Modeling U.S. Butter Consumption With Zero Observations

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A heteroscedastic double-hurdle model is used to investigate household butter consumption in the United States. Results suggest that failure to incorporate heteroscedastic errors may lead to unreliable elasticity estimates. Decomposition of the effects of variables leads to insightful information and makes the double-hurdle model a more useful tool in micro demand analysis. Larger and higher-income households are more likely to consume butter than others and also consume more, but income elasticity is very small. Age, region, and seasonality are among the other significant determinants of household butter consumption.

Modeling demand relationships with microdata presents well-known problems. In particular, the sample often contains a significant proportion of zero observations. In this case, ordinary least squares estimation based on all or positive observations produces biased parameter estimates (Amemiya, pp. 366–367). In addition, excluding the zero observations also causes the loss of efficiency.

In estimating economic relationships with limited dependent variables, the Tobit model (Tobin) has been a natural choice. Unlike the 1960's and 1970's, the current availability of numerous state-of-the-art statistical software programs has made Tobit estimation a more viable tool than ever. Despite its popularity, however, the Tobit model is often found too restrictive. This is because the parameterization of the model implies that the variables and parameters that determine the probability of consumption also determine the level of consumption and, more seriously, determine it in the same fashion. Thus, any variable that increases (decreases) the likelihood of consumption must also increase (decrease) the level of consumption. This is not a desirable property in an empirical demand model.

Generalizations of the Tobit model have become increasingly popular recently, and the empirical literature in this area has continued to grow. Many of these studies are based on the double-hurdle model, a generalization of the Tobit model with separate parameterization of the participation and consumption level decisions. In the analysis of U.S. food demand, the more recent applications of the double-hurdle model include Blisard and Blaylock, Haines et al., Popkin et al., and Reynolds.

In most previous studies, the double-hurdle model has been estimated with the assumption of homoscedastic errors. However, maximum-likelihood (ML) estimates based on the homoscedasticity assumption are inconsistent when the errors are heteroscedastic. In this paper, heteroscedasticity of errors is incorporated and the consequence of error misspecification explored. We examine household consumption of butter in the United States, using data from the Bureau of Labor Statistics' (BLS) 1989 and 1990 Consumer Expenditure Diary Surveys. We explore the empirical results further by calculating and decomposing the elasticities of consumption, a procedure that is often overlooked by previous users of the double-hurdle model, and demonstrate that such decomposition of elasticities is crucial to a proper assessment of the effects of variables on consumption.

The Double-Hurdle Model

The idea behind the double-hurdle model is that two hurdles have to be overcome by the consumer for a positive expenditure to occur: to participate in the market (i.e., to be a potential consumer) and to actually consume. The model, due to Atkinson et al., Blundell and Meghir, Cragg, and Deaton and

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Irish, features a probit equation to explain participation and a Tobit mechanism for nonconsumption among potential consumers.

For observation t, define d_t as the dummy variable for participation and d_t^* as its corresponding latent (unobserved) variable. Likewise, let y_t be the tth observation on the dependent variable and y_t^* be the corresponding latent variable. The two latent variables are described by the regression equations

$$
\begin{aligned}
d_t^* &= x_t\alpha + u_t \\
y_t^* &= x_t\beta + v_t,
\end{aligned} \tag{1}
$$

where x_t is a vector of exogenous variables, α and β are conformable parameter vectors, and u_t and v_t are independent random errors such that $u_t \sim N(0,1)$ and $v_t \sim N(0,\sigma_t)$. The observed consumption y_t relates to the latent variables y_t^* and d_t^* such that

$$
\begin{aligned}
y_t &= y_t^* \quad \text{if } d_t^* > 0 \text{ and } y_t^* > 0 \\
&= 0 \quad \text{otherwise.}
\end{aligned} \tag{2}
$$

Note that, in principle, the two latent variables in (1) could be specified as functions of separate (not necessarily exclusive) sets of regressors. In empirical studies, however, researchers have often struggled with specifications of the two equations because the list of variables is typically limited and theory provides no guidance. Though we use identical sets of variables in the two equations, the different parameter vectors allow the flexibility in modeling the participation and consumption decisions.

