The Relative Effects of Climate Change on Agricultural Production with Priority or Share-based Water Rights

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1 Introduction

Average global temperatures are predicted to increase over the next century, causing shifts in the amount of precipitation at high elevations falling as rain rather than as snow (IPCC, 2014). This shift leads to smaller snowpacks at higher elevations, as well as seasonal shifts in surface water flows with mountain snowpacks melting earlier to result in lower flow rates later in the growing season (Moursi et al., 2017). Agriculture in many arid and semi-arid regions depends on runoff from melting mountain snowpacks for crop irrigation and livestock consumption (Leonard & Libecap, 2019). Because crop watering requirements are highest during growing seasons, a shift in the sub-annual timing of water availability could reduce crop yields or render certain crops nonviable in certain regions (Qin et al., 2020). The effects of earlier snowmelt can therefore be compared to that of a drought even if the total annual water supply remains unchanged.

Winter wet-day minimum temperatures have increased by 1.1 degrees Celsius in the western U.S. between 1920 and 2004 (Knowles et al., 2006). Accompanying these warmer temperatures, snowpacks have begun melting as much as four weeks sooner (Stewart et al., 2005), and a decrease in the percentage of precipitation falling as snow (Knowles et al., 2006). Over the 21st century, the Intergovernmental Panel on Climate Change predicts that temperatures will continue to increase globally (IPCC, 2014) and is expected to cause snowpacks to melt by a further 30 to 40 days earlier in the year (Stewart et al., 2004). Furthermore, Berghuijs et al. (2014) find evidence that the percentage of precipitation falling as snow and the average annual streamflow in the U.S. are positively correlated. This indicates that we should expect streams to supply less water to downstream users, including crop producers, in the near future especially in areas that depend heavily on snowmelt.

In most of the western U.S., where many snowmelt dependent water basins are located, surface water is allocated to users under the Prior Appropriation doctrine, a priority based water rights system which grants the right to use water based on the priority order associated with each water right determined by the date of first beneficial use, known as "first in time, first in right" (Libecap, 2007). This system arose due to the scarcity of surface water in the West which made riparian water rights, common in the eastern U.S., impractical for agricultural irrigation and use in early mining operations which often required water to be conveyed over long distances. During droughts under priority based water rights, water allocations are reduced to account for the reduction in the water supply beginning with the most junior rights holder and proceeding to more senior right holders until the amount of water allocated matches the anticipated water supply. Due to the heterogeneous risk among water users, Prior Appropriation may lead to an inefficient allocation of water under certain conditions (Burness & Quirk, 1979).

In many areas, farmers have formed cooperative organizations called Irrigation Districts which allow members to pool their priority based water rights and subsequently distribute water proportionally to all members, creating a share based water rights institution (Ji & Cobourn, 2018). According to Ji and Cobourn (2018), these organizations reduce risk to their members due to the diverse set of water rights they hold, spread risk among all members, and can facilitate water transfers among members with lower transaction costs. In a drought, farmers with proportional share water rights would have all of their allocations reduced proportionally to the shares they own thereby reducing and spreading the risk of drought.

The well-known work by Burness and Quirk (1979) found that proportional share water rights are optimal and priority water rights are inefficient under the specific condition that firms have identical production functions and assuming that water availability only varies at the annual level, but has not been expanded upon to explore complications such as sub-annual streamflow variation, uncertainty of streamflow, incomplete information, introduction of reservoirs, and others. We suspect that the variability of sub-annual streamflow and heterogeneity in firms' abilities to produce with water at different times of year can result in different relative efficiency between Prior Appropriation and Proportional Shares than that found by Burness and Quirk (1979). Burness and Quirk (1979) also find that proportional shares can be achieved under Prior Appropriation if there are efficient markets for water right and advocate for the easing of restriction on water right trading. Recent work has identified water basins worldwide that are most at risk of having water supplies fall short of meeting demand based on projected climate scenarios and hydrologic data (Mankin et al., 2015; Qin et al., 2020). Research on the economic effects of within year seasonal timing of snowmelt (even without appreciable changes in annual precipitation) does not currently exist.

To our knowledge, there has been no research on how changes in the timing of snowmelt may affect water users under different water rights institutions. We begin to fill this gap in the literature by presenting a theoretical economic model of agricultural production with heterogeneous firms subject to a change in sub-annual timing of water availability to demonstrate factors that influence the welfare impacts of a change in snowmelt timing in the context of priority based and share based water rights as a function of output prices, priority order, and other factors. We explore how heterogeneity of marginal revenue and relative intensity of seasonal water use leads to further heterogeneity in the effects of changes in seasonal water availability and the decision to sell water rights away from agricultural production to municipal and industrial use. Our work differs from the research by Burness and Quirk (1979) in that we do not consider the risk to farmers of not receiving their full allocation of water, but rather the risk of earlier sub-annual availability of water holding the quantity of water received constant. The results will answer the questions: does early snowmelt as a result of warming cause a greater loss of welfare under priority based or share based water rights?

We find that when we relax the assumption that firms are homogeneous there are conditions in which losses to agricultural profits due to earlier snowmelt are greater under Proportional Sharing than Prior Appropriation. We also find conditions which result in greater aggregate profits under Prior Appropriation than Proportional Sharing. Specifically, these results occur when the shift toward earlier snowmelt is small and either the senior firm's production is relatively more intensive in early season water use, or their output price is significantly higher than the junior firm's. However, when firms are assumed to be homogeneous, our model does produce results consistent with past research.

2 The Model

We propose that the relative efficiency of priority based and share based water rights depends on the randomness of snowmelt timing, the relative intensity of sub-annual water inputs to agricultural production, and the relative prices of outputs between firms. We consider the case of two agricultural firms using water from a stream with a normal annual amount of water $\bar{w}$ which is fed by snowmelt and is the only source of water. Each firm i owns an existing right to use a portion of the water in the stream $\hat{w}_i$ where $\hat{w}_A = \psi\bar{w}$ and $\hat{w}_B = (1-\psi)\bar{w}$ per year. The water source is fully allocated such that the total normal annual water supply $\bar{w} = \hat{w}_A + \hat{w}_B$. There are no reservoirs or other water storage methods available to the two firms and they are not able to trade water between each other. There are two seasons corresponding to the firms' planting (1) and growing (2) seasons which they can choose to use a portion of their annual water allocation. However the quantity $\bar{x} = \rho\bar{w}$ and $\bar{y} = (1-\rho)\bar{w}$, is available for use in seasons 1 and 2 respectively which constrains the firms' abilities to spread their allocations across seasons. For simplicity, we assume that crops are harvested immediately following the growing season and thus no water is required afterward.

Suppose that, as a result of climate change, there is a possibility that snowpack supplying water to the stream melts earlier in the year resulting in a shift in seasonal water availability such that more water is available for use in the first season $(1+\phi)\bar{x}$ and less in the second $(1-\phi)\bar{y}$ where ϕ is the percentage of water shifted from season 2 to season 1. We assume that probability of early snowmelt is θ and the probability of normal snowmelt is $1-\theta$. The total amount of water and the distribution of it across the two seasons is revealed to both firms prior to making their production decisions. Assuming their production functions are well behaved, then an increase in season 1 water will decrease the marginal productivity of season 1 water, and the corresponding decrease in season 2 water will increase its marginal productivity. Based on the past assumption that the choices of the firms do not affect input or output prices, we know that the ratio of input prices will not change. However, the ratio of marginal products of the seasonal water inputs has changed, and therefore cannot be optimal.

We treat the firms' watering decisions as a static problem for a given year based on snowpack measurements made prior to the planting season. The firms do not choose how much water to use in total, but rather they choose how much of their total water allocation to use in each season. In this way, we isolate the effect of a shift in the timing of water availability from the effect of water shortage. Let x_i and y_i be the amount of water that firm i chooses to use in season 1 and 2, respectively. We will assume that the production of firm A is $x_A^\alpha y_A^\beta$ and the production of firm B is $x_B^\delta y_B^\gamma$ with both output prices set to 1. This functional form reflects our assumptions that water used in each season is complementary in crop production. To begin, we do not assume that the production functions are identical, but later explore some different possible parameter values including the case where they are identical.

2.1 Case 1: Prior Appropriation

Firm A is a senior rights holder with a right to $\hat{w}_A = \psi\bar{w}$ units of water annually. Firm B is a junior rights holder with a right to $\hat{w}_B = (1-\psi)\bar{w}$ units of water annually.

