The Estimation of Farm Business Inefficiency in the Presence of Debt Repayment

by Steele C. West

The estimation of farm business inefficiency in the presence of debt repayment

Dr. Steele C. West
The University of Western Australia

Contributed paper prepared for presentation at the 31st Triennial International Conference of Agricultural Economists, Online, 17-31 August 2021

The estimation of farm business inefficiency in the presence of debt repayment

Abstract

Farm businesses often use debt to purchase inputs and meet operational costs. Numerous studies have investigated the impact of debt use on farm performance as measured using technical efficiency. However, no prior studies in agriculture have considered treating the created debt repayment obligation as a by-product of production based on purchasing discretionary inputs. This study employs nonparametric directional distance function models to quantify the impact of debt repayment as a by-product of input use on farm-level partial inefficiency, using a panel data of 54 mixed enterprise broadacre farms in Western Australia from 2002-2011. Results show that the inclusion of debt repayment obligations generated as a by-product in the production model results in higher estimates of partial inefficiency being obtained. The study's results imply that failure to account for repayment obligations created in the production process may lead to bias in estimated inefficiency measures of farm businesses.

Key words: Farm business, debt, partial inefficiency, by-production

JEL Codes: G32, Q14, Q12, D24

Globally, the efficiency with which farms use production inputs to generate output has been the subject of extensive study because of its importance to farm enterprise viability and food security (Serra et al. 2008; Mugera and Langemeier 2011; Guesmi and Serra 2015; Henderson 2015; Abdul-Salam and Phimister 2017; Pieralli et al. 2017). Amongst this vast literature, several studies have examined the impact of debt use and credit availability on the productive performance of farms, using various production efficiency measures (Taylor et al. 1986; Giannakas et al. 2001; Lambert and Bayda 2005; Zhengfei and Oude Lansink 2006; Briggeman et al. 2009b; Mugera and Nyambane 2014; Brewer and Featherstone 2017). Despite the insights offered by these studies on the relationship between debt use and various measures of production efficiency, none of these studies have defined their production models to include debt repayment risk as a by-product of input use.

It is important to acknowledge from the outset that access to credit is widely accepted as important to the growth and productive performance of firms not just in agriculture, but the broader economy (Heil 2018). Nevertheless, recent studies in financial economics have exposed the existence of a U-shape relationship between debt and productive performance, whereby a trade-off exists between investment that promotes enhancement in productive performance and over-investment into sub-optimal activities (Coricelli et al. 2012; Jin et al. 2019). These studies suggest that debt use needs to be optimised by firms seeking to maximise productive performance so as not to induce *potentially* adverse ('undesirable') consequences to firms.

A limitation of production models in investigating the optimization of farm business debt use is that the inclusion of debt as an input in these models would lead to a 'double-accounting', since farm businesses use a combination of debt and cash on hand to acquire the inputs that would be included in production models. To overcome this limitation, this study factors debt repayment obligations created as by-products of production input use into production models to examine the nexus between productive performance of farm businesses and their debt use. To understand debt repayment as an by-product of input use, consider that each time a farm business borrows money to buy specific inputs, such as pesticides or fertilisers, it generates an obligation to repay the money borrowed. If farm businesses cannot meet these repayment obligations, this can have adverse outcomes including potential bankruptcy of the farm enterprise. This severe consequence occurred on a mass scale in the United States in the 1980s when a collapse in land prices and inability to meet repayment obligations resulted in over 200,000 farm foreclosures (Briggeman et al., 2009a; Cowley and Clark, 2016). Episodes of prolonged drought in some regions of Australia also pose similar threats to farm businesses. Beyond enterprise risks, debt obligations have been shown to have significant adverse consequences on the mental health and wellbeing of borrowers (Hojman et al., 2016; Hiilamo and Grundy, 2018), with a growing body of literature identifying a link between farm operator suicide and indebtedness (Rajan and Ramcharan, 2015; Merriott, 2016; Logstein, 2016; Crnek-Georgeson et al., 2017; Perceval et al., 2018). Such risks and stresses from repayment obligations will likely increase in the presence of growing uncertainty caused by events such as climate change and adverse trade policies.

The redefinition of the production function to incorporate undesirable outputs and by-products has occurred extensively in the environmental and agricultural economics literature. Many studies have investigated the efficiency of energy input use and the production of air pollutants as by-products of economic output (Seiford and Zhou 2002; Zhou and Ang 2008; Bian and Yang 2010; Shi et al. 2010; Yang and Pollitt 2010; Wang et al. 2011; Meurty et al. 2012; Wang et al. 2013; Vlontzos et al. 2014; Lozano 2015; Lin and Fei 2015; Chen et al. 2017; Pham & Zelenyuk 2019). Some studies have considered other by-products such as nitrogen or pesticide effluent that lead to water and soil contamination (Gollop and Swinand 1998; Fernández et al. 2002; Ball et al. 2004; Rezek and Perrin 2004; Ball et al. 2005; Kuosmanen 2005; Tamani et al. 2012; Skevas et al., 2014; Njuki and Bravo-Utera 2015; Njuki et al. 2016; Mamardashvili et al. 2016; Huang and Bruemmer 2017; Dakpo & Oude Lansink 2018; Malikov et al. 2018). A key finding of these studies, with implications to the modeling of by-products more broadly, is that their exclusion in analyses can lead to bias in the estimation of productivity and efficiency measures (Hailu & Veeman 2001; He et al. 2013). In financial economics, a growing number of studies have sought to investigate the impact of undesirable outputs in the form of nonperforming loans on bank efficiency (Fukuyama & Weber, 2010; Barros et al., 2012; Lozano, 2016; Qayyum and Riaz, 2018; Fujii et al., 2018). Understanding the impact of nonperforming loans on bank performance is essential because borrowers' failure to repay loans adversely impacts the profitability and cash flow of banks, which can lead to insolvency in severe cases (Pham and Zelenyuk, 2018). This study seeks to examine the effects of repayment obligations generated as a by-product of input use on the performance of farm businesses by applying recent advances in the modelling of outputs sought to be minimised in production processes as described in the productivity and efficiency analysis literature. The farm capital market is mainly dominated by debt as most farms are sole proprietorships that cannot raise funds through the stock market. Therefore, the two main sources of capital for investment are debt or retained earnings. Adding debt to a farm business creates a repayment obligation in the form of interest expenses and principal payments in each year regardless of whether production takes place or not. This may affect a farm's cash flow and increase its riskiness causing financial distress. Following Ray et al. (2018), this study applies a single optimization model to investigate directional inefficiency between farm production, as an output, and production inputs, with repayment obligations as a by-product of the use of these inputs.

To the best of the author's knowledge, this study is the first in the agricultural economics literature to define a production function model that includes the repayment obligation as an by-product of input use. By extension, the study is also the first to examine how the reduction of repayment obligations impacts production output, showing that the inclusion of repayment obligations, as a by-product of input use in production models, leads to higher estimates of partial inefficiency estimates. It is proposed that this the study's approach will provide a useful approach for stakeholders to better understand the efficiency of working capital debt use by farm businesses.

The paper proceeds with a discussion of repayment obligations as a by-product of input use. The empirical method is then presented, followed by a description of the study data and presentation of the results. The paper concludes with a discussion of the paper's findings and potential considerations for further research.

Repayment obligations as a by-product of input use

To understand how a repayment obligation may be classified as a by-product of input use, consider the process of how a farm business acquires its inputs necessary to produce its outputs. Farm businesses must buy their inputs without precise knowledge of what their actual output will be due to production season variability caused by climate, pestilence, and diseases (Quaye et al. 2017). Because of this, farm businesses must buy their inputs to grow a targeted output projected from their knowledge of anticipated production conditions based on past experiences (Briggeman et al. 2009b).

The investment strategies of farm businesses have been shown by past empirical studies to be strongly affected by access to credit (Bierlen and Featherstone 1998; Chaddad et al. 2005; O'Toole et al. 2013). Borrowed funds are often added to existing cash on hand to purchase farm inputs to produce more outputs. The repayment obligation is incurred when money is borrowed and is hence a by-product of input use, as opposed to an undesirable output of production. The repayment obligation is incurred regardless of how much or little production actually occurs (Gerber 2013, Ray et al. 2018). The risk exists that the revenue received from the production output will not be enough to repay the money borrowed (Cochrane and Thornton 2016; Bampasidou et al. 2017), hence making the repayment obligation a by-product of input use that farm businesses would seek to minimize.