Denote the univariate standard normal distribution and density functions as $\Phi(\bullet)$ and $\phi(\bullet)$, respectively. Then, (1) and (2) suggest the following sample likelihood function for the double-hurdle model (Blundell and Meghir, eq. (17)):

$$
\begin{aligned}
L = &\prod_{y_t=0} [1 - \Phi(x_t\alpha)\Phi(x_t\beta/\sigma_t)] \\
&\times \prod_{y_t>0} \Phi(x_t\alpha)\sigma_t^{-1}\Phi[(y_t - x_t\beta)/\sigma_t].
\end{aligned} \tag{3}
$$

It is obvious that the double-hurdle model reduces to the Tobit model when the probit mechanism (i.e., $d_t^* > 0$) is absent in (2). This is also seen in the likelihood function (3) when $\Phi(x_t\alpha) = 1$. Thus, the two models are nested, and selection between the specifications can be done conveniently by the likelihood-ratio (LR) test.

When the double-hurdle model is estimated with homoscedastic error specification, that is, with $\sigma_t = \sigma$, ML estimation produces biased and inconsistent parameter estimates when the errors are heteroscedastic (Arabmazar and Schmidt). Heteroscedasticity can easily be incorporated by allowing the standard deviation σ_t to vary across observations. In particular, the standard deviation σ_t is parameterized as follows:

$$\sigma_t = \exp(w_t\gamma), \tag{4}$$

where w_t, a subset of x_t, is a vector of exogenous variables and γ is a conformable parameter vector. The exponential specification in (4) has the desirable property that the standard deviation σ_t will be strictly positive.[1]

In assessing the appropriateness of the double-hurdle model in modeling demand with zero observations, one might note that household data are typically collected in a relatively short sample period. For commodities that are purchased relatively less frequently, zero observations may be the consequence of infrequency of purchase. Based on a sample from the 1989 Diary Survey, Blisard and Blaylock concluded that the infrequency-of-purchase model is preferable to the double-hurdle model in modeling U.S. butter demand.[2] Thus, the infrequency-of-purchase model cannot be dismissed as a possible account for the zero observations. However, since households consuming more of a commodity are more likely to report consumption during a given period than others, the probability $\Phi(x_t\alpha)$ in the double-hurdle model, when carefully interpreted, should also reflects the probability of purchase.[3] Therefore, it has been argued that the double-hurdle model is also appropriate for modeling demand with zeros resulting from nonconsumption, infrequency of purchase, or a mixture of both (Yen, p. 887). Nevertheless, while we focus on the double-hurdle model in this study, the model will be tested against the infrequency-of-purchase model, using the nonnested LR test procedure of Vuong. The development of the likelihood function for the infrequency-of-purchase model and the nonnested LR test procedure are presented in the appendix.

[1] The other specifications considered include $\sigma_t = w_t\gamma$, $\sigma_t^2 = \exp(w_t\gamma)$, $\sigma_t = \sigma \exp(w_t\gamma)$, and $\sigma_t^2 = \sigma^2 \exp(w_t\gamma)$, where σ is a constant. Maddala (chap. 6) and Greene (chap. 21) discuss some of the specifications that have been considered in the literature.

[2] In Blisard and Blaylock, both the double-hurdle and the infrequency-of-purchase models were estimated with homoscedastic errors. Thus, the LR test might be testing one misspecified model against another and therefore the implication of the test is not clear.

[3] When the probability of participation $\Phi(x_t\alpha)$ is constant, the double-hurdle model is observationally equivalent to a model of infrequency of purchase with corner solution; see Deaton and Irish. When, $\Phi(x_t\alpha)$ is allowed to vary across observations, however, the two models are only intimately related, in that they both nest the standard Tobit model.

Elasticities

Most empirical applications of the double-hurdle model to date have reported only parameter estimates of the model. This is not entirely informative, because the probability, and therefore the unconditional mean, of consumption depends on both the first-hurdle and second-hurdle regressors. This is not surprising because both the probit and Tobit mechanisms determine the zero (and positive) outcomes. The net effect of an explanatory variable on consumption becomes particularly ambiguous when the variable has conflicting signs on the participation and consumption equations. In addition, the specification of heteroscedasticity also complicates such effects. Therefore, for the double-hurdle model, it is important to examine the effects of explanatory variables more carefully. Our decomposition of elasticities is slightly more complicated than that of McDonald and Moffitt for the standard Tobit model due to the double-hurdle parameterization and the heteroscedasticity specification.