2.1.1 Normal Streamflow

Under the normal streamflow condition firm A chooses x_A, y_A to solve:

$$\underset{x_A, y_A}{\arg\max} \quad x_A^{\alpha} y_A^{\beta} \tag{1}$$

$$s.t. \;\; x_A + y_A \leq \hat{w}_A \tag{2}$$

$$x_A \leq \bar{x} \tag{3}$$

$$y_A \leq \bar{y} \tag{4}$$

which results in the following optimal water use for firm A:

$$x_{A,N,P} \equiv x_A^* = \frac{\alpha}{\alpha + \beta} \hat{w}_A \tag{5}$$

$$y_{A,N,P} \equiv y_A^* = \frac{\beta}{\alpha + \beta} \hat{w}_A \tag{6}$$

Firm B chooses x_B, y_B to solve:

$$\underset{x_B, y_B}{\arg\max} \quad x_B^{\delta} y_B^{\gamma} \tag{7}$$

$$s.t. \;\; x_B + y_B \leq \hat{w}_B \tag{8}$$

$$x_B \leq \bar{x} - x_A \tag{9}$$

$$y_B \leq \bar{y} - y_A \tag{10}$$

If the normal streamflow is fully allocated such that $x_A + x_B = \bar{x}$ and $y_A + y_B = \bar{y}$, then firm B's second and third constraints bind and they simply use the remaining water in each season:

$$y_{B,N,P} \equiv x_B^* = \bar{x} - x_{A,N} = \bar{x} - \frac{\alpha}{\alpha+\beta}\hat{w}_A \tag{11}$$

$$y_{B,N,P} \equiv y_B^* = \bar{y} - y_{A,N} = \bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A \tag{12}$$

2.1.2 Early Streamflow

Under the early streamflow condition firm A chooses x_A, y_A to solve:

$$\underset{x_A, y_A}{\arg\max} \quad x_A^{\alpha} y_A^{\beta} \tag{13}$$

$$s.t. \;\; x_A + y_A \leq \hat{w}_A \tag{14}$$

$$x_A \leq (1+\phi)\bar{x} \tag{15}$$

$$y_A \leq (1-\phi)\bar{y} \tag{16}$$

Under the assumption that each firm is allocated half of normal streamflow, so long as $\phi < 0.5$ firm A will not be constrained by the decrease in season 2 water and its optimal usage will not change.

$$x_{A,E,P} \equiv x_A^* = \frac{\alpha}{\alpha+\beta}\hat{w}_A \tag{17}$$

$$y_{A,E,P} \equiv y_A^* = \frac{\beta}{\alpha+\beta}\hat{w}_A \tag{18}$$

If we allow $\phi \geq 0.5$ then firm A uses all of the season 2 water and whatever remains of its allocation is used in season 1.

$$x_{A,E,P} \equiv x_A^* = \hat{w}_A - (1-\phi)\bar{y} \tag{19}$$

$$y_{A,E,P} \equiv y_A^* = (1-\phi)\bar{y} \tag{20}$$

Firm B now chooses x_B, y_B to solve:

$$\underset{x_B, y_B}{\arg\max} \quad x_B^\delta y_B^\gamma \tag{21}$$

$$s.t. \;\; x_B + y_B \leq \hat{w}_B \tag{22}$$

$$x_B \leq (1+\phi)\bar{x} - x_A \tag{23}$$

$$y_B \leq (1-\phi)\bar{y} - y_A \tag{24}$$

Again assuming full allocation, firm B's first and third constraints bind and they use whatever amount of water that firm A doesn't use in season 2 and whatever remains of their own allocation in season 1:

$$x_{B,E,P} \equiv x_B^* = \hat{w}_B - x_A^* = \hat{w}_B - [(1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A] \tag{25}$$

$$y_{B,E,P} \equiv y_B^* = (1-\phi)\bar{y} - y_A^* = (1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A \tag{26}$$

If $\phi \geq 0.5$, then firm B will receive no water in season 2 and use as much of it's allocation in season 1 as it can.

$$x_{B,E,P} \equiv x_B^* = \hat{w}_B - x_A^* = \hat{w}_B - (\hat{w}_A - (1-\phi)\bar{y}) \tag{27}$$

$$y_{B,E,P} \equiv y_B^* = (1-\phi)\bar{y} - y_A^* = 0 \tag{28}$$

2.1.3 Expected Welfare

Firm A's expected profits when $\phi < 0.5$ are:

$$
\begin{aligned}
E(\pi_{A,P}) &= (1-\theta)x_{A,N,P}^{\alpha}y_{A,N,P}^{\beta} + \theta x_{A,E,P}^{\alpha}y_{A,E,P}^{\beta} \\
&= \left[\frac{\alpha}{\alpha+\beta}\right]^{\alpha}\left[\frac{\beta}{\alpha+\beta}\right]^{\beta}\hat{w}_A^{\alpha+\beta}
\end{aligned} \tag{29}
$$

Firm B's expected Profits are:

$$
\begin{aligned}
E(\pi_{B,P}) &= (1-\theta)x_{B,N,P}^{\delta}y_{B,N,P}^{\gamma} + \theta x_{B,E,P}^{\delta}y_{B,E}^{\gamma} \\
&= (1-\theta)\left[\bar{x} - \frac{\alpha}{\alpha+\beta}\hat{w}_A\right]^{\delta}\left[\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A\right]^{\gamma} \\
&\quad + \theta\left[\hat{w}_B - [(1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A]\right]^{\delta}\left[(1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A\right]^{\gamma}
\end{aligned} \tag{30}
$$

Total expected welfare under priority ordering $E(W_P)$ is:

$$
E(W_P) = E(\pi_{A,P}) + E(\pi_{B,P}) \tag{31}
$$

It is clear, given the complementary nature of seasonal water in this production function, that in the case of $\phi \geq 0.5$ firm B's profit is zero.

Differentiating $E(W_P)$ with respect to ϕ yields the change in welfare as a result of an increase (decrease) in the proportion of water flowing in season 1 (2):

$$\frac{\partial E(W_P)}{\partial \phi} = \theta\bar{y}(\delta x^{\delta-1}_{B,E,P} y^{\gamma}_{B,E,P} - \gamma x^{\delta}_{B,E,P} y^{\gamma-1}_{B,E,P}) \tag{32}$$

$$= \theta\bar{y} x^{\delta}_{B,E,P} y^{\gamma}_{B,E,P} \left(\frac{\delta}{\hat{w}_B - [(1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A]} - \frac{\gamma}{(1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A} \right) \tag{33}$$

We can use the fact that $\hat{w}_A$ and $\hat{w}_B$ can be expressed as some percentage ψ and $1-\psi$ of $\bar{w}$, and similarly $\bar{x}$ and $\bar{y}$ as ρ and $1-\rho$ percent of $\bar{w}$ then this can be rewritten as:

$$= \theta(1-\rho)\left[(1-\psi)\bar{w} - [(1-\phi)(1-\rho)\bar{w} - \frac{\beta}{\alpha+\beta}\psi\bar{w}]\right]^{\delta} \left[(1-\phi)(1-\rho)\bar{w} - \frac{\beta}{\alpha+\beta}\psi\bar{w}\right]^{\gamma}$$
$$\left(\frac{\delta}{(1-\psi) - [(1-\phi)(1-\rho) - \frac{\beta}{\alpha+\beta}\psi]} - \frac{\gamma}{(1-\phi)(1-\rho) - \frac{\beta}{\alpha+\beta}\psi} \right) \tag{34}$$

The term in parentheses determines the sign of the derivative. Under the conditions that each firm's allocation is $\hat{w}_A = \hat{w}_B = \frac{1}{2}\bar{w}$ and normal seasonal streamflow is $\bar{x} = \bar{y} = \frac{1}{2}\bar{w}$, then $\psi = \rho = \frac{1}{2}$ and the above expression can be rewritten as:

$$= \theta\left[\frac{1}{2}\bar{w}\left(\phi + \frac{\beta}{\alpha+\beta}\right)\right]^{\delta}\left[\frac{1}{2}\bar{w}\left(1 - \left\{\phi + \frac{\beta}{\alpha+\beta}\right\}\right)\right]^{\gamma}\left(\frac{\delta}{\phi + \frac{\beta}{\alpha+\beta}} - \frac{\gamma}{1 - \left(\phi + \frac{\beta}{\alpha+\beta}\right)}\right) \tag{35}$$

$$1 - \phi < \frac{\beta}{\alpha+\beta}$$

Now the sign of the derivative is solely determined by the difference between the ratios of firm B's output elasticities of seasonal water to the percentage of water shifted to and from each season plus firm A's relative intensity of season 2 water. If the difference is positive then $\frac{\partial E(W_P)}{\partial \phi} > 0$ and vice versa. Thus, it is theoretically possible for an increase in season 1 water to increase or decrease welfare under Prior Appropriation.

Differentiating $E(W_P)$ with respect to θ gives the change in welfare due to an increase in the probability of early streamflow occurring.

$$\frac{\partial E(W_P)}{\partial \theta} = \left[\hat{w}_B - [(1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A]\right]^\delta \left[(1-\phi)\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A\right]^\gamma$$
$$- \left[\bar{x} - \frac{\alpha}{\alpha+\beta}\hat{w}_A\right]^\delta \left[\bar{y} - \frac{\beta}{\alpha+\beta}\hat{w}_A\right]^\gamma \tag{36}$$
$$= \pi_{B,E,P} - \pi_{B,N,P} \tag{37}$$

2.2 Case 2: Proportional Shares

Now we assume that the firms cooperate to form an irrigation district which distributes water to the firms proportionally. In this case, firm A is entitled to q percent of the water available in each season and firm B is entitled to $1-q$ percent.

2.2.1 Normal Streamflow

Under the normal streamflow condition firm A receives:

$$x_{A,N,S} = q\bar{x} \tag{38}$$
$$y_{A,N,S} = q\bar{y} \tag{39}$$

while firm B receives:

$$x_{B,N,S} = (1-q)\bar{x} \tag{40}$$
$$y_{B,N,S} = (1-q)\bar{y} \tag{41}$$

Due to the proportional sharing rule each firm gets an equal share of water.

