**Reproduction, Food Provision and Sustainability in Peasant Economies: On Modelling of Joint Resource Use and Valuation?**

by Ernst-August Nuppenau

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# Reproduction, Food Provision and Sustainability in Peasant Economies: On Modelling of Joint Resource Use and Valuation?

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Abstract:

There is an ongoing debate on better linking food production to reproduction of biological resources and eco-systems. Currently food production is part of profit led agribusiness, which is income oriented in farming and is weak on system matters with nature. In contrast, in peasant economies objectives were broader, including reproduction of humans and nature. We use this notification for revision and work on drudgery and food provision sustaining population sizes. The hypothesis is as follows: (1) Reproduction and production (as objectives) are to be reconnected. (2) The paper explores how one can apply programming techniques to derive behavioural equations based on energy spending and shadow prices as incentives for reproduction. (3) It shows how conditional behavioural equations can be readjusted in a system analysis.

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## 1 Introduction

If we think about food production in a broader sense of ecological systems, a crucial issue is getting objective functions right. Usually, in the analysis of farm behaviour (as part of agricultural economics) an ad-hoc assumption is profit maximization. A question is: are there alternatives? Admittedly there is a long discussion on utility maximization vs. profit maximization in peasant economies, and as the frequently mentioning of food needs in farm-household modelling shows: pure profit maximization is a special case of utility maximization (de Janvry et al. 1991). But what is utility? Is it food consumption and leisure only? Apparent solutions were found (ibid); but these solutions ignore often reproduction and drudgery (Christensen 1989).

Our starting hypothesis is: academic wisdom has departed from a deeper analysis of reproduction, eventually for good reason. It looks as if peasant oriented behaviour is not good for growth; rather capital inflow and imbedding farming in a market economy as commercialization counts (Mundlak, 2000). Reproduction seems not to fit into farm business concepts. So to say, it might be right to postulate that peasant behaviour is "romantic", backward and traditional, at least as concept; but it is also said that peasantry included reproduction. There is quiet a "peasantry" as concept (Sahnin,1987). But how can we describe it in terms of reproduction and nature reliance? Against that background it is the aim of this paper to explore the joint sense of production, reproduction and behaviour towards nature in a formalized approach. Further, we relate the problem to objective function recognition. The context of objective function formulation is offered and we provide a conceptual outlay for more sustainable farming based on peasantry.

The paper is organized in five chapters. (1) We talk about the background and concept. (2) A modelling framework of programming for behavioural functions is introduced. (3) The equilibrium is explained and production is modified along reproduction. (4) Empirical grounding is discussed and (5) scopes for application are deliberated. It is admittedly a theoretical paper, but shows a rigours way on making empirical research feasible built on peasantry.

## 2 Challenges

This paper challenges the view of current food economics in regards to focusing on linear processes (from input to output and looking at value chains only), i.e. with one objective: profit. Especially there seems to be a deficit in theories on linkages between food economics and biological reproduction (Biesecker and Hofmeister 2010). The challenge for this paper is to model nature and labour interactions with a focus on palatable energy. It is reckoned as an alternative understanding for reproduction (Georgecu-Reogen 1960) in the context of energy, entropy and recovering. Our thinking is that food pricing should be linked to both, labour productivity and reproduction, rather than see fossil energy prices as anchor (Dorward 2013). Natural recovering is seen as requisite for peasants, but we have also drudgery. Ihe issue of drudgery comes into perspective (Banaji 1975) when we have to acknowledge efforts and returns on effort. Drudgery is a mode to describe labouring for food and caring for survival in subsistence. We explicitly look at drudgery for reproduction at family and village level seeking justification. Hereby we to notify Georgescu-Roegen (1993) who offered a background on knowledge about energy flows between reproduction and production and will explore effort minimization for reproduction.

## 3 State of the art in modelling

We will try to model nature and labour interactions having a focus on palatable energy and balances. Hereby, firstly, we refer to Chayanov (1966) who emphasized natural returns, yet as food derived from labour (income) as position of peasantry, being important in survival. Secondly modelling of drudgery matters as said (Banaji, (1975). Drudgery can be measured as expenses of energy. Then we have to look at net energy (or entropy Georgescu-Roegen, 1993).

In contrast we have to integrate and generalized on utility (Just et al. 2008) and work on welfare.

## 4 Concept

We think scarcity measurement is a core problem for exemplifying needs for reproduction. How to do that? A frequently mentioned version is objective function stating and expanding welfare analysis to steady state equilibrium with reproduction as objective; so we need revelation of reproduction and equilibria for reproduction. Equilibria request a quantity (being for instance "supply" and "demand" of something (at markets commodities) to be equalized and ask for a trigger which enables adjustment (price). It has been the invention of a "price", which delivered coordination at markets. A price has several functions. It is (i) a medium, which can adjust, (ii) it is an incentive, (iii) it is a calibration unit for financial statements and (iv) is a "valuation" tool. Prices show what issues in objective are giving priorities as revealed preference. We will expand to reproduction and population.

For peasants it can be argued that "population" density and drudgery are aspects which they want to put in equilibrium. Increased population has the price of drudgery. (However, admittedly this is not a decision in a framework of methodological individualism.) We model population as “collective decision” taken as “proliferation demand” and drudgery is the price.

**5 Formalization of reproduction**

How to formalize? First, variables in a game (equilibrium), which will be our method (yet on reproduction) have to be specified concerning their affiliation with behaviour. (i) We see a village population, which is split in different occupations (families) as equilibrium variable. We further introduce reproduction (of the peasant community as sizes) in vector mode “[n]:= [$n_1$, …, $n_n$]´ ” (i.e. number of members of a peasant community in different social units-occupations). (ii) Energy is spent (mathematically as product of hours multiplied by energy per hour spent). Particularly energy is spent in different activities as related to seeding (or offspring), proliferation, etc. For all measurements energy spent per hour is the anchor. Then at “technology” level (iii) seeds (offspring) are core for survival. Seeds are imbedded in activities (time and energy spent) of proliferation (naturally or by assistance of humans) and subject outcome, yet in natural systems. In case of human connectivity, when it comes to the agrarian activities, input seeds are further categorized by labouring to improve germination probabilities.