Based on the double-hurdle structure (1) and (2) and the normality assumptions of the error terms, the probabilities of participation and consumption are, respectively,

$$P(d_t^* > 0) = \Phi(x_t\alpha), \tag{5}$$

$$\begin{aligned} P(y_t > 0) &= P(d_t^* > 0, y_t^* > 0) \\ &= \Phi(x_t\alpha)\Phi(x_t\beta/\sigma_t). \end{aligned} \tag{6}$$

The second equality in (6) holds because of the independence assumption between the error terms u_t and v_t. Because the dependent variable y_t is truncated at zero, the conditional mean of y_t is (Amemiya, p. 367; Maddala, p. 158)

$$E(y_t \mid y_t > 0) = x_t\beta + \sigma_t\left[\frac{\phi(x_t\beta/\sigma_t)}{\Phi(x_t\beta/\sigma_t)}\right]. \tag{7}$$

Thus, the unconditional mean of y_t is

$$\begin{aligned} E(y_t) &= P(y_t > 0)E(y_t \mid y_t > 0) \\ &= \Phi(x_t\alpha)\Phi(x_t\beta/\sigma_t) \\ &\quad \times \left\{x_t\beta + \sigma_t\left[\frac{\phi(x_t\beta/\sigma_t)}{\Phi(x_t\beta/\sigma_t)}\right]\right\}. \end{aligned} \tag{8}$$

For generality, consider the marginal responses of x_{tj} (the jth element of x_t, with associated parameters α_j and β_j), which is also used in the heteroscedasticity equation (4). The derivative of the participation probability (5) with respect to x_{tj} is

$$\frac{\partial P(d_t^* > 0)}{\partial x_{tj}} = \phi(x_t\alpha)\alpha_j. \tag{9}$$

The marginal responses of participation have rarely been considered in empirical studies. Such marginal responses, however, can be important pieces of information for food marketers in targeting potential consumer groups and evaluating the effectiveness of certain marketing strategies. The marginal effect of x_{tj} on the probability of consumption is[4]

$$\begin{aligned} \frac{\partial P(y_t > 0)}{\partial x_{tj}} &= \Phi(x_t\beta/\sigma_t)\phi(x_t\alpha)\alpha_j \\ &\quad + \Phi(x_t\alpha)\phi(x_t\beta/\sigma_t)\sigma_t^{-1} \\ &\quad \times \left[\beta_j - (x_t\beta/\sigma_t)\frac{\partial\sigma_t}{\partial x_{tj}}\right]. \end{aligned} \tag{10}$$

From (6) and (10), it is obvious that the probability of consumption depends upon both the first-hurdle parameters (α) and second-hurdle parameters (β). Thus, the common practice among users of the double-hurdle model of reporting only the parameter estimates explains participation only but not probability of consumption. This is not entirely informative. The derivative of the conditional mean with respect to x_{tj} is

$$\begin{aligned} \frac{\partial E(y_t \mid y_t > 0)}{\partial x_{tj}} &= \beta_j + \left[\frac{\phi(x_t\beta/\sigma_t)}{\Phi(x_t\beta/\sigma_t)}\right]\frac{\partial\sigma_t}{\partial x_{tj}} \\ &\quad - \left[\frac{\phi(x_t\beta/\sigma_t)}{\Phi(x_t\beta/\sigma_t)}\right]\left[\beta_j - (x_t\beta/\sigma_t)\frac{\partial\sigma_t}{\partial x_{tj}}\right] \\ &\quad \times \left[(x_t\beta/\sigma_t) + \frac{\phi(x_t\beta/\sigma_t)}{\Phi(x_t\beta/\sigma_t)}\right]. \end{aligned} \tag{11}$$

The derivative (11) suggests that, for variables which are used in both the latent consumption equation and the heteroscedastic equation, the marginal effects on the conditional mean may not be directly related to the corresponding consumption coefficient. Note that for the homoscedastic specification or for variables not used in the heteroscedastic equation, the term $\partial\sigma_t/\partial x_{tj} = 0$ and the derivatives (10) and (11) can be simplified; see footnote 4. In this case, the derivative (11) reduces

[4] For this study, $w_{tj} = x_{tj}$ for $j = 1,2$. Thus, $\partial\sigma_t/\partial x_{tj} = \sigma_t\gamma_j$ for $j = 1,2$; $= 0$ for $j > 2$, where γ_j is the jth element of γ.

to the expression considered in Maddala (p. 160) for the homoscedastic Tobit model.

