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# Estimating shadow price for symbiotic nitrogen and technical efficiency for legume-based conservation agriculture in Malawi

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Determining the value of legumes as soil fertility amendments can be challenging, yet this information is required to guide public policy and to incentivise prescribed land-management practices such as conservation agriculture. We use a directional input distance function (DIDF) to estimate shadow prices for symbiotic nitrogen and the technical efficiency for mixed maize-legume production systems in Malawi. The shadow prices reflect the trade-off between fertiliser nitrogen and symbiotic nitrogen required to achieve a given quantity of output. Our results reveal considerable technical inefficiency in the production system. The estimated shadow prices vary across farms and are, on average, higher than the reference price for commercial nitrogen. The results suggest that it would be beneficial to redesign the current price-support programs that subsidise chemical fertilisers and indirectly crowd-out organic soil amendments such as legumes.

**Key words:** Africa, biological nitrogen fixation, directional distance function, efficiency and productivity, sustainable agricultural intensification.

## 1. Introduction

Legumes are an important component of smallholder farming systems in sub-Saharan Africa (Sanginga 2003; Giller *et al*. 2009). Besides crop outputs, legume-based cropping systems (henceforth LBCS) supply a variety of indirect benefits that are essential for sustainable agricultural intensification (Giller *et al*. 2009; Jensen *et al*. 2012; Preissel *et al*. 2015). For example, LBCS can help to suppress parasitic weeds and pest/disease incidence recurring from the use of monoculture. In addition to breaking the disease cycle and controlling weeds, LBCS maintain soil fertility through nutrient recycling and prevention of soil erosion (Giller *et al*. 2009; Preissel *et al*. 2015). Thus, the organic soil amendments supplied through LBCS can reduce the need, at least partly, for commercial fertiliser

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application and can hence lower farm investment costs (Pannell and Falconer 1988; Sanginga 2003; Mafongoya *et al.* 2007). Furthermore, as part of a soil-nitrogen management plan, LBCS represent a cheap form of abatement to reduce nitrogen leachates associated with excessive fertiliser use (Jensen *et al.* 2012). However, the values of LBCS benefits, particularly the nutrient-recycling function, have not been adequately studied. This is partly because the nitrogen derived from legume association is an intermediate resource, which is neither directly observable nor traded in commodity markets, and thus difficult to value through direct market prices. Instead, valuing biological nitrogen derived from legume-based symbiotic fixation process (LBSF-N) requires the application of indirect methods, such as shadow pricing (Piot-Lepetit and Vermersch 1998; Reinhard *et al.* 1999; Färe *et al.* 2009).

Valuing soil fertility benefits can help ascertain the economic importance of LBCS and justify the role of legumes in conservation agriculture and sustainable environmental management. Currently, legume intensification is being promoted in sub-Saharan Africa as one of the strategies available under conservation agriculture (Giller *et al.* 2009; Thierfelder *et al.* 2013). For example, in Malawi, the Government has included legume seed as part of the targeted farm input support program. The farm-subsidy program promotes both chemical and biological (legumes) fertilisers. Coincidentally, the impact of conservation agriculture practices is not well researched in the case of Malawi and other African countries (Giller *et al.* 2009; Thierfelder *et al.* 2013). Therefore, accurate information on the economic benefits of LBSF-N will be useful for policy interventions that promote conservation agriculture across Africa.

A few studies have attempted to value LBSF-N (Pannell and Falconer 1988; Döbereiner 1997; Smil 1999; Herridge *et al.* 2008; Chianu *et al.* 2011). Apart from Pannell and Falconer (1988) and Schilizzi and Pannell (2001) who use bio-economic modelling approaches to value LBSF-N, previous studies have mainly applied the replacement-cost method to estimate the value of LBSF-N (Smil 1999; Herridge *et al.* 2008; Chianu *et al.* 2011). The replacement-cost method is based on an assumption that the two alternative inputs being valued are perfectly substitutable; thus, the estimates arising from this method do not capture changes in the degree of input substitutability. We address this limitation by adopting an econometric approach, and use the ratio of marginal products to determine the degree of substitutability between a pair of inputs. In our approach, we apply the directional input distance function (DIDF) technique to estimate shadow prices for LBSF-N. The estimated shadow price reflects the trade-off between nitrogen from commercial fertilisers and LBSF-N, required to produce a given quantity of output. Accordingly, this shadow price represents the benefit of using LBSF-N as a production input.