2.2.2 Early Streamflow

Under the early streamflow condition firm A receives:

$$x_{A,E,S} = q(1+\phi)\bar{x} \tag{42}$$

$$y_{A,E,S} = q(1-\phi)\bar{y} \tag{43}$$

while firm B receives:

$$x_{B,E,S} = (1-q)(1+\phi)\bar{x} \tag{44}$$

$$y_{B,E,S} = (1-q)(1-\phi)\bar{y} \tag{45}$$

2.2.3 Expected Welfare

Each firm's expected profits under proportional shares is:

$$E(\pi_{A,S}) = (1-\theta)\left[(q\bar{x})^{\alpha}(q\bar{y})^{\beta}\right] + \theta\left[(q(1+\phi)\bar{x})^{\alpha}(q(1-\phi)\bar{y})^{\beta}\right] \tag{46}$$

$$E(\pi_{B,S}) = (1-\theta)\left[((1-q)\bar{x})^{\delta}((1-q)\bar{y})^{\gamma}\right] + \theta\left[((1-q)(1+\phi)\bar{x})^{\delta}((1-q)(1-\phi)\bar{y})^{\gamma}\right] \tag{47}$$

Expected welfare under proportional shares $E(W_S)$ is:

$$\begin{aligned} E(W_S) = (1-\theta)&\left[(q\bar{x})^{\alpha}(q\bar{y})^{\beta} + ((1-q)\bar{x})^{\delta}((1-q)\bar{y})^{\gamma}\right] \\ &+ \theta\left[(q(1+\phi)\bar{x})^{\alpha}(q(1-\phi)\bar{y})^{\beta} + ((1-q)(1+\phi)\bar{x})^{\delta}((1-q)(1-\phi)\bar{y})^{\gamma}\right] \end{aligned} \tag{48}$$

Differentiating $E(W_S)$ with respect to ϕ and substituting $\bar{x} = \rho\bar{w}$ and $\bar{y} = (1-\rho)\bar{w}$ yields the change in welfare as a result of an increase (decrease) in the proportion of water flowing in season 1 (2).

$$\frac{\partial E(W_S)}{\partial \phi} = \theta \left[[q(1+\phi)\rho\bar{w}]^{\alpha}[q(1-\phi)(1-\rho)\bar{w}]^{\beta} \left(\frac{\alpha}{1+\phi} - \frac{\beta}{1-\phi} \right) \right. \\ \left. + [(1-q)(1+\phi)\rho\bar{w}]^{\delta}[(1-q)(1-\phi)(1-\rho)\bar{w}]^{\gamma} \left(\frac{\delta}{1+\phi} - \frac{\gamma}{1-\phi} \right) \right] \tag{49}$$

Assuming that $q = \rho = \frac{1}{2}$:

$$= \theta \left\{ \left[\frac{1}{4}(1+\phi)\bar{w} \right]^{\alpha} \left[\frac{1}{4}(1-\phi)\bar{w} \right]^{\beta} \left(\frac{\alpha}{1+\phi} - \frac{\beta}{1-\phi} \right) \right. \\ \left. + \left[\frac{1}{4}(1+\phi)\bar{w} \right]^{\delta} \left[\frac{1}{4}(1-\phi)\bar{w} \right]^{\gamma} \left(\frac{\delta}{1+\phi} - \frac{\gamma}{1-\phi} \right) \right\} \tag{50}$$

The two terms in parentheses are the differences between each firm's output elasticity of season 1 and 2 water relative to the percentage of water in each season respectively. If a firm's output elasticity of seasonal water relative to the percentage of seasonal water is the same across seasons then the change in that firm's contribution to total welfare due to a change in ϕ will be zero. However, as with the case of Prior Appropriation, the sign of ${}^{\partial E(W_S)}/_{\partial \phi}$ is ambiguous without further assumptions regarding the values of the production parameters.

Differentiating $E(W_S)$ with respect to θ gives the change in welfare due to a increase in the probability of early streamflow occurring.

$$\frac{\partial E(W_S)}{\partial \theta} = \left[q^{\alpha+\beta}\{(1+\phi)\bar{x}\}^{\alpha}\{(1-\phi)\bar{y}\}^{\beta} + (1-q)^{\delta+\gamma}\{(1+\phi)\bar{x}\}^{\delta}\{(1-\phi)\bar{y}\}^{\gamma}\right]$$
$$- \left[q^{\alpha+\beta}\bar{x}^{\alpha}\bar{y}^{\beta} + (1-q)^{\delta+\gamma}\bar{x}^{\delta}\bar{y}^{\gamma}\right] \quad (51)$$

$$= \pi_{B,E,S} - \pi_{B,N,S} < 0 \quad (52)$$

2.3 Comparison

To evaluate under which allocation system shifting water from season 2 to season 1 has a larger negative effect on welfare, we can take the difference of the derivatives of the two welfare functions W_P and W_S with respect to ϕ. Assuming $0 < \theta, \phi, q < 1$:

$$D = \frac{\partial E(W_P)}{\partial \phi} - \frac{\partial E(W_S)}{\partial \phi} \quad (53)$$
$$= \theta\left\{\bar{y}x^{\delta}_{B,E,P}y^{\gamma}_{B,E,P}\left(\frac{\delta}{x_{B,E,P}} - \frac{\gamma}{y_{B,E,P}}\right)\right.$$
$$\left.- \left[x^{\alpha}_{A,E,S}y^{\beta}_{A,E,S}\left(\frac{\alpha}{1+\phi} - \frac{\beta}{1-\phi}\right) + x^{\delta}_{B,E,S}y^{\gamma}_{B,E,S}\left(\frac{\delta}{1+\phi} - \frac{\gamma}{1-\phi}\right)\right]\right\} \quad (54)$$

Using the assumptions that $\psi = \rho = \frac{1}{2}$ this can be written as:

$$= \theta\left\{x^{\delta}_{B,E,P}y^{\gamma}_{B,E,P}\left(\frac{\delta}{\phi + \frac{\beta}{\alpha+\beta}} - \frac{\gamma}{1 - \left(\phi + \frac{\beta}{\alpha+\beta}\right)}\right)\right.$$
$$\left.- \left[x^{\alpha}_{A,E,S}y^{\beta}_{A,E,S}\left(\frac{\alpha}{1+\phi} - \frac{\beta}{1-\phi}\right) + x^{\delta}_{B,E,S}y^{\gamma}_{B,E,S}\left(\frac{\delta}{1+\phi} - \frac{\gamma}{1-\phi}\right)\right]\right\} \quad (55)$$

Substituting the optimal values of seasonal water use and simplifying, D becomes:

$$D = \theta \Bigg\{ \left(\frac{\bar{w}}{2}\right)^{\delta+\gamma} \left(\phi + \frac{\beta}{\alpha+\beta}\right)^{\delta} \left(1 - \left(\phi + \frac{\beta}{\alpha+\beta}\right)\right)^{\gamma} \left(\frac{\delta}{\phi + \frac{\beta}{\alpha+\beta}} - \frac{\gamma}{1 - \left(\phi + \frac{\beta}{\alpha+\beta}\right)} \right)$$
$$- \left[\left(\frac{\bar{w}}{4}\right)^{\alpha+\beta} (1+\phi)^{\alpha}(1-\phi)^{\beta} \left(\frac{\alpha}{1+\phi} - \frac{\beta}{1-\phi} \right) + \left(\frac{\bar{w}}{4}\right)^{\delta+\gamma} (1+\phi)^{\delta}(1-\phi)^{\gamma} \left(\frac{\delta}{1+\phi} - \frac{\gamma}{1-\phi} \right) \right] \Bigg\} \tag{56}$$

If we simplify further by assuming constant returns to scale for both firms, then this becomes:

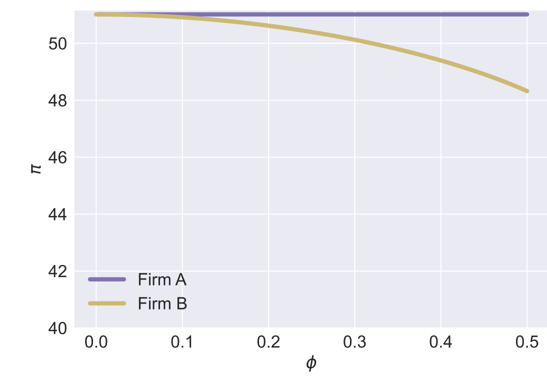
$$= \theta \frac{\bar{w}}{2} \Bigg\{ (\phi+\beta)^{\delta} (1-(\phi+\beta))^{\gamma} \left(\frac{\delta}{\phi+\beta} - \frac{\gamma}{1-(\phi+\beta)} \right)$$
$$- \frac{1}{2} \left[(1+\phi)^{\alpha}(1-\phi)^{\beta} \left(\frac{\alpha}{1+\phi} - \frac{\beta}{1-\phi} \right) + (1+\phi)^{\delta}(1-\phi)^{\gamma} \left(\frac{\delta}{1+\phi} - \frac{\gamma}{1-\phi} \right) \right] \Bigg\} \tag{57}$$

The sign of equation 57 is again ambiguous without making assumptions about the production parameters of the two firms. We are most interested in exploring the implications of the model under different heterogeneity conditions of the two firms. There are theoretically possible parameter values that could result in $D > 0$, and even in higher welfare under Prior Appropriation than Proportional Shares.

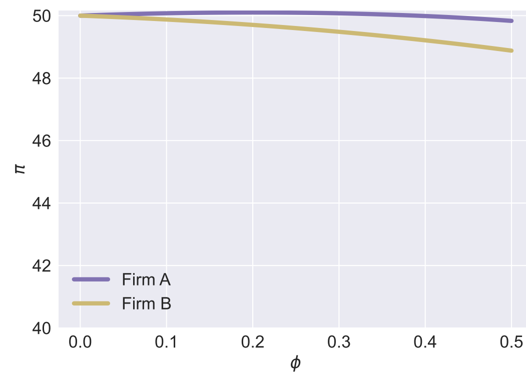
To illustrate the possible outcomes presented by equation 57 we consider three cases of different assumptions. Here we present selected simulation results of the three cases and leave full simulation results to the appendix. First, we can alter our assumption regarding the relative intensity of seasonal water use by assuming, for example, that firm A's production is more relatively intensive in season 1 water (i.e. $\alpha > \beta$). This would make $W_P > W_S \mid \phi < 0.5$. A numerical example is shown in Figure 1 in the case where $\alpha = 0.6$ and $\beta = 0.4$.

Second, if we allow the price of firm A to be different from firm B's, then there are sufficiently large values of the relative price of the two outputs ${}^{p_A}/_{p_B}$ that could make $D > 0$ and $W_P > W_S$. In Figure 2, the case of ${}^{p_A}/_{p_B} = 5$ is shown.