Based on the marginal responses (9), (10), and (11), the corresponding elasticities are straightfoward. By construction, the elasticities of probability and conditional level of consumption add up to the elasticity of unconditional level of consumption; see (8). These elasticities allow a thorough examination of the effects of variables on various components of consumption. For instance, the elasticity of participation measures the effect of a variable on the likelihood to participate in the market, whereas the elasticity of probability of consumption reflects the effect of the variable on the probability to actually consume. Conditional on consumption (i.e., given that a decision to consume has been made), the elasticity of the conditional level of consumption measures the effect of a variable on consumption. Finally, the elasticity of the unconditional level of consumption (i.e., the total elasticity) provides an overall assessment of the effect of a variable on consumption.

For statistical inferences, the standard errors for elasticities can be derived by mathematical approximation. Denote the parameters vector characterizing the model as $\theta = [\alpha,\beta,\gamma]'$, with ML estimator $\hat{\theta}$ and variance-covariance matrix $\hat{\Sigma}$, the kth elasticity (a scalar) as $\hat{E}_k = \hat{E}_k(\hat{\theta})$, and the Jacobian of transformation from $\hat{\theta}$ to $\hat{E}_k$ as J_k. Then, by the delta method (Rao), the variance of $\hat{E}_k$ can be approximated by

$$\text{(12)} \qquad \mathrm{Var}(\hat{E}_k) \approx J_k' \, \hat{\Sigma} \, J_k,$$

where J_k can be evaluated at the ML estimates and at the sample means of exogenous variables.

Since several of the regressors are binary variables, the effects of these variables cannot strictly be expressed in terms of elasticities. The effects of each variable on each component of consumption can be more appropriately calculated as the difference in this component as the value of the variable changes from zero to one, *ceteris paribus*.

Data

The sample for the present study was drawn from the BLS' 1989 and 1990 Consumer Expenditure Diary Surveys (U.S. Department of Commerce 1989, 1990). Each year the Diary Survey was conducted on each sample consumer unit during two consecutive one-week periods. The data tapes include households which completed both one and two weeks of the surveys. One common practice among users of the Diary Survey data is to treat replicates of the same household as separate observations. The major problem with this approach is that, for households with complete two-week information, the values of explanatory variables do not change from one week to the other. Consequently, variations in weekly consumption are picked up by the error terms, causing correlations among the errors.[5] To avoid such problems, we include only households with complete two-week information.

Household expenditure on butter during the two-week period was used as the dependent variable. As common in other cross sectional data, information on prices was not available in the Diary Surveys. However, the regional and seasonal dummies are likely to account for some of the regional and temporal price variations. Drawing on Blisard and Blaylock's earlier study of butter demand, the explanatory variables included household size, education, income, and dummy variables indicating age of the household head, regions, race, year (1990 or 1989), and quarter during which the interviews were conducted. Households with missing information for any of these variables were excluded. This resulted in a final sample of 8083 observations, of which 4313 came from the 1989 Diary Survey and 3770 from the 1990 Diary Survey. Only 1498 households (or 18.53 per cent) reported expenditure of butter during the two-week period. It is particularly noteworthy that the high proportion of zero observations prevents the use of any statistical procedure which does not accommodate the limited dependent variable. The detailed definitions of all variables used and the sample means for the full, consumer, and nonconsumer samples are presented in Table 1.

Results

The double-hurdle model was estimated by maximizing the logarithm of the likelihood function (3).[6] Numerical optimization was carried out with the quadratic hill-climbing algorithm (Goldfeld et al.). The Hessian matrix was derived by numerically differentiating the analytic gradient, and was inverted to derive the variance-covariance matrix of the estimated parameters.

In preliminary estimation, different combinations of continuous variables (household size, in-

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[5] We thank one referee for pointing out this problem.

[6] The analytic derivatives of the log-likelihood functions for the double-hurdle and infrequency-of-purchase models are available from the authors.

Table 1. Sample Means: U.S. Butter Consumption[a]