Technology and Assumptions

In this study, we consider the case where a producer *j* transforms *n* inputs, that require the use of short-term debt (working capital) and *m* inputs that do not require working capital, into *p* desirable outputs. The combination of a vector of non-working capital dependent inputs $x_1 \in R_+^m$, a vector of working capital dependent inputs $x_2 \in R_+^n$ and a desirable output vector inputs $g \in R_+^p$ is feasible if g can be produced from x_1 and x_2. In the use of the *m* inputs that comprise the vector input x_2 (otherwise referred to as 'selected inputs' in the remainder of this study), a by-product is generated in the form of a repayment obligation (b), which is viewed as a by-product of purchasing specific inputs on borrowed money, as it creates a financial risk to the farm enterprise. The only means by which b is reduced is to use less x_2, which assumes joint disposability between x_2 and b. By contrast, both the desired production output g and the vector of non-working capital dependent inputs, x_1, and the vector of working capital dependent inputs, x_2, are freely disposable. The assumption of free disposability means that, given inputs x_1 and x_2, it is possible to produce any amount of g less than that observed (i.e. it is possible to reduce g by any amount free of charge). Following Murty et al. (2012) and Ray et al. (2018), the free disposability of g, x_1 and x_2, coupled with the joint disposability of b as a by-product of x_2, gives rise to an overall production possibility set as specified in equation 1.1, which corresponds to two technological subsets, as specified in equations 1.2 and 1.3:

(1.1) $$T^O = \left\{(x_1, x_2; g, b) : (x_1, x_2; g) \in T^g \wedge \left(x_2; b\right) \in T^b\right\}$$

(1.2) $$T^g = \left\{(x_1, x_2; g) : F^g(x_1, x_2; g) \le 0; \frac{\partial F^g}{\partial x_i} < 0 (i = 1, 2); \frac{\partial F^g}{\partial g} > 0\right\}$$

(1.3) $$T^b = \left\{(x_2; b) : F^b(x_2; b) \ge 0; \frac{\partial F^b(kx_2; kb)}{\partial k} > 0\right\}$$

Equation 1.2 defines the production correspondence, F^g, such that the production of g using inputs x_1 and x_2 is relative to a maximum correspondence (level), whereby increases in inputs will reduce the relative correspondence and increases in output increase the relative correspondence. The technology subset identified in equation 1.3 states that the undesired by-product can only increase or decrease in direct proportion with an increase or decrease in the selected inputs. Equation 1.1 represents the combined production possibility set.

From the technology subsets identified in equations 1.1-3, and following Ray et al. (2018), the corresponding non-parametric production possibility set for N total producers is:

(2.1) $$S^O = \left\{(x_1, x_2; g, b) : (x_1, x_2, g) \in S^g_{BP} \wedge (x_2, b) \in S^b_{BP}\right\}$$

(2.2) $$S^g_{BP} = \left\{\begin{array}{l}(x_1, x_2; g) : g \le \sum_{j=1}^{N} \lambda_j g_j; x_1 \ge \sum_{j=1}^{N} \lambda_j x_1^j; x_2 \ge \sum_{j=1}^{N} \lambda_j x_2^j; \\ \sum_{j=1}^{N} \lambda_j = 1;\ \lambda_j \ge 0; (j = 1, 2, ..., N)\end{array}\right\}$$

(2.3) $$S^b_{BP} = \left\{\begin{array}{l}(x_2; b) : x_2 \le \alpha \sum_{j=1}^{N} \lambda_j x_2^j; b = \alpha \sum_{j=1}^{N} \lambda_j b_j; 0 \le \alpha \le 1; \sum_{j=1}^{N} \lambda_j = 1; \\ \lambda_j \ge 0; (j = 1, 2, ..., N)\end{array}\right\}$$

The technology assumptions applying to the overall production possibility set, S^O, are detailed in Appendix 1. The production possibility set in equation 2 is subject to variable returns to scale when $\sum_{j=1}^{N}\lambda_j = 1$ and $0 \leq \alpha \leq 1$, where α, the disposability parameter, is the proportional reduction in the jointly disposable undesirable output and working capital dependent inputs, while λ_j are the intensity variables used to construct the best-practice frontier (Färe and Grosskopf 2009). In the case of constant returns to scale, α is set equal to unity and $\lambda_j \geq 0$ (Färe and Grosskopf 2003). In the variable returns to scale models estimated in this study, α is imposed as a single global scaler for computational simplicity via a linear programming model to provide various estimates of the underlying production technology. It is noted that though techniques have been proposed to estimate the optimal technology under VRS, due to the presence of the scaling parameters in their constraints, the optimization problems associated with estimators of technology such as those proposed by Färe and Grosskopf (2003), Kuosmamen (2005), and Podinovski and Kuosmanen (2011) are nonlinear and hence less preferable to evaluate inefficiency, per the objective of this study (Pham and Zelenyuk, 2019). In this study, we applied an iterative approach to estimate α, determining that α for the the underlying technology is closely approximated at 0.5.

Measuring partial inefficiency in the presence of a by-product

The directional distance function, as proposed by Ray et al. (2018) in advancing the earlier work of Chambers et al. (1996), Murty et al. (2012) and Lozano (2015), can be used to estimate the partial inefficiency, β, in the context of by-products as follows:

$$(3) \qquad \vec{D}(g_0, b_0 | x_1, x_2) = \max \beta : \big((1+\beta)g_0, (1-\beta)b_0\big) \in P(x_1^0, x_2^0)$$

where P is the output set for a given input bundle and g_0 and b_0 are the output and by-product levels for a specific base observation *0*. The directional inefficiency measure in equation 3 considers the largest proportion by which the desirable output may be increased, and the selected inputs and their undesirable output, as a jointly disposable by-product, may be reduced. Several empirical studies have applied the directional distance function to investigate undesirable outputs from production processes, including Njuki & Bravo-Utera (2015), Huang & Bruemmer (2017), Pham & Zelenyk (2018), Pham & Zelenyk (2019).

A graphical illustration of directional inefficiency is provided in figure 1.

In figure 1, producers Q_1, Q_2, and Q_3 are benchmarked to be along the production frontier and are therefore technically efficient, whilst Q_4 is below the frontier and therefore inefficient.

The objective of directional inefficiency estimation, as per equation 3, is to measure the distance of the increase in output and reduction of the by-product required to project an inefficient producer to the frontier. To reach the frontier, Q_4 is projected the shortest possible radial distance, τ, to the frontier, which is point G. To measure the proportional reduction of the selected inputs, x_2, and their by-product, b, achieved through a movement to G, a contraction of equivalent distance τ is made along line OI_4 to point K.

The directional inefficiency may be hence measured as:

$$(4) \qquad \vec{D}\left(g_4, b_4 \mid x_1, x_2\right) = \frac{GQ_4}{OI_4} = \frac{KI_4}{OI_4}$$

The data envelopment analysis (DEA) problem to be optimized assumes a single ('unified') decision making process because the decision making agent within the farm business (i.e., the farm manager/owner) presides over the production and financial decisions.

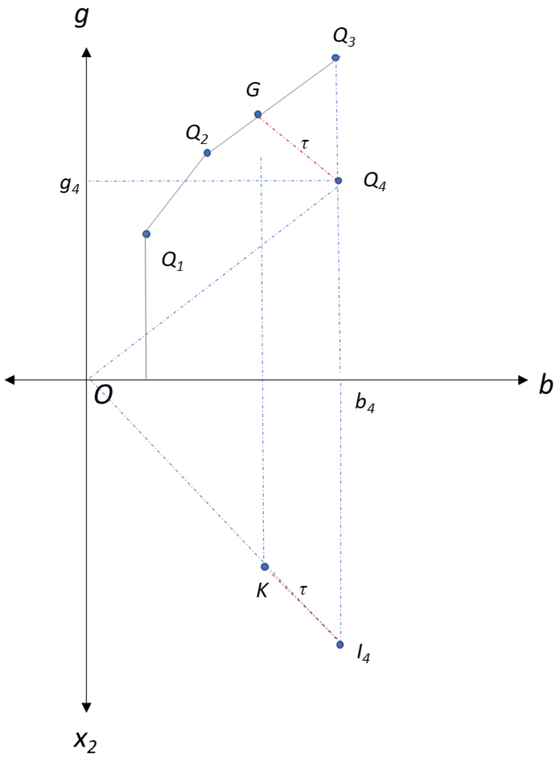


Figure 1. Directional inefficiency in the presence of a by-product to be minimised

