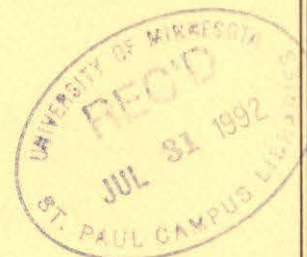# The University of Western Australia

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A METHOD FOR EVALUATING SUPPLY RESPONSE

TO PRICE UNDERWRITING[*]

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A METHOD FOR EVALUATING SUPPLY RESPONSE
TO PRICE UNDERWRITING

Abstract

This paper presents a method for evaluating
the supply response of individual producers to a
price underwriting scheme. The method includes
precise formulae to take account of the impact of
price underwriting on the producer's uncertain
conditions. The Australian Wheat Board's
Guaranteed Minimum Price Scheme is taken as a
specific example of price underwriting in
practice. Results show the scheme to lead to only
relatively small supply responses. The paper also
demonstrates the impact on producer behaviour of
an increase in price uncertainty in the presence
of an underwriting scheme.

The contribution of Newbery and Stiglitz (1981) to the welfare
evaluation of price stabilisation schemes represents an important
milestone in the assessment of government policies which influence the
riskiness of market participation. Expanding the focus of the welfare
effects of such policies to include not just changes in consumer and
producer net returns (or surplus), but also changes in the riskiness of
those returns, clearly results in a more complete welfare evaluation (see
also Gilbert 1985, and Hinchy and Fisher 1985). An acknowledged limitation
of the Newbery-Stiglitz methodology is, however, that it is based on an
assumption of no supply response by producers to the policy (see Fraser
1986).

Price support schemes are more widespread in agricultural marketing
than price stabilisation schemes, yet their welfare evaluation, at least
from the perspective of producers, seems to have received less attention
(see, however, Gallagher 1978, Quiggin 1983, Martin and Urban 1984, and
Hinchy 1987). This is perhaps because, as far as producers are concerned,
the nature of the welfare impact of a price support scheme would seem less
contentious than that of a price stabilisation scheme. In particular, not
only does a price support scheme generally act to increase expected per-
unit price, it also acts to reduce the variability of that price
(typically by eliminating unusually low price outcomes), both of which
result in favourable welfare effects on a risk averse producer. By
contrast, the claim that such schemes encourage unjustified output
expansion and have negative market consequences has been widely stated
(see, for example, Anderson and Tyers 1986, Sarris and Freebairn 1985).
However, such claims are typically derived from an aggregated view of
supply response, rather than from the supply response of individual
producers to price support. Moreover, it can be argued not only that the
latter perspective is what is required for the multiple effects of price
support on supply in uncertain conditions to be adequately captured but
also that, since aggregate behaviour is by definition the sum of the
behaviour of individuals, an understanding of individual supply responses
represents a foundation for determining the aggregate response. It is the
principal objective in this paper to present a method for evaluating the
supply response of individual producers to a price support scheme, and
specifically that of price underwriting, thereby providing a basis for a
more complete assessment of such schemes. An additional objective in the
paper is to investigate the way in which the presence of a price
underwriting scheme affects the response of both risk neutral and risk
averse producers to increased uncertainty of that price.

The paper is organised as follows. The first section sets out the model to be used in analysing producer behaviour. The second section presents a method for incorporating the effects of a price underwriting scheme into this model. The third section discusses the information requirements of the model and introduces the Australian Wheat Board's Guaranteed Minimum Price Scheme as a specific example of price underwriting. The fourth section presents and discusses the example results. The fifth section examines the theoretical and empirical consequences of increased price uncertainty when such a scheme is in place. It is followed by a conclusion.

The Model

The model of producer behaviour used in this paper is developed in Fraser (1984 1986). It assumes that the only input to production is the farmer's own labour, ℓ, and that a single output is produced which is subject to multiplicative risk:

$$x = \theta f(\ell)$$

where:
    $f(\ell)$ = planned output $[f'(\ell) > 0, f''(\ell) < 0]$
      θ = multiplicative risk term $[E(\theta) = 1]$

    x = uncertain actual output $[E(x) = \bar{x} = f(\ell)]$.

With price also uncertain, the producer's random income (y) is thus given by:

$$y = px$$

where:

        p = uncertain price $[E(p) = \bar{p}]$

It is further assumed that the producer's utility is (additively) separable in income and leisure so that his objective is to maximise by choice of labour input:
(1) $E[U(px)] - w\ell$

where:

        w = (constant) marginal disutility of labour[1]
    $U(px)$ = utility of random income $(U' > 0 , U'' \leq 0)$.
    It is shown in Fraser (1984) that using a second-order Taylor series expansion (1) may be approximated by:

$$(2) \quad U(\bar{p}\bar{x}) + 0.5U''(\bar{p}\bar{x})\bar{x}^2(\sigma_p^2 + \sigma_\theta^2\bar{p}^2) - \sigma_{p\theta}\bar{x} \, U'(\bar{p}\bar{x})(R-1) - w\ell$$

where:

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1. The previous assumption of diminishing returns to labour means that this assumption of constancy does not in effect restrict the analysis, but does simplify its presentation. Also, see Newbery and Stiglitz (1981): "With separable utility, there is little difference between diminishing returns to effort or increasing disutility of effort" (p.307).