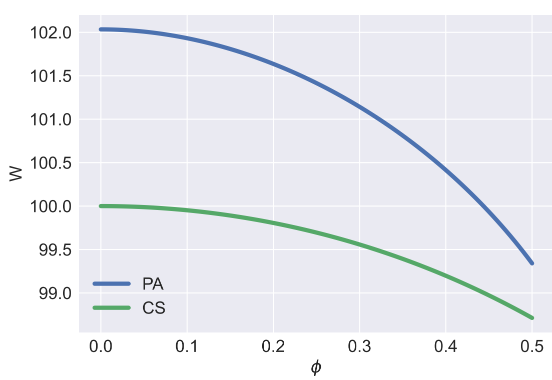
Figure 1: Case 1



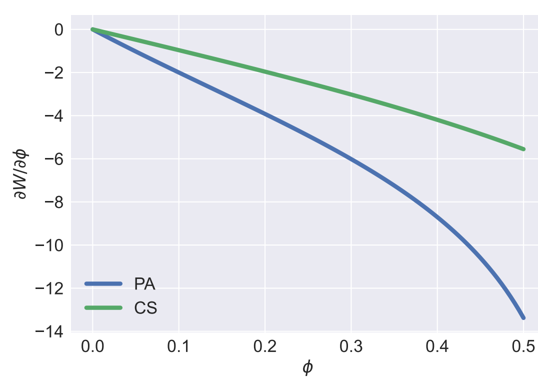
(a) Profit under Prior Appropriation



(b) Profit under Proportional Sharing



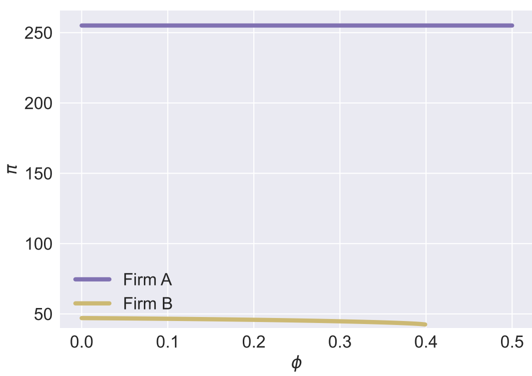
(c) Total Welfare under PA and PS



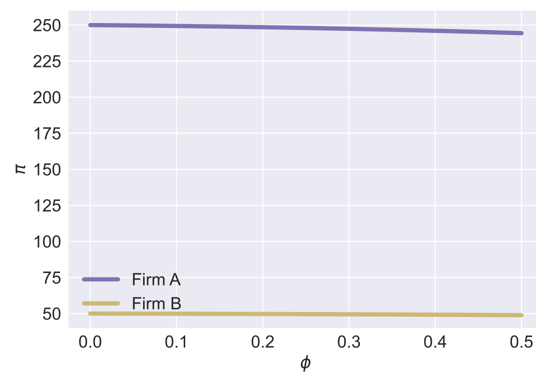
(d) $\partial W/\partial\phi$ under PA and PS

Note: $\bar{w} = 200$, $\alpha = 0.6$, $\beta = 0.4$, $\delta = 0.4$, $\gamma = 0.6$, $\theta = 0.1$.

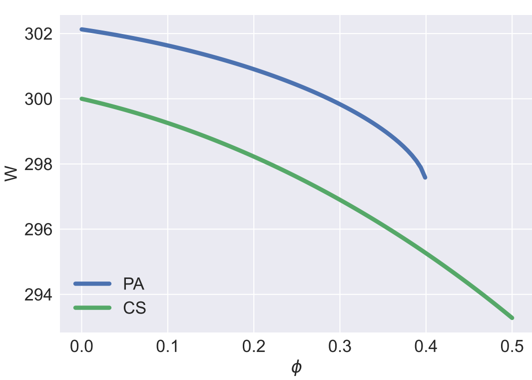
Figure 2: Case 2



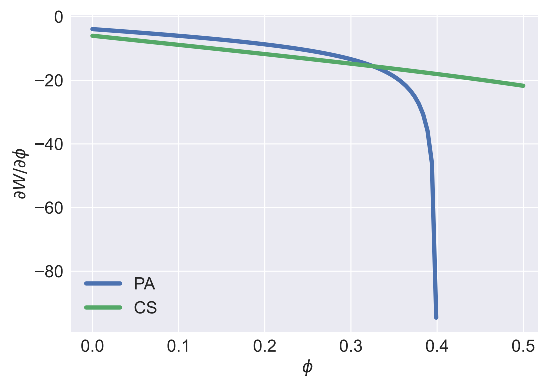
(a) Profit under Prior Appropriation



(b) Profit under Proportional Sharing



(c) Total Welfare under PA and PS



(d) $\partial W/\partial \phi$ under PA and PS

Note: $\bar{w} = 200$, $\alpha = 0.4$, $\beta = 0.6$, $\delta = 0.4$, $\gamma = 0.6$, $\theta = 0.1$, $p_a = 5$, $p_b = 1$.

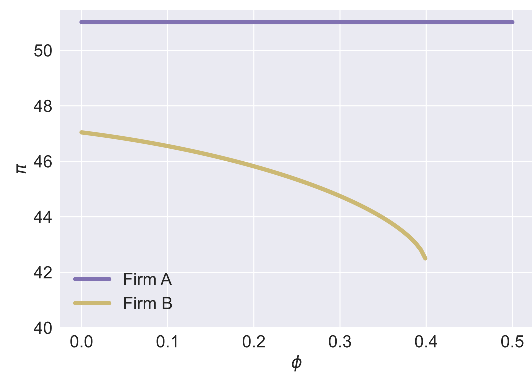
The case where firm A needs more season 1 water than season 2 is exemplified by a farm that adopts a crop with an earlier growing season while other firms do not. This could become a more common occurrence as farmers observe earlier streamflows and try to adapt their production practices to a perceived "new normal". The case where firm A has a substantially greater output price can easily be exemplified by a firm with a much more valuable crop than another firm. These results clearly show that with reasonable assumptions outcomes where Prior Appropriation obtains greater welfare than Proportional Sharing for agricultural producers are possible. Thus, Proportional Sharing may not be a universally superior water allocation institution than Prior Appropriation.

If instead we make the assumption that the two firms are homogeneous in production then the model obtains results consistent with past research. Figure 3 shows numerical results for some of the equations derived above with homogeneous firms which require more season 2 water than season 1 water. Under Prior Appropriation, firm A is completely shielded from the affect of the shift of water toward season 1 whereas firm 2 receives less water, and makes less profit, as the percentage of water shifted from season 2 to season 1 increases. However, under the shared based allocation with equal shares, each firm's profits are equal and reduced equally as would be expected. This also leads to greater welfare under the share based system than the priority based system which agrees with the research by Burness and Quirk (1979). We also find that the shifting of water toward the earlier season decreases welfare more rapidly under priority based water rights than it does under share based.

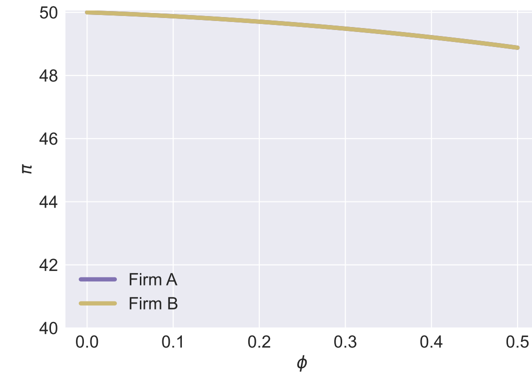
3 Conclusion

We have presented a model of agricultural production with two seasonal water inputs to demonstrate the effects of a shift in the timing of water availability on agricultural firm welfare as a result of a warming climate. In a departure from previous research, we allow firms to be heterogeneous in their output prices and relative intensity of inputs. The model

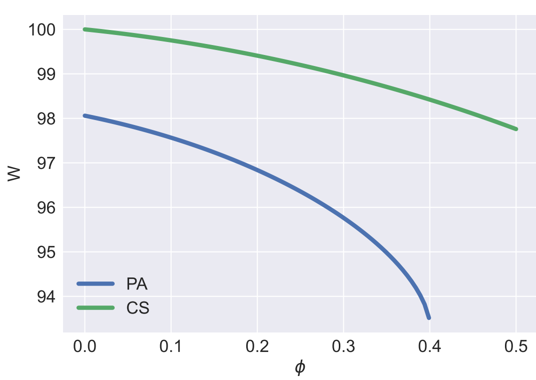
Figure 3: Case 3



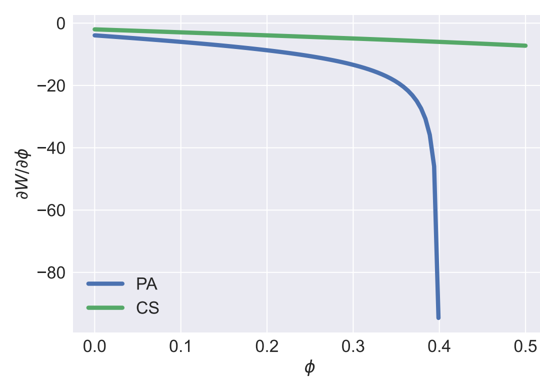
(a) Profit under Prior Appropriation



(b) Profit under Proportional Sharing



(c) Total Welfare under PA and PS



(d) $\partial W/\partial \phi$ under PA and PS

Note: $\bar{w} = 200$, $\alpha = 0.4$, $\beta = 0.6$, $\delta = 0.4$, $\gamma = 0.6$, $\theta = 0.1$, $p_A = 1$, $p_B = 1$.

shows that when (1) the senior firm is relatively intensive in season 1 water, (2) both firms are relatively intensive in season 1 water, and (3) the senior firm has a significantly higher output price a priority based water allocation mechanism such as Prior Appropriation can yield higher welfare and greater resilience to earlier water availability than a proportional shares mechanism. However, it also shows that proportional shares can be more efficient than prior appropriation in agreement with past research when similar assumptions made. Our results indicate that switching to a proportional shares system of water rights may not be a universal improvement over Prior Appropriation and that the quantity and quality of firm heterogeneity in a region should be considered when making such a judgement.