Consider the following two-directional distance function models presented in equation 5. The first, equation 5.1, is the directional distance function, which excludes the undesirable output, while the second equation, equation 5.2, includes the undesirable output. The optimization model presented in equation 5.1 assumes free disposability of inputs and outputs, whereas equation 5.2 assumes joint disposability, with input vector, x_2 including a by-product.

$$\begin{aligned}
&\max_{\lambda,\beta} \beta \text{ subject to:}\\
&\sum_{j=1}^{N} \lambda_j g_j \geq (1+\beta) g_0;\\
(5.1) \qquad &\sum_{j=1}^{N} \lambda_j x_1^j \leq x_1^0;\\
&\sum_{j=1}^{N} \lambda_j x_2^j \leq (1-\beta) x_2^0;\\
&\sum_{j=1}^{N} \lambda_j = 1;\ \lambda_j \geq 0; (j = 1, 2, ..., N)
\end{aligned}$$

$$\begin{aligned}
&\max_{\lambda,\beta} \beta \text{ subject to:}\\
&\sum_{j=1}^{N} \lambda_j g_j \geq (1+\beta) g_0;\\
&\sum_{j=1}^{N} \lambda_j x_1^j \leq x_1^0;\\
(5.2) \qquad &\alpha \sum_{j=1}^{N} \lambda_j x_2^j = (1-\beta) x_2^0;\\
&\alpha \sum_{j=1}^{N} \lambda_j b_j = (1-\beta) b_0;\\
&0 \leq \alpha \leq 1; \sum_{j=1}^{N} \lambda_j = 1;\ \lambda_j \geq 0; (j = 1, 2, ..., N)
\end{aligned}$$

In equations 5.1 and 5.2, β is an estimate of the maximum amount that the good output may be expanded and the specific inputs and the by-product they create may be contracted. The directional distance functions are constructed such that the maximization of β yields a unit expansion of the good output and a unit contraction of the by-product and the input that creates it. The estimates obtained from the optimization model are sensitive to the choice of the directional vector (Fukyama and Weber, 2017). The models presented in equations 5.1 and 5.2 are estimated subject to an assumption of variable returns to scale ($VRS = \sum_{j=1}^{N} \lambda_j = 1$).

Following Ray et al. (2018), equation 5.2 is transformed by setting $\gamma = \alpha\lambda_j$ to enable linearization of the constraints via a first-order Taylor approximation. The model can now be represented as:

$$\max_{\lambda,\beta} \beta \text{ subject to:}$$

$$\begin{aligned}
&\sum_{j=1}^{N} \gamma_j g_j \geq (\alpha+\beta) g_0; \\
&\sum_{j=1}^{N} \gamma_j x_1^j \leq \alpha x_1^0; \\
&\sum_{j=1}^{N} \gamma_j x_2^j \leq (1-\beta) x_2^0; \\
&\sum_{j=1}^{N} \gamma_j b_j = (1-\beta) b_0; \\
&0 \leq \alpha \leq 1; \sum_{j=1}^{N} \gamma_j = \alpha;\ \gamma_j \geq 0; (j=1,2,\ldots,N)
\end{aligned} \tag{6}$$

Application to Western Australia's Wheatbelt

This study draws on a balanced data panel of 54 farms, located in Western Australia's Wheatbelt region between 2002 and 2011. The region is in the south-west corner of Western Australia and covers a total area of around 197,300 square kilometres, of which over 60% is used for agriculture. The farm sizes ranged from 1,150 to 12,730 hectares, with an average size of 3,990 hectares and average annual revenue of A$990,600. The survey group may be considered representative of farms in Western Australia's Wheatbelt: the Planfarm Bankwest Benchmarks (2011) for 2010/2011, the largest comprehensive survey of financial and production performance measures of over 500 farms in Western Australia, shows average farm size in 2010/11 at 4,185 hectares and average annual revenue at A$1.03 million. Revenue and expenditure items are presented in Australian Dollars (A$). Where normalization of variables occurs, indexation uses 2002 as the base year. Production output (g) is the desirable output and is calculated as the sum of total receipts from crop and livestock production that have been normalized by the Australian Bureau of Agricultural and Resource Economics Sciences ('ABARES') index of producer receipt prices with 2002 as the base year (ABARES 2012). The inputs not purchased with short-term working capital are growing season rainfall (x_{11}) and land (x_{12}). Growing season rainfall is the rainfall between April and November, measured in millimetres, the growing season for crop producers in the study region. The land is the total land area in hectares used for crop and livestock production. The selected inputs purchased, using short-term loans that generate a repayment obligation as a by-product of a single production cycle, are labour (x_{21}), crop inputs (x_{22}), livestock production inputs (x_{23}), and operational inputs (x_{24}). Labour is calculated as the number of man-weeks of both permanent and casual labour used in a calendar year. The crop input measure is constructed as the sum of expenditure on fertilizer, normalised by the ABARES fertilizer cost index, chemicals normalized by the ABARES chemicals cost index, and seeds normalised by the ABARES seed cost index. The livestock production input measure is the sum of expenditure on livestock purchased, normalised by the ABARES livestock purchase price index, and livestock production costs, normalised by the ABARES livestock production costs index.

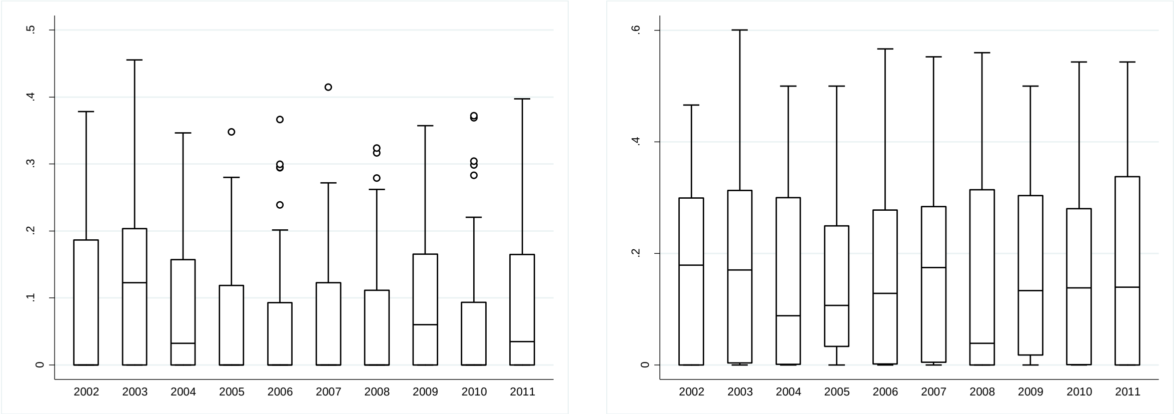
Table 1. Variable means by year

	Obs.	**Repayment Obligation (AUD)**	**Revenue (AUD)**	**Growing Season Rainfall (mm)**	**Land (Ha)**	**Labour (Man Weeks)**	**Crop Inputs (AUD)**	**Livestock Inputs (AUD)**	**Operational costs (AUD)**
Average	540	225,120.36	990,601.15	212.12	3,871.41	120.65	305,523.10	15,960.12	123,568
2002	54	87,494.19	722,733.75	152.59	3,455.96	119.01	293,563.77	19,707.71	109,059
2003	54	118,554.56	954,131.30	268.77	3,536.81	123.05	295,002.89	19,625.01	116,885
2004	54	77,259.54	975,447.93	235.12	3,635.52	121.05	359,301.00	11,477.10	139,296
2005	54	123,252.96	1,013,178.63	272.08	3,734.37	120.67	342,414.38	17,210.32	126,998
2006	54	153,975.50	635,561.33	129.36	3,941.74	120.90	275,543.53	19,535.46	109,485
2007	54	263,101.54	900,809.30	166.29	3,859.50	117.73	290,484.83	15,672.12	99,352
2008	54	272,604.12	1,160,509.71	217.33	4,061.72	126.44	240,527.25	7,300.05	122,735
2009	54	268,513.56	1,105,412.76	244.84	4,052.09	121.74	279,080.37	13,105.01	129,413
2010	54	410,594.00	1,165,561.71	153.68	4,306.15	119.22	316,952.25	13,376.79	128,055
2011	54	475,853.61	1,827,643.88	291.68	4,362.13	123.65	550,201.39	22,591.60	159,801

Operational cost is calculated as the sum of the expenditure on contract services normalised by the ABARES contractor cost index, repair, and maintenance normalised by the ABARES maintenance cost index, and fuel normalised by the ABARES fuel price index relative to the study base year. Short-term debt (b), the aggregation of loans with a term of less than one calendar year, as is the case of working capital for non-land and plant production inputs is a proxy for working capital debt used for the purchase of the selected production inputs (i.e. inputs x_{21}, x_{22}, x_{23} and x_{24}). The repayment obligation is normalised by the ABARES interest cost index relative to the study base year. In the absence of specific information on the allocation of debt in production, it is assumed that the short-term debt use is allocated proportionately across all x_2 inputs.