σ_p^2 = variance of p

σ_θ^2 = variance of θ

$\sigma_{p\theta}$ = covariance of p,θ

$R = -U''(\bar{p}\bar{x}) \cdot \bar{p}\bar{x}/U'(\bar{p}\bar{x})$ = the producer's coefficient of relative risk aversion (evaluated at $\bar{p},\bar{x}$).

Note from (2) that whether a covariance of a given sign contributes positively or negatively to utility depends on whether R exceeds or is less than unity.

It is shown in Fraser (1986) that differentiating (1) with respect to ℓ gives the producer's first order condition as:

(3) $E[U'(px)p\theta]f'(\ell) = w$.

which, using a second-order Taylor series expansion, may be approximated by:

(4) $U'(\bar{p}\bar{x})\left[\bar{p}+0.5(\sigma_p^2/\bar{p}+\sigma_\theta^2\bar{p})[R(R-1)-\bar{p}\bar{x}R']+\sigma_{p\theta}[(R-1)^2-\bar{p}\bar{x}R']\right]f'(\ell) = w$.

Price Underwriting

From Quiggin (1983 p.200) "The crucial characteristic of an underwriting scheme is the formulation of a guaranteed minimum price. If the market price falls below this minimum, government payments are used to make up the difference". Or more technically, from Hinchy (1987 p.2) "Underwriting involves winsorisation of the probability distribution of price, shifting probability mass below the underwritten price to the underwritten price" (see Figure 1 in the Appendix).

Also from Hinchy (1987 p.2) "It is intuitively clear that underwriting will raise the mean, reduce the variance and increase the positive skewness of the price distribution for most plausible forms of probability distributions". However, in order to incorporate the precise impact of a price underwriting scheme into the model of section 1, specific formulae for characterising this impact are needed.[2] Unfortunately, as demonstrated by Martin and Urban (1984) in a model where there is no price-output covariation, the derivation of such formulae is not a simple procedure.[3] For this reason, the analysis was confined to the case where price and output were assumed to be initially jointly normally distributed. A formal derivation of the formulae listed below is contained in the Appendix: -

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2. Note that because the model considers only the first two moments of the price and output distributions, no formula for the skewness impact is required.

3. Note also that Martin and Urban (1984) restrict their attention to the formulae for the first two moments of a standard normal price variate.

$$(5) \quad E(p_u) = F(\hat{p})\hat{p} + [1-F(\hat{p})]\left[\bar{p}+\sigma_p Z(\hat{p})/[1-F(\hat{p})]\right]$$

$$(6) \quad \text{Var}(p_u) = [1-F(\hat{p})]\sigma_p^2\left[1-[Z(\hat{p})/(1-F(\hat{p}))]^2 + [(\hat{p}-\bar{p})/\sigma_p][Z(\hat{p})/(1-F(\hat{p}))]\right]$$

$$+ F(\hat{p})[\hat{p}-E(p_u)]^2 + [1-F(\hat{p})][\epsilon_2 - E(p_u)]^2$$

$$(7) \quad \text{Cov}(p_u,x) = [1-F(\hat{p})]\rho(\sigma_x/\sigma_p)\sigma_2^2 + \rho\sigma_x Z(\hat{p})[\epsilon_2 - \hat{p}]$$

where:

$\hat{p}$ = underwritten price

$Z(\hat{p}) = (1/\sqrt{2\pi})\exp\left[-0.5[(\hat{p}-\bar{p})/\sigma_p]^2\right]$

$F(\hat{p})$ = cumulative probability of $p \le \hat{p}$

$E(p_u)$ = expected price with underwriting

$\text{Var}(p_u)$ = variance of price with underwriting

$\epsilon_2 = \bar{p} + \sigma_p Z(\hat{p})/[1-F(\hat{p})]$

$\text{Cov}(p_u,x)$ = covariance of the underwritten price with output

ρ = correlation coefficient of the underlying joint normal distribution

$\sigma_2^2 = \sigma_p^2\left[1-[Z(\hat{p})/(1-F(\hat{p}))]^2 + [(\hat{p}-\bar{p})/\sigma_p][Z(\hat{p})/(1-F(\hat{p}))]\right]$.

The impact of a price underwriting scheme on a producer's welfare and level of output can be found by substituting $E(p_u)$, $\text{Var}(p_u)$ and $\text{Cov}(p_u,x)$ for $\bar{p}$, σ_p^2 and $\sigma_{p\theta}\bar{x}$ in (2) and (4).