Furthermore a variable similar to “price” here as drudgery” is introduced; it is measured as energy spent per hour and can adjust. Drudgery has an upper and lower limit. We could start for calibration with the upper level, which is energy (from food) equal to energy produced in extreme survival i.e. without any energy surplus in human life. Yet, the underlying concept and measurement is a physical input-output tableau. This is a “world” of complete misery and drudgery; though it can serve as reference. However, even peasants will find and found equilibria above misery. So, how to model it? We introduce an artificial surplus like producer and consumer surplus in food. Note, in economic theories supply, demand, equilibrium, profit and utility serve as utensils to receive surplus and behaviour equations. Behavioural equations can be constructed like in regular economics. At supply side (revenue minus cost) producer surplus is the aim. And on the demand side utility minus expenditure is consumer surplus. We modify it.

**6 Modelling**

**6.1 Surplus**

To start we assume that a reference vector of “n” (population size/mix) is declared as a physical objective and we minimize costs of generating it. Hereby we state costs are energy spent measured in working for “n”. The technology to achieve “n” is known to the peasants; it is: $n \geq A\, h_r$. Where "$h_r$" is hours as well as energy spent per hour in different occupations (The mathematical formulation is below). Then, for the idea of peasantry to maximize an objective "n" while minimizing energy spent, we resume (linear) programming. The underlying concept is those of recovering behavioural equations (Paris and Howitt, 2000) from limited data using maximum entropy (Heckelei and Wolff 2003, Golan et al. 1996) which works with programming. Programming operates with primal and dual solutions. Duals offer shadow prices (Nuppenau, 2014). Shadow prices are obtained from mirrored optimization (costs or revenues). The research is a statistical one in its outlay, aiming at a recovering of coefficients which describe behaviour. But we aim at flexible descriptions of technologies and response to change.

"$h_r$" as energy spent, is energy loss and it makes sense to postulate that it is to be minimized. Further we indicate $h_{r,w}$ for reproduction as wish. For primal programming the setup is:

$$\text{Min } e' \cdot h_{r,w} \quad \text{s.t.} \quad n_w \geq A_r\, h_{r,w} \tag{1}$$

where vectors are: $h_{r,w}$ := working time
$e$ := energy spent per working time unit
$n_w$ := population wish,
$A$ := technology matrix

The result is a vector "$h_{r,w}$*". Equivalently the dual is maximizing weights for the constraint.

$$\text{Max } \lambda_{e,h}{}' \cdot n_w \quad \text{s.t.} \quad e \leq A_r{}'\, \lambda_{e,h} \tag{2}$$

where: $\lambda_{e,h}$ := shadow price vector

In fact the representation offers a calculation of the shadow price such as:

$$\lambda_{e,h} = A_r{}'^{-1}\, e \tag{3}$$

The equation (3) underlies the problem of value detection. But, it does not tell us something about size of "e". It must be flexible for corresponding shadow price evaluation, so far it is fixed. In fact from programming and maximum entropy we construct a synonymous of revenue minus cost. This serves analogous demonstration of "peasant knowledge" on reproduction. Net revenue is surplus in which shadow price is $\lambda_{e,h}$, for "valuation" of "n" is based on drudgery. Taking surplus (4) we can model how to get surplus of peasant in terms of reproduction.

$$SP = \lambda_{e,h} \cdot n_w - e' \cdot h_{r,w} \tag{4}$$

Note the anchor for assessment is the cost of drudgery "$e' \cdot h_{r,w}$". It can be considered the price equivalent for a balancing requesting reproduction, i.e. along needs and offers to reproduce. Since it is drudgery humans have to pay for they optimize. (It is not a "traded price" at intended supply and demand to be brought into equilibrium; but it can serve as reference.) For communication, the idea is: drudgery might be similar to market evaluation and we have drudgery as input. In input market valuation, for example, the price for feed determines the output price, there meat. If feed prices translate into meat prices (by livestock industry), they are observable as mark-up; so why not drudgery translated into valuation of population as indictor for reproduction? I.e. for the moment we seek "commodification" (tradable output); then a price for reproduction from drudgery prevails (on (de) commodification: Gerber and Gerber, 2017). Formally the next step is construction of the counterpart: nature "supply".

Before the counter of reproduction as demand "supply as request for equilibrium" is modelled, we have to outline behaviour. Note "n" is a vector of achieved $n_0$. By duality $\lambda_{e,h}$ (statistical evidence) it is feasible to expand the problem of drudgery and reproduction in a quadratic version. This version is based on optimal h* and $\lambda_{e,h}$* (shown and technically done: Paris and Howitt, 2001). Equation (5) bears similarities to an indirect "profit" function" expressed as consequence of optimized activities and, in fashion of Shepard's lemma, "price" is reliant.

$$SP=\lambda_{e,h}\cdot n- e'\cdot h_{r,w}=.5\lambda_{e,h}'Q_{11}\lambda_{e,h} +.5\ e'\ Q_{12}\ e + \lambda_{e,h}'Q_{13}\ e + e'Q_{14}\ x_1 + \lambda_{e,h}'Q_{15}x_1 \quad (5)$$

Then, for the generalized approach (5), derived from initial programming (1) and (2), gives optimization towards shadow prices (Shepard's Lemma); (5) gives "demand" for population. This "demand", though it is not equated so far, is a market result in behavioural equation (5a).

$$n_d = Q_{11}\ \lambda_{e,h} + Q_{13}\ e + Q_{15}\ x_1 \quad (5a)$$

At the same time if we optimize towards the drudgery, a shadow price prevails and we get:

$$h_{r,w} = Q_{12}\ e + Q_{13}\ \lambda_{e,h} + Q_{14}\ x_1 \quad (5b)$$

In fact the "prices" e and $\lambda_{e,h}$ could become linked and a behavioural equation appears which is given by a uniformed price, i.e. if e = A' $\lambda_{e,h}$; and we get.

$$n_w = Q_{13}^{**}\ \lambda_{e,h} + Q_{14}^{**}\ x_1 \quad (6)$$

In general the result is a flexible request for reproduction. It is the first condition (pull) for equilibria. It spells out a type of flexibility in peasant community which results in a "wish for a vector (number) of families (sizes) for instance as drivers. This vector "n" will be part of the behavioral concept of consumption and production as that concept relies on no. of humans.