Results

In this section, we report the estimates of the partial inefficiency models from directional distance functions. The beta estimates are reported with and without the repayment obligation as a by-product subject to the assumption of a variable returns to scale ('VRS') technology. In the presence of the by-product, α (the disposability parameter) is set equal to 0.5, which was selected by an iterative process.



A. No by-product

B. By-product, α=0.5

Figure 2. Box plots of directional inefficiency

Table 2. Summary statistics for models with and without by-products

	No by-product					By-product, α=0.5				
	Mean	Median	Max	Std dev	Min	Mean	Median	Max	Std dev	Min
2002	0.0864	0.0000	0.3783	0.1074	0.0000	0.1625	0.1787	0.4658	0.1548	0.0000
2003	0.1263	0.1229	0.4550	0.1286	0.0000	0.1781	0.1702	0.6007	0.1581	0.0000
2004	0.0818	0.0321	0.3465	0.0987	0.0000	0.1537	0.0881	0.5000	0.1618	0.0000
2005	0.0577	0.0000	0.3481	0.0840	0.0000	0.1442	0.1068	0.5000	0.1342	0.0000
2006	0.0562	0.0000	0.3664	0.0940	0.0000	0.1707	0.1286	0.5669	0.1727	0.0000
2007	0.0568	0.0000	0.4145	0.0915	0.0000	0.1774	0.1744	0.5527	0.1627	0.0000
2008	0.0594	0.0000	0.3238	0.0929	0.0000	0.1578	0.0386	0.5600	0.1851	0.0000
2009	0.0991	0.0598	0.3573	0.1094	0.0000	0.1875	0.1332	0.5000	0.1763	0.0000
2010	0.0586	0.0000	0.3718	0.1038	0.0000	0.1805	0.1379	0.5430	0.1796	0.0000
2011	0.0946	0.0351	0.3971	0.1226	0.0000	0.1682	0.1396	0.5432	0.1737	0.0000

Table 2 reports the summary statistics while figure 2 shows the box plots of the models estimated. Where the by-product is included, the mean beta estimate for the survey period decreases to 0.0557. The inclusion of the by-product in the models estimated results in increased measures of partial inefficiency. Where there is no-by-product, it is observed that the median is zero for six of the ten years in the study period; the average median observed is 0.0250, while the mean inefficiency estimate is 0.0777. A beta value of the mean estimate implies that a producer could increase farm production by 7.77% and contract their use of the selected inputs and the by-product generated by 7.77% at the same time. Where the by-product is included, the median and mean beta are generally found to be 0.1296 and 0.1681 respectively. A beta value of the mean estimate implies that a producer could increase farm production by 16.81% and contract their use of the selected inputs and the by-product generated by 16.81% at the same time. The full estimates for each model are listed in Appendix 2.

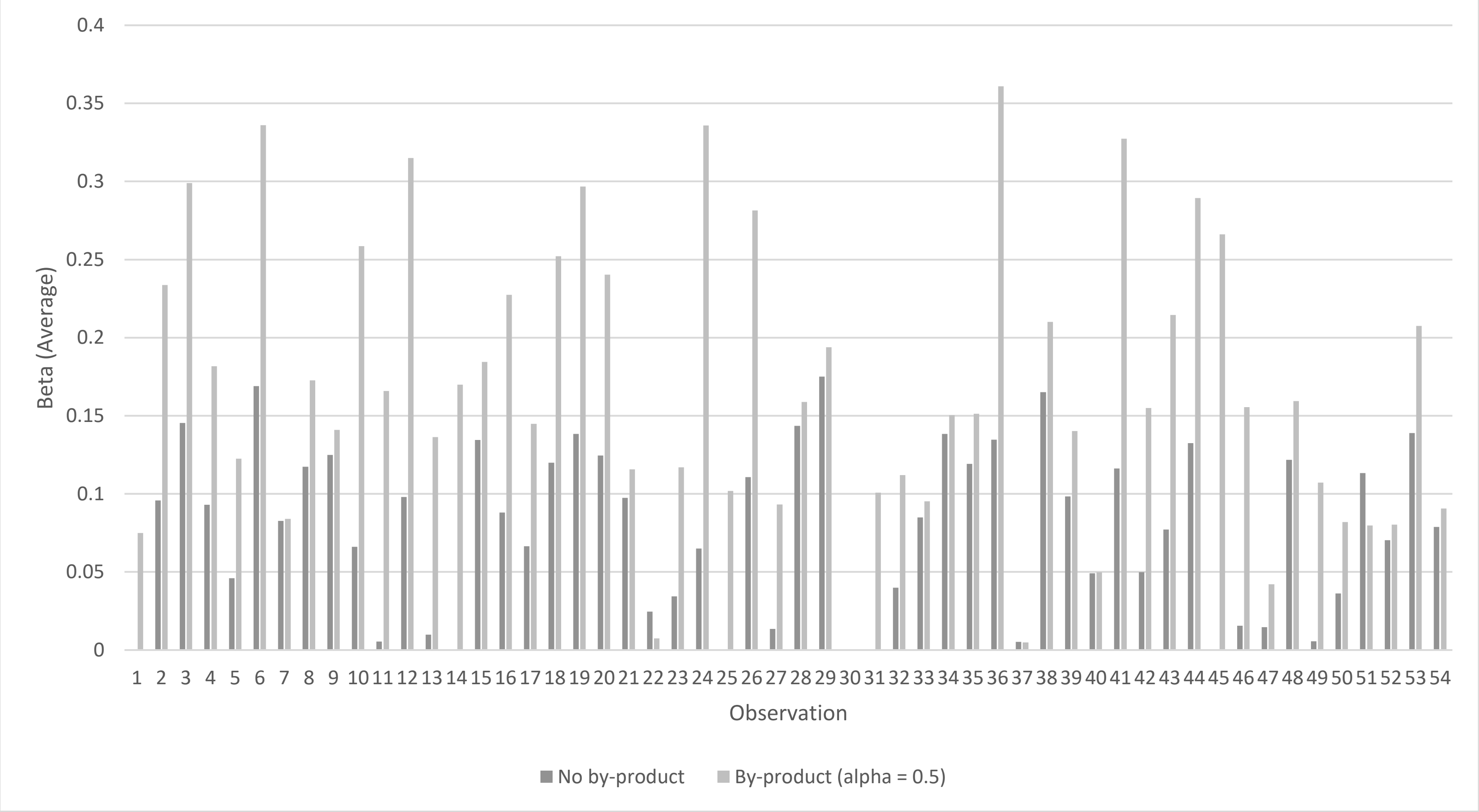


Figure 3. Mean estimates of directional inefficiency by farm

At the farm level, we observe that for the survey period the mean beta estimate is greater in the presence of the by-product for 51 out of 54 farm businesses. For 24 observations, the difference in the average beta estimate increased by 0.1 or more where the by-product was introduced.