Information Requirements

In order to be able to use (4) to evaluate a producer's supply response to a price underwriting scheme, three broad types of information are required:

(a) a specification of the producer's risk aversion as characterised by his utility function;

(b) a specification of the producer's initial economic circumstances. In what follows this is taken to comprise:[4]

   (i) $f(\ell)$

   (ii) σ_p^2

   (iii) σ_θ^2

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4. The assumption that w is constant for a producer means that information about its value will not be required.

(iv) $\sigma_{p\theta}$

(v) $\bar{p}$

(c) a specification of the percentage mean price at which the underwriting scheme operates--which in this case is taken to refer to the Australian Wheat Board's Guaranteed Minimum Price Scheme.

It is assumed that the producer's attitude to income risk can be adequately represented by the constant relative risk aversion function:

$$U(px) = (px)^{(1-R)}/(1-R)$$

where $R \neq 1$.

It should be noted that this assumption simplifies (4) by eliminating the terms related to whether R is increasing or decreasing (R'). In addition, note that this function implies the producer exhibits decreasing absolute risk aversion. In what follows, a range of values of R consistent with empirical estimates is considered (see Newbery and Stiglitz 1981, Chap.7).

The specification of the producer's initial economic circumstances requires a mixture of assumptions and actual industry data. The already-simplified relationship between the producer's labour input and his output $[f(\ell)]$ requires further simplification to a precise functional form. In what follows it is assumed that this form is given by:

$$\bar{x} = \ell^{m}$$

where it is also assumed that m lies in the range 0.5 to unity, and ℓ is given a positioning value equal to unity ($\bar{x} = 1$).[5] The producer's information about the relative size of σ_p^2, σ_θ^2 and $\sigma_{p\theta}$ is based on actual industry data with the additional main assumption that the producer has rational expectations (ie, his beliefs about $\bar{p}$, σ_p^2, σ_θ^2 and $\sigma_{p\theta}$ are correct). With the Australian Wheat Board's Guaranteed Minimum Price Scheme to be used as the example price underwriting scheme, suitable details of the breakdown of the overall income variation in the Australian wheat industry are provided in Harris et al (1974). Using this breakdown, which is based on the following approximation:[6]

$$\sigma_y^2 = \bar{x}^2\sigma_p^2 + \bar{p}^2\sigma_x^2 + 2\bar{p}\bar{x}\,\sigma_{px}$$

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5. Note that for $\bar{x} = 1$ to satisfy (4) over a range of values of R, the value of w must be assumed to be (precisely) inversely related to the value of R. However, as the results are calculated in percentage change terms, this additional assumption is not felt to be particularly restrictive. Note also that m = 0.5 corresponds to a deterministic supply elasticity of unity and that this elasticity tends towards infinity as m tends towards unity.

6. See Harris et al (1974) pp.304-305.

and setting a positioning value for income variability of:[7]

$$\sigma_y^2 = 10$$

gives:

$$\bar{x}^2 \sigma_p^2 = 0.34$$

$$\bar{p}^2 \sigma_x^2 = 10.52$$

$$2\bar{p}\bar{x} \, \sigma_{px} = -0.86$$

which, recalling that initially:

$$\ell = 1$$

gives:

$$\sigma_p^2 = 0.34 \, .$$

However, further specification of this breakdown requires an initial setting of $\bar{p}$. A positioning value of:

$$\bar{p} = 10.75$$

was chosen with a view to establishing an initial coefficient of variation (CV) of each of the random variables which corresponded closely to the actual industry values calculated by Harris et al (1974 p.302). With this initial setting:

$$\sigma_x^2 = \bar{x}^2 \sigma_\theta^2 = 0.091$$

$$\sigma_{px} = \sigma_{p\theta} = -0.08$$

so that (with actual industry values in parentheses):

$$CV_p = 5.4\% \ (\ 5.5\%)$$

$$CV_x = 30.2\% \ (30.5\%)$$

$$CV_y = 29.6\% \ (29.3\%)$$

Note also that these initial settings give:

$$E(y) = \bar{p}\bar{x} + \sigma_{px} = 10.67$$

as the initial value of expected income.

Finally, in what follows the price underwriting scheme is considered to operate with a range of underwritten prices between 85 and 95 per cent of the mean price.

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7. Recalling note 3, σ_y^2 would also vary over a range of values of R but for the setting of $\bar{x} = 1$ for all producers.

Results and Discussion

On the basis of the information detailed in the previous section, the formulae for the impact of an underwriting scheme on the producer's uncertain conditions given in the second section and the model of producer behaviour outlined in the first section, it is possible to determine the supply response of individual producers to the introduction of a price underwriting scheme for a range of underwritten prices and attitudes to risk. Examples of these responses are presented in Table 1.