### 6.2 Condition anticipated from nature and energy balance

For the second condition (push, i.e. reluctance of nature or willingness) we need an inclusion of nature. It shall be simulated as a provision delivering reproduction potentials (incl. those of nature). I.e. from the perspective of peasants who recognize interdependency with nature, i.e. the equilibrium has to be seen as a compensating on acquiescence for nature. The argument is as follows: peasants may think of injecting seeds $s_h$ and $s_{n.1}$ in nature and recognizing natural

$s_{n.2}$. Then they expect a certain volume of biomass from seeds. The outcome is organic matter produced, which split between human and nature. We take an organic matter balance such as:

$$b_h + b_n \leq B_1 [s_{n,1} + s_h] + B_2 s_{n,2} + B_3 x_2 \quad (7)$$

where: $s_h$ := seeds by humans
$s_n$ := seeds by nature
$b_h$ := organic matter by humans
$b_n$ := organic matter by nature

For a further explanation: let us look at 2 levels of explanations: first technical and then content-wise. Technically biomass is a vector resulting from different species. Peasants may just look at their seeds. But it might not be only crops in agriculture for which nature seeks reproduction. Natural seeds serve as benchmark; this will be later explored. A general version takes biomass and seeds explicitly and gives them a double meaning.

$$\begin{bmatrix} b_h \\ b_n \end{bmatrix} \leq B_{11} [s_{n,1}] + B_{12} [s_h] + B_2 s_{n,2} + B_3 x_2 \quad (7')$$

Firstly some are part of biomass, $b_h$, is free for humans. It is extraction of biomass lost to nature. Second biomass $b_n$ is claimed. For the moment let us work with peasants' perception. They (should) see generation or production of palatable biomass by nature as an indirect effect of actions. A certain proportion of biomass $b_h$ is then used to support humans: the other biomass is for nature $b_n$; respectively. At the moment we do not qualify for $b_n$. It is later an interface. I.e. tolerance for remaining palatable biomass is reasonable for human population. From the perspective of peasant the remaining biomass is a concession. Then this corresponds to a willingness to concede a certain number of humans (population, human reproduction in case of a certain biomass granted) to nature as requested by nature from humans. Volumes depend on seed availability for natural system which is constrained and recognized by humans:

$$C_2 s_{n,2} + C_3 [n_s - n_{max}] = C_1 b_h \quad (8)$$

In equation (8) we postulate a special knowledge of peasants on biomass based on natural seeds and extraction (in balance). I.e. if a natural seed (species) vector is recognized and nature "accepts" amounts of humans, biomass can be extracted for the purpose of feeding humans. However, the system is likewise limited by biomass which is not taken by humans. A consequence for nature management is: seed injection is giving equation (8) as dependent on energy spent and highlighted in (9). In other words seeding by humans requires expenses

$$s_h = D_1 h_{hr} \quad (9)$$

where: $h_r$ := realized labour for reproduction

Inserting the condition (8) and (9) in (7) gives an outline on what can be obtained from a "patient" nature (with humans) in the case of simple biomass substitution possibilities.

$$C_1^{-1}[C_2\, s_{n,2} + C_3\, [n_s\; \text{-}\; n_{max}\,]] \;+ b_n \leq\; B_1\, [s_{n,1} + D_1\, h_h\,] + B_2\, s_{n,2} + B_3\, x_2 \qquad (10)$$

where: $n_s$ := population supplied by nature

Equation (10) outlines a frame where humans (a peasant community) can reach reproduction with nature. If the condition is accepted (empirically) it helps balancing, nature with humans in reproduction. To show it, next, system equations (10) have to be translated into a constrained optimization. Getting this behavioural equation in programming, peasants are facing:

$$n_s \leq\; C_1{*}\, h_r + C_2{*}\, b_n + C_3{*}\, s_n + C_4{*}\;\; x_2 \qquad (11)$$

Equation (11) is a condensed version of equation (10), i.e. we calculated joint matrices. Then, if we invert it, a greater equal condition appears for programming which still contains n and h.

$$s_n \;\geq\; C_1{**}\, h_r + C_2{**}\, n_s + C_3{**}\, x{*} \qquad (11')$$

To further reduce the number of variables, additionally peasants may check the balance of their existence and link it to labouring. This is similar to above notion but now in the perspective of labour. Labour has to be delivered in nature interaction and it spells out as provision of physical activity for reproduction. It shall be an energy balanced provision which gives the condition for labouring in energy which is coming from population size as energetic basis.

$$n_s = B\, [h_r + h_p] \qquad (11'')$$

And this gives

$$s_n \;\geq C_1{***}\, h_r + C_2{***}\, h_h + C_3{***}\, x_2{*} \qquad (12)$$

Note as condition, (12) is supporting population and delivers needed labour for production.

For a further understanding of the equilibrium: equation (12) is based on a supposition that nature is tolerating humans though humans extract palatable biomass. Compensation is by seed provision at expense of labour (drudgery). It may include planting of wild species, etc.

### 6.3 Nature tolerance and "objective"

Apparently, at this point, some remarks on rationale for the above outline are needed. We are not going further into a deeper analysis on nature functioning. However, two aspects matter if one wants to simplify for a system analysis. First, we assume human anticipation of "nature" response. Second we ask, does compensation and labouring for nature fit to "choice"? As "true" type of calculus competing with commercial farm calculus of "revenue minus cost" labour only might not be as accomplishable for numeration; but it is approximated as "near supply price" for reproduction. Our next step is to establish a calculus at peasant community level. After having done that, a simple comprehensive depiction of an extended peasant objective is given, and we explore reproduction using a nature related approach. For the moment this is the number of humans in energy classes and drudgery. It means given the valuation by shadows price it is a “need”. In the logic of “supply” (for cost equivalents) we seek benefits; again, to “simulate” nature response.

$$E = \lambda_{e,h} \cdot n_w = \lambda_{e,h} \cdot A' h_{r,o} \qquad (13)$$

Equation (13) is a benefit equivalent. Humans are willing to pay for nature. In other word: meeting the “correct” understanding of nature as an incentive response unit like an agent from the perspective of a principal (human), there is a benefit of nature and it can be understood as cost as well as exceeded intakes; here calibrated in “quantity” multiplied by “price” nature is calculable. (Note, perhaps it is a matter of “natural” selection or evolutionary approach to get such response; Bobulescu, 2015; here it serves simulation). By establishing (13) it is presumed that peasant communities have “tested” nature, and “best” anticipation justifies a behavioural concept (eventually tested again) of nature. In modelling we get equilibriums of push and pull with nature, being a main aspect in human analysis of peasants.