Variable	Definition	Full Sample	Consumers	Non-consumers
Expend	Butter expenditure ($/two weeks)	0.384 (1.056)	2.074 (1.584)	0.000
Size	Household size	2.527 (1.508)	2.930 (1.609)	2.436 (1.469)
Income	Household income ($00/two weeks)	10.357 (8.640)	11.699 (8.812)	10.052 (8.571)
Education	Education index of household head	3.420 (1.398)	3.446 (1.416)	3.414 (1.394)
Dummy variables (yes = 1; 0 otherwise)				
Head ≥65	Household head is 65 or over	0.214	0.184	0.220
Head ≤30	Household head is 30 or under	0.222	0.163	0.235
Rural	Rural household (reference)	0.119	0.136	0.114
Northeast	Urban household in the northeast	0.182	0.229	0.172
Midwest	Urban household in the midwest	0.230	0.223	0.231
South	Urban household in the south	0.261	0.214	0.272
West	Urban household is in the west	0.208	0.198	0.211
Non-black	Household is non-black	0.904	0.915	0.906
Spring	Survey occurred in spring	0.251	0.237	0.255
Summer	Survey occurred in summer	0.253	0.237	0.256
Fall	Survey occurred in fall	0.237	0.237	0.237
Winter	Survey occurred in winter (ref.)	0.259	0.289	0.252
Year 1990	From the 1990 Diary Survey	0.466	0.456	0.469
Year 1989	From the 1989 Diary Survey (ref.)	0.534	0.544	0.531
Sample size		8083	1498	6585

SOURCE: Compiled from BLS' Consumer Expenditure Diary Surveys, 1989 and 1990.
[a]In parentheses are standard deviations of continuous variables.

come, education) were experimentally included in various forms of the heteroscedasticity equation; see footnote 1. By the Akaike Information Criterion (Amemiya, pp. 146–47), the exponential form (4) was chosen, with household size being significant at the 0.01 level. Thus, the assumption of homoscedastic errors was rejected.

To test the double-hurdle model against the infrequency-of-purchase model, the latter was also estimated, with household size in the heteroscedastic equation (significant at 0.01 level), which leads to a lower log-likelihood function value (−6256.44). Based on the nonnested LR test procedure of Vuong (see appendix), the standard normal statistics was calculated as 1.02. Thus, contrary to the findings of Blisard and Blaylock, there is no basis for preferring one model to the other. The ML estimates for the infrequency-of-purchase model are presented in the appendix.

To explore the consequence of misspecification, the double-hurdle model was also estimated with the homoscedastic specification (i.e., with only a constant in the heteroscedastic equation). Based on the log-likelihood values of the two models estimated, the LR test result (χ^2 = 47.24, d.f. = 1) suggested rejection of the homoscedastic model at a significance level of 0.05. This result confirms the significance of household size in the heteroscedasticity equation and lends support to the heteroscedastic specification.[7]

The estimation results of both models are presented in Table 2. In assessing the parameter estimates of the heteroscedastic model, household size and income are both significant (at the 0.10 level or lower) and have conflicting signs in the participation and consumption equations. Opposite signs are also observed for income in the homoscedastic model. Blisard and Blaylock also reported offsetting regression coefficients for these variables in the participation and consumption equations.[8] These opposite effects of variables are likely to be masked by the restrictive parameterization of the Tobit model, and therefore they highlight the importance of the double-hurdle parameterization.[9]

[7] The homoscedastic infrequency-of-purchase model (log-likelihood = −6273.09) was also tested against the homoscedastic double-hurdle model. The result ($\hat{z}$ = 0.37; see Table A1) suggests that neither model is preferable to the other. Such inference should be taken with cautions, however; see footnote 4.

[8] In Blisard and Blaylock, eight of the twelve regressors have conflicting signs on the participation and consumption equations. Further comparisons of results from the current study with those of Blisard and Blaylock are not possible because elasticities are not reported in the latter.

[9] Indeed, our estimation results for the Tobit model, not reported here due to space limit, interestingly suggest the insignificance of income. The Tobit model was tested against the double-hurdle model by the LR test and was rejected at a significance of less than 0.01.

Parameter estimates for the homoscedastic model are very different from those of the heteroscedastic model. For instance, in contrast to results of the heteroscedastic model, income is not significant in the participation equation and household size is not significant in the consumption equation.

As suggested by the marginal responses discussed above, while the participation parameters exclusively determine the direction of effects on participation, because of the conflicting signs of these variables on the probability and the conditional level of consumption can be opposite, and because household size was used in the heteroscedastic equation, the effects of these variables on the probability and conditional level of consumption are not clear. To gain more insight into the effects of these explanatory variables and the differences caused by the different specifications (across models), we must turn to the elasticities.