Table 1: Impact of Price Underwriting on Supply (per cent change in output; m = 0.5)

Underwritten Price [% of E(p)]	R						
	0.0	0.3	0.6	0.9	1.2	1.5	1.8
85.0	0.007	0.004	0.002	0.000	-0.001	-0.002	-0.002
87.5	0.028	0.014	0.006	0.001	-0.002	-0.004	-0.005
90.0	0.094	0.048	0.021	0.004	-0.007	-0.014	-0.019
92.5	0.270	0.139	0.061	0.012	-0.020	-0.042	-0.056
95.0	0.661	0.340	0.151	0.030	-0.049	-0.103	-0.140

The first point to note about these responses is that in all cases their magnitude represents less than one per cent of initial output. The possibility that this unresponsiveness was due to relatively unproductive labour input was examined by recalculating the responses for m=0.99 (ie, almost constant returns to labour). A comparison with the results in Table 1 (m=0.5) is given in Table 2. It can be seen from this table that, although more productive labour leads to much larger responses for R=0, for R ≥ 0.3 the magnitude of the responses remains generally small despite the increase in labour productivity.[8] Rather, the explanation of these small magnitudes lies in recognising that, as the information about CVs in the third section shows, the producer's price is relatively less uncertain than his output so that even underwriting this price to 95 per cent of the mean level has only a small impact on his initial economic circumstances.[9]

Within the range of values of R and the underwritten price there is, however, a considerable variation in not only the relative magnitude but also the direction of the responses. For values of R<1, all responses are

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8. Note that although m = 0.99 implies an unrealistically large deterministic supply elasticity of 99, an overestimate of supply response from this source has been introduced in an attempt to balance the underestimate implied by the conclusion of Hinchy (1987) "that the producer benefits from underwriting are greater than those implied by the expected utility model" (Abstract).

9. Note that underwriting to 95 per cent of mean price involves 17.83 per cent of price outcomes.

Table 2: Impact of More Productive Effort on Supply Responses to Price
        Underwriting (per cent change in output)

Underwritten Price		R						
[% of E(p)]	m	0.0	0.3	0.6	0.9	1.2	1.5	1.8
85	0.5	0.007	0.004	0.002	0.000	-0.001	-0.002	-0.002
	0.99	0.742	0.016	0.004	0.001	-0.001	-0.002	-0.002
90	0.5	0.094	0.048	0.021	0.004	-0.007	-0.014	-0.019
	0.99	9.697	0.200	0.054	0.009	-0.012	-0.023	-0.028
95	0.5	0.661	0.340	0.151	0.030	-0.049	-0.103	-0.140
	0.99	91.961	1.435	0.396	0.063	-0.090	-0.171	-0.216

positive reflecting the favourable welfare impact of the scheme.[10] In addition, this impact is positively related to the size of the underwritten price as can be seen from the increasing magnitude of responses down the table. This feature reflects the increasing impact of the scheme on expected price (positive) and the variation of price (negative) as the underwritten price is increased. Moreover, for a given underwritten price, the magnitude of the response decreases as R increases towards unity reflecting the inhibiting impact of increased risk aversion on the willingness of a producer to increase supply in response to improved economic circumstances. It should be noted that this result is a specific example of the more general result that increased risk aversion is typically associated with more cautious behaviour. Other closely related examples of this general result are the demonstrations by Newbery and Stiglitz (1981) that both the elasticity of supply under uncertainty and the supply response to price stabilisation are inversely related to the coefficient of relative risk aversion (pp.307 and 310 respectively). Since it has already been noted that price underwriting represents a mixture of increased mean price and price stabilisation, it is not surprising that the inverse relationship which holds for the two changes separately also holds jointly.

For values of R>1, price underwriting also has a favourable welfare impact. However, as shown in equation (4), the qualitative impact of such a scheme on a producer's optimal supply is negative for R>1. As a consequence, a favourable welfare impact is indicated by a reduction in supply for R>1 as compared to an increase in supply for R<1. Once again,

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10. For $\hat{p}$ = 85 per cent of $\bar{p}$ and R = 0.9, the response is positive but less than 0.0005.

this result is a specific example of a more general result.[11] In particular, Newbery and Stiglitz (1981) also demonstrate both that the elasticity of supply under uncertainty and the supply response to price stabilisation are positive or negative as R is less than or greater than unity (pp.307 and 310 respectively, but see also p.82), and they explain this latter result by pointing out that individuals who are very risk averse (R>1) are worried "about the worst possible contingencies (e.g. starvation)" (p.82) so that when something unfavourable occurs "they have to work harder to avoid these extreme contingencies" (p.82). That the negative supply responses in Tables 1 and 2 also increase in magnitude both with the size of the underwritten price and with the size of R can be explained in the same way. Nevertheless, the significance of these negative supply responses should not be overstated, particularly as results reported in Bond and Wonder (1980) suggest values of R among Australian farmers are typically below unity.[12]

Finally, it should be noted these results suggest that, with the Australian Wheat Board operating a Guaranteed Minimum Price of approximately 95 per cent of the mean price, although the aggregate welfare impact of the scheme is unambiguously favourable, both the magnitude and the direction of the aggregate supply response to the scheme will depend on the distribution of attitudes to risk among producers, with the evidence of Bond and Wonder (1980) suggesting a small positive response is to be expected.