The paper makes a contribution to the literature in two ways. First, we demonstrate the application of a DIDF to value the contribution of LBSF-

N as a factor of production. To the best of our knowledge, this is the first study to apply DIDF to value LBSF-N. Although a nonfrontier production function could be used to determine shadow prices, as in Barbier (1994) and Magnan *et al.* (2012), such an approach does not account for inefficiencies. Given the extent of inefficiency routinely reported in studies evaluating the performance of smallholder agriculture, it is more appropriate to use more general frameworks that allow for the estimation of both inefficiency and shadow prices. Compared to other frontier-based approaches, the DIDF represents a flexible technique that can derive an inefficiency measure that accounts for possible input reductions. With the DIDF, it is possible to set a uniform translation vector that evaluates the extent to which the technology can achieve input (cost) savings (Färe and Grosskopf 2004; Färe *et al.* 2009; Hailu and Chambers 2012). A key advantage of such an approach is that the set of individual firm's inefficiencies can be compared across farms and summed into an aggregate measure of industry inefficiency (Färe and Grosskopf 2004; Färe *et al.* 2008, 2009; Hailu and Chambers 2012). An alternative distance function specification is the radial approach, where the directional vector for input contraction or output expansion is data-driven (dictated by the input or output mix for each observation) and therefore unknown to the analyst (Färe *et al.* 2008; Hailu and Chambers 2012). Radial input (output) distance function values reflect the highest (lowest) possible proportionate reduction in inputs (outputs) and thus provide only relative measures of inefficiency (Hailu and Veeman 2000). Further, the DIDF, like its radial distance function counterpart, can be used to represent multi-input, multi-output production technologies (Hailu and Veeman 2000). Our second contribution is on the application of a bootstrapping technique, within the DIDF method, to test the robustness of the estimates (Canty 2002; Canty and Ripley 2015). By using the bootstrapping technique, we are able to get a sense of the variability surrounding shadow price and technical efficiency estimates.

The rest of the paper is organised as follows. In the next section, we present a theoretical representation of the DIDF, followed by empirical estimation procedures. We describe the study site and data in Section 4. Finally, empirical results are discussed in Section 5 and conclusions are presented in Section 6.

## 2. Theoretical model

The productive efficiency of a firm is determined by comparing its inputs and outputs against the boundaries of the best-practice frontier. A firm's measure of inefficiency is given by how far that firm is from the frontier boundary. Denote production inputs by $x = (x_1, x_2, \ldots.., x_N) \in \Re_+^N$ and outputs by $y = (y_1, y_2, \ldots.., y_M) \in \Re_+^M$. Then, a production technology, which maps out all feasible input–output combinations, can be defined as an input requirement set $L(y) = \{x : x\, can\, produce\, y\}$.

The directional input distance function, $\mathrm{DIDF}(x, y; g_x) = \sup_{\beta}\{\beta : (x - \beta g_x, y) \in L(y)\}$, represents the technology and helps to measure a firm's level of inefficiency. The vector, $g(g_x \in \Re^N_+)$, is the translation metric which maps the direction in which inputs are scaled. Thus, the translation vector seeks to achieve input contraction in the $g_x$-direction. For any firm on the frontier boundary, DIDF($x$, $y$; $g_x$) = 0, indicating that it is technically infeasible to translate the input bundle in any direction. Conversely, any firm below the technology frontier has a positive distance value, DIDF($x$, $y$; $g_x$) > 0, such that the observed input bundle can be translated in the direction given by $g(g_x)$.

The DIDF inherits the standard properties imposed on the production technology $L(y)$ (Chambers *et al*. 1996; Färe *et al*. 2008). We assume that $L(y)$ is a closed, convex, nonempty set with inputs and outputs freely disposable (Färe *et al*. 2008). Other important properties of the DIDF technology include the following: (i) representation, which implies that all technologically feasible input–output combinations have non-negative directional distance function value, and vice versa; (ii) translation, which denotes that adding a multiple of the direction vector to the input–output bundle reduces the distance function by that multiple; (iii) monotonicity, which indicates that the function is nondecreasing in inputs and nonincreasing in outputs; and (iv) the function is concave in the input–output vector (Chambers *et al*. 1996; Färe *et al*. 2008).