Returning back to the construction of “nature” as a response unit (i.e. in the eyes of the peasant community) the payment (cost) is mirrored by a (intake) benefit function of nature. It is not a real benefit rather a hypothetical helping peasant to balance their population wishes with nature. The benefit of nature can be exemplified as being similar to revenue. So “revenue (species reproduction) minus cost (humans)” is considered an equivalent surplus for nature.

$$SN = R - E \qquad (14)$$

In this frame, expressed with already given $\lambda_{e,n}$ and in a first round of programming (but now qualified $\lambda_{e,n}{*}= \lambda_{e,h}{\cdot}A'$) further qualification of drudgery “price” enables a new “objective”:

$$SN = \lambda_{e,s} \cdot s_{n,2} - \lambda_{e,n}{*} \cdot h_{r,o} \qquad (15)$$

In equation (15) we have a sequence type of solving the problem, i.e. an explicit quantitative notion for “revenue” of nature: it is used where species prevalence in natural systems ($s_{n,2}$ multiplied by $\lambda_{e,s}$ the shadow) are retrievable. $\lambda_{e,s}$ is the shadow price. For that quantification an acceptance (supply) $s_n{*}$ is realized (accepted by nature). So thinking in terms of a supply function, it means costs of an agent prevail (nature is an agent of peasantry). It is presumed that peasants need to know how their incentives (payments for “s” by laboring for reproduction) translate into “action” of nature (supporting population). In action: nature tolerates humans; i.e. tolerance is active and rewards must be lined into nature’s benefits knowing (12). Again in our analysis programming constitutes behavior. For empirical parts to be constructed statistically $s_n{*}$ is given (for technique, Nuppenau 2014). Then primal programming of behavior serves to find the behavior as reconstruction if we use:

$$\text{Min } \lambda^{*}_{e,n} \cdot h_{r,o} \qquad (16)$$

$$s_n{*} \geq C_l{**}\, h_{r,o} + x{*}$$

and from the primal $h_r$ (as offer to stimulate acceptance of humans) can be stated and found. Then the dual is

$$\text{Max } \lambda_{e,s} \cdot s_n^* \quad (17)$$

$$\lambda_{e,n}{}^* \leq C_1{}^{**'} \lambda_{e,s}$$

Repeating that the construction of quadratic objective functions is possible, i.e. having a data basis of $n_s$ $h_{r,o}$ and $\lambda_{e,s}$ (using Heckelei and Wolff, 2003) we can finally come to the surplus function expressed as a quadratic function of "prices" (assignments of scarcity $\lambda_{e,n}$ and $\lambda_{e,s}$):

$$SN= \lambda_{e,s} \cdot s_{n,2} - \lambda_{e,n} \cdot n_s = F(s_n , \lambda_{e,n}, \lambda_{e,s}, x_n) =$$

$$.5\, s_n' Q_{21}\, s_n + .5\, \lambda_{e,n}' Q_{22}\, \lambda_{e,n} + .5\, \lambda_{e,s}'\, Q_{13}\, \lambda_{e,s} + \lambda_{e,n}\, 'Q_{14}\, s_n + \lambda_{e,s}\, Q_{15}\, s_n + \lambda_{e,n}\, 'Q_{16}\, x_n + \lambda_{e,s}\, Q_{17}\, x_n \quad (18)$$

From optimization we can derive: a system of behavioural equation ending up in acceptance

$$h_{r,o} = Q_{21}\, \lambda_{e,n} + Q_{22}\, s_n + Q_{23}\, \lambda_{e,s} + Q_{15}\, x_n^* \quad (19)$$

The parallel of this "offer" (acceptance) to the previous function of "human request" is manifested in the same shadow price $\lambda_{e,n}$; it can be specified as equilibrium price. It means if we know $s_n$, $n_s$ and $\lambda_{e,n}$, we also get an internal valuation of seeds for natural species as done by humans with the reference to laboring for nature. Natural seeds (species) are a constraint to the system of reproduction and shadow prices adjusts. So there is a need for an interface between species in nature, and laboring for seeds, as part of a reproduction strategy as well human reproduction as part of the humans system matters! This interface will be further explored when we look at production. For the moment, i.e. in the current specification of nature as an element for reproduction and in case of coexistence, the "explanation" of the population size of the peasantry as a part of nature controlled equilibrium is fine. Taking the perspective of seeds, seeds are a reproduction equivalent observable for peasants. Seeds are collected and proliferated in the wilderness as part of drudgery, but necessary to get equilibrium with nature. The natural proliferation and reproduction is in balance with humans. Then taking population as "system aim" in a steady state the solution for drudgery in equilibrium, which means $h_{r,w} = h_{r,o}$, delivers the population size which is given at that equilibrium:

$$n^* = A\, h_r^*$$

Hence human reproduction, population size, and recognition of nature's reproduction are in balance. Moreover the balance depends in information on seeds for nature and production. For production in the human sphere we have to parallel the reproduction with production (below).

## 7 Interim Discussion on reproduction in nature-human-interaction

### 7. Seeds

In between, further words must be made in the direction of how taking nature, reproduction, offspring inclusion, seed and long run viability into account. The reproduction issue was introduced as a biomass accounting in which seeds determine biomass for humans and nature.

$$\Delta S = B_1 [s_{n,1} + s_h] + B_2 s_{n,2} + B_3 x - b_h \quad (20)$$

From a human perspective biomass is determining the population because it contains the needed food. Finally for humans it should be evident that they (humans) have to work for seed proliferation (as compensation, notably they will do it voluntarily and have done this for centenaries). Seed highlight the dependency of humans in the long run as based on equilibrium between human and natural reproduction, seeds. As dependent on palatable biomass and its functioning, nature is simultaneously a source and a conflicting partner in biomass use and reproduction. This aspect is especially important in case of foreseeable degradation. In cases of restoration actually a dynamic process of recovery is needed. Then, what is the role of reproduction in managing nature? It can be spelt out along seed proliferation. However this is beyond this brief piece of work. Actually behind the equation (20) is a dynamic concept of change in standing biomass. Recursively the biomass dynamics: $S_t = [I- \delta]S_{t-1} + C\, s_n$ is backing seeds for new developments. In this paper it is assumed that dynamic systems come as steady state. In the steady state the provided seeds in nature are in equilibrium with amounts of seed needed for the “best” reproduction of nature (the system). Human reproduction is part of the “game” from a natural science perspective. The extraction of biomass (for sure) changes the “natural equilibrium”; but should not endanger resilience. But flexibility is requested.