Increased Price Uncertainty

Consider a situation where a price underwriting scheme is in operation and that producers experience an increase in the uncertainty of the underlying price distribution. If the increase in uncertainty takes the form of a symmetrical increase in the variance of this price (σ_p^2) then it follows that the underlying mean price would be unchanged but that the magnitude of the covariance of the underlying price with output would be increased.[13] However, to determine a producer's supply response to this increase in price uncertainty, it is necessary to determine the impact of the increase on the probability distribution of the underwritten price. For example, differentiating equation (5) with respect to σ_p gives:

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11. See Meyer and Ormiston (1983) for a demonstration of the general result, in which it is shown that the direction of response of the choice variable in such a decision model will only be the same for all risk averse decision makers for a very restrictive class of payoff functions (e.g. the optimal choice is independent of the random variable).

12. Note that this statement assumes a typical value of R based on Bond and Wonder's (1980) mean estimates of the coefficient of absolute risk aversion (-2A) and mean net monetary return (x^*). Note also that Newbery and Stiglitz's (1981) survey of empirical evidence about attitudes to risk concludes that R typically varies between 0.5 and 1.2.

13. Recall that $Cov(p,x) = \rho \sigma_p \sigma_x$.

$$(8) \quad \partial E(p_u)/\partial\sigma_p = [\partial F(\hat{p})/\partial\sigma_p]\hat{p} + Z(\hat{p}) + \sigma_p[\partial Z(\hat{p})/\partial\sigma_p] - \bar{p}[\partial F(\hat{p})/\partial\sigma_p]$$

$$= Z(\hat{p}) + \sigma_p[\partial Z(\hat{p})/\partial\sigma_p] + [\partial F(\hat{p})/\partial\sigma_p][\hat{p} - \bar{p}]$$

If the shift in probability weight is confined to the tails of the underlying price distribution, so that there is no change in $F(\hat{p})$ and $Z(\hat{p})$, then it is clear from (8) that the mean underwritten price is increased by such an increase in uncertainty:

$$\partial E(p_u)/\partial\sigma_p = Z(\hat{p}) > 0 .$$

However, while the likely impact of an increase in σ_p on $Z(\hat{p})$ is unclear, $F(\hat{p})$ would typically be increased by this increase for $\hat{p} < \bar{p}$. In other words, although it would seem reasonable to expect an incease in σ_p also to increase $E(p_u)$, the outcome is not algebraically unambiguous. The same ambiguity applies to $Var(p_u)$ and $Cov(p_u,x)$. Nevertheless, it seems unlikely that $Var(p_u)$ would not be increased by an increase in σ_p, and that the indirect impact on σ_{px} of this increase would not be strong enough to be transmitted to $Cov(p_u,x)$.

These expectations are to a large extent confirmed in the results presented in Table 3. This table shows the impact of the increase in price uncertainty on the underwritten price distribution over the range of underwritten prices. A comparison of columns (1) and (4) shows that the increase in σ_p typically increases $E(p_u)$ with the magnitude of this effect increasing with the underwritten price. Similarly, columns (2) and (5) show that the increase in σ_p also increases $Var(p_u)$. However, the magnitude of this effect decreases as the underwritten price is increased reflecting the increasing proportion of relatively low prices which is being underwritten.

Table 3: Impact of Increased Price Uncertainty on the Distribution of the Underwritten Price (underlying σ_p increased by 10 per cent)

Underwritten Price [% of E(p)]	Before			After		
	(1) $E(p_u)$	(2) $Var(p_u)$	(3) $Cov(p_u,x)$	(4) $E(p_u)$	(5) $Var(p_u)$	(6) $Cov(p_u,x)$
85.0	10.751	0.338	-0.080	10.751	0.406	-0.087
87.5	10.752	0.334	-0.079	10.754	0.397	-0.086
90.0	10.757	0.321	-0.077	10.762	0.378	-0.084
92.5	10.772	0.293	-0.073	10.782	0.341	-0.079
95.0	10.806	0.246	-0.066	10.822	0.284	-0.070

Finally, columns (3) and (6) show that the indirect impact on σ_{px} is transmitted to $Cov(p_u,x)$ but that, as with $Var(p_u)$, the magnitude of this effect decreases as the underwritten price increases.

Turning to the impact of these changes in the underwritten price distribution on supply, Table 4 presents the results once again for a range of values of R and underwritten prices. As in the case of introducing the underwriting scheme itself, the increase in price uncertainty leads to a generally small supply response reflecting the relative insignificance of this price uncertainty in the producer's initial economic circumstances.