We derive shadow prices for LBSF-N by exploiting the duality relationship between the DIDF and the cost function. Let $w = \left(w_1, w_2, \ldots.., w_N\right) \in \Re^N_+$ denote the vector of input prices for which the shadow cost function is given by:

$$C(y, w) = \underset{x}{inf}\{wx : x \in L(y)\} \tag{1}$$

Equation (1) gives the minimum cost that can be achieved, given input price ($w$) and output vector $y \in \Re^M_+$. It follows from the principle of cost minimisation that $C(y, x) \le wx \forall x \in L(y)$. That is, the minimum cost cannot exceed the actual cost of producing $y \in \Re^M_+$. Achieving input-efficiency implies that $\{x - [\mathrm{DIDF}(y, x : g_x).g_x] \in L(y)\}$. Thus, for any production situation where technical inefficiency can be reduced or eliminated, the minimum cost ought to be lower than the actual costs as follows:

$$C(y, x) \le w\{x - [\mathrm{DIDF}(y, x : g_x).g_x]\} = wx - wg_x\mathrm{DIDF}(y, x : g_x) \tag{2}$$

By rearranging Equation (2), the duality relationship between the cost function and the DIDF can be expressed as (Chambers *et al*. 1996; Färe *et al*. 2009):

$$\mathrm{DIDF}(y, x : g_x) = \min_{w} \left\{ \frac{wx - C(y, x)}{wg_x} \right\} \tag{3}$$

Applying the envelope theorem to Equation (3) yields the following normalised input price vector:

$$w_n = wg_x \frac{\partial \mathrm{DIDF}(x, y; g)}{\partial x_n} \quad n = 1, 2, \ldots ..N \tag{4}$$

Provided that the DIDF is differentiable, one can estimate partial derivatives and use these to derive shadow prices. For any two different inputs, $n$ and $n^{'}$, it follows that their price ratio equals the corresponding ratio of distance function derivatives. The ratio of distance function derivatives indicates the marginal rate of technical substitution (MRS) expressed by (Chambers *et al.* 1996):
[Correction added on 17 July 2017, after first online publication: Equation 5 has been corrected]

$$\frac{w_n}{w_{n'}} = \frac{\partial \mathrm{DIDF}(x, y; g_x)/\partial x_n}{\partial \mathrm{DIDF}(x, y; g_x)/\partial x_{n'}} = \frac{MPx_n}{MPx_{n'}} = MRS_{x_n x_{n'}} \quad \forall n, n' \tag{5}$$

where $w_n$ is the price for the $n$th production factor ($x_n$), and $MP_x$ is the marginal product derived with respect to factor $x$. Thus, the knowledge of one factor price ($w_n$) can be used to compute the price of the unknown inputs, in this case, LBSF-N.

In addition to the marginal products, one can also obtain technical (in)efficiency measures, if a frontier-based technical relationship is specified. For a directional distance measure of inefficiency, zero distance indicates that a firm is fully efficient, and a positive distance function value shows the level of inefficiency as follows:

$$TI(x, y) = \mathrm{DIDF}(x, y; g_x) \tag{6}$$

## 3. Empirical estimation

Both nonparametric approaches (e.g. data envelopment analysis (DEA)) and parametric methods (e.g. stochastic frontier analysis) can be used to estimate the DIDF parameters. We specify the DIDF as a parametric function, which is differentiable, and thus, it is easy to recover shadow prices from it. Such a parametric specification can be estimated either as a deterministic frontier or as a stochastic function. In our approach, we estimate the DIDF as a deterministic frontier using mathematical programming techniques to impose representation, monotonicity and translation properties on the function easily

(Färe and Grosskopf 2004; Färe *et al.* 2009; Hailu and Chambers 2012; Bostian and Herlihy 2014). The stochastic frontier models could also be estimated using maximum-likelihood methods (e.g. Coelli and Perelman 2000) or using Bayesian methods (e.g. O'Donnell and Coelli 2005; Hailu and Chambers 2012). Although one could impose monotonicity and other conditions using Bayesian methods, the choice of proper priors on the parameters of frontier models is not straightforward, and the use of improper priors could affect the accuracy of the posterior estimates (Fernández *et al.* 1997, 2000). Bayesian estimation is also computationally intensive and more difficult. Alternatively, theoretical regularity restrictions can be imposed more easily using mathematical programming techniques, as is the case in this study. This approach is straightforward, given the central importance of constraints in defining the feasible space in constrained optimisation problems.