The emphasis here is that of an analysis in which the metabolism of human and nature matters; it delineates palatable organic matter extraction in conjunction of biomass and reproduction. Reproduction can be detected by quantitative measures (surface functions). However, it is not a straight production of seeds which is meant (this is another subject). In “production” of reproduction human labour and population, both are linked to production of food and seeds. By no chance, existence of individuals is addressed. So we have to generalize and see correlations.

### 7.2 Reproduction, constraints and interfaces

As seeds are the focus in reproduction their fate is essential. The question is can seeds (in the system) reproduce. Reproduction, from the side of peasants as well as equilibriums is envisaged. To comment on achievements: in the opinion of the author concise interfaces could be established which will enable us to depict natural constraints for food production more precisely than a pure production economics approach if we reference to seeds.

Essential, as we aim at a recursive representation of production and reproduction the levels of seed and harvest are to be distinguished. Though they are dependent on each other, i.e. simultaneously, reproduction is dependent on natural seeds. Further, there is nature's response to farming, here in terms of its performance i.e. to bring eco-system service as expected (hoped) to flower; farmers have to work. For a complete model nature cannot be modelled separately.

### 7.3 Peasants and Seeds

Some words also to peasants' behaviour should be made: in the eyes of peasants, who are defined as nature oriented farmers, it might be sufficient to have an empirically grounded response model to nature. Admittedly, the response (model) of nature is an ad-hoc one. Nevertheless we have discussed its plausibility and showed how peasants can infer nature behaviour by numerical correlations between seeds and their biomass acquisition potential.

The advantage of our approach is that biological processes can be integrated (at a minimum) into decision making in peasant economies. This brings us back to what is meant by "peasant economy". It is about: (i) ways of dealing with the concept "peasant and nature"; (ii) it looks at behaviour under resource scarcity within nature's reproduction, here seeds. And (iii) we specifically suggest documenting peasant responses to nature's needed to sustain carrying capacities (Kremen, et al. 2012). In principle, we claim, (iv) peasants are defined as both, food production and nature caring units who use a holistic objective instead of profit maximization.

## 8 Production and consumption

### 8.1 Reference to market and welfare economics and integration

A rude way to integrate reproduction and production is working with producer and consumer surplus (Just, Hueth, Schmitz, 2002) and taking into account resource constraints. In the above outline reproduction was based on energy spent for reproduction on basis of drudgery multiplied by hours spent. Now we must add laboring for production. Reproduction and production are competing for energy produced and spent on laboring for food and reproduction.

Actually we could start with utility maximization of peasants (de Janvry, et al. 1991) and see leisure as anti-drudgery. But to be concise, it would be inappropriate to take a functional approach such as utility and work with leisure in Cobb-Douglas functions. A specific mode, chosen here, is separating consumption decisions and add them in a programing as well as extract energy from production programming; i.e. needed food. This raises the question on what "utility" is? And how it determines reproduction? We substitute utility by a taste index and see production as serving taste (as part of consumer identity of peasant). For a taste index peasants will minimize energy expenses or look at exchanging commodities on the basis of market prices.

### 8.2 Extended production economics

Taking a version of peasant interaction with market exchange of foods which is the common commercial perspective, a version of decoupling production and consumption decisions is usual presumed (de Janvry et al. 1991). It is primarily, as said before, sales as revenues (labor income: Chayanov, 1966) which counts and peasant can buy food at local markets. For a recoupling of reproduction into this perspective we do it in two steps. First peasants have other costs and seek revenues. Then in a second step on a more complex world of transaction costs and trade involvement may complete the story. For the production we formulate objective function (21) such as revenue maximizing and energy balancing by a Lagrange approach:

$$PP = p_p \cdot [q_{p,p} - q_{c,p}] - \lambda_{e,p} [e' \cdot h_p + e' \cdot h_r - n_{e,c}' \cdot c_c] \quad (21)$$

where $q_{p,p}$ := production

$q_{p,c}$ := own consumption

$\lambda_{e,p}$:= shadow price

To make things short: the production is delineated by a market price driven calculus

$$PP = p_p \cdot q_{p,n} - \lambda_{e,p} [e' \cdot h_p - c_{nc}] \quad (21')$$

where $c_{n,c}$ := fixed given energy requirement for reproduction, off farm, etc. by population

$q_{p,n}$ := fixed given energy by population

And a technology prevails which is labor oriented:

$$q_{p,n} \geq T_{p,1} h_p \quad (22a)$$

as well as production is constraint by factors such as land, etc. (the usual programming):

$$x_p \geq T_{p,2} h_p \quad (22b)$$

This translates revenues and efforts into a quadratic revenue function, which is an indirect revenue function. Since it contains programming solutions which can be retrieved from statistical methods such as Maximum Entropy, it offers (23). In principle the producer surplus is:

$$RP = F(p_c, , n\, c_c - e' \cdot h_r, x_p) =$$
$$.5 p_p' Q_{32}\, p_p + .5[n\, c_c - e' \cdot h_r]' Q_{33} [n\, c_c - e' \cdot h_r] + p_p' Q_{34} [n\, c_c - e' \cdot h_r] + p_p' Q_{34}\, x_p + [n\, c_c - e' \cdot h_r]' Q_{35}\, x_p \quad (23)$$

We use revenue and producer surplus synonym and it gives supply, if we take derivatives.

$$[q_{p,p} - q_{c,p}] = Q_{32}\, p_c + Q_{34} [n\, c_c - e' \cdot h_r] + Q_{34}\, x_p \quad (24a)$$

As well the shadow prices for the constraint incl. the energy constraint can be derived for it:

$$\lambda_{e,p} = Q_{33}[n\, c_c - e' \cdot h_r] + Q_{34}\, p_c' + Q_{35}\, x_p \quad (24b)$$

To get a final version of production and consumption for equilibrium and pricing of drudgery, linked to reproduction, we need residual supply $[q_{p,p} - q_{c,p}]$. It can be equated with the demand

from outside or we assume a fix price for the consumer welfare (yet of non-peasants). But interdependency should be given, because n and $h_r$ are not exogenous variables to production.
At the scale of production, function (24a) can be considered a "demand" for physical energy in production (own consumption; see below in closing the model). Importantly, at this level production delivers essentially energy of needed laboring. For that, to be concise, we have to take the net effects of consumption. Talking about net effects brings us to a debate on what consumption and production mean. In the given framework of peasants we can define consumption $q_{c,p}$ as surplus in consumption over energy requirements to reproduce labor. In fact, since the focus is not on absolute necessary production rather taste matters, a surplus can be realized. This surplus above pure reproduction should include energy lost due to meat consumption, production of tasty vegetables, etc.; i.e. beyond pure, energy, vitamin and mineral needs. So we have to distinguish consumption from reproductive service. In fact, for any food item the balance has to be established between "spent" and "obtained" energy (Pimental and Pimental, 2008) and exogenous energy can be included in $x_p$. This does not mean that peasants do not have "taste" (well-being from producing tasty food) and we can not only work for "obtaining" energy (drudgery of survival); rather we are interested in the balance depiction.