We have made many assumption in this model in the interest of analytical tractability which may limit the applicability of our findings. We have used a Cobb-Douglas production function with constant returns to scale to easily obtain analytical and numerical results, but Anderson et al. (1996) recommend that researchers perform empirical tests for appropriate functional form before selecting one. The choice of functional form also reflects our assumption that water use across seasons is complementary rather than substitutable in production which may not be true with all crops.

We have not allowed firms to use alternate water sources, when in fact agricultural producers in snowmelt dependent water basins do supplement surface water inputs with groundwater when necessary. Additionally, many large agricultural producers individually possess a portfolio of surface water rights with various seniority dates that enable them to hedge against the risk of drought and changes in snowmelt timing without joining cooperative organizations such as irrigation districts. However, with increasing drought severity groundwater reserves are already being strained and will only worsen with rising temperatures making it a less reliable alternative in a future with much earlier snowmelt. Also, The model only applies to producers in areas with no upstream storage like a reservoir which would mitigate the effects of changes in snowmelt timing by delaying any early snowmelt until it is needed albeit with some water loss due to evapotranspiration.

We plan to make further adjustments to the model to increase the applicability of it and allow it to answer additional questions. The first change will be to generalize the production functions which will remove the strict assumptions associated with the Cobb-Douglas form that the model currently uses. Second, we would like to incorporate trading of water rights to determine the effect of earlier snowmelt on the sales of water right from agriculture to municipal and industrial uses. Finally, including reservoirs with evapotranspiration loss would be an interesting modification to the model, but may require a dynamic rather than static model.