The quadratic functional form is used because it is a flexible form that allows for the global imposition of the translation property (Hailu and Chambers 2012). We choose the unit directional vectors, with negative elements for inputs and zero elements for output, so that the projection to the frontier of an observed point seeks to contract inputs while holding the output vector constant. Since the data are normalised by mean values, the use of the unit vector for direction is equivalent to the use of the average sample direction for the translation. By assigning the unit vectors, the directional input distance function gives an estimate of the maximum unit reduction in inputs that is feasible for a given amount of output (Färe and Grosskopf 2004; Hailu and Chambers 2012). This approach is valuable because it makes it easier to interpret the estimated measure of technical inefficiency as proportions of the sample mean values. Further, when a common directional vector is chosen for all firms, the directional distance function measure of inefficiency for an individual firm can be aggregated to a measure of industry inefficiency (Färe and Grosskopf 2004; Färe *et al.* 2008, 2009; Hailu and Chambers 2012). The quadratic DIDF is specified as follows:

$$\begin{aligned}\mathrm{DIDF}(x, y; -1, 0) = \beta_0 + \sum_{n=1}^{N} \beta_n x_n + \sum_{m=1}^{M} \beta_m y_m + 0.5 \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_{nn'} x_n..x_{n'} \\ + 0.5 \sum_{m=1}^{M} \sum_{m'=1}^{M} \beta_{mm'} y_m.y_{m'} + \sum_{m=1}^{M} \sum_{n=1}^{N} \beta_{mn} y_m.x_n + u\end{aligned} \tag{7}$$

where $y_m$ represents the production output under consideration, $x_n$ is a vector of inputs, and $u$ represents the inefficiency term. Equation (7) is estimated in R, using APEAR package (Hailu 2013).

The underlying distance function and parameter values ($\beta_s$ and $u$) are based on a true, but unobserved, technology frontier. Typically, the estimated frontier and parameters depend on a sample of observations, usually believed to be representative of the true population. Thus, one empirical challenge is to mitigate the potential bias that could result from small samples and sampling variability. Therefore, we apply nonparametric bootstrapping techniques to explore the variability of our sample estimates. Using the bootstrapping procedure explained in Canty (2002) and Canty and Ripley (2015), the estimation of the function in Equation (7) was bootstrapped a thousand times, whereby each pseudo (bootstrap) sample was drawn with replacement from the original sample.

We start by assuming that a random sample, replicated $r$-times and where $r$ is large enough, can be used to construct the unknown population distribution from which the original sample was drawn. Application of Equation (7) to the bootstrap sample gives distance function parameters $\hat{\psi}$, and the distribution function $\hat{F}$, which are considered to approximate the true population parameters, $\psi$ and $F$. A bootstrapping algorithm generates $r$-pseudo samples which can yield consistent sample parameters $\hat{\psi}_r^*$ and $\hat{F}$. The bootstrap sample parameters are related to the true unobserved population parameters in the following way:

$$\psi = f(F) \Leftrightarrow \hat{\psi} = f(\hat{F}) \tag{8}$$

Equation (8) shows that given a representative sample, the estimated sample parameters can be used to recover population parameters. Through resampling, an approximate empirical distribution can be obtained for any statistics of interest, for example the variance and standard error. The statistics that are generated through bootstrapping in this analysis are as follows:

$$\begin{aligned} &\text{bias} = E(\hat{\psi}_r^*) - \hat{\psi} = \bar{\hat{\psi}}_r^* - \hat{\psi}; \\ &\sigma = \frac{1}{r-1}\sum_{r=1}^{R}\left(\hat{\psi}_r^* - \bar{\hat{\psi}}_r^*\right)^2; \quad \text{SE} = \left[\sqrt{\frac{1}{r-1}\sum_{r=1}^{R}\left(\hat{\psi}_r^* - \bar{\hat{\psi}}_r^*\right)^2}\right]^{0.5} \end{aligned} \tag{9}$$

where $\sigma$ and SE represent the variance and standard errors of individual parameters, respectively. For a detailed description of this bootstrapping procedure, the reader is referred to Canty (2002) and Canty and Ripley (2015).

## 4. Study area and data description

### 4.1 Study area

We use survey data collected from Kasungu and Mzimba districts in Malawi. The survey was conducted in the 2013/2014 crop season. These districts are part of the medium-altitude agro-ecological zone of the country. A subdivision of this agro-ecological zone is the Kasungu–Lilongwe plain. This zone is one of the areas where LBCS are most dominant in Malawi.

The survey followed a three-stage random sampling approach. First, the study zones were selected based on Ministry of Agriculture administrative demarcations, known as extension planning areas (EPAs). The EPAs are district-level administrative units established to coordinate and oversee the execution of extension services across the country. The second step involved the choice of an EPA section. An EPA section is the lowest unit of administration in the Ministry of Agriculture hierarchy. Subsequently, maize-legume producers were identified for each EPA section. This procedure was done in order to establish a sampling frame. Third, after the selection of EPA sections and the enumeration of maize-legume producers was completed, face-to-face interviews were conducted with a set of randomly chosen respondents (household heads).