The elasticities of participation, probability, conditional level, and unconditional level of consumption with respect to the continuous explanatory variables were evaluated at the sample means of all explanatory variables. The results are presented in Table 3. Based on the heteroscedastic model, the elasticities with respect to household size suggest that larger households are more likely to participate (i.e., to consider consuming butter) and are more likely to consume butter than others. Contrary to what the negative and significant coefficient (−0.728) would suggest, the elasticity of the conditional level of consumption is positive and insignificant. The insignificant effect of household size on the conditional level is obviously caused by the conflicting signs of this variable in the latent consumption equation and the heteroscedasticity equation (see eq. (11)); it also highlights one of the important reasons for calculating and decomposing the elasticities, especially when heteroscedasticity of the errors is accommodated. Overall, the elasticity of unconditional level of consumption (i.e., the total elasticity) suggests that as the size of the household increases by one percent, *ceteris paribus*, the consumption of butter

Table 2. ML Estimation of the Double-Hurdle Model: U.S. Butter Consumption[a]

	With Heteroscedastic Errors			With Homoscedastic Errors		
Variable	Particip.	Consump.	Het.	Particip.	Consump.	Het.
Constant	−1.339***	1.887***	0.524***	−0.284	−1.007	0.986***
	(0.224)	(0.561)	(0.074)	(0.333)	(0.818)	(0.064)
Size	0.481***	−0.728***	0.140***	0.135**	0.092	
	(0.082)	(0.116)	(0.018)	(0.056)	(0.100)	
Income	−0.009*	0.043***		−0.008	0.053***	
	(0.005)	(0.012)		(0.006)	(0.016)	
Education	0.008	0.014		−0.016	0.047	
	(0.027)	(0.068)		(0.039)	(0.093)	
Head ≥65	−0.031	−0.119		−0.062	−0.021	
	(0.092)	(0.255)		(0.150)	(0.363)	
Head ≤30	−0.200**	−0.425*		−0.169	−0.452	
	(0.096)	(0.242)		(0.145)	(0.349)	
Northeast	0.140	−0.025		0.128	−0.004	
	(0.120)	(0.289)		(0.186)	(0.395)	
Midwest	−0.011	−0.341		−0.135	−0.066	
	(0.121)	(0.293)		(0.172)	(0.394)	
South	−0.116	−0.515*		−0.207	−0.386	
	(0.127)	(0.308)		(0.183)	(0.429)	
West	−0.092	−0.224		−0.204	−0.042	
	(0.124)	(0.303)		(0.172)	(0.406)	
Non-black	0.063	−0.017		0.085	−0.037	
	(0.134)	(0.321)		(0.190)	(0.450)	
Spring	−0.041	−0.336		−0.133	−0.064	
	(0.089)	(0.222)		(0.117)	(0.300)	
Summer	−0.071	−0.279		−0.050	−0.325	
	(0.087)	(0.217)		(0.129)	(0.310)	
Fall	0.034	−0.476**		0.158	−0.688**	
	(0.093)	(0.223)		(0.160)	(0.331)	
Year 1990	−0.056	0.294		−0.105	0.385	
	(0.075)	(0.187)		(0.125)	(0.294)	
LLF	−6247.177			−6270.797		

[a]Asterisks indicate levels of significance: *** = 0.01, ** = 0.05, * = 0.10.

Table 3. Effects of Variables on Participation, Probability, Conditional Level, and Unconditional Level of Consumption[a]

Variable	With Heteroscedastic Errors				With Homoscedastic Errors			
	Participation	Probability	Cond. Level	Uncond. Level	Participation	Probability	Cond. Level	Uncond. Level
	Elasticities with respect to continuous variables							
Size	1.172***	0.562***	0.004	0.567***	0.321***	0.399***	0.039	0.438***
	(0.123)	(0.063)	(0.036)	(0.075)	(0.122)	(0.057)	(0.043)	(0.053)
Income	−0.086*	0.060*	0.084***	0.144***	−0.077	0.108***	0.092***	0.200***
	(0.046)	(0.035)	(0.024)	(0.049)	(0.056)	(0.034)	(0.025)	(0.049)
Education	0.027	0.043	0.009	0.052	−0.052	0.002	0.027	0.029
	(0.089)	(0.064)	(0.044)	(0.085)	(0.125)	(0.065)	(0.053)	(0.084)
	Average effects of binary variables[b]							
Head ≤30	−0.076	−0.063	−0.150	−0.147	−0.065	−0.054	−0.146	−0.130
South	−0.045	−0.056	−0.188	−0.144	−0.081	−0.058	−0.126	−0.138
West	−0.035	−0.033	−0.085	−0.083	−0.080	−0.039	−0.014	−0.082
Spring	−0.016	−0.030	−0.126	−0.086	−0.051	−0.028	−0.022	−0.062
Summer	−0.027	−0.032	−0.105	−0.086	−0.019	−0.028	−0.110	−0.078
Fall	0.013	−0.026	−0.175	−0.087	0.062	−0.019	−0.224	−0.079