### 7.3 Peasant consumption of own crops and purchased food

For the consumption depiction of peasants we distinguish between food use coming from own production and purchase. Own production reduces the cash needs and, then, in an internal equilibrium of peasant consumption surplus can be derived based on cash expenditures. Cash is just a conversion tool (not like in economics an aim). The surplus is "utility" minus cash spent. There is a distinction between $p_c$ and $p_p$. We first specify expenditure and then surplus.

$$CP = p_c \cdot q_{c,c} - \lambda_{c,p} [y - p_c q_{c,c} + p_p [q_{c,p} - q_{s,p}]] \quad (25)$$

This statement for the consumption side of peasants contains two elements: minimization of expenditures (purchases of food) from the market and maintenance of a cash flow as a constraint, not more. Cash for food increases budgets; but income by laboring can be gotten.
In function (25) consumption is represented by two variables, own food and purchased food and we need preferences. Preferences may be detected by an index.
The construction of the index works along three aspects. (1) In the index, which shall show a sort of satisfaction, we introduce a minimum food requirement defined by population size. Each food item and population group is linked by a fixed requirement. (2) There is a possibility as deviation from local and on farm produced food to food bought. (3) By the choice of coefficients in a matrices (experiments or econometrics) we can simulate (stimulate) a preference order over local foods. Since index "i" has to be fulfilled, the choice set of food stuffs

behind (i.e. as bought and or produced) gives information on strength of community preferences. Note "i" can be also a norm, indeed as assessment of not being a peasant. Stretching from 0 to 1 and constructed by weights as preferences, for example a big difference in satisfying minimum needs from local food like coarse grains (porridge from millet, etc., i.e. a preference against local satisfaction) vs. off-set by buying foods (like wheat bread; for European standards) is a choice of peasants. But buying special food is costly and it has to be balanced with other foods. If equation (26) is a preference (ranking) it must be bound to population size and nutrition requirements; i.e. the index shall display a deviation from a "norm".

$$i_{c,c} \geq i'[I_c\, q_{c,c} - I_p\,[q_{c,p} - c_n'\, n_e]] \qquad (26)$$

Equation (26) implies we can calibrate the function against good and bad situations reducing the degree of freedom for peasants. For instance meeting exactly the food needs by local produce can be indexed 0 p.c. and completely buying food is 100 p.c.; implying purchased food is more costly and better processed has a preference and more satisfaction improves welfare. Finally equation (26) serves for the minimization of costs. As a measure retrieved from a consumer preference it is indicting behavioral directions. The corresponding programming for the primal is a problem of simultaneous planning of $q_{c,c}$, $q_{c,c}$ and $\lambda_{c,p}$. As the primal is stated:

$$\text{Min!}\quad p_c \cdot q_{c,c} - \lambda_{c,p}[y - p_c\, q_{c,c} - p_p\,[q_{c,p} - q_{s,p}\,]]$$

$$\text{s.t.}\quad i^*_{c,c} \geq x_c'[I_c\, q_{c,c} - I_p[q_{c,p} - c_n'\, n_e]]$$

The received consumption from own and purchased food as well as information on shadow price can be supplemented with the dual. For shadow prices of the index constraint the dual is:

$$\text{Max}\ \lambda_{c,i,1}\, i^*_{c,c,1} + \lambda_{c,i}\, i^*_{c,c,2} + \lambda_{c,i}\, y$$

$$p_c \cdot q_{c,c} \leq I_c'\, \lambda_{c,i}$$

$$[q_{c,p} - c_n'\, n_e] \leq I_p'\, \lambda_{p,i}$$

$$y \leq [p_c\, q_{c,c} + p_p\,[q_{c,p} - q_{s,p}\,]]\, \lambda_{c,p}$$

The outcome of primal and dual programming offers again numerical representation such as:

$$CS = \lambda_{c,i,1}\, i^*_{c,c,1} + \lambda_{c,i}\, i^*_{c,c,2} - p_c \cdot q_{c,c} + \lambda_{c,p}\,[y - p_c\, q_{c,c} - p_p\,[q_{c,p} - q_{s,p}\,]]\lambda_{c,i} \qquad (25)$$

The presentation (25) depicts both, choice and revealed preference. Technically the following quadratic description corresponds to the calculation of equation (25) as surplus. And the minimization of expenditure due to self-production gives an expenditure function as indirect:

$$CS = F(p_c, , y+p_p\, q_{s,p}, x_c) =$$

$$.5\ i_c'Q_{41}i_c + .5p_c'Q_{42}p_c + .5[y+p_pq_{s,p}]'Q_{43}[y+p_pq_{s,p}\ ] + p_c'Q_{44}[y+p_pq_{s,p}\ ] + i_c'Q_{45}p_c + i_c'Q_{46}[y+p_p\ q_{s,p}\ ]$$

$$+p_c'Q_{47}\, x_p + [y+p_p\, q_{s,p}\,]'Q_{48}\, x_c + i_c'Q_{49}\, x_p \qquad (26)$$

However, these are only the calculi without the constraints on population, biomass, etc. Having included constraints quadratic functions incl. the constraints are feasible.

To complete the approach, consumption can be determined by the purchase of food and own consumption which is consecutively determined by the purchase price, income and production.