## 4.2 Data description

The survey collected information on farm and household characteristics. We use primary data from a sample of 135 plots, representing the mixed maize-legume cropping system. Fifty-five per cent of the sample (74 plots) was from Kasungu and the remainder (45 per cent) came from Mzimba district (61 plots). We recorded one maize-legume intercropped plot per producer hence the number of producers is the same as the number of plots. The sample represents 13 per cent and 12 per cent of the total number of maize-legume producers enumerated in Mzimba district (453) and Kasungu district (596), respectively. Therefore, the number of farms sampled in the two districts is proportionately represented. However, we realise that our sample size (135 out of 1049 farms) is relatively small; hence, we employ bootstrapping procedures to ameliorate some potential problems associated with sampling variability. Typically, farmers intercropped maize with common beans (*Phaseolus vulgaris*), groundnuts (*Arachis hypogaea*), or soya bean (*Glycine max*). Table 1 summarises the data.

**Table 1** Descriptive statistics for the variables used in the estimation ($n$ = 135)

| Variable name | Unit | Mean | SD | Min. | Max. |
|---|---|---|---|---|---|
| *Output* | | | | | |
| Crop output ($y$) | Output index/acre | 0.32 | 0.24 | 0.01 | 1.31 |
| *Inputs* | | | | | |
| Fertiliser nitrogen ($x_1$) | kg N/acre | 27.33 | 19.78 | 0.00 | 109.00 |
| Symbiotic nitrogen ($x_2$) | kg N/acre | 0.74 | 1.13 | 0.00 | 11.06 |
| Labour ($x_3$) | AEU/acre | 1.88 | 1.28 | 0.18 | 8.05 |
| Other variable inputs ($x_4$) | Input index/acre | 1.74 | 1.15 | 0.23 | 8.94 |

Note: AEU, Adult equivalent units (1 Male-adult = 1 AEU, 1 Female = 0.8 AEU, 1 Child = 0.5 AEU).

Total crop output is an implicit output-quantity index aggregated using a multilateral Tornqvist approach, where the contribution of each commodity is weighted by its relative share in total crop value and the corresponding natural logarithm of outputs (Caves *et al.* 1982).[1] Similarly, the aggregate value for other variable inputs represents an implicit input-quantity index comprising purchased seeds, nonfertiliser chemicals and herbicides. This implicit input-quantity variable is deflated by a weighted price index of expenditure share and the corresponding natural logarithm of inputs. A total of four production inputs are included in the estimation of the input distance function. These inputs are quantity of commercial fertiliser, family labour, farm expenses, and symbiotic nitrogen. The quantity of commercial fertiliser is expressed in kilograms of total nitrogen as the major active ingredient contained in the fertilisers used in production. The main mineral fertilisers used in the maize-based production in Malawi are Urea, Calcium Ammonium Nitrate, and 23:21:0 + 4S. These three fertilisers, respectively, contain 46 per cent, 27 per cent, and 23 per cent nitrogen (N). Family labour is converted to adult equivalent units (AEU), which represents farm labour supply measured in terms of full-time equivalent employees. The computation of AEU is based on the conversion factors as follows: one adult male (15 years of age and over, working on a full day-basis) represents one AEU, whereas one adult female working for a full day represents 0.8 AEU, and one child (5–14 years) working for a full day represents 0.5 AEU. Overall, the data presented in Table 1 show wide variations in terms of the input–output combinations, which reflects farm heterogeneity regarding resource endowment and managerial ability, among other factors.

Symbiotic nitrogen (LBSF-N) is included as an additional source of nitrogen, which is available to the component crops through intercropping and legume rotation. LBSF-N values are not directly observable. However, the literature currently contains abundant LBSF-N estimates that are obtained using reliable measurement methods, such as $^{15}$N-based techniques (Peoples *et al.* 2009; Ronner and Franke 2012). We use published LBSF-N estimates to compute the amount of symbiotic N fixed on agricultural land. The quantity of symbiotic nitrogen was computed based on the harvest-index method, which relates plant biomass, nitrogen concentrations, and the proportion of atmospheric nitrogen fixed (Høgh-Jensen *et al.* 2004; Herridge *et al.* 2008; Peoples *et al.* 2009). Alternatively, one could use total area under leguminous crops to quantify the amount of farm-level symbiotic nitrogen fixation. However, we prefer to use grain-weight because the quantity and quality of grain harvested also reflects the growth conditions in which the crop developed and matured (Stern 1993). Thus, crop productivity (yield) is

[1] We thank an anonymous reviewer for this suggestion. The translog multilateral output (input) index for the $k$th farm is given by: $\ln \delta_k = 0.5 \sum \left(R_i^k + \bar{R}_i\right)\left(\ln q_i^k - \overline{\ln q_i}\right)$, where $R$ represents revenue (expenditure) share for the $i$th commodity (input) and $q$ is the corresponding output (input) quantity.

used as an indirect measure of soil quality (fertility) and captures potential differences in soil quality across farms. Computation details for LBSF-N are provided in the appendix and summarised in Table S1.

