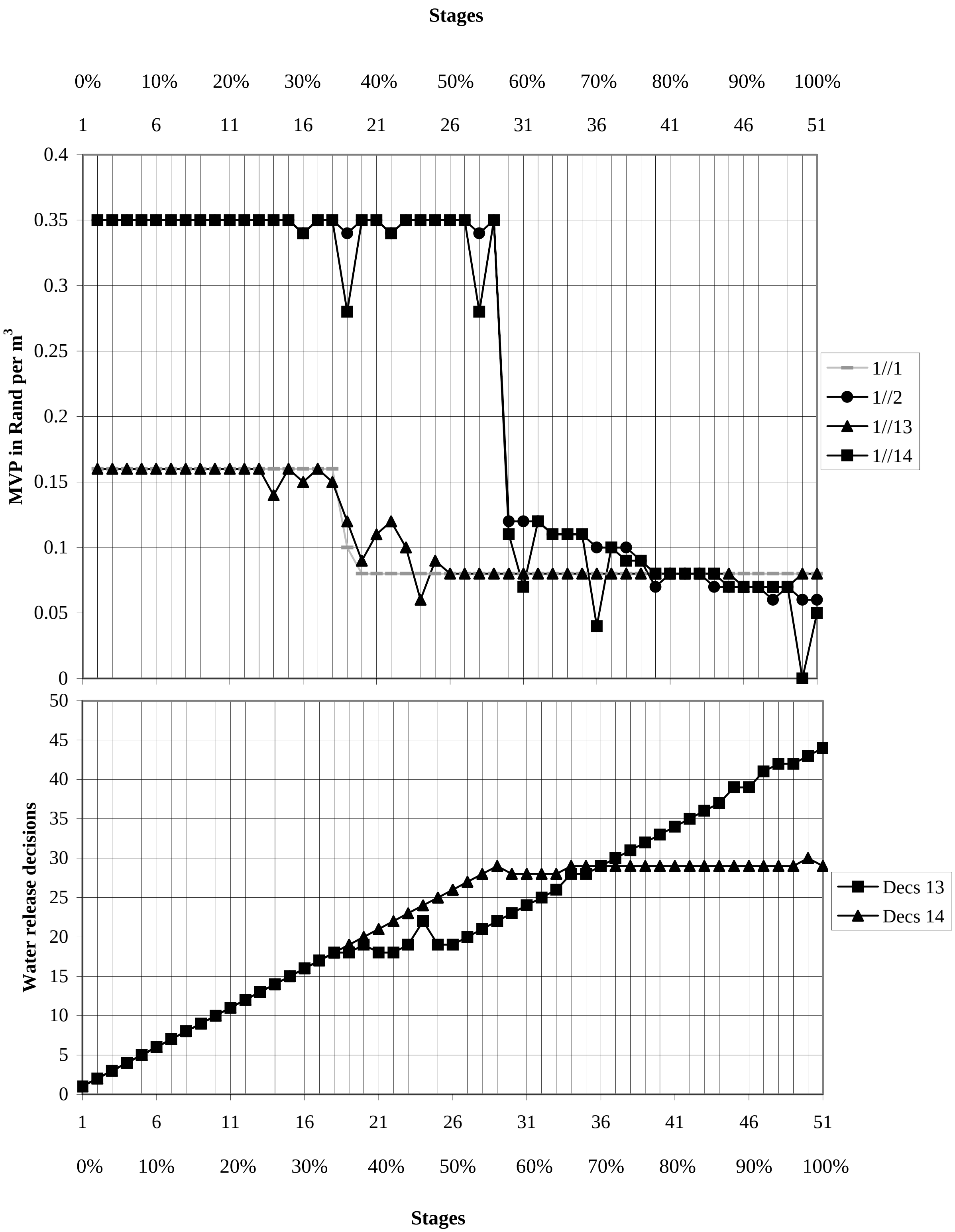

Figure 3: MVP in Rand per m³ and optimal water release decisions for LMW base case (75 ha farm, Ramah Canal, Vanderkloof Dam, 2000)

situations in the Fish-Sundays River Scheme, using MOTAD and allowing for potential risk. The report indicated that the MVPs of water were zero for three farm types and varied between R0.0003 and R0.2115/m^3 for the rest of the farms. Louw (2001) developed a positive mathematical programming model to study the impact of water markets in the Berg River Basin. From the median capitalised market value of water of R1.60 per cubic metre, at a capitalisation rate of 13 per cent, he found the rental rate (short term MVP) to be R0.21/m^3, if no trade is assumed, and R0.30/m^3 if trade is assumed.

The water release decision of Figure 3 shows that all the available reservoir CS water is applied to wheat in winter, until the maximum area allocated to the crop is fully utilised. Thereafter, no more water is applied to wheat production. This implies that all the available water is first applied to wheat until its MVP drops dramatically to a low level, after which any additional water is saved. In summer, the water is released until the area, which is planted to lucerne has been fully irrigated. Subsequently, variable quantities of water are saved and applied to the maize crop. However, quantities of water saved for future use remain constant at about 14 per cent of the farmer's reservoir capacity.

Rotating crops on a piece of land is very important for the area employed in this alternation, for a number of reasons. Firstly, the different crops in the enterprise (maize, lucerne and wheat) have different economic values. Secondly, there are two seasons (summer and winter) and the crops are season specific – maize in summer, and wheat in winter. Lucerne is a perennial crop that spans over both seasons. Lastly, crop rotation helps with the control of weeds, pests, and diseases. Note that lucerne is included in the crop enterprise mix because it has a high market value as fodder for livestock and therefore boosts the farm income. Thus, many farmers set aside a piece of their land (15 ha in 75 ha farm) for lucerne cultivation to avoid possible competition for land with the two staple crops – wheat and maize.