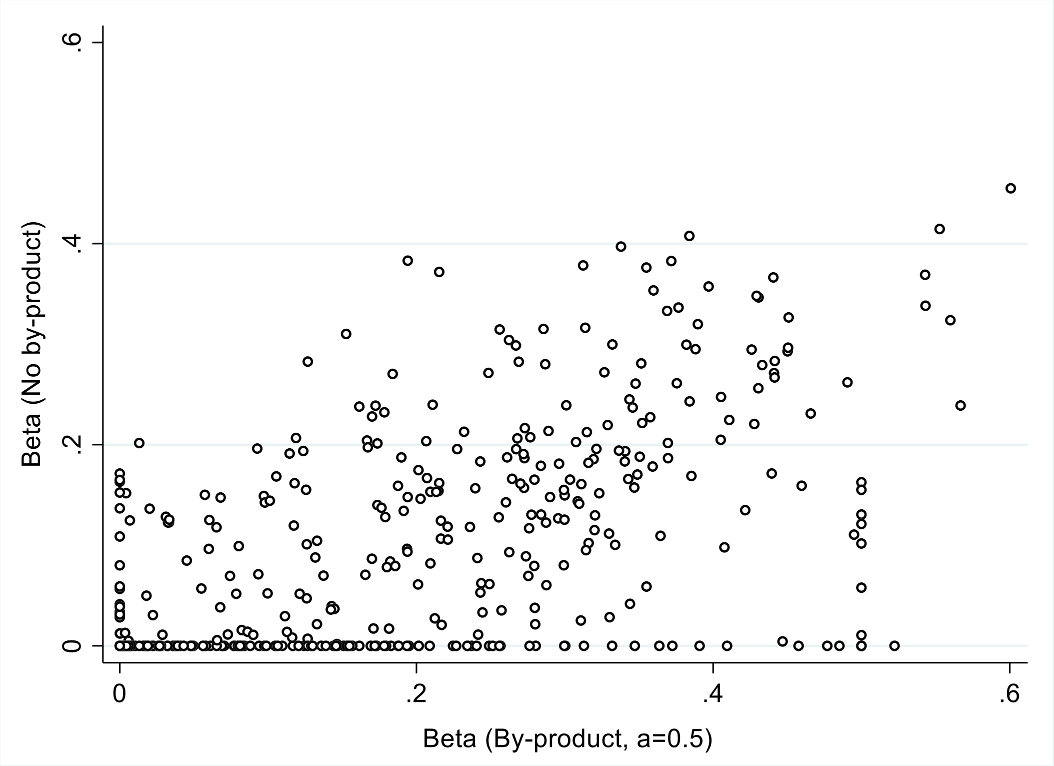


Figure 4. Plot of directional inefficiency estimates with and without a by-product

Figure 4 suggests a moderate correlation between the estimates of the models that include and exclude the by-product, confirmed by the pearson correlation coefficient of 0.5417 that exists between the estimates. In examination of the relationship between farm output size as measured by farm revenue and the beta estimates with and without the inclusion of the by-product, only a very weak negative relationship is observed with correlation coefficients of -0.1767 and -0.1322 observed respectively. A low positive correlation between the size of repayment obligations and beta estimates are observed where the by-product is included (0.2871).

Concluding Remarks

This article used a directional distance function approach to investigate the effect of debt repayment obligation, as a by-product of using debt to purchase farm inputs, on the performance of farm businesses with an application to broadacre mixed enterprise farm businesses in Western Australia's Wheatbelt. To the best of the author's knowledge, no prior study in the agricultural economics literature has sought to define production function models whereby debt repayment obligations are a by-product of production input use. We build upon the existing literature by proposing a new application of recent advancements in the modelling of production by-products to examine the partial inefficiency adjusted for debt used in production. The application is proposed to provide new insights for policymakers and other stakeholders on how debt repayment obligations, as a by-product of agricultural production input use, impact partial inefficiency at the farm level.

The study shows that the inclusion of repayment obligations as a by-product of input use in production models has important implications for the analysis of production inefficiency at the farm level. The study finds that under the assumption of variable returns to scale, the inclusion of a by-product leads to higher estimates of partial inefficiency relative to the model that does not include the by-product. Only a moderate positive correlation is observed between the partial inefficiency estimates of the models that include repayment obligations as a by-product of input use and the model that did not. A low positive correlation is observed between the size of repayment obligations and the partial inefficiency estimates obtained by the model that included the by-product.

The main finding of the study is that analysts could benefit from defining production models to include short-term debt repayment obligations incurred in the purchase of production inputs since it provides a more complete picture of farm enterprise viability and production efficiency. An implication of this study for policymakers and farm business managers is that improved efficiency in the use of production inputs can have the effect of reducing the potential adverse impacts of debt repayment obligations on farm businesses generated as a by-product of using debt to acquire the production inputs.

The study was limited by geography and sample size. Further studies that include debt repayment as a by-product of production input use would benefit from greater information on the specific use of debt and the terms of debt repayment to determine the impact on partial efficiency of purchasing certain inputs that depend on debt finance. Future studies may also adopt a dynamic approach to examine how adjustment costs of farm business capital structures influence partial efficiency estimates and output elasticity in the context of ongoing repayment obligations over time.

References

ABARES. *Australian Commodity Statistics*, (Canberra, Australia: Australian Bureau of Agricultural and Resource Economics Statistics, 2012).

Abdul-Salam, Y., and E. Phimister. 2017. Efficiency Effects of Access to Information on small-scale agriculture: Empirical evidence from Uganda using stochastic frontier and IRT models. *Journal of Agricultural Economics* 68 (2): 494-517.

Bampasidou, M., A. K. Mishra, and C. B. Moss. 2017. Modeling debt choice in agriculture: the effect of endogenous asset values. *Agricultural Finance Review* 77 (1): 95-110.

Bankwest. *Planfarm Bankwest Benchmarks 2010–11*, (West Perth, Australia: Bankwest, 2011).

Barros, C. P., S. Managi, and R. Matousek. 2012. The technical efficiency of the Japanese banks: Non-radial directional performance measurement with undesirable output. *Omega,* 40:1-8.

Bian, Y. W., and F. Yang. 2010. Resource and environment efficiency analysis of provinces in China: A DEA approach based on Shannon's entropy. *Energy Policy* 38: 1909-1917.

Bierlen, R., and A. Featherstone. 1998. Fundamental Q, cash flow, and investment: Evidence from farm panel data. *The Review of Economics and Statistics* 80: 427–435.

Brewer, B., and A. M. Featherstone. 2017. Agency cost of debt: evidence from Kansas farm operations. *Agricultural Finance Review* 77 (1): 111-124.

Briggeman, B. C., M. A. Gunderson, and B. A. Gloy. 2009a. The Financial Health of Agricultural Lenders. *American Journal of Agricultural Economics* 91 (5): 1406-1413.

Briggeman, B. C., C. A. Towe, and M. J. Morehart. 2009b. Credit constraints: Their existence, determinants, and implications for U.S. farm and nonfarm sole proprietorships. *American Journal of Agricultural Economics* 91 (1): 275-289.

Chambers, R., Y. Chung, and R. Färe. 1996. Benefit and distance functions. *Journal of Economic Theory* 70: 407-419.

Chen L., Y. Wang. and F. Lai. 2017. Semi-disposability of undesirable outputs in data envelopment analysis for environmental assessments. *European Journal of Operational Research* 260: 665-664.

Cochrane, L., and A. Thornton. 2017. A socio-cultural analysis of smallholder borrowing and debt in southern Ethiopia. *Journal of Rural Studies* 49: 69-77.

Coricelli, F., N. Driffield, S. Pal, and I. Roland. 2012. Why does leverage hurt productivity growth? A firm level analysis. Journal of International Money and Finance 31: 1674-1694.

Cowley, C., and M. Clark. 2016. Farm Debt Accumulating. *Federal Reserve Bank of Kansas City- Ag Credit Survey*.

Crnek-Georgeson, K.T., L. A. Wilson, and A. Page. 2017. Factors influencing suicide in older rural males: a review of Australian studies. *Rural and Remote Health* 17, 4020.

Dakpo, K. H., and A. O. Lansink. 2019. Dynamic pollution-adjusted inefficiency under the by-production of bad outputs. *European Journal of Operational Research* https://doi.org/10.1016/j.ejor.2018.12.040

Färe, R., and S. Grosskopf. 2009. A comment on weak disposability in nonparametric production analysis. *American Journal of Agricultural Economics,* 91 (2): 535–538.

Färe, R., and S. Grosskopf. 2003. Nonparametricproductivity analysis with undesirable outputs: Comment. *American Journal of Agricultural Economics* 85 (4): 1070–1074.

Farrell, M. J. 1957. The measurement of technical efficiency. *Journal of the Royal Statistical Research Society Series A General* 120 (3): 253-281.

Fernández, C., G. Koop, and M. F. J. Steel. 2002. Multiple-Output Production with Undesirable Outputs. *Journal of the American Statistical Association* 97: 432-442.

Fujii, H., S. Managi, R. Matousek, and A. Rughoo. 2018. Bank efficiency, productivity, and convergence in EU countries: a weighted Russell directional distance model. *The European Journal of Finance* 24 (2): 135-156.

Fukuyama, H., and W. L. Weber. 2017. *Advances in DEA Theory and Applications: With Extensions to Forecasting Models*. 1st Ed. Hoboken, New Jersey: John Wiley & Sons

Fukuyama, H., and W. L. Weber. 2010. A slacks-based inefficiency measure for a two-stage system with bad outputs. *Omega* 38: 398-409.