Legume-based symbiotic fixation process was estimated for two cropping practices, namely, intercropping and crop rotation. LBSF-N from intercropping was estimated using the harvest-index method (see Table S1), whereas that from crop rotation was estimated using coefficients obtained from fitting a crop-response model (Stauber and Burt 1973; Stauber *et al.* 1975; Frank *et al.* 1990). The crop-response model was specified as a quadratic function to allow for diminishing marginal productivity as follows:

$$y_t = \alpha_0 + \alpha_1 N_t + \alpha_2 x_t + \alpha_3 N_t^2 + \alpha_4 x_t^2 + \alpha_5 N_t . x_t + \alpha_6 N_{t-1} + \varepsilon_t \tag{10}$$

where $y_t$ is output in the current season $t$, $N_t$ is the quantity of nitrogen applied in the current season, $N_{t-1}$ is the carry-over nitrogen from the previous season, $x$ represents other factors of production, and $\varepsilon$ is the random error term. Table S2 shows the results of the crop-response model.

In our sample, crop rotations had been adopted on 61 farms (45 per cent). Out of these 61 farms, 72 per cent were legume-maize rotations and most of the remaining (23 per cent) were tobacco-maize rotations. Further, 52 per cent of the sample (135 farms) practised *in-situ* crop residue retention, a practice aimed at enhancing soil productivity through residue mineralisation. We used this plot-history data to test whether crop rotation has significant effects on land productivity (i.e. crop yield) and the evidence suggests positive incremental effects (Table S2). The net residual nitrogen was then estimated implicitly as a ratio of input elasticities for the intercropped LBSF-N and the rotation variables.

## 5. Results and discussion

The coefficient estimates of the directional input distance function are reported in Table S3. The estimated first-order coefficients have the expected signs: they are positive for inputs and negative for output variables, with generally small standard errors.

### 5.1 Estimated measure of technical efficiency

Recall that technical inefficiency is given by the relative distance to the frontier and the shorter the distance, the more efficient the production unit is. Table 2 presents the results of the DIDF measure of technical inefficiency (TI). The DIDF estimates reveal a considerable level of inefficiency for the sample farms. Only 31 farms (representing 23 per cent of the total sample) were operating at maximum efficiency. The mean TI obtained from the bootstrap model is 0.52, compared to 0.47 from the base model. The TI obtained using the base (nonbootstrap) model is significantly lower than that

**Table 2** Estimated directional input distance function measure of technical inefficiency (TI)

| | Mean inefficiency | SE | [95 per cent confidence interval] |
|---|---|---|---|
| Base model | 0.47 | 0.03 | 0.41 to 0.53 |
| Bootstrap model | 0.52 | 0.06 | 0.40 to 0.64 |

obtained using the bootstrap model ($P < 0.1$). Therefore, the base model underestimates technical inefficiency. Generally, it is the bootstrap estimates that are considered plausible and robust (Mugera and Ojede 2014). Thus, the TI estimate of 0.52 implies that if the average farmer operated efficiently, she or he could produce the same output with an input bundle that is smaller by 52% of the mean input values reported in Table 1. For example, a producer who used 27.33 kg per acre of nitrogen fertiliser could have used 13.12 kg per acre of this input to produce the average output.

Our findings are comparable with previous studies conducted in the region, although few studies have applied directional distance functions on African agriculture. For example, using a directional distance function, Singbo and Lansink (2010) reported mean inefficiency of 0.20 for the Beninese rice and vegetable production system. In another study, Singbo *et al*. (2014) evaluated the performance of vegetable production in Benin and reported TI of 0.14 and marketing inefficiency of 0.25. Mulwa and Emrouznejad (2013) evaluated the performance of sugarcane production in Kenya and estimated TI to be 0.14. Collectively, this evidence shows that there is substantial scope for improving performance in the studied production systems and African agriculture in general.