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A Figures

Figure 4: Case 1: Profits under Prior Appropriation

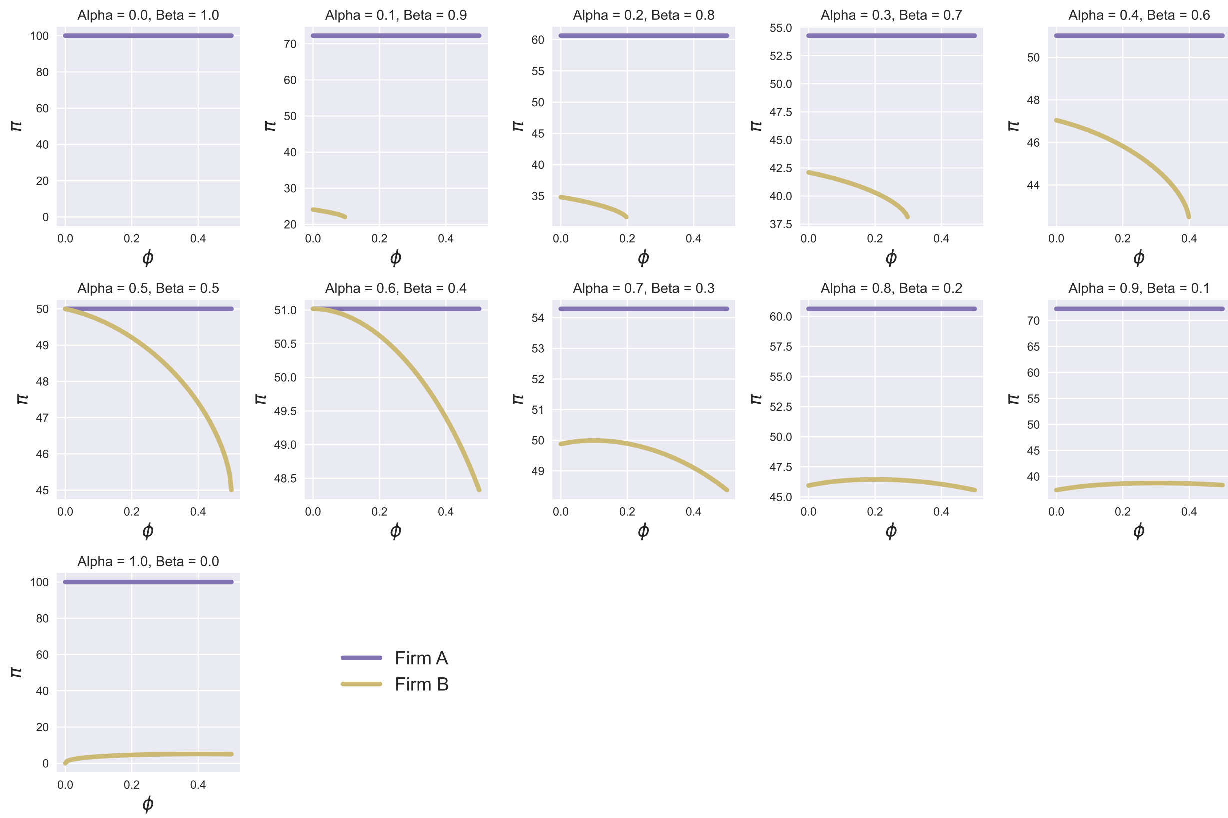

Figure 5: Case 1: Profits under Proportional Shares

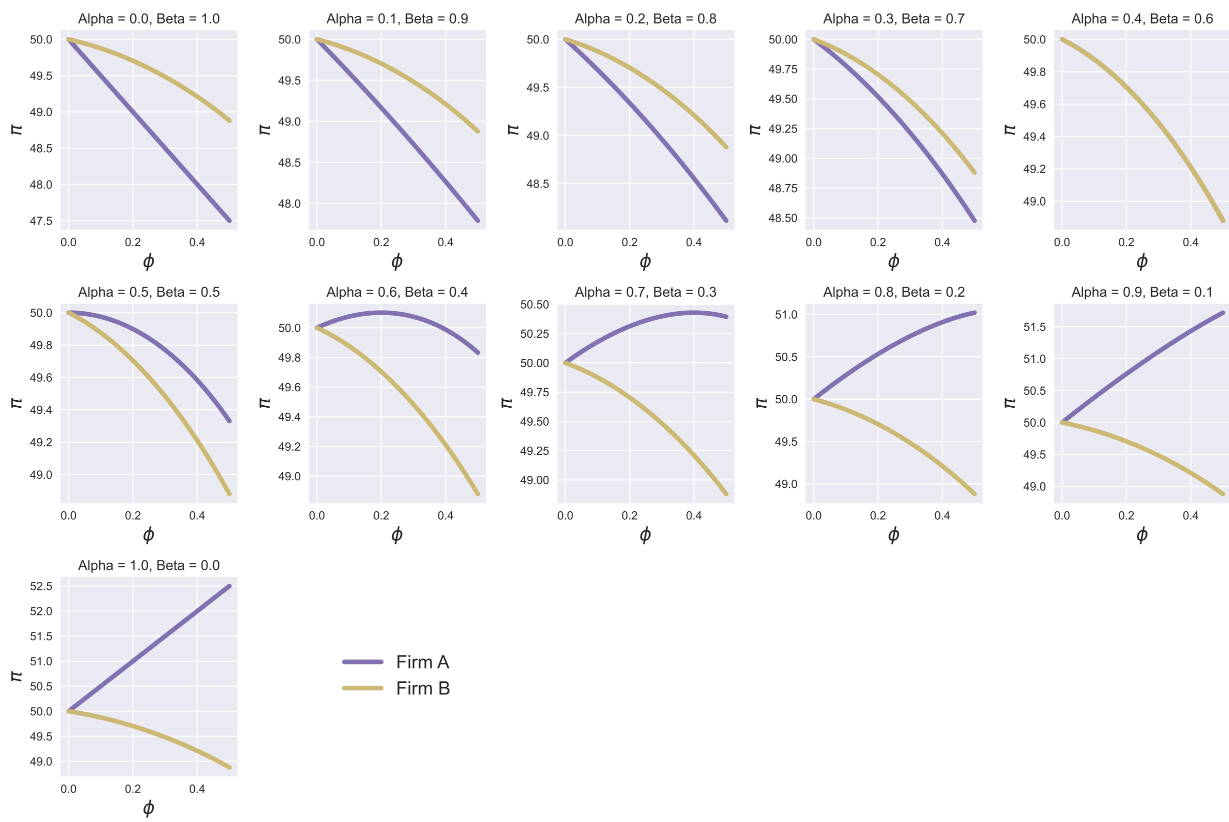

Figure 6: Case 1: Total Welfare

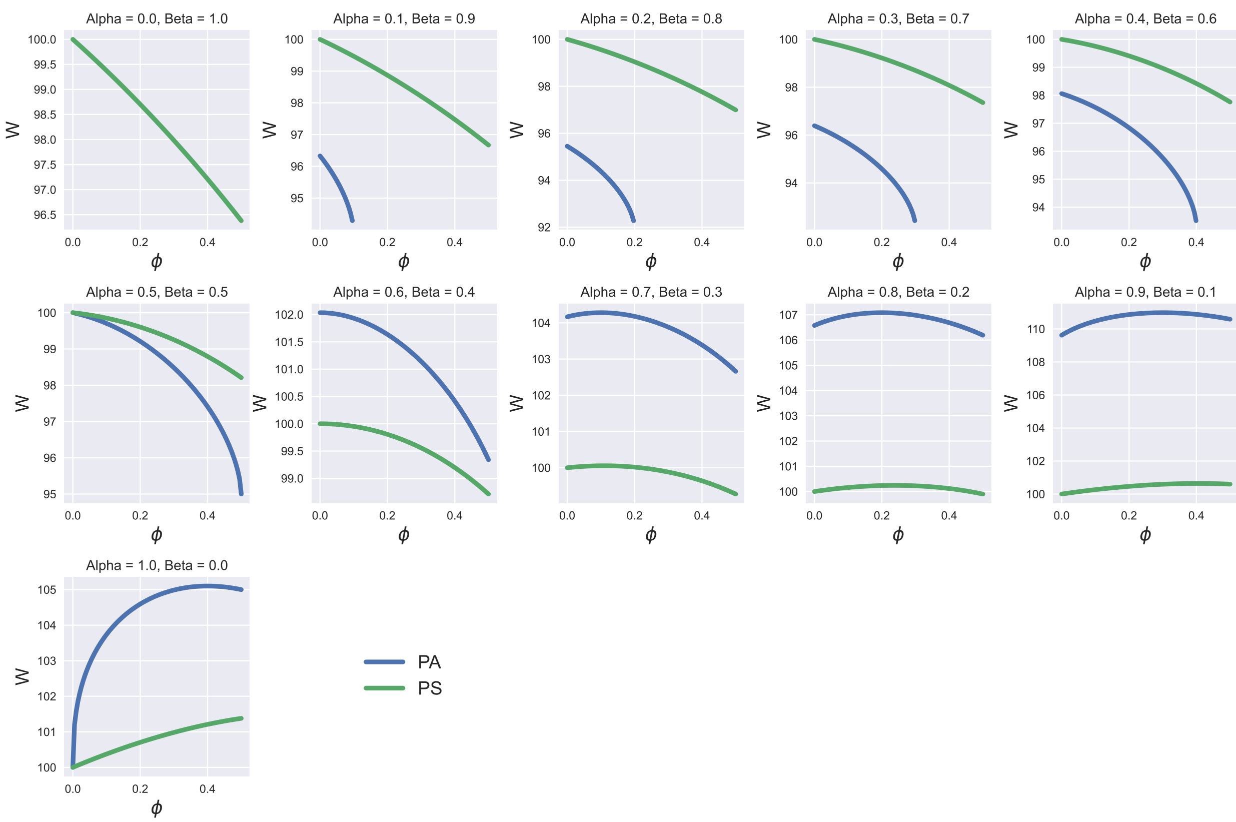

Figure 7: Case 1: Change in Total Welfare Due to Earlier Streamflow