4. CONCLUSIONS

The study shows that stochastic dynamic programming is a versatile tool for the optimal allocation of water under stochastic conditions in an irrigated farming and mixed enterprise system. The model looks at the value of water from the combined crop enterprises by considering the capacity share, the season inflows and releases of water from the reservoir, and from the outcomes of policy decisions regarding the amount of water release. The results show that although financial returns increased with increase in CS content and release of water, the marginal value product of water decreased with increase in water supply. This phenomenon agrees with the conventional economic law of decreasing marginal utility of a resource as more of the resource is supplied.

The MVPs facilitate efficient use of water. In addition, under CS arrangement and a mixed crop enterprise, crops with relatively high MVPs for water, will cause the release of all or most of the reservoir contents for use in the immediate season, rather than save the water for future use. On the other hand, when MVPs are low, water is saved for future use. The MVPs, which determine the farmer's ability to produce capital, is an important element for determining water prices in both the present and the future. The study further shows that capacity sharing is a viable and optimal institutional arrangement in the management of scarce water resources for irrigation by controlling stochastic water flows and making releases to crops in accordance with their MVPs.

The paper recommends that to adopt CS in South Africa, two main institutions – Catchment Management Associations (CMAs) and Water Users Associations (WUAs) will play important roles in ensuring its success. CMAs will be involved in bulk water management. As bulk managers, CMAs will try to acquire and safeguard an equitable allocation of the resource to bulk shareholders, and ensure compliance with all rules and regulations that govern water allocation. CMAs can monitor and measure stream inflows, rainfall collection in the reservoirs, and record of losses due to evaporation, seepage and reservoir overflow spills, in order to update the stakeholders on size of users, use patterns and water availability. On the other hand, CMAs can secure the bulk share for emerging small farmers and delegate WUAs to carry out the administrative, supply-side, as well as the demand-side management on their behalf, until the farmers are able to make such decisions themselves. WUAs will administer water use at the retail level, where a large number of small shareholders, such as households and small-irrigated farms, are involved.

The South African government must establish appropriate institutional provisions that will guarantee exclusive water user rights – transferable through trade and enforced by law – to all farmers (water users) in the management of water. The SDP model will provide farmers with adequate information to manage their respective water allocation efficiently. The individual farmer, by envisaging the capacity of his reservoir contents at the beginning of a season, will be able to determine whether his water requirements for the forthcoming season are adequately met. The farmer can either buy water from a willing supplier, when in deficit or sell water in a case of surplus, ahead of inflows to avoid possible spills.

REFERENCES

Backeberg GR (1997). Water institutions, markets and decentralised resource management: Prospects for innovative policy reforms in irrigated agriculture. *Agrekon* 36(4):350-384.

Conradie B (2002). *The value of water in the Fish-Sundays Scheme of the Eastern Cape*. WRC Report No 987/1/02.

DEAT (1999). *State of the environment - South Africa: Freshwater systems and resources.* http://www.ngo.grida.no/soesa/nsder/issues/water/pressure.htm.

Dudley NJ (1988). A single decision maker approach to irrigation reservoir and farm management. *Water Resources Research* 24(5):633-640.

Dudley NJ (1999a). A new revenue trade-off for irrigation: Stabilization or maximization. *Agrekon* 38(4):559-565.

Dudley NJ (1999b). *An introduction to a computer model Sim-Dy-Sim, for Deterministic and Stochastic Dynamic Programming*. Armidale, NSW Australia.

Dudley NJ & Bryant MJ (1995). *The concept of capacity sharing and responses to questions posed on implications for irrigators*. Centre for Water Policy Research, University of New England, Armidale, NSW, Australia.

Dudley NJ & Hearn AB (1993). Systems modelling to integrate river valley water supply and irrigation decision making under uncertainty. *Agricultural Systems* 42:3-23.

Dudley NJ, Reklis DM & Burt OR (1976). Reliability, trade-offs and water resource development modelling with multiple crops. *Water Resources Research* 12(6):1101-1108.

DWAF (1998). *White Paper on Water Policy*. Department of Water Affairs and Forestry, Pretoria, South Africa.

FAO (2002). *Water resources - Water availability per person per year*. AQUASTAT, FAO Rome.

Gakpo EFY (2002). *Alternative institutional arrangements towards optimal water allocation*. University of the Free Sate, Bloemfontein, South Africa.

Gakpo EFY & Du Plessis LA (2001). Alternative water institutional framework to meet changing agricultural environment in South Africa. *Agrekon* 40(4):755-769.

Gakpo EFY, Du Plessis LA & Viljoen MF (2001). Towards institutional arrangements to ensure optimal allocation and security of South Africa's water resources. *Agrekon* 40(1):40-87.

Louw DB (2001). *The development of a methodology to determine the true value of water and the impact of a potential water market on the efficient utilisation of water in the Berg River Basin*. WRC Report No 943/1/02.

Nieuwoudt WL, Backeberg GR & Du Plessis HM (2004). The value of water in the South African economy: Some implications. *Agrekon* 43(2):162-183.

Reca J, Roldan J, Alcaide M, Lopez R & Camacho E (2001). Optimisation model for water allocation in deficit irrigation systems: Application to the Bembezer irrigation system. *Agricultural Water Management* 48:117-132.

Viljoen MF, Dudley NJ & Gakpo EFY (2000). A methodological perspective on valuing water: A comparison between linear programming and stochastic dynamic programming *Agrekon* 39(4):471-483.