$q_{c,c} = Q_{42}\, p_c + Q_{44}\, [y+p_p\, q_{p,p}] + Q_{45}\, i_c' + Q_{47}\, x_c$ (26a)

$q_{c,p} = Q_{44}\, p_c + Q_{43}\, [y+p_p\, q_{p,p}] + Q_{46}\, i_c' + Q_{48}\, x_c$ (26b)

Additionally we can specify the shadow prices as endogenous. Including food retention, whereby sales and expenditures equate and surplus is generated by the sales, we get the actual consumption. For the equilibrium between production and own consumption shadow prices will equate and we receive shadow prices which reflects the scarcity assessment of the peasant (community) in production dependent on resources devoted to reproduction; the shadow price for production is dependent on population and labor inputs in reproduction. Though pricing is still a market solution, i.e. as if households buy from the production-unit, which offers a unified welfare function. This welfare function, in the next step, must be balanced with reproduction.

## 9 Equilibrium between reproduction and production

### 9.1 Idea

Now let us work on balancing reproduction, production and consumption, expanding the idea of equilibrium and communicate it with meanings of equilibrium between reproduction and production, i.e. by adjusting shadow prices. In the above set-up two shadow prices emerged: $\lambda_{e,r}$ and $\lambda_{e,p}$. Both are given within constraints in optimization. A "natural" thinking would be that constraints equate and by equating on regulates energy use towards reproduction vs. production; hereby energy is in effort units. Unluckily, scaling differs. In the case of reproduction we worked along energy minimization for reproduction (drudgery). In the case of production, consumption (well-being) money was the scale as well as it is given by pricing. So how can we match the two scaling? A simple theoretical answer would be "no", mission impossible. Since the two conceptual frameworks differ, equating would mean to compare apples with pears!?

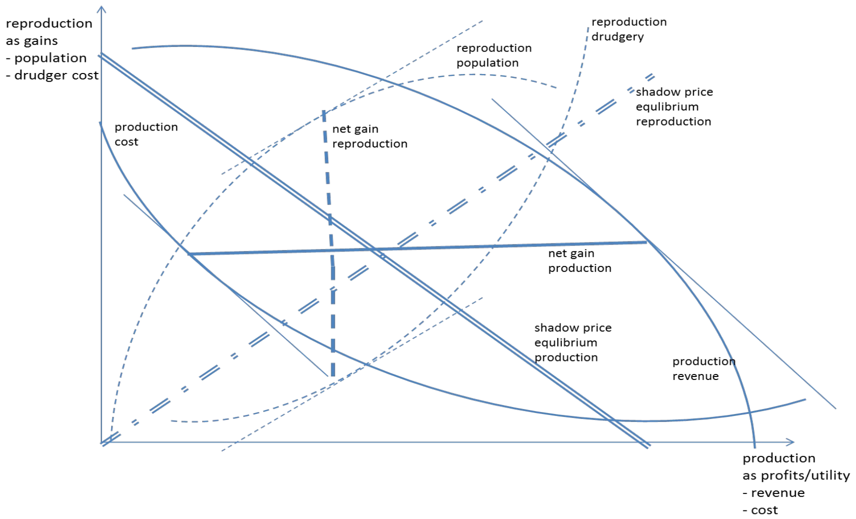
Yet there is a possibility for overcoming the divide. For strict neoclassical thinking, it may not work; but we can work with duality. Duality is a concept already used for other purposes in science; also in economics. In economics there are primal solutions which may be physical, for example, in ordinary planning of farms: cropping pattern, etc. Then the dual optimization is an equivalent of price fixing, i.e. one gets prices as a per unit values for the same optimization. For generalization, production can be considered a value oriented maximization of which the energy aspect is minimization. Then for reproduction maximization of human offspring is a physical goal given "utility" as minimizing the dual cost per unit. At the core lies an interpretation of shadow prices. A shadow price is usually a marginal value for a constraint. Hence also marginal products can equate, and we receive an indirect maximization. In Diagram 1 for the line in the production space (which is equal to marginal costs and benefit, i.e. in cost-benefit space) this shadow prices is an optimal shadow price. Then for the purpose of equating we can invert the slope of the shadow price. And we get a similar representation in the space of energy and costs.

To make an interpretation of this concept, which can be called “shadow price line departure” in “economics” (cost-benefits-analysis): firstly and then from a point of view of drudgery (effort-minimization-line departure) it equates prices. Secondly, our peasant (community) is con-fronted with finding the balance between reproduction and production in the dual space: welfare and energy (Diagram 1). Because peasants do not neglect the duality of production and reproduction, they should be better adapted finding the compromise line. Nevertheless the critic could be that it is not sufficient proving that the solution is found? Is really optimal?

So what are the alternatives against which the suggested version must compete in a contest?

A perspective is to look into a joint maximization of production surplus and reproduction surplus taking the notion of inverse lining (joint at 90 grades contrast, but still being dual).

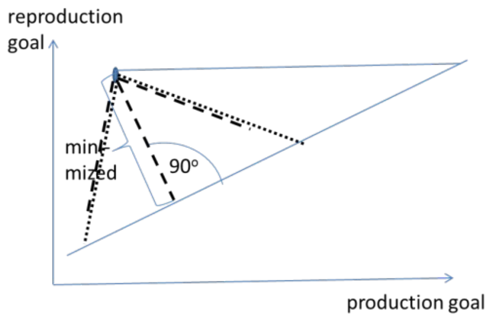
Diagram 1: Equilibrium between Reproduction and Production




Source: own design

Diagram 2 helps understanding the topic further. On the horizontal axis the distance between the utility function and first slope (as consumer surplus) and the distance of the second slope (shadow price to cost function as producer surplus) is given as well as slopes of shadow price and utility function should be equal, i.e. maximal. At the vertical axis the distance between the reproduction achievement function (marginal achievement as shadow price) and, again, the slope (which is now inverse) is the reproduction surplus as well as this distance between slope (equalized shadow price) and the energy spent function is the surplus of nature. The two distances, over again, should be maximized. This happens if the inverted values are equal.

Diagram 2: Minimization of secondary objective function




Source: own design

For intuitive proof: distances can be expressed in geometry as quadratic units if triangles prevail. The optimal position implies that derivatives on corresponding utility functions and energy functions equate and, for instance, the tow slopes are orthogonal. From geometric a triangle is the sum of angles and must be 180 degrees, which is $\pi$. Then, counter angle is $\pi$- $\alpha$.