### 5.2 Morishima elasticity of input substitution

The Morishima elasticity of substitution (MES) provides complete information about input substitutability in cases where a production technology has more than two inputs (Blackorby and Russell 1989). The MES measures the degree of curvature of the isoquant or the relative change in shadow prices associated with a unit change in the ratio of the corresponding inputs (Grosskopf *et al*. 1995). We calculate the indirect Morishima elasticities to get a sense of the ease with which one input can be substituted for another in the production process. Equation (11) shows the MES derived from the distance function approach (Blackorby and Russell 1989; Grosskopf *et al.* 1995):

$$MES_{nn'} = x^*_{n'}\left[\frac{DIDF_{nn'}}{DIDF_n} - \frac{DIDF_{n'n'}}{DIDF_{n'}}\right] \tag{11}$$

where $x^*_n$ is the frontier value of input (input level adjusted for inefficiency), and $\mathrm{DIDF}_n$ and $\mathrm{DIDF}_{nn}$ are the first-order and second-order derivatives of

**Table 3** Estimates of the indirect Morishima elasticity of input substitution

| | $f$ | $s$ | $l$ | $o$ |
|---|---|---|---|---|
| $f$ | −0.89 [−2.21 to 0.43] | 1.11 [−0.23 to 2.45] | 1.06 [−0.29 to 2.40] | 1.76 [0.20 to 3.33] |
| $s$ | 0.26 [0.10 to 0.42] | −0.13 [−0.20 to −0.07] | 0.01 [0.01 to 0.02] | 0.27 [0.14 to 0.39] |
| $l$ | 0.30 [0.23 to 0.36] | 0.03 [0.02 to 0.04] | −0.19 [−0.24 to −0.15] | 0.70 [0.28 to 1.13] |
| $o$ | 3.79 [0.11 to 7.48] | 2.08 [0.58 to 3.58] | 2.10 [0.59 to 3.61] | −3.70 [−7.33 to −0.08] |

Note: $f$, fertiliser nitrogen, $s$, symbiotic nitrogen, $l$, family labour; $o$, other input costs.

the directional input distance function, respectively. The MES estimates are reported in Table 3.

The sign and size of the MES are important: the elasticity sign helps to classify inputs as substitutes or complements, whereas the size of the elasticity indicates the degree of substitutability or complementarity. Inputs $n$ and $n'$ are considered to be Morishima substitutes if $MES_{nn'} < 0$, or complements if $MES_{nn'} > 0$. High MES values show a low degree of substitution while low values indicate relative ease of substitution (Grosskopf *et al*. 1995). As shown in Table 3, the MES elasticities are generally asymmetric ($MES_{nn'} \neq MES_{n'n}$). For example, the substitution of commercial fertiliser for symbiotic nitrogen gives an elasticity value of 1.11, whereas the reverse yields 0.26. The size and sign of the elasticities suggest that the two inputs are partially substitutable; thus, increasing fertiliser nitrogen to replace symbiotic nitrogen is relatively difficult, but the reverse is relatively easy.

## 5.3 Estimates of LBSF-N shadow prices

We apply Equation (5) to obtain shadow prices for LBSF-N. Table 4 gives a summary of shadow prices for the base and bootstrap models. The value of LBSF-N is estimated as a fraction of the average market price for commercial nitrogen. The price of commercial N is US$2.11/kg N, based on the 2013/14 crop-season price of fertiliser nitrogen (Urea) which was selling at MWK17,000 per 50 kg bag (1 US$ = 350 MWK). The shadow price values are positive and range from 1.01 to 22.23 US$/kg, when evaluated using the 95 per cent confidence interval.[2] The mean shadow price for LBSF-N is estimated to be US$5.26/kg for the bootstrap model, and US$20.1/kg for the base model. The difference between the two mean shadow prices is statistically significant ($P < 0.01$). We note that both the base and the

[2] The possibility that shadow prices could be sensitive to heterogeneities in soil quality has been pointed out by an anonymous reviewer. However, in the absence of explicit soil quality data, we believe that crop yield is a good proxy indicator to reflect potential differences in soil quality attributes across farms. In addition, if farmers aim to optimise investment returns, then their input allocation decisions, for example, fertiliser application will tend to approximate the prescribed agronomic recommendations which are based on soil quality differences across the country (Benson 1997). Since we only have input and output data, we consider that using the ratio of marginal products (input substitution effects) is the most reasonable approach that accounts for the differences in farm characteristics as well as farmer characteristics. As a result, the estimated shadow prices correspondingly vary across farms.