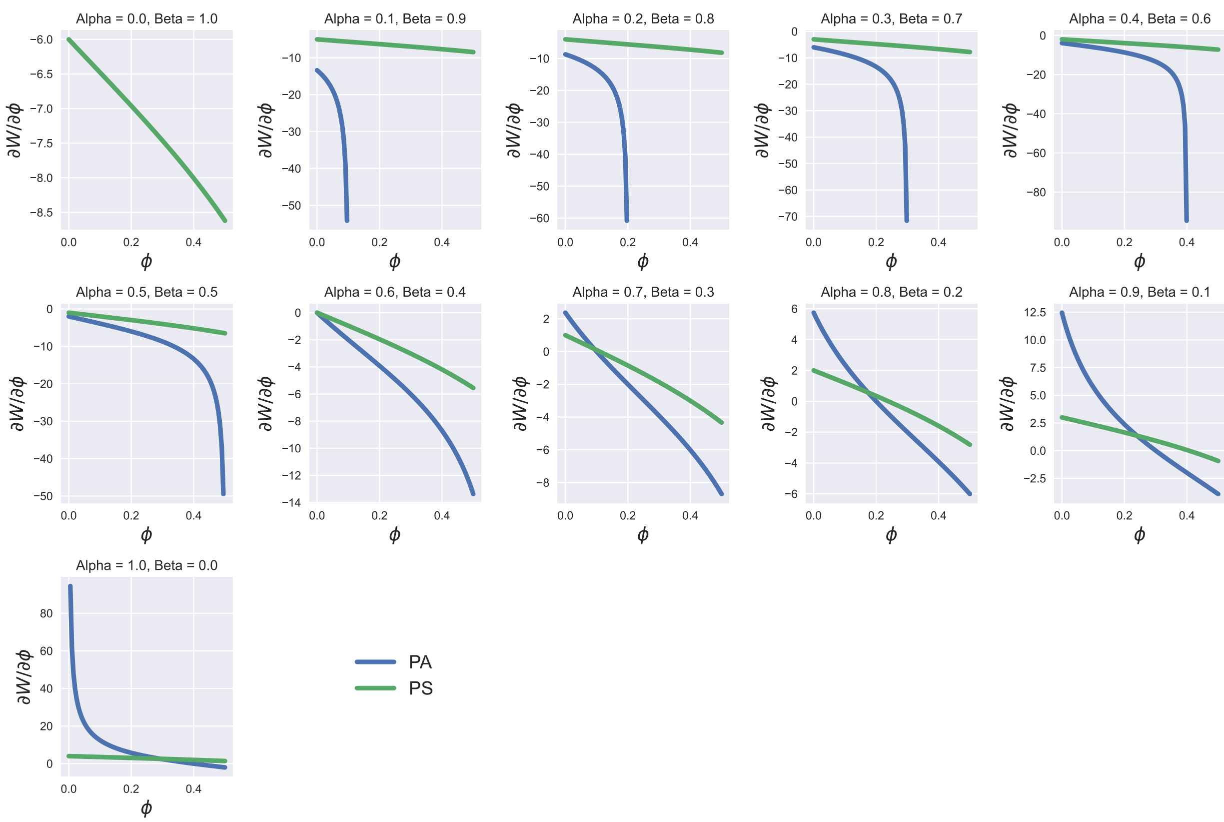

Figure 8: Case 2: Profits under Prior Appropriation

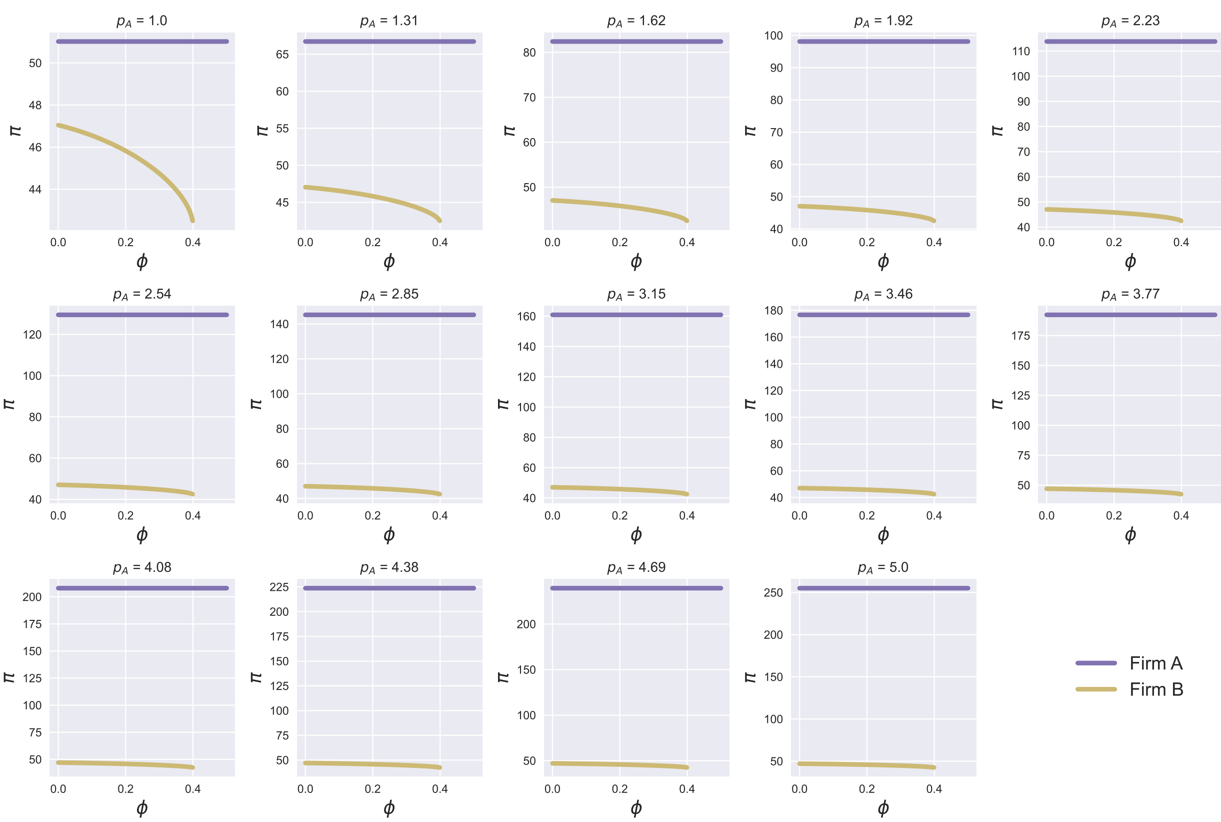

Figure 9: Case 2: Profits under Proportional Shares

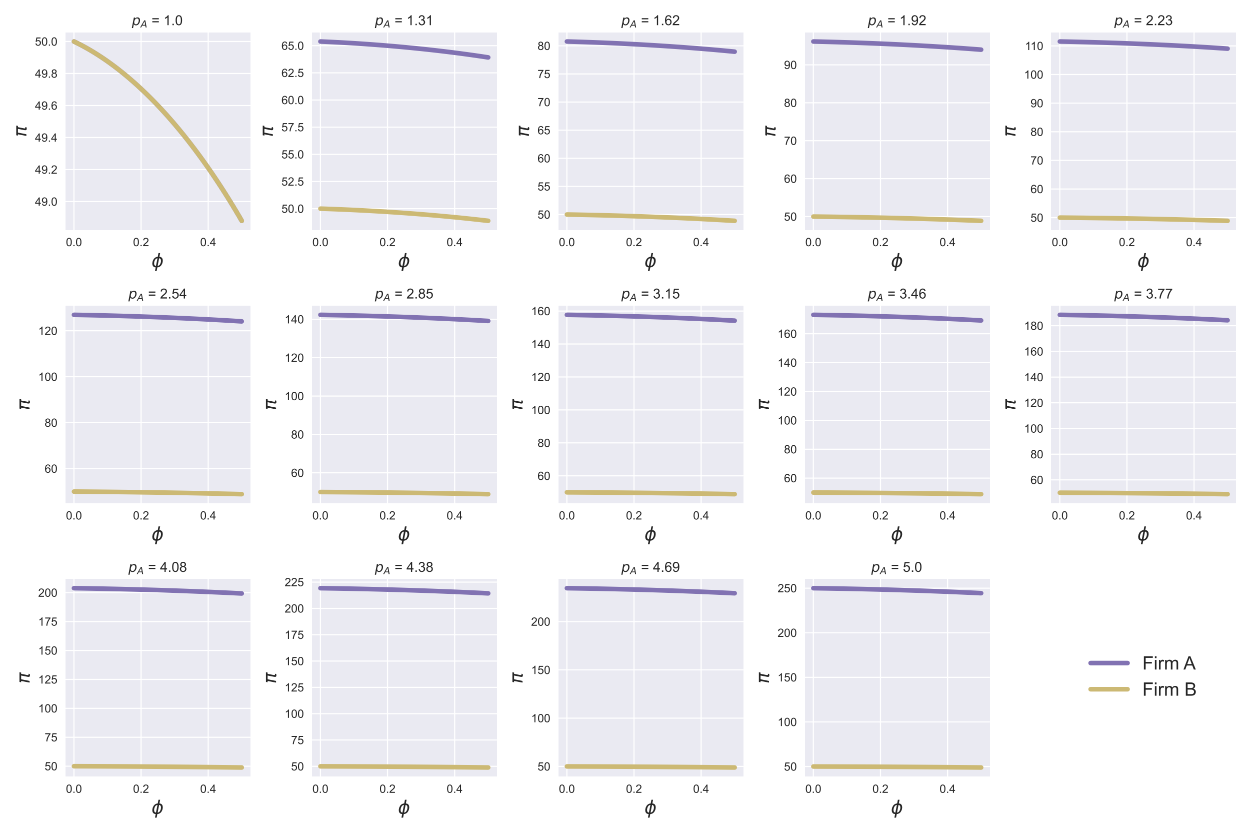

Figure 10: Case 2: Total Welfare

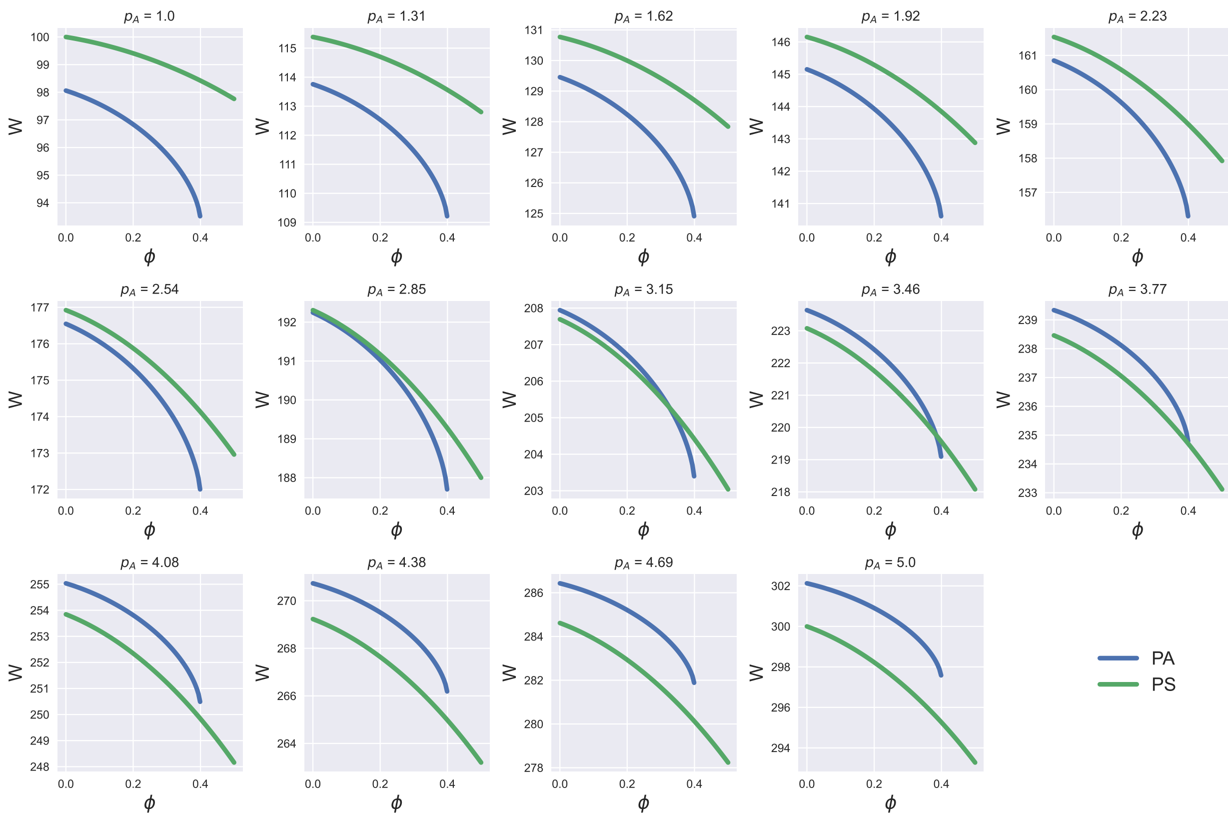

Figure 11: Case 2: Change in Total Welfare Due to Earlier Streamflow

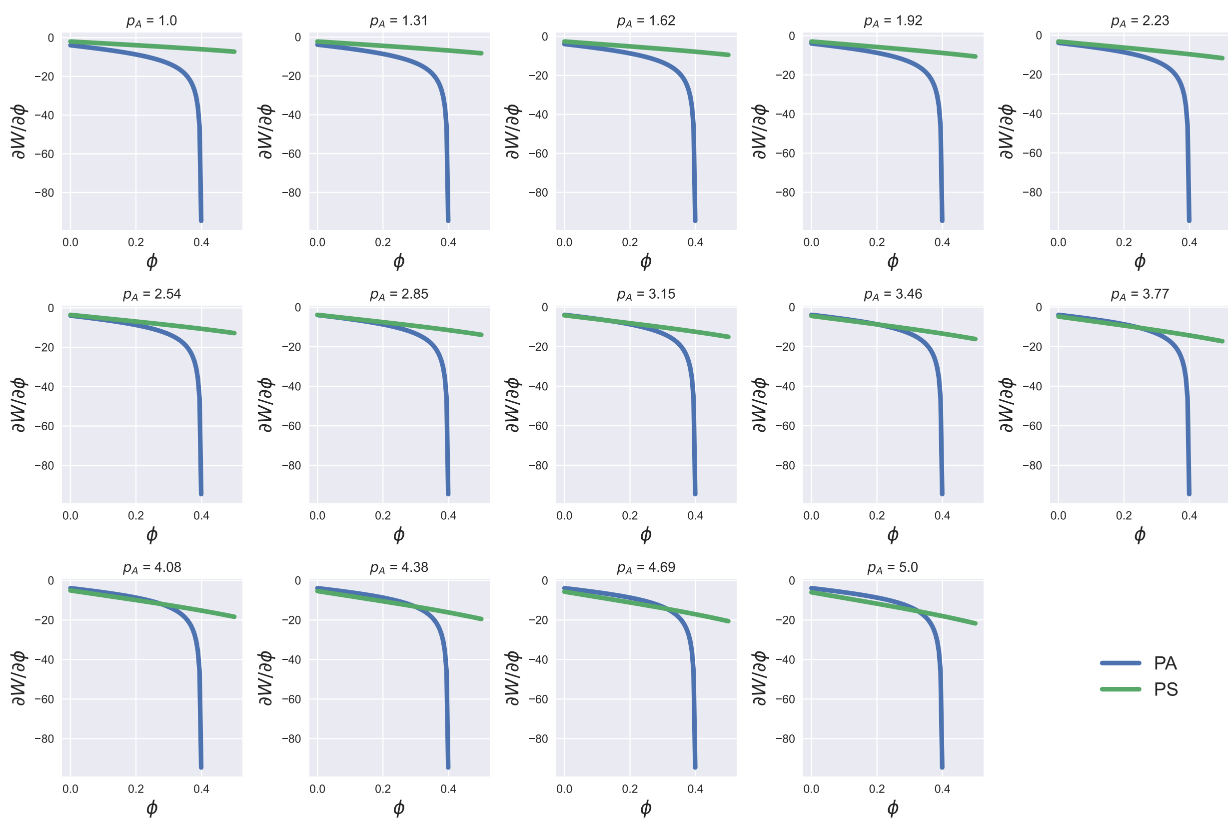

Figure 12: Case 3: Profits under Prior Appropriation

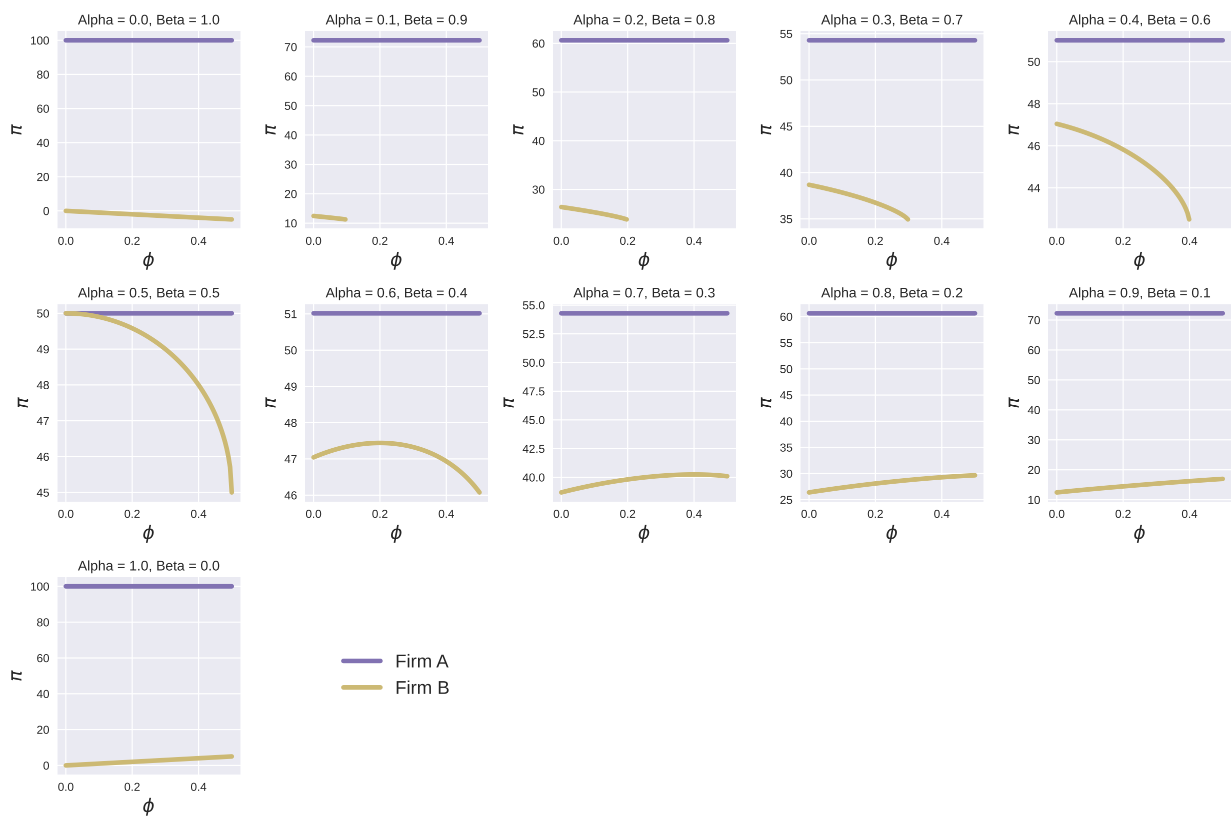

Figure 13: Case 3: Profits under Proportional Shares

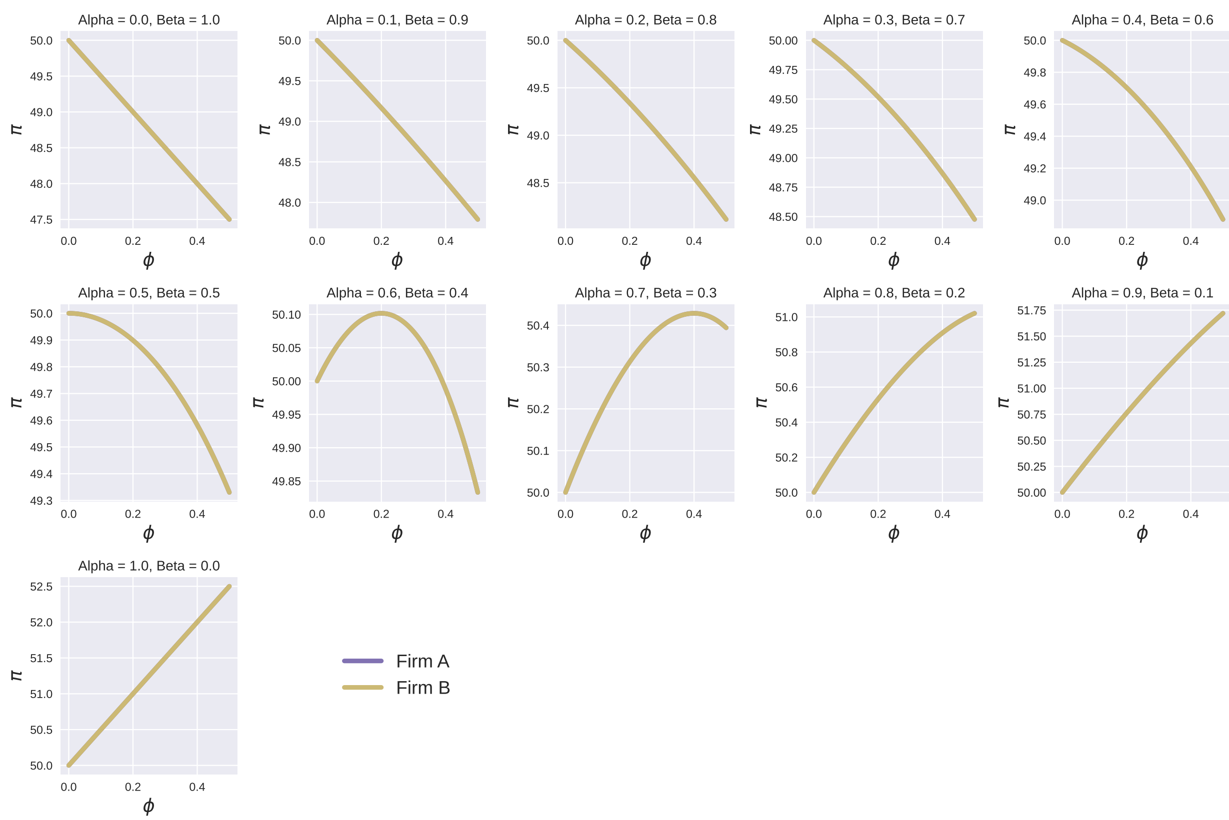

Figure 14: Case 3: Total Welfare

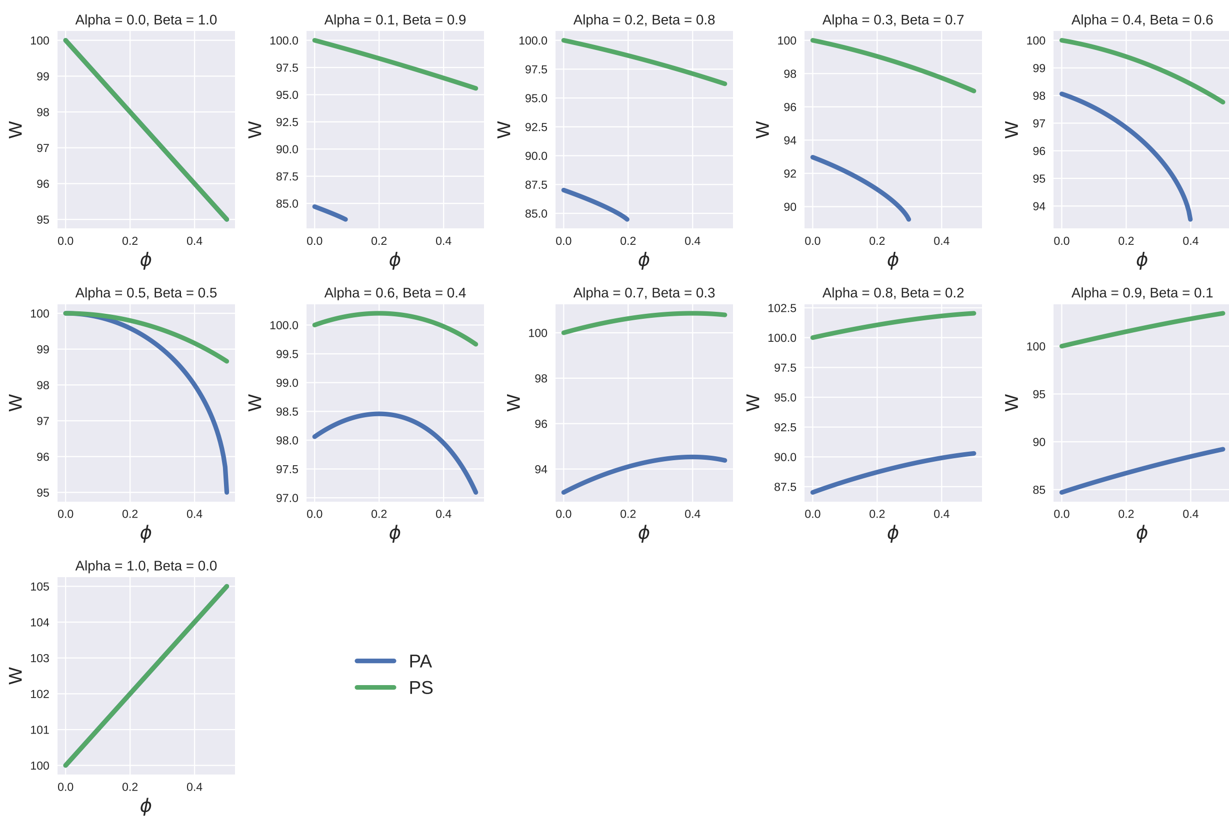

Figure 15: Case 3: Change in Total Welfare Due to Earlier Streamflow