Table 4: Impact of Increased Uncertainty of an Underwritten Price on Supply (per cent change in output) (underlying σ_p increased by 10 per cent)

Underwritten Price	R						
[% of E(p)]	0.0	0.3	0.6	0.9	1.2	1.5	1.8
85.0	-0.064	-0.028	-0.010	-0.001	0.002	0.000	-0.003
87.5	-0.047	-0.019	-0.006	0.000	0.000	-0.003	-0.007
90.0	-0.008	-0.001	0.002	0.001	-0.003	-0.008	-0.014
92.5	0.041	0.027	0.014	0.003	-0.007	-0.017	-0.027
95.0	0.102	0.060	0.029	0.006	-0.012	-0.028	-0.042

There are, however, some interesting changes in magnitude and direction of these responses across the table. For example, in the case of R=0, only the response for an underwritten price of 90 per cent of $\bar{p}$ or less reproduces the conventional negative response in the absence of price underwriting (ie, a negative covariance reduces expected income so that an increase in its magnitude has an unfavourable welfare impact). For underwritten prices above 90 per cent of $\bar{p}$, the increase in σ_p causes a large enough increase in $E(p_u)$ for this to dominate the covariance effect and result in a favourable welfare impact and a positive supply response. Moreover, this favourable impact is reinforced by the decline in the magnitude of the covariance effect as the underwritten price is increased.

For values of R greater than zero, the two conflicting effects on expected income are joined by two conflicting effects on the variance of income: the increase in $Var(p_u)$; and the increase in the magnitude of Cov (p_u,x). Table 4 shows that, for an underwritten price of 85 per cent of $\bar{p}$, the welfare balance of these four effects is unfavourable for R<1.8, but favourable for R=1.8. This result implies that, of the two effects on the variance of income, the covariance effect becomes increasingly important as R increases. Note that the importance of this covariance effect may in part be attributed to the dominance of output uncertainty in determining the variance of income. Moreover, Table 4 shows that, at the 87.5 and 90 per cent levels of underwritten price, the welfare balance of the four

effects turns favourable at lower levels of R. This result is consistent with the greater rate of decline in the magnitude of the $Var(p_u)$ effect compared with the $Cov(p_u,x)$ effect as indicated in Table 3. Table 4 also shows that, for the levels of underwritten price above 90 per cent of $\bar{p}$, the welfare balance is favourable regardless of the level of R, leading to a positive supply response for R<1 and a negative supply response for R>1. In other words, if a price underwriting scheme is in operation with an underwritten price of greater than 90 per cent of $\bar{p}$, the actual supply responses to increased price uncertainty are the reverse of the conventional responses.

Finally, it should be noted these results suggest that, with the Australian Wheat Board operating on a Guaranteed Minimum Price Scheme of 95 per cent of the mean price, an increase in the uncertainty of the underlying price distribution will have a favourable welfare impact on all producers regardless of their attitude to risk. This is principally because of the positive effect on the expected price with underwriting of such an increase in uncertainty. However, as in the case of the introduction of the underwriting scheme itself, both the magnitude and the direction of the associated aggregate supply response will depend on the distribution of attitudes to risk among producers, although the evidence of Bond and Wonder (1980) once again suggests a small positive response.

Conclusion

The main objective in this paper has been to present a method for evaluating the supply response of individual producers to a price underwriting scheme. This method required the development of precise formulae to take account of the impact of price underwriting on the producer's uncertain conditions. The Australian Wheat Board's Guaranteed Minimum Price Scheme was taken as a specific example of price underwriting in practice. Individual supply responses, although indicating a favourable welfare impact of the scheme, were shown in general to be quite small reflecting the relative (to output) insignificance of price uncertainty in the Australian wheat industry. Nevertheless, both the magnitude and the direction of individual responses were shown to vary depending on the level of the price underwriting and the degree of risk aversion of the producer. The productivity of labour was also shown to determine the magnitude of the individual producer's supply response.

An additional objective in the paper was to investigate the way in which the presence of a price underwriting scheme affects the response of producers to increased price uncertainty. It was shown that an increase in price uncertainty affects the underwritten price distribution in three ways: (a) by increasing the expected price with underwriting, (b) by increasing the variance of price with underwriting, and (c) by increasing the magnitude of the covariance between the underwritten price and output. The strength of the first effect was shown to dominate for a level of underwriting above 90 per cent of the mean price, thus implying the unconventional result that an increase in price uncertainty has a favourable welfare impact for all producers. This in turn leads to a positive or negative supply response depending on the degree of risk aversion of each producer.

Finally, in relation to the Australian Wheat Board's Guaranteed Minimum Price Scheme it was clear, for both the introduction of the scheme (at 95 per cent of mean price) and an increase in price uncertainty in the presence of the scheme, that not just the magnitude but also the direction

of the aggregate supply response depends on the distribution of risk attitudes among producers. Nevertheless, on the basis of the Bond and Wonder (1980) estimates of attitudes to risk among Australian farmers, a small positive aggregate supply response is to be expected in each case.