**Table 4** Estimated shadow prices for symbiotic nitrogen (US$/kg N)

| | Mean | SE | [95 per cent confidence interval] |
|---|---|---|---|
| Base model | 20.10 | 1.07 | 17.98 to 22.23 |
| Bootstrap model | 5.26 | 2.15 | 1.01 to 9.51 |

bootstrap models yield average shadow prices that are higher than the reference market price of US$2.11/kg N. The estimated shadow values can be interpreted as the opportunity cost of using LBSF-N in terms of foregone commercial nitrogen, keeping output constant. An alternative interpretation of shadow price values is to regard them as surrogate or implicit prices for a nonmarket good (LBSF-N). In this regard, the commercial fertiliser market could serve as a proxy market for LBSF-N. Thus, our estimated mean shadow price of US$5.26/kg could serve as an appropriate accounting value for fertiliser cost-savings, achieved as a result of substituting LBSF-N for fertiliser N. This shadow price represents the per-unit benefit that a producer would gain by using LBSF-N as a substitute for fertiliser nitrogen (Bond and Farzin 2007). For example, a farm operating without any unit of fertiliser N would increase crop output-value by US$5.26 if an extra kilogram of LBSF-N were available in the soil.

Because shadow prices for LBSF-N are not readily available in the literature, we compare our estimates against somewhat related environmental services. Table 5 presents these studies. The analysis of sustainable intensification or best management practice most closely related to ours is an application by Bond and Farzin (2007), which deals with the effect of low input production system using legume cover-crops, herbicides, and air pollution in California, USA. Unfortunately, due to limitations in their soil quality data, the study did not include shadow prices for nonmarketable inputs (legume-fertiliser effects). Other studies that have treated nitrogen leachate from agricultural sources as a bad output are Shaik *et al.* (2002), who studied the effect of organic and inorganic fertilisers in the USA; Reinhard *et al.* (1999), who assessed the Dutch dairy industry; and Piot-Lepetit and Vermersch (1998), who analysed the French pork sector. From these studies, the estimated shadow prices for excess nitrogen are reported to be in the range of US$2.00–4.77/kg for the US study, US$1.86/kg for The Netherlands, and US$0.14–1.02/kg for the French pork sector. Recently, Hou *et al.* (2015) estimated the cost of soil erosion and nitrogen loss in the Chinese Ansai region. From this study, the cost of soil erosion and nitrogen loss are estimated to be US$0.02 per kg/ha and US$0.06 per kg/ha, respectively. The reviewed studies show variations in the estimated shadow price values. The variation in the shadow price estimates in the above studies is not surprising because each study dealt with a different subsector that could differ in a number of ways, including operational scale and local environmental conditions at the study locations. Nevertheless, our results are close to

**Table 5** Selected studies using the distance function approach to value environmental goods and services

| Environmental good/service | Shadow price/value | Reference price/market | Estimation method | Efficiency score | Study region | Reference |
|---|---|---|---|---|---|---|
| Organic N (hog slurry) | \$0.14–1.02/kg | Commercial fertiliser | RDF/DEA | 0.94–0.96 | France | Piot-Lepetit and Vermersch (1998) |
| Organic N (dairy slurry) | \$1.86/kg | Dairy output | SFA | 0.89 | Netherlands | Reinhard *et al*. (1999) |
| Soil N (N pollution) | \$2.00–\$4.77/kg | Desirable output (crop and livestock products) | RDF | – | Nebraska, USA | Shaik *et al*. (2002) |
| Soil N (N pollution) | \$3.81-\$4.30/kg | Input cost (crop and livestock production) | RDF | – | Nebraska, USA | Shaik *et al*. (2002) |
| Fertiliser N | \$1.38 $\times$ $10^{-3}$–\$2.17 $\times$ $10^{-3}$ | 1 | DEA (bootstrap) | 0.64–0.88 (0.31–0.78) | Benin | Singbo *et al*. (2015) |
| Symbiotic N | \$20.10/kg | Commercial fertiliser price | DIDF | 0.47 | Malawi | This study |
| Symbiotic N | \$5.26/kg | Commercial fertiliser price | DIDF (bootstrap) | 0.52 | Malawi | This study |

Note: DIDF, directional input distance function; DEA, data envelopment analysis; RDF, radial distance function; SFA, stochastic frontier analysis.

the estimates for the US and the Netherlands (Reinhard *et al.* 1999; Shaik *et al.* 2002).

## 6. Conclusions

The challenge of maintaining or improving agricultural productivity in sub-Saharan Africa is enormous. As such, agricultural researchers and policy-makers are constantly looking for technologies that are economically attractive and environmentally sound. The best