Gerber, J. 2013. The Hidden Consequences of Credit: An Illustration from Rural Indonesia. *Development and Change* 44 (4): 839-860.

Giannakas, K., R. Schoney, and V. Tzouvelekas. 2001. Technical efficiency, technological change and output growth of wheat farms in Saskatchewan. *Canadian Journal of Agricultural Economics* 49 (1): 135-152.

Gollop, F. M., and G. P. Swinand. 1998. From Total Factor to Total Resource Productivity: An Application to Agriculture. *American Journal of Agricultural Economics* 80 (3): 577-583.

Guesmi, B., and T. Serra. 2015. Can we improve arm performance? The determinants of farm technical and environmental efficiency. *Applied Economic Perspectives and Policy* 37 (4), 692-717.

Hailu A., and T. S. Veeman. 2001. Non-parametric Productivity Analysis with Undesirable Outputs: An Application to the Canadian Pulp and Paper Industry. *American Journal of Agricultural Economics* 83 (3), 605-616.

He, F., Q. Zhang, J. Lei, W. Fu, and X. Xiaoning. 2013. Energy efficiency and productivity change of China’s iron and steel industry: Accounting for undesirable outputs. *Energy Policy* 54: 204-213.

Heil, M. 2018. Finance and Productivity: A literature review. *Journal of Economic Suverys* 32 (5): 1355-1383.

Henderson, H. 2015. Considering technical and allocative efficiency in the inverse size-productivity relationship. *Journal of Agricultural Economics* 66 (2): 442-469.

Hiilamo, A., and E. Grundy. 2018. Household debt and depressive symptoms among older adults in three continental European countries. *Ageing and Society*, https://doi.org/10.1017/S0144686X18001113

Hojman, D. A., A. Miranda, and J. Ruiz-Tagle. 2016. Debt trajectories and mental health. *Social Science and Medicine* 167: 54-62.

Huang, W., and B. Bruemmer. 2017. Balancing economic revenue and grazing pressure of livestock grazing on the Qinghai–Tibetan–Plateau. *Australian Journal of Agricultural and Resource Economics* 61 (4): 645-662.

Jin, M., S. Zhao, and S. C. Kumbhakar. 2019. Financial constraints and firm productivity: Evidence from Chinese manufacturing. *European Journal of Operational Research* 275: 1139-1156.

Kuosmanen, T., 2005. Weak Disposability in Nonparametric Production Analysis with Undesirable Outputs. *American Journal of American Economics* 87 (4): 1077-1082.

Lambert, D.K., and V. V. Bayda. 2005. The impacts of farm financial structure on production efficiency. *Journal of Agricultural and Applied Economics* 37 (1): 277-289.

Lin, B., and R. Fei. 2015. Regional differences of CO_2 emissions performance in China’s agricultural sector: A Malmquist index approach. *European Journal of Agronomy* 70: 33-40.

Logstein, B. 2016. Farm-Related Concerns and Mental Health Status Among Norwegian Farmers. *Journal of Agromedicine* 21 (4): 316-326.

Lozano, S. 2016. Slacks-based inefficiency approach for general networks with bad outputs: An application to the banking sector. *Omega* 60: 73-84.

Lozano, S. 2015. A joint-inputs Network DEA approach to production and pollution-generating technologies. *Expert Systems Applications* 42: 7960-7968.

Njuki, E., B. E. Bravo-Utera, and D. Mukherjee. 2016. The Good and the Bad: Environmental Efficiency in Northeastern U.S. Dairy Farming. *Agricultural and Resource Economics Review* 45 (1): 22-43.

Njuki, E., and B. E. Bravo-Utera. 2015. The economic costs of environmental regulation in U.S. dairy farming: A directional distance function approach. *American Journal of Agricultural Economics* 97 (4): 1087-1106.

Malikov, E., R. Bokusheva, and S. C. Kumbhakar. 2018. A hedonic-output-index-based approach to modeling polluting technologies. *Empirical Economics* 54 (1): 287-308.

Mamardashvili, P., G. Emvalomatis, and P. Jan. 2016. Environmental Performance and Shadow Value of Polluting on Swiss Dairy Farms. *Journal of Agricultural and Resource Economics* 42 (2): 225-246.

Merriott, D. 2016. Factors associated with the farmer suicide crisis in India. *Journal of Epidemiology and Global Health* 6 (4): 217-227

Minviel, J. J., and T. Sipiläinen. 2018. Dynamic stochastic analysis of the farm subsidy-efficiency link: evidence from France. *Journal of Productivity Analysis* 50: 41-54.

Mugera, A. W., and G. G. Nyambane. 2014. Impact of debt structure on production efficiency and financial performance on Broadacre farms in Western Australia. *Australian Journal of Agricultural and Resource Economics* 56 (1): 1-17.

Mugera, A. W., and M. R. Langemeier. 2011. Does farm size and specialization matter for productive efficiency? Results from Kansas. *Journal of Agricultural and Applied Economics* 43 (4): 515-528.

Murty, S., R. Russell, and S. B. Levkoff. 2012. On modelling pollution-generating technologies. *Journal of Environmental and Economic Management* 64 (1): 117-135.

O'Toole, C. M., C. Newman, and T. Hennessy. 2013. Financing Constraints and Agricultural Investment: Effects of the Irish Financial Crisis. *Journal of Agricultural Economics* 65 (1): 152-176.

Perceval, M., K. Kõlves, V. Ross, P. Reddy, and D. De Leo. 2018. Environmental factors and suicide in Australian farmers: A qualitative study. *Archives of Environmental and Occupational Health*, https://doi.org/10.1080/19338244.2018.1453774.

Pham, M. D., and V. Zelenyuk. 2019. Weak disposability in nonparametric production analysis: A new taxonomy of reference technology sets. *European Journal of Operational Research* 274: 186-198.

Pham, M. D., and V. Zelenyuk. 2018. Slack-based directional distance function in the presence of bad outputs: theory and application to Vietnamese banking. *Empirical Economics* 54: 153-187.

Pieralli, S., S. Hüttell, and M. Odening. 2017. Abandonment of milk production under uncertainty and inefficiency: the case of western German Farms. *European Review of Agricultural Economics* 30 (1): 1-30.

Podinovski, V. V., and T. Kuosmanen. 2011. Modelling weak disposability in data envelopment analysis under relaxed convexity assumptions. *European Journal of Operational Research* 211 (3): 577–585.

Qayyum, A., and K. Riaz. 2018. Incorporating Credit Quality in Bank Efficiency Measurements: A Directional Distance Function Approach. *Journal of Risk and Financial Management* doi:10.3390/jrfm11040078.

Quaye, F. M., D. Nadolnyak, and V. Hartarska. 2017. Factors Affecting Farm Loan Delinquency in the Southeast. *Research in Applied Economics* 9 (4): 75-92.

Rajan, R., and R. Ramcharan. 2015. The Anatomy of a Credit Crisis: The Boom and Bust in Farm Land Prices in the United States in the 1920s. *American Economic Review* 105 (4): 1439-1477.

Ray S. C., K. Mukherjee, and A. Venkatesh. 2018. Nonparametric measures of efficiency in the presence of undesirable outputs: a by-production approach. *Empirical Economics* 54 (1): 31-65.

Ray S. C., and A. Ghose. 2014. Production efficiency in Indian agriculture: An assessment of the post green revolution years. *Omega* 44: 58-69.

Rezek, J. P., and R. K. Perrin. 2004. Environmentally Adjusted Agricultural Productivity in the Great Plains. *Journal of Agricultural and Resource Economics* 29 (2): 346-369.

Seiford, L. M., J. and Zhu. 2002. Modeling undesirable factors in efficiency evaluation. *Economic Letters* 28 (1): 16-28.

Serra, T., D. Zilberman, and J. Gil. 2008. Farms' technical inefficiencies in the presence of government programs. *Australian Journal of Agricultural and Resource Economics* 52 (1): 57-76.

Shi, G.M., J. Bi, and J. N. Wang. 2010. Chinese regional industrial energy efficiency evaluation based on a DEA model of fixing non-energy inputs. *Energy Policy* 38 (10): 6172–6179.

Skevas, T., S. E. Stefanou, and A. O. Lansink, 2014. Pesticide use, environmental spillovers and efficiency: A DEA risk-adjusted efficiency approach applied to Dutch arable farming. *European Journal of Operational Research* 237 (2): 658-664.