APPENDIX

Winsorising a Normal Distribution
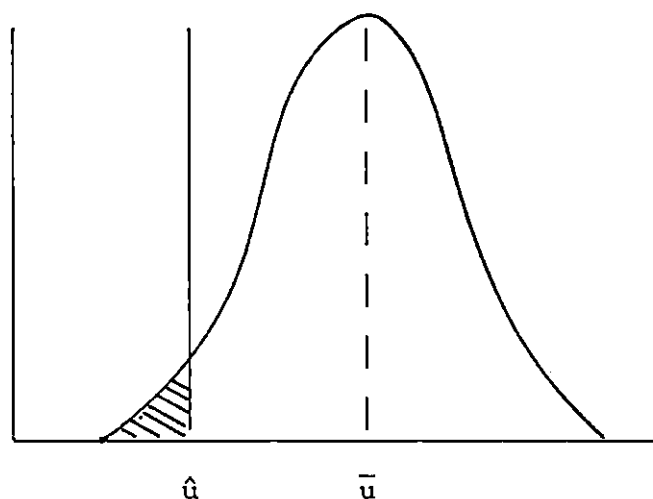
Figure 1: Winsorising a Normal Distribution

Let $\hat{u}$ be the point of Winsorising and $\bar{u}$ be the original mean (ϵ), σ_u^2 the original variance of a normal distribution.

Then this is equivalent to mixing two distributions in the proportion $F(\hat{u})$, $(1-F(\hat{u}))$; where:

$$\text{for } u \le \hat{u} : \quad \epsilon_1 = \hat{u} \quad , \quad \sigma_1^2 = 0$$

and:

$$\text{for } u > \hat{u} : \quad \epsilon_2 = E(u|u>\hat{u}) \quad , \quad \sigma_2^2 = \text{Var}(u|u>\hat{u}) \quad .$$

The second of these is a truncated normal distribution where:

(A1) $\epsilon_2 = E(u|u>\hat{u}) = \bar{u} + \sigma_u Z(\hat{u})/[1-F(\hat{u})]$

(A2) $\sigma_2^2 = \text{Var}(u|u>\hat{u}) = \sigma_u^2 \left(1 - \left[Z(\hat{u})/[1-F(\hat{u})] \right]^2 \right.$

$$\left. + \left[(\hat{u}-\bar{u})/\sigma_u \right] \left[Z(\hat{u})/[1-F(\hat{u})] \right] \right)$$

(see Johnson and Leone 1964 p.128).

Note the following formulae for a mixture (x):

$$E(x) = \sum_{i=1}^{k} p_i \epsilon_i$$

$$Var(x) = \sum_{i=1}^{k} p_i \sigma_i^2 + \sum_{i=1}^{k} p_i (\epsilon_i - \bar{\epsilon})^2$$

where:

$$\bar{\epsilon} = \sum_{i=1}^{k} p_i \epsilon_i = E(x) \quad .$$

For k=2 and the above information:

(A3)
$$E(x) = F(\hat{u}) \cdot \hat{u} + [1-F(\hat{u})]\left[\hat{u}+\sigma_u Z(\hat{u})/[1-F(\hat{u})]\right]$$

(A4)
$$Var(x) = [1-F(\hat{u})]\sigma_u^2\left(1 - \left[Z(\hat{u})/[1-F(\hat{u})]\right]^2\right.$$

$$+ \left[(\hat{u}-\bar{u})/\sigma_u\right]\left[Z(\hat{u})/[1-F(\hat{u})]\right]\Big)$$

$$+ F(\hat{u})[\hat{u}-E(x)]^2 + [1-F(\hat{u})][\epsilon_2 - E(x)]^2$$

(see Johnson and Leone 1964 p.129).

To assess the impact of Winsorising on the covariance between u and some other normally distributed variable v, let ρ be the correlation coefficient and $\rho\sigma_u\sigma_v$ the initial covariance.

$$\text{Then } Cov(x,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [x-E(x)][v-E(v)]f_{uv}\,du\,dv$$

$$= \int_{-\infty}^{\hat{u}}\int_{-\infty}^{\infty} [\hat{u}-E(x)][v-E(v)]f_{uv}\,du\,dv$$

$$+ \int_{\hat{u}}^{\infty}\int_{-\infty}^{\infty} [u-E(x)][v-E(v)]f_{uv}\,du\,dv \quad .$$

Consider the first term:

$$= [\hat{u}-E(x)] \int_{-\infty}^{\hat{u}} f(u)\,du \int_{-\infty}^{\infty} [v|u-E(v)]f(v|u)\,dv$$

$$= [\hat{u}-E(x)]F(\hat{u})[\bar{v}_u-E(v)]$$

where:

$$\bar{v}_u = E(v|u<\hat{u})$$

$$= E(v) + \rho \ (\sigma_v/\sigma_u)\left[\bar{u}-\sigma_u[Z(\hat{u})/F(\hat{u})]-\bar{u}\right]$$

(see Mood, Graybill and Boes 1974 p.167, and Maddala 1983 p.367),

so that $\qquad = [\hat{u}-E(x)]F(\hat{u})[-\rho\sigma_v \cdot Z(\hat{u})/F(\hat{u})]$.