[a]Standard errors in parentheses. Asterisks indicate levels of significance: *** = 0.01, ** = 0.05.
[b]Calculated as the changes in components of consumption as each variable changes from zero to one, ceteris paribus.

increases by 0.57 per cent. Judging from the elasticities with respect to income, higher-income households are less likely to participate in the market, but are more likely to consume; conditional on consumption, these households also consume more butter than others. Overall, the total elasticity with respect to income suggests that butter is a normal good, though the elasticity is very small. That is, as income increases (decreases) by one percent, *ceteris paribus,* consumption of butter increases (decreases) by only 0.14 percent. Education is not a significant determinant of butter consumption.

The elasticities derived from the homoscedastic model suggest that failure to accommodate heteroscedasticity can produce very different results. For instance, contrary to the heteroscedastic model, the homoscedastic model suggest that income does not play a significant role in participation. Most other elasticities are qualitatively similar (in terms of signs and significance) to the corresponding elasticities suggested by the heteroscedastic model. However, there are notable quantitative differences. For instances, the elasticity of the probability of consumption with respect to household size (0.32) is about seven standard deviations below that calculated from the heteroscedastic model (1.17).

Also presented in Table 3 are the effects of significant dummy variables.[10] These effects suggest that, relative to others, households with members under 30 years of age are about 8 per cent less likely to participate in the market, are 6 per cent less likely to consume butter, and, conditional on consumption, consume about 0.15 lb. less than others during the two-week period. Overall, the effect on the unconditional level of consumption suggests that, these households consume only about 0.147 lb. less than others during the two-week period. The effects of other dummy variables can be interpreted in the same manner. The homoscedastic model suggests quite different effects of these dummy variables.

Concluding Remarks

The high proportion of zero observations in the current sample precludes the use of standard econometric procedures such as the ordinary least squares. The double-hurdle model is a useful generalization of the Tobit model in that it allows the participation and consumption decisions to be determined by separate sets of parameters. In addition, the specification of heteroscedastic errors further reduces the possibility of misspecification and avoids inconsistency of the parameter estimates. Our results suggest that failure to account for heteroscedasticity in the errors can lead to unreliable elasticity estimates, which could have misleading policy and marketing implications.

With the increasing availability of micro survey data, the double-hurdle model has become more popular than ever. We demonstrate that results of the double-hurdle model can be exploited further

[10] A dummy variable is considered significant if the corresponding "elasticities" (not reported) are significant at the 0.10 level or lower.

by decomposing the effects of explanatory variables. We find that larger households are more likely to consume butter than others and also consume more. Higher-income households are less likely to participate in the market than others but, overall, are more likely to consume butter and also consume more. Butter is a normal good, but the income elasticity is very small.

Appendix

This appendix presents briefly the infrequency-of-purchase model, the nonnested model specification test, and parameter estimates of the infrequency-of-purchase models.

The Infrequency-of-Purchase Model

Define latent purchase s_t^* and latent consumption y_t^* as linear functions of exogenous variables x_t (along with conformable parameter vectors θ and β):

$$s_t^* = x_t\theta + \epsilon_t,$$

$$y_t^* = x_t\beta + v_t,$$

where ϵ_t and v_t are independent random errors such that $\epsilon_t \sim N(0,1)$ and $v_t \sim N(0,\sigma_t)$. The observed consumption y_t is such that (Blundell and Meghir)

$$y_t = y_t^*/Pr(s_t^* > 0) \quad \text{if } y_t^* > 0 \text{ and } s_t^* > 0$$
$$= 0 \quad \text{otherwise.}$$

Thus, similar to the double-hurdle model, a zero observation occurs if the household does not purchase or does not consume. The sample likelihood function for the infrequency-of-purchase model is (Blundell and Meghir, Table 1).

$$L = \prod_{y_t=0} [1 - \Phi(x_t\theta)\Phi(x_t\beta/\sigma_t)] \times \prod_{y_t>0} [\Phi(x_t\theta)]^2\sigma_t^{-1}\phi[(\Phi(x_t\theta)y_t - x_t\beta)/\sigma_t].$$

Table A1. ML Estimates of the Infrequency-of-Purchase Model[a]