Tamani, L. D., B. Larue, and G. West. 2012. Technical and environmental efficiencies and best management practices in agriculture. *Applied Economics* 44 (13): 1659-1672.

Taylor, T., H. E. Drummond, and A. T. Gomes. 1986. Agricultural credit programs and production efficiency: An analysis of traditional farming in Southeastern Minas Gerais, Brazil. *American Journal of Agricultural Economics* 68 (1): 110-118.

Wang, K., S. Yu, and W. Zhang. 2013. China's regional energy and environmental efficiency: A DEA window analysis based dynamic evaluation. *Mathematical and Computer Modelling* 58 (5-6): 1117-1127.

Wang, Q., P. Zhou, and D. Zhou. 2011. Efficiency measurement with carbon dioxide emissions: the case of China. *Applied Energy* doi:10.1016/j.apenergy.2011.02.022

West, S.C. 2020. *Toward A Better Understanding of Farm Business Viability: Essays on Production Benchmark Performance and Capital Structure Choice.* Doctoral Dissertation. University of Western Australia.

White, J. 2018. The (almost) $70 billion investment in Australian agriculture. Retrieved from https://www2.deloitte.com/au/en/pages/consumer-industrial-products/articles/agribusiness-rural-debt-funding.html#

Yang, H., and M. Pollitt. 2010. The necessity of distinguishing weak and strong disposability among undesirable outputs in DEA: Environmental performance of Chinese coal-fired power plants. *Energy Policy* 38: 4440-4444.

Zhou, P., and B. W. Ang. 2008. Linear programming models for measuring economy-wide energy efficiency performance. *Energy Policy* 36: 2011-2016.

Appendix 1

The production possibility set identified in equation 2 is subject the following technological assumptions:

Assumption 1: All observed input-output bundles are feasible,

(A1-1) $(x_1^j, x_2^j; g^j, b^j) \in S^O (j = 1, 2, ..., N)$;

Assumption 2: The production output (g) and inputs (x_1 and x_2) are freely disposable,

(A1-2-1) $(x_1, x_2; g, b) \in S^O \wedge \tilde{g} \le g \Rightarrow (x_1, x_2; \tilde{g}, b) \in S$, and

(A1-2-2) $(x_1, x_2; g, b) \in S^O \wedge \tilde{x}_1 \ge x_1 \Rightarrow (\tilde{x}_1, x_2; g, b) \in S$, and

(A1-2-3) $(x_1, x_2; g, b) \in S^O \wedge \tilde{x}_2 \ge x_2 \Rightarrow (x_1, \tilde{x}_2; g, b) \in S$.

Assumption 3: The repayment obligation by-product (b) is a by-product of the specific working capital dependent inputs (x_2) in that the only way it is possible to reduce b is by a proportional reduction in x_2,

(A3-1) $(x_1^0, x_2^0; g^0, b^0) \in S^O \wedge \left(x_2^1, b^1\right) = \left(\alpha x_2^0, \alpha b^0\right); 0 \le \alpha \le 1 \Rightarrow (x_1^0, x_2^1; g^0, b^1) \in S$;

In the case where no by-product is assumed from production, Assumption 3 is not applicable and all inputs and outputs are assumed to be freely disposable.

Assumption 4: The overall production possibility set is convex,

(A4-1)
$$\begin{array}{l} (x_1^1, x_2^1; g^1, b^1) \in S^O \wedge \left(x_1^2, x_2^2; g^2, b^2\right) \in S^O \wedge 0 \le \lambda \le 1 \Rightarrow \\ \left(\lambda (x_1^1, x_2^1; g^1, b^1) + (1-\lambda)\left(x_1^2, x_2^2; g^2, b^2\right)\right) \in S^O \end{array} .$$

Appendix 2

Table A2-1. Beta estimates without undesirable output (variable returns to scale)

Obs.	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.1437	0.1571	0.2049	0.0000	0.0044	0.0000	0.0000	0.1443	0.1689	0.1342
3	0.2216	0.2997	0.1574	0.0795	0.0000	0.0000	0.0000	0.3573	0.0000	0.3382
4	0.0839	0.1632	0.0273	0.0353	0.0000	0.1427	0.0000	0.1550	0.2833	0.0389
5	0.2712	0.1881	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.2310	0.1517	0.2965	0.1185	0.2390	0.0000	0.1834	0.0473	0.1608	0.2611
7	0.1268	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0128	0.3041	0.3830
8	0.3783	0.0794	0.3465	0.0000	0.2995	0.0000	0.0000	0.0698	0.0000	0.0000
9	0.2129	0.1179	0.0000	0.0071	0.0000	0.4145	0.0000	0.1245	0.3718	0.0000
10	0.0000	0.1480	0.1685	0.0000	0.1094	0.0530	0.1714	0.0000	0.0000	0.0111
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0285	0.0000	0.0000	0.0252	0.0000
12	0.1867	0.1278	0.2388	0.1496	0.0964	0.0000	0.0000	0.1212	0.0000	0.0589
13	0.0000	0.0979	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.2322	0.3364	0.1748	0.0614	0.0518	0.0295	0.0952	0.1518	0.1591	0.0522
16	0.0000	0.0000	0.3199	0.0000	0.3664	0.0000	0.1247	0.0000	0.0000	0.0695
17	0.0000	0.2195	0.1668	0.1659	0.0000	0.0000	0.1116	0.0000	0.0000	0.0000
18	0.0000	0.2809	0.0873	0.0172	0.0890	0.1856	0.1151	0.1866	0.0000	0.2370
19	0.1004	0.2669	0.2017	0.2074	0.0000	0.1305	0.2620	0.1225	0.0215	0.0707
20	0.1703	0.2017	0.2562	0.0361	0.0000	0.0018	0.0623	0.0000	0.3690	0.1480
21	0.1961	0.2042	0.0000	0.0000	0.1304	0.0000	0.0000	0.2608	0.0306	0.1531
22	0.0000	0.0000	0.0000	0.1227	0.0000	0.1226	0.0000	0.0000	0.0000	0.0000
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0124	0.3266	0.0000	0.0056
24	0.2825	0.0000	0.2273	0.0879	0.0418	0.0000	0.0000	0.0106	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0801	0.1400	0.1254	0.0083	0.0208	0.0000	0.3238	0.1661	0.0000	0.2431
27	0.0000	0.0000	0.0000	0.0000	0.1349	0.0000	0.0000	0.0000	0.0000	0.0000
28	0.0000	0.0049	0.0000	0.1652	0.2950	0.2027	0.1540	0.1283	0.1088	0.3762
29	0.2398	0.3330	0.0000	0.3481	0.1462	0.0783	0.0339	0.1913	0.1833	0.1974
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
31	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
32	0.0000	0.0000	0.0000	0.0000	0.0000	0.1935	0.0000	0.0000	0.0801	0.1255
33	0.0395	0.1552	0.1368	0.0383	0.0000	0.2165	0.0000	0.0499	0.0000	0.2137
34	0.2929	0.1938	0.1532	0.1490	0.0140	0.1617	0.1045	0.3151	0.0000	0.0000
35	0.1281	0.3102	0.0215	0.0517	0.0000	0.0000	0.2792	0.3146	0.0000	0.0866
36	0.0000	0.1956	0.1627	0.1613	0.1018	0.1306	0.3163	0.0000	0.2206	0.0578
37	0.0000	0.0000	0.0000	0.0413	0.0000	0.0000	0.0110	0.0000	0.0000	0.0000
38	0.0000	0.4076	0.0000	0.2801	0.2013	0.0000	0.2475	0.1182	0.0000	0.3971
39	0.0000	0.0848	0.0964	0.0000	0.0171	0.0000	0.0000	0.2827	0.1196	0.3827
40	0.2063	0.0000	0.0570	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2281