**9.2 Equilibrium as well as selection of equation, macro-closure and variables for solution**

From the above outline of behavioral equations, i.e. in the compartments of reproduction and production, as well as looking at the necessary links through laboring we receive a set of equations and variable which are to be brought into equilibrium. At the same time it has to clarified, which are relevant variables? At the core the shadow prices for drudgery and labor should adjust (whereas they are inverse to each other, they are to be approximated linearly):

$$\lambda_{e,p} = 1/\lambda_{e,r} = 1/\lambda_{e\,r,0}[1-\lambda_{e\,r}] \qquad (27)$$

Then the labor-energy constraint (either as $h_r + h_r = Cf\, n_e$ or $1'h_r + 1'h_r = c\ 1'n_e$ : i.e. as linkage between population and spending energy for reproduction vs. production) counts. Further at the level of production and consumption the balance $q_{p,p} - q^s_{p,p} = q_{c,p}$ and $q_{c,p} + q^d_{c,p} = q_{c,c}$ must hold which requires a further external definition of non-community demand (as dependent on external sales, both at farm prices). By this approach the above behavioral functions have to be supplemented. Another issue is that factor costs are only represented by shadow prices. These shadow prices, at the other hand are the trigger to adjust the system. Finally revenues must be equal to costs plus income and also, if no surplus profit exists in peasant economies for cash flow (assumed as behavior), accounting delivers cash flow as income per labor unit:

$$p_p\, q_p + y_i - y_o - p_p\, q_c = \lambda_{e,\,p}\, h_t \qquad (28)$$

To supplement this accounting, firstly, the production technology q = A h can be inserted and secondly we make reference to the already introduced interaction of biomass and population. Thirdly for a closure net income and off-farm labour add in another equation. Assuming that net off-farm income which can contribute too food purchases depends on labouring, drudgery can be extended by one additional activity of farm labouring and $y_i$ - $y_o$ = $\varsigma_o$ $h_o$ and the total labor is $h_t$= $h_r$ + $h_p$ + $h_o$; as well as the total labor corresponds to consumption is food dependent: $h_t$= $\varsigma_c'[q_{p,p}+q_{c,c}]$. Which gives a measurement for $y_i$ - $y_o$ = $\varsigma_o[\varsigma_c'[q_{p,p}+q_{c,c}] - h_r - h_p]$. And: $p_p\, q_{pp} + \varsigma_o[\varsigma_c'[q_{p,p}+q_{c,c}]- h_r - h_p] - p_p\, q_c = \lambda_{e,p}\,[h_r + h_p - \varsigma_c'[q_{p,p}+q_{c,c}]]$

Further, in brief, the relationship $q_c$ = ? $q_p$ links biomass $b_h$ and $b_h$ (as $q_p$ can be expressed in biomass and the same applies to $q_c$). So an appropriate intermediary is the bio-mass of crops $q_c = Z_1$ b and $q_p = Z_2$ b whence $q_c = Z_2^{-1}Z_1\, q_p$

Equipped with this information and further assuming that market charges a mark-up $p_p$= $p_p$+$c_c$ (partly justified by transport and marketing costs) as well as production and consumption are regulated by size of a peasant community, the price patterns can be included in the above outline of equilibrium between reproduction and production. In that perspective a quantification of links between prices and reproduction is feasible by taking internal mark-ups as quasi if income can be neglected or statistical information on net income prevails as related to:

$$p_p\, q_p - [p_c + c]\, Z_2^{-1}Z_1\, q_p - q_p\, A^{-1}\, q_p\, \lambda_{e,p} - y_i - y_o \quad (29)$$

$$p_p = [1- c]\, Z_2^{-1}Z_1]^{-1}A^{-1}\, q_p\, \lambda_{e,p} - Z^*\, p_p \quad (30)$$

Now we have reduced all information to flexible "pricing" in (re-) and production and can commence to set up the final conditions for equilibrium? The equilibrium is imbedded in a macro-closure which offers coefficients to bring in the reality of market integration of a peasant in a modern world. However, we have reduced the number of variables. Basically four equation systems appear and we have to work a new adjustment for joint equilibrium:

$$\mathbf{n_e} = q_{10} + Q_{11}\,\boldsymbol{\lambda_e} + Q_{13}\, e + Q_{15}\, x_1$$
$$\mathbf{n_e} = q_{20} + Q_{21}{}^*\boldsymbol{\lambda_e} + Q_{22}\, s_n + Q_{23}\,\mathbf{h_p} + Q_{15}\, x_n{}^* \quad (30)$$
$$\mathbf{q_{p,s}} = Q_{32}\, Z\, [\lambda_{e,0}/[I - \boldsymbol{\lambda_e}] - y_p] + Q_{34}\, [c_c\, \mathbf{n_e} - e'A\, \mathbf{h}_r] + Q_{34}\, x_p$$
$$\mathbf{q_{p,s}} = [Q_{42}-Q_{44}][\lambda_{e,0}/[I-\boldsymbol{\lambda_e}]+c-y_p]+[Q_{44}-Q_{43}][y+C^*Q_{21}]\lambda_{e,0}/[I - \boldsymbol{\lambda_e}]+ [Q_{45} - Q_{46}]\,i_c' + Q_{47}{}^*\, x_c$$

In this equilibrium 4 variable sets (vectors and scalars) appear as being endogenous $n_e$, $q_{p,s}$= $[q_{p,p}-q_{c,p}]$, $h_p$, and $\lambda_e$ whereas $[h_p+h_r+h_o]$= $\Xi$ $n_e$. The other variables are given and further explanatory equations like external sale and demand are reduced form equations included. The system can now be solved; apparently it is under the assumption of peasant welfare maximization which serves as reference system. Other modelling of institutions is also feasible.

**10 Summary**

A novel approach was presented in which we used shadow price analysis to link peasant behaviour of production to reproduction by specifying interfaces. Peasant production was referenced as labour intensive and drudgery. Peasants maximized food. And population. Using quadratic objective function as primal and dual programming, indirect objective functions were stated. Equating of shadow prices as well as labour (effort) allocation for reproduction and production served establishing joint equilibrium. In a special section a novel procedure was suggested how peasants might infer such equilibrium of sustaining population and maximize income.

In final as outcome it was shown, that an equation system offers an optimal population size of peasant community in reproduction, which fits within carrying capacities of natural systems. Consumption and production surpluses interacted with reproduction drudgery and objectives for sustaining natural populations. Hereby the ad-hoc assumption in agricultural economics of profit maximizing was revised and a generalized concept of objective function derivation can be conceptualized for nature-human-interaction. The concept put reproduction and production into a broad perspective of survival and welfare. To a certain extent the envisaged peasant community can be characterized as aiming at sustainable farming and food systems maintenance.

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