	With Heteroscedastic Errors			With Homoscedastic Errors		
Variable	Particip.	Consump.	Het.	Particip.	Consump.	Het.
Constant	−0.912***	0.135	−0.795***	−0.200	−0.642**	0.242
	(0.120)	(0.126)	(0.182)	(0.159)	(0.269)	(0.149)
Size	0.143***	0.031	0.197***	0.011	0.172***	
	(0.021)	(0.023)	(0.030)	(0.011)	(0.028)	
Income	0.001	0.010***		0.001	0.014***	
	(0.002)	(0.003)		(0.002)	(0.005)	
Education	−0.005	0.010		−0.001	0.008	
	(0.011)	(0.015)		(0.014)	(0.023)	
Head ≥65	0.006	−0.052		0.005	−0.078	
	(0.036)	(0.053)		(0.052)	(0.082)	
Head ≤30	0.011	−0.284***		0.041	−0.399***	
	(0.039)	(0.057)		(0.054)	(0.087)	
Northeast	0.084*	0.074		0.082	0.103	
	(0.045)	(0.070)		(0.060)	(0.101)	
Midwest	0.034	−0.119*		0.008	−0.161	
	(0.046)	(0.070)		(0.060)	(0.101)	
South	−0.064	−0.210***		−0.068	−0.325***	
	(0.046)	(0.073)		(0.061)	(0.107)	
West	−0.044	−0.114		−0.048	−0.194*	
	(0.046)	(0.073)		(0.061)	(0.105)	
Non-black	0.085*	0.014		0.052	0.057	
	(0.047)	(0.070)		(0.063)	(0.102)	
Spring	0.006	−0.117**		−0.026	−0.150**	
	(0.034)	(0.051)		(0.044)	(0.076)	
Summer	−0.019	−0.130***		−0.007	−0.195***	
	(0.033)	(0.050)		(0.045)	(0.074)	
Fall	0.083**	−0.145***		0.121**	−0.224***	
	(0.036)	(0.053)		(0.050)	(0.078)	
Year 1990	0.020	0.055		0.022	0.062	
	(0.028)	(0.041)		(0.038)	(0.060)	
LLF	−6256.442			−6273.092		
$\hat{z}$[b]	1.015			0.368		

[a]Asterisks indicate levels of significance: *** = 0.01, ** = 0.05, * = 0.10.
[b]Test statistics for testing the double-hurdle model against the infrequency-of-purchase model; $\hat{z}$ is asymptotically $N(0,1)$.

Nonnested Model Specification Test

Denote the log-likelihood function of the double-hurdle model and the infrequency-of-purchase model (or whatever competing model) as, respectively,

$$\log L(\delta) = \sum_{t=1}^{n} \log f(y_t \mid x_t, w_t; \delta),$$

$$\log L(\tau) = \sum_{t=1}^{n} \log g(y_t \mid x_t, w_t; \tau),$$

where δ and τ are the parameter vectors characterizing the two models, and n is the sample size. Let $\hat{\delta}$ and $\hat{\tau}$ be the ML estimators of δ and τ, respectively, and define the statistics

$$\hat{\omega}^2 = \frac{1}{n} \sum_{t=1}^{n} \left\{ \log \left[\frac{f(y_t \mid x_t, w_t; \hat{\delta})}{g(y_t \mid x_t, w_t; \hat{\tau})} \right] \right\}^2 - \left\{ \frac{1}{n} \sum_{t=1}^{n} \log \left[\frac{f(y_t \mid x_t, w_t; \hat{\delta})}{g(y_t \mid x_t, w_t; \hat{\tau})} \right] \right\}^2$$

$$LR(\hat{\delta}, \hat{\tau}) = \log L(\hat{\delta}) - \log L(\hat{\tau}) = \sum_{t=1}^{n} \log \left[\frac{f(y_t \mid x_t, w_t; \hat{\delta})}{g(y_t \mid x_t w_t; \hat{\tau})} \right].$$

Vuong suggests that, under the null hypothesis that there is no difference between the two models, the following statistic is asymptotically $N(0,1)$:

$$\hat{z} = \frac{LR(\hat{\delta}, \hat{\tau})}{\sqrt{n}\, \hat{\omega}}.$$

A positive value of $\hat{z}$(e.g., $\hat{z} > Z_{\alpha/2}$) where α is the level of significance) would suggest that the double-hurdle model is preferred to the infrequency-of-purchase model, whereas a negative value of $\hat{z}$ (e.g., $\hat{z} < -Z_{\alpha/2}$) would suggest the reverse. When $|\hat{z}| \leq Z_{\alpha/2}$, there is no difference between the two models.

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