41	0.1783	0.4550	0.0000	0.0000	0.0604	0.0000	0.0695	0.1820	0.1169	0.1010
42	0.2067	0.0000	0.1022	0.1619	0.0000	0.0000	0.0284	0.0000	0.0000	0.0000
43	0.0000	0.0000	0.0378	0.1959	0.0931	0.0333	0.0000	0.2246	0.0000	0.1874
44	0.0000	0.2714	0.1298	0.0000	0.2946	0.1941	0.1592	0.1653	0.0000	0.1106
45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
46	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1551	0.0000	0.0000
47	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1475	0.0000	0.0000
48	0.2125	0.2392	0.0368	0.1957	0.0712	0.2720	0.0139	0.0113	0.0000	0.1649
49	0.0000	0.0000	0.0567	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.1811	0.0992	0.0000	0.0000	0.0820	0.0000	0.0000	0.0000	0.0000
51	0.1364	0.3535	0.0157	0.1251	0.0000	0.1425	0.1373	0.1524	0.0389	0.0312
52	0.0000	0.1874	0.1713	0.0000	0.0000	0.0000	0.0123	0.2379	0.0936	0.0000
53	0.1066	0.0611	0.1413	0.1056	0.1567	0.1907	0.1790	0.1502	0.2987	0.0000
54	0.0000	0.2036	0.0000	0.0000	0.0000	0.0591	0.0000	0.2704	0.0109	0.2451
Mean	0.0864	0.1263	0.0818	0.0577	0.0562	0.0568	0.0594	0.0991	0.0586	0.0946
Max	0.3783	0.4550	0.3465	0.3481	0.3664	0.4145	0.3238	0.3573	0.3718	0.3971
Std dev	0.1074	0.1286	0.0987	0.0840	0.0940	0.0915	0.0929	0.1094	0.1038	0.1226
Min	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A2-2. Beta estimates with undesirable output (variable returns to scale, α = 0.5)

Obs.	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	0.3003	0.1440	0.0000	0.0384	0.0321	0.2343	0.0000	0.0000	0.0006	0.0000
2	0.3088	0.2725	0.4052	0.1731	0.4468	0.0048	0.0485	0.1011	0.3854	0.1914
3	0.3522	0.3321	0.3469	0.2794	0.0000	0.4094	0.1509	0.3970	0.1782	0.5432
4	0.1823	0.0000	0.2124	0.2574	0.1359	0.2603	0.0271	0.2994	0.4416	0.0000
5	0.4411	0.3506	0.0000	0.0000	0.2346	0.1692	0.0055	0.0189	0.0007	0.0045
6	0.4658	0.3232	0.4505	0.2210	0.5669	0.1796	0.3405	0.1260	0.3112	0.3756
7	0.2956	0.0037	0.0104	0.0693	0.0000	0.0000	0.0000	0.0036	0.2624	0.1941
8	0.3124	0.1857	0.4307	0.0974	0.3820	0.0796	0.0000	0.1373	0.0581	0.0430
9	0.2321	0.0652	0.0000	0.1266	0.0000	0.5527	0.0000	0.2165	0.2153	0.0018
10	0.0769	0.2901	0.1055	0.0840	0.3646	0.2431	0.4395	0.5000	0.2405	0.2416
11	0.0000	0.0000	0.1730	0.2763	0.1567	0.3302	0.1095	0.1467	0.3109	0.1548
12	0.2728	0.2555	0.1724	0.2999	0.1937	0.1876	0.3909	0.5000	0.5223	0.3551
13	0.0000	0.4077	0.0940	0.0040	0.0020	0.2514	0.2567	0.0000	0.3472	0.0000
14	0.0371	0.1241	0.0358	0.0687	0.0880	0.1167	0.4770	0.5000	0.0988	0.1531
15	0.1784	0.3767	0.2014	0.2492	0.1213	0.1114	0.3143	0.0043	0.1876	0.0996
16	0.0000	0.0170	0.3897	0.0610	0.4406	0.5000	0.0068	0.2997	0.4852	0.0743
17	0.1878	0.3290	0.2071	0.3428	0.0320	0.0157	0.3299	0.0000	0.0049	0.0000
18	0.0000	0.3516	0.2411	0.1710	0.2738	0.3194	0.3201	0.3696	0.1300	0.3458
19	0.3340	0.4415	0.3695	0.2767	0.0391	0.2840	0.4906	0.2874	0.2801	0.1656
20	0.3489	0.0131	0.4305	0.1423	0.2372	0.1464	0.2437	0.1048	0.5430	0.1941
21	0.0927	0.1667	0.0060	0.0004	0.2775	0.0278	0.0018	0.3477	0.0223	0.2133
22	0.0000	0.0000	0.0000	0.0331	0.0076	0.0325	0.0000	0.0000	0.0013	0.0000
23	0.3725	0.2804	0.0000	0.0000	0.0000	0.0000	0.0000	0.4509	0.0000	0.0656
24	0.2690	0.0000	0.3576	0.1318	0.3440	0.5000	0.5000	0.5000	0.2559	0.5000
25	0.0005	0.0000	0.0000	0.5000	0.5000	0.0190	0.0000	0.0000	0.0000	0.0000
26	0.2994	0.1737	0.2998	0.1162	0.2170	0.0000	0.5600	0.2645	0.5000	0.3842
27	0.0181	0.0000	0.0000	0.0000	0.4217	0.1389	0.1266	0.0000	0.2268	0.0000
28	0.0000	0.0059	0.0059	0.2795	0.3882	0.3076	0.2149	0.0311	0.0000	0.3550
29	0.2109	0.3690	0.0216	0.4293	0.2028	0.1801	0.0000	0.1145	0.2430	0.1672
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
31	0.0000	0.0000	0.0000	0.0076	0.0000	0.0000	0.0000	0.5000	0.5000	0.0000
32	0.0000	0.3637	0.0773	0.1936	0.0048	0.3408	0.0259	0.0811	0.0000	0.0333
33	0.1428	0.1256	0.0000	0.0677	0.0227	0.2731	0.0131	0.0180	0.0000	0.2890
34	0.4502	0.1237	0.2094	0.0971	0.0861	0.1179	0.1330	0.2857	0.0000	0.0000
35	0.1790	0.1526	0.1329	0.0785	0.0476	0.0564	0.4331	0.2561	0.0075	0.1700
36	0.2015	0.2672	0.5000	0.2702	0.5000	0.5000	0.3138	0.1291	0.4278	0.5000
37	0.0000	0.0000	0.0181	0.0000	0.0000	0.0000	0.0288	0.0000	0.0013	0.0000
38	0.0000	0.3840	0.2089	0.2868	0.1737	0.0000	0.4054	0.2361	0.0689	0.3379
39	0.0000	0.0452	0.0600	0.5000	0.1816	0.0000	0.0000	0.1267	0.1173	0.3718
40	0.2681	0.0041	0.0550	0.0000	0.0013	0.0000	0.0000	0.0000	0.0000	0.1701
41	0.3592	0.6007	0.5000	0.2016	0.2876	0.3319	0.2754	0.3160	0.2760	0.1261
42	0.1188	0.0000	0.3161	0.2153	0.0821	0.0000	0.0000	0.1719	0.1458	0.5000

43	0.2490	0.0799	0.2798	0.3213	0.2626	0.2445	0.0000	0.4109	0.1065	0.1898
44	0.0086	0.2485	0.3204	0.0704	0.4260	0.3365	0.4597	0.3035	0.2247	0.4950
45	0.0000	0.0000	0.0000	0.1615	0.5000	0.0000	0.5000	0.5000	0.5000	0.5000
46	0.0000	0.0033	0.0010	0.0000	0.0000	0.5000	0.0026	0.5000	0.5000	0.0481
47	0.0000	0.0000	0.0023	0.0410	0.0000	0.1880	0.0000	0.0680	0.0653	0.0561
48	0.3149	0.3010	0.1447	0.2275	0.0934	0.3267	0.1127	0.0728	0.0000	0.0000
49	0.4576	0.3129	0.0000	0.0000	0.1535	0.0000	0.0000	0.0268	0.0002	0.1206
50	0.1950	0.2962	0.0802	0.0384	0.0000	0.2094	0.0000	0.0000	0.0000	0.0000
51	0.0201	0.3598	0.0822	0.0604	0.0000	0.0978	0.1764	0.0000	0.0000	0.0000
52	0.0031	0.2611	0.0000	0.0000	0.0000	0.1828	0.0002	0.1615	0.1939	0.0011
53	0.2165	0.2011	0.3097	0.2212	0.2397	0.2722	0.2839	0.0574	0.2671	0.0059
54	0.0000	0.2066	0.0324	0.0000	0.0496	0.0000	0.0000	0.1840	0.0900	0.3436
Mean	0.1625	0.1781	0.1537	0.1442	0.1707	0.1774	0.0751	0.1425	0.1805	0.1682
Max	0.4658	0.6007	0.5000	0.5000	0.5669	0.5527	0.4644	0.8691	0.5430	0.5432
Std dev	0.1548	0.1581	0.1618	0.1342	0.1727	0.1627	0.1124	0.1945	0.1796	0.1737
Min	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000