Next consider the second term:

$$= \int_{\hat{u}}^{\infty} \int_{-\infty}^{\infty} (u-\epsilon_2)[v-E(v)]f_{uv}dudv$$

$$+ \int_{\hat{u}}^{\infty} \int_{-\infty}^{\infty} [\epsilon_2-E(x)][v-E(v)]f_{uv}dudv$$

$$= Cov(x,v|u>\hat{u})[1-F(\hat{u})]$$

$$+ [\epsilon_2-E(x)][1-F(\hat{u})][E(v|u>\hat{u})-E(v)]$$

$$\text{(Note: } f(x) = f(u)/[1-F(\hat{u})]\text{)}$$

$$= \rho(\sigma_v/\sigma_u) \cdot \sigma_2^2[1-F(\hat{u})]$$

$$+ [\epsilon_2-E(x)][1-F(\hat{u})]\left[\rho\sigma_v Z(\hat{u})/[1-F(\hat{u})]\right]$$

(see Johnson and Kotz 1972 p.112).
Bringing together the first and second terms:

$$Cov(x,v) = -[\hat{u}-E(x)]F(\hat{u})[\rho \cdot \sigma_v Z(\hat{u})/F(\hat{u})]$$

$$+ [1-F(\hat{u})]\rho \cdot (\sigma_v/\sigma_u) \cdot \sigma_2^2$$

$$+ [1-F(\hat{u})][\epsilon_2-E(x)]\rho \cdot \sigma_v Z(\hat{u})/[1-F(\hat{u})]$$

(A5) $\qquad = [1-F(\hat{u})]\rho \cdot (\sigma_v/\sigma_u) \cdot \sigma_2^2$

$$+ \rho\sigma_v Z(\hat{u})[\epsilon_2-\hat{u}] \quad .$$

References

Anderson, K. and Tyers, R. (1986), 'Agricultural policies of industrial countries and their effects on traditional food exporters', Economic Record 62(179), 385-399.

Bond, G. and Wonder, B. (1980), 'Risk attitudes amongst Australian farmers', Australian Journal of Agricultural Economics 24(1), 16-34.

Fraser, R.W. (1984), 'Risk aversion and covariances in agriculture: a note', Journal of Agricultural Economics 35(2), 269-271.

Fraser, R.W. (1986), 'Supply responses, risk aversion and covariances in agriculture', Australian Journal of Agricultural Economics 30(2/3), 153-156.

Gallagher, P. (1978), 'The effectiveness of price support policy--some evidence for US corn acreage response', Agricultural Economics Research 30(4), 8-14.

Gilbert, C.L. (1985), 'Futures trading and the welfare evaluation of commodity price stabilisation', Economic Journal 95(379), 637-661.

Harris, S., Crawford, J.G., Gruen, F.H. and Honan, N.D. (1974), Rural Policy in Australia, A Report to the Prime Minister by a Working Group. Canberra, AGPS.

Hinchy, M. (1987), 'Evaluating the Producer Benefits from Underwriting', Paper presented to the 31st Annual Conference of the Australian Agricultural Economics Society, Adelaide, February 1987.

Hinchy, M. and Fisher, B. (1985), 'An Assessment of Producer Gains from Wool Price Stabilisation', Paper presented to the 29th Annual Conference of the Australian Agricultural Economics Society, Armidale, February 1985.

Johnson, N.L. and Leone, F.C. (1964), Statistics and Experimental Design, Vol I, John Wiley and Sons, New York.

Johnson, N.L. and Kotz, S. (1972), Distributions in Statistics, Continuous Multivariate Distributions, John Wiley and Sons, New York.

Maddala, G.S. (1983), Limited-Dependent and Qualitative Variables in Econometrics, Cambridge University Press.

Martin, W. and Urban, P. (1984), 'Modelling Producer Response Under Support Price and Stabilisation Schemes'. Paper presented to the 28th Annual Conference of the Australian Agricultural Economics Society, Sydney, February 1984.

Meyer, J. and Ormiston, M.B. (1983), 'The comparative statistics of cumulative distribution function changes for a class of risk averse agents', Journal of Economic Theory 31(1), 153-169.

Mood, A.M., Graybill, F.A. and Boes, D.C. (1974), Introduction to the Theory of Statistics, 3rd edn, McGraw-Hill, Kogakusha.

Newbery, D.M.G. and Stiglitz, J.E. (1982), The Theory of Commodity Price Stabilisation, Clarendon Press, Oxford.

Quiggin, J. (1983), 'Underwriting agricultural commodity prices', Australian Journal of Agricultural Economics 27(3), 200-211.

Sarris, A.H. and Freebairn, J. (1983), 'Endogenous price policies and international wheat prices', American Journal of Agricultural Economics 65(2), 214-224.