Allocatable Fixed Inputs and Jointness in Agricultural Production: More Implications

Samuel Asunka and C. Richard Shumway

The presence of allocatable fixed inputs may cause truly joint technologies to appear nonjoint in the short run as well as truly nonjoint technologies to appear joint. This paper demonstrates theoretically why this can happen and then documents that it actually occurs in a significant way in aggregate U.S. agricultural production. A simple testing procedure is used that requires no data on input allocations. The important finding is that failure to reject true (apparent) nonjointness does not justify modeling short-run (long-run) supply independent of alternative output prices.

The subject of joint multioutput production has received significant and increasing theoretical and empirical attention in recent years. In 1972, Lau developed simple dual tests for joint production under price taking, profit maximizing behavior. He demonstrated that when multiple outputs are produced by joint technologies, the profit function is not additively separable in output prices.[1] Hence, one or more off-diagonal elements in the output price submatrix of the profit function's hessian is nonzero. Or equivalently, output supplies are not independent of alternative output prices. Soon after, Sakai (1974) showed that outputs cannot be gross substitutes when the multioutput technology is normal (i.e., when a price-taking, profit-maximizing producer has an incentive to voluntarily produce more than one output). For such a technology, outputs are jointly produced, and output supply responds positively to changes in some alternative output price and never responds negatively.

A decade later, Shumway, Pope, and Nash (1984) showed that an allocatable input can cause short-run supplies of technically independent outputs to depend on alternative output prices. If the allocatable input is fixed, it can give the appearance of jointness in production even for such outputs. Shumway, Pope, and Nash (1988) and Chambers and Just (1989) distinguished theoretically between this "apparent" (i.e., short-run) jointness and "true" (i.e., long-run) jointness caused by technically interdependent production. The latter also devised a test for true nonjointness based on the parameter estimates of the restricted (or short-run) profit function. While the nature of product interdependence caused only by a constraining allocatable input may be different from that caused by technical interdependence, its effect on the specification of the choice equations is the same—the exogenous price of each interdependent output appears in the output supply equations. The supply equations are short run if one or more inputs are fixed and long run if all inputs are variable.

Moschini (1989) demonstrated that a normal multioutput technology does not rule out the possibility that outputs are gross substitutes in the short run when outputs are joint only because of allocatable fixed inputs. Some short-run output supplies can decrease with an increase in an alternative output price and can increase with an increase in input price. Leathers (1991) documented conditions under which fixed allocatable inputs create an incentive for a firm to produce multiple outputs. The incentive is short-run economies of scope (i.e., complementary outputs in the short run) and/or short-run diseconomies of size for at least one output.

While it is evident that the presence of allocatable fixed inputs has important implications for economic modeling, some important implications

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[1] Lau referred to technologies for which the profit function is not additively separable in output prices as being joint in inputs. We shall refer to them as joint and their complement as nonjoint.

have not yet been identified and exploited. This paper presents a simple but rigorous treatment of the allocatable input problem. In particular, the impact of an allocatable input on economic modeling is identified for three lengths of run—long run, short run, and very short run. Two equivalent ways of testing for true nonjointness are presented. Some new testable hypotheses for normal multioutput technologies are derived. They show why an allocatable input causes a true nonjoint technology to exhibit apparent jointness and a true joint technology to exhibit apparent nonjointness. The paper concludes with an empirical application for U.S. agriculture.

The Theory

Consider a multioutput firm that produces m outputs, $y = (y_1, \ldots, y_m)$ with prices $p = (p_1, \ldots, p_m)$, uses n variable inputs, $x = (x_1, \ldots, x_n)$ with prices $w = (w_1, \ldots, w_n)$, and t fixed allocatable inputs, $z = (z_1, \ldots, z_t)$ with prices when purchased of $r = (r_1, \ldots, r_t)$. The quantities of x and z used in the production of y_i are denoted by x^i and z^i, respectively, and both are assumed to be weakly essential. The firm's indirect profit function can be distinguished between three lengths of run: long run when all inputs are variable—$\pi^L(p,w,r)$, short run when the allocatable inputs are fixed in total availability—$\pi^S(p,w,z)$, and very short run when the individual allocations of the fixed inputs are also fixed—$\pi^V(p,w,z^1, \ldots, z^m)$.

The short-run and very short-run functions can be embedded within the long-run profit function as follows:

$$(1) \quad \pi^L = \pi\{p,w,z^1[p,w,z(p,w,r)], \ldots, z^m[p,w,z(p,w,r)]\},$$

where $z(p,w,r)$ is the vector of long-run demand equations for allocatable inputs that are fixed in the short run, and $z^i(p,w,z)$ is the vector of short-run allocation equations of the fixed inputs used in output i. This depiction facilitates derivations for the various lengths of run. By application of the envelope theorem, $\partial\pi/\partial p_i = y_i^*$. For the very short run, this implies:

$$(2a) \quad \partial\pi^V(p,w,z^1, \ldots, z^m)/\partial p_i = y_i^{V*}(p,w,z^1, \ldots, z^m),$$

where y_i^{V*} is the very short-run supply. For the short run, it is:

$$(2b) \quad \partial\pi^S(p,w,z)/\partial p_i = y_i^{S*}[p,w,z^1(p,w,z), \ldots, z^m(p,w,z)] = y_i^{S*}(p,w,z),$$

where y_i^{S*} is the short-run supply. For the long run, it is:

$$(2c) \quad \partial\pi^L(p,w,r)/\partial p_i = y_i^{L*}\{p,w,z^1[p,w,z(p,w,r)], \ldots, z^m[p,w,z(p,w,r)]\} = y_i^{L*}(p,w,r),$$

where y_i^{L*} is the long-run supply.

$$(3) \quad \partial^2\pi^L(p,w,r)/\partial p_i\partial p_j = \partial y_i^{L*}(p,w,r)/\partial p_j = \partial y_i^{V*}(p,w,z^1, \ldots, z^m)/\partial p_j + \sum_{h=1}^{m}\sum_{k=1}^{t}[\partial y_i^{V*}(p,w,z^1, \ldots, z^m)/\partial z_k^h][\partial z_k^h(p,w,z)/\partial p_j] + \sum_{h=1}^{m}\sum_{k=1}^{t}\sum_{u=1}^{t}[\partial y_i^{V*}(p,w,z^1, \ldots, z^m)/\partial z_k^h][\partial z_k^h(p,w,z)/\partial z_u][\partial z_u(p,w,r)/\partial p_j] = A_{ij} + B_{ij} + C_{ij},$$

where $A_{ij} = \partial y_i^{V*}(p,w,z^1, \ldots, z^m)/\partial p_j$,

$$B_{ij} = \sum_{h=1}^{m}\sum_{k=1}^{t}[\partial y_i^{V*}(p,w,z^1, \ldots, z^m)/\partial z_k^h][\partial z_k^h(p,w,z)/\partial p_j], \text{ and}$$

$$C_{ij} = \sum_{h=1}^{m}\sum_{k=1}^{t}\sum_{u=1}^{t}[\partial y_i^{V*}(p,w,z^1, \ldots, z^m)/\partial z_k^h][\partial z_k^h(p,w,z)/\partial z_u][\partial z_u(p,w,r)/\partial p_j].$$

Differentiating (2c) with respect to $p_{j \neq i}$ by the chain rule, we can recover the cross-price output supply effects for all three lengths of run:

Equation (3) decomposes the effect of a change in p_j on the output decision of y_i into three separate effects. The first is the change in y_i induced by the change in p_j when the allocation of z is constant and all other prices are constant. The second is the change in y_i associated with the optimal reallocation of the fixed inputs in response to the price change while the total amount of each fixed input remains constant. The third is the response in y_i to the price change when more or less of the fixed inputs can be "purchased" from the market at price r. This decomposition permits us to distinguish the three lengths of run:

(a) In the very short run, both the total level and the allocation of z are constant, so $B_{ij} = C_{ij} = 0$, and

$$\text{(4a)} \quad \partial^2\pi^V(p,w,z^1, \ldots, z^m)/\partial p_i \partial p_j = A_{ij}.$$

(b) In the short run, the allocation of z is variable but its total level is constant, so $C_{ij} = 0$, and

$$\text{(4b)} \quad \partial^2\pi^S(p,w,z)/\partial p_i \partial p_j = A_{ij} + B_{ij},$$

(c) In the long run, both the fixed input vector z and its allocations are variable, so

$$\text{(4c)} \quad \partial^2\pi^L(p,w,r)/\partial p_i \partial p_j = A_{ij} + B_{ij} + C_{ij}.$$

Nonjointness

Nonjointness entails null cross-price second derivatives of the profit function in output prices (Lau 1972), i.e.,

$$\text{(5)} \quad \partial^2\pi/\partial p_i \partial p_j = 0, \ \forall j \neq i.$$

For different lengths of run, these cross-price derivatives are different. Nonjointness can be tested for each length of run, but it may mean something different in each case. Because no inputs are fixed in the long run, long-run (or true) nonjointness of y_i implies technical independence and requires that $A_{ij} + B_{ij} + C_{ij} = 0, \forall j \neq i$. Short-run (or apparent) nonjointness of output y_i imposes the restriction that $A_{ij} + B_{ij} = 0, \forall j \neq i$. Very-short-run nonjointness of output y_i imposes the restriction that $A_{ij} = 0, \forall j \neq i$.

Lau demonstrated that long-run nonjointness is implied by a price-taking firm that maximizes long-run profit with technically independent production functions, $y_i = f_i(x^i,z^i)$:

(6)

$$\text{Max}_{x,z}\ \pi = \sum_{i=1}^{m} \pi_i$$

$$= \sum_{i=1}^{m} \left[p_i f_i(x^i,z^i) - \sum_{g=1}^{n} w_g x_g^i - \sum_{k=1}^{t} r_k z_k^i \right].$$

Satisfaction of first and second-order conditions for (6) renders each output supply equation as:

$$\text{(7)} \quad y_i = y_i^*(p_i,w,r),$$

and the profit function as the sum of the individual output profit functions:

$$\text{(8)} \quad \pi = \pi^*(p,w,r) = \sum_{i=1}^{m} \pi_i^*(p_i,w,r).$$

We present three propositions and corollaries regarding long-run and short-run nonjoint production. The first proposition is a simple restatement of a finding by Chambers and Just (1989) and is presented here for completeness. The remaining propositions and corollaries are new, although the first corollary is obvious from proposition 1.

PROPOSITION 1: If output y_i is long-run (truly) nonjoint, $A_{ij} = 0, \forall j \neq i$, so very-short-run nonjointness implies and is implied by long-run nonjointness.

PROOF: See Chambers and Just (1989, p. 989).

COROLLARY 1: If output y_i is long-run (truly) nonjoint, $B_{ij} + C_{ij} = 0, \forall j \neq i$.

PROOF: Since $A_{ij} + B_{ij} + C_{ij} = 0$ and $A_{ij} = 0$, $\forall j \neq i$, then $B_{ij} + C_{ij} = 0, \forall j \neq i$.

PROPOSITION 2: If output y_i is long-run (truly) nonjoint, $B_{ij} \leq 0, \forall j \neq i$.

PROOF: If y_i is long-run (truly) nonjoint,

$$B_{ij} = \sum_{h=1}^{m}\sum_{k=1}^{t} [\partial y_i^{V*}(p,w,z^1, \ldots, z^m)/\partial z_k^h] [\partial z_k^h(p,w,z)/\partial p_j]$$

$$= \sum_{k=1}^{t} [\partial y_i^{V*}(p_i,w,z^i)/\partial z_k^i][\partial z_k^i(z_k,z_k^j)/\partial z_k^j] [\partial z_k^j(p_j,w,z)/\partial p_j],$$

where $\partial z_k^i(z_k,z_k^j)/\partial z_k^j$ is the marginal rate of substitution in the allocation of z_k between output j and output i when the total availability of z_k is fixed.[2]

[2] Other allocations of z_k are omitted from the parentheses in this term to note that they are endogenous. The allocation z_k^j is also endogenous, but it is parameterized by means of the chain rule in this term since its endogeneity is noted in the last term. Long-run nonjointness is sufficient to remove the summation over h from the second line since the supply of output i would be dependent neither on any output price except i nor on any fixed input allocations except those to output i. The same logic

Assuming positive marginal productivities, concave production functions, and normal inputs, $\partial y_i^V*(p_i,w,z^i)/\partial z_k^i > 0$ and $\partial z_k^j(p_j,w,z)/\partial p_j \geq 0$. Since z is fixed in the short run, an increase in z_k^j in response to an increase in p_j would induce a decrease in the optimal allocation of z_k to some other output and would create no incentive to increase the allocation to any other output. Hence, $\partial z_k^i(z_k,z_k^j)/\partial z_k^j \leq 0, \forall j \neq i$.

COROLLARY 2: If output y_i is long-run (truly) nonjoint, y_i tends to be short-run (apparently) joint.

PROOF: From propositions 1 and 2, $A_{ij} + B_{ij} \leq 0, \forall j \neq i$, if output y_i is long-run (truly) nonjoint.

PROPOSITION 3: The presence of an allocatable fixed input may cause multioutput normal outputs that are long-run (truly) joint to be short-run (apparently) nonjoint.

PROOF: Defining multioutput normal outputs for competitive firms as those whose marginal cost does not increase as the quantity of the other output increases, Sakai (1974) proved they cannot be gross substitutes. For such outputs, this means that $A_{ij} + B_{ij} + C_{ij} \geq 0, j \neq i$. If commodities i and j are also long-run joint, the relationship is a strict inequality. To complete the proof that $A_{ij} + B_{ij}$ could be zero, it is sufficient to show that $C_{ij} \geq 0, j \neq i$. For convenience, the definition of C_{ij} is repeated:

$$C_{ij} = \sum_{h=1}^{m} \sum_{k=1}^{t} \sum_{u=1}^{t} [\partial y_i^V*(p,w,z^1, \ldots, z^m)/\partial z_k^h] [\partial z_k^h(p,w,z)/\partial z_u][\partial z_u(p,w,r)/\partial p_j].$$

Holding all prices constant except p_j, and assuming (a) positive marginal productivities of all inputs, (b) concave production functions,[3] (c) normal inputs, and (d) normal multioutput joint production, then an increase in z_k^h induces an increase in y_h which in turn induces an increase in y_i, so the first term is positive. Likewise, an increase in p_j does not induce a decrease in z_k^j, y_j, or the marginal physical productivity of z_k^h in y_h, so there is no incentive to use less z_k^h to restore satisfaction of the first-order conditions. Therefore, an increase in p_j is not accompanied by a decrease in z_k^j, z_k^h, or z_u, so the second and third terms are nonnegative. Because $A_{ij} + B_{ij} + C_{ij} \geq 0$ and $C_{ij} \geq 0, j \neq i, A_{ij} + B_{ij} \gtreqless 0, j \neq i$.

Empirical Application

It is apparent that an allocatable fixed input can make technology that is truly nonjoint appear joint in the short run, and technology that is truly joint appear nonjoint in the short run. However, a truly nonjoint technology remains truly nonjoint in the very short run when the allocations of the fixed input are also nonvarying. Most empirical tests of nonjointness using profit functions (and even some cost functions) have, perhaps unwittingly, focused on short-run nonjointness (e.g., Shumway 1983, Ball 1988, Weaver 1983, Ray 1982, Lopez 1984). It appears that the only profit function test that focused on long-run nonjointness was by Chambers and Just (1989). They conducted the test using the very short-run implication that $A_{ij} = 0$ for a long-run nonjoint technology. However, that test requires data on the allocations of the fixed inputs.

In our empirical test of long-run nonjointness, we construct a long-run (unrestricted) profit function and perform the equivalent test that $A_{ij} + B_{ij} + C_{ij} = 0$. This test does not require any knowledge of the input allocations. We also test for short-run nonjointness, $A_{ij} + B_{ij} = 0$, to empirically evaluate the hypothesis that allocatable fixed inputs tend to cause a long-run nonjoint technology to appear joint and a long-run joint technology to appear nonjoint.

We perform these tests for aggregate U.S. agricultural production using annual data for the period 1949–91. The data used are updated and extensively revised price and quantity series of data appearing in Ball (1988). They incorporate several important improvements over earlier data series in terms of reliably measuring aggregate prices and quantities. Based on highly detailed output and input data, the aggregates used in this application are the same as those used by Ball (1988). They include five output categories (livestock, milk, grain, oilseeds, and other crops) and seven input categories (durable equipment, farm-produced durables, hired labor, energy, other purchased inputs, real estate, and self-employed labor). All input categories are measured as service flows. Price aggregates are constructed as Tornqvist indexes. Output aggregates are obtained by dividing category receipts or expenditures by the aggregate price.

Real estate and self-employed labor are most often regarded as fixed inputs in short-run models. In some models, capital is also treated as a fixed

applies to the last term of the second line since the allocation of any fixed input to output j would not depend on any output price except j.

[3] In the case of long-run joint production, individual production functions that are independent of all other outputs cannot be written. Here the term production function is used in the broader sense of a transformation function that relates the quantity of one output to the allocation of inputs used directly in its production, levels of unallocated inputs, and other output level(s).

input. Therefore, in our tests we will consider two short-run scenarios, one (model A) with real estate and self-employed labor fixed and the other (model B) with durable equipment and farm-produced durables also fixed.

Based on Lim and Shumway's finding (1996) that the normalized quadratic functional form is strongly preferred to the translog and is slightly preferred to the generalized Leontief for these data, the normalized quadratic is used to approximate the true functional form of the profit and restricted profit functions:

$$\pi = \alpha + P'\beta + .5P'\gamma P + P'\lambda T, \tag{9}$$

where π is profit for the long-run model and receipts less variable costs (restricted profit) for the short-run model, P is (p,w,r) for the profit function and (p,w,z) for the restricted profit function, T is time and is included as a proxy for disembodied technical change,[4] and α, β, γ, and λ are conformable parameters. The variables π, p, w, and r are all normalized (divided) by the price of durable equipment to maintain linear homogeneity of the profit function in prices. Expected output prices, p, are represented by the lagged prices. All quantities are measured as netputs (positively measured for outputs and negatively measured for inputs).

Estimation is carried out by invoking the envelope theorem to obtain the system of linear netput supply equations from (9):

$$\partial\pi/\partial P = Y = \beta + \gamma P + \lambda T, \tag{10}$$

where Y is (y,x,z) for the long-run model and (y,x) for the short-run model. Because the price of durable equipment is used to normalize profit and prices, its equation is quadratic and its quantity is not included in the vector x. Thus, the estimation system consists of eleven equations for the long-run model, nine equations for short-run model A, and seven equations for short-run model B. Symmetry of cross-partial derivatives of π is maintained by linear restrictions on the parameters of the system.

Because of the possibility of simultaneity in supply and demand of the inputs, instruments are developed for input prices and quantities specified as regressors in the estimated equations. The instruments are fitted values from linear regressions of these variables on their lagged values, lagged expected output prices, and current values of variables assumed exogenous in a more complete but unspecified model of U.S. input markets. That model includes population, per capita income, consumer price index, manufacturing price index, price index of primary inputs, prime rate, GNP implicit price deflator, nonagricultural wage index, inflation rate, and government purchases of agricultural commodities.

An additive and normally distributed error term is appended to each equation. It is assumed to be uncorrelated across observations but possibly correlated across equations both because of interrelated production decisions and because of the cross-equation restrictions. Estimation is accomplished by iterative 3SLS. This is equivalent to maximum likelihood estimation.

Tests for nonjointness were conducted for each output category for each model. They involved joint nullity restrictions on elements of γ, i.e.,

$$\gamma_{ij} = 0, \forall j \neq i, j \text{ and } i \text{ outputs.} \tag{11}$$

The results of these F tests are reported in table 1 for the long-run and both short-run models. In the long-run model, nonjointness is rejected at the 5% significance level for two output categories—livestock and grain. In the short-run model A, it is not rejected for any output category. In the short-run model B, it is rejected for other crops.

Our findings suggest that livestock and grain are truly joint outputs, but the presence of allocatable fixed inputs makes these outputs appear nonjoint in both short-run scenarios. Both real estate (at least the land portion) and self-employed labor are inputs that are clearly allocated among most outputs produced. Machinery time and many of the farm-produced durables (inventories) are also clearly allocated among most outputs. Thus, when they are constraining, as they often are for a single production period, they impose binding restrictions on profit-maximizing production. By their offsetting effects on the cross-price output supply coefficients, the effect of these binding restrictions on

Table 1. Long-Run and Short-Run Nonjointness Tests

	F-Statistic for Nonjointness		
	Long Run	Short Run	
Output		Model A	Model B
Livestock	4.50	0.25	1.18
Dairy	1.25	1.17	1.31
Grain	6.56	0.56	1.44
Oilseeds	1.32	1.86	0.85
Other crops	1.09	1.28	2.73
Critical value, $F^{.05}_{4,DF}$	2.39	2.40	2.41
DF	385	306	231

[4] Estimates of embodied technical change have already been measured in the construction of several of the input categories, especially labor.

aggregate U.S. agricultural production is to remove the short-run appearance of true jointness from both joint outputs. When all these inputs are constraining, they also add the appearance of short-run jointness to a category that is truly nonjoint.

Dairy and oilseeds are estimated to be the only truly nonjoint output categories that are also nonjoint in both short-run scenarios. This suggests that the combined offsetting effect of family labor, real estate, and capital constraints on these output categories is too small to show up as being significant.

Conclusions

Increasing attention has been given in recent years to the impact of allocatable fixed inputs on technology and economic relationships. This paper adds to that body of literature by showing that the presence of allocatable fixed inputs may cause truly joint technologies to appear nonjoint in the short run as well as truly nonjoint technologies to appear joint. It demonstrates theoretically why this can happen. Then using aggregate U.S. data for five output categories, it documents empirically that this phenomenon actually occurs in a significant way in U.S. agricultural production. The empirical documentation is accomplished using a simple testing procedure that, unlike previously implemented tests, does not require any data on input allocations. The important finding of this study is that apparent nonjointness cannot be inferred from true nonjointness, so justification for modeling short-run supply independent of alternative output prices is not provided by failure to reject true (or long-run) nonjointness. Likewise, justification for modeling long-run supply without regard to changes in alternative output prices is not provided by failure to reject apparent (or short-run) nonjointness. Consequently, the test must be conducted for the same length of run as the supply equations to be estimated. Further, it is not sufficient to test for apparent nonjointness holding one subset of inputs fixed while estimating supply equations with a larger or smaller subset of fixed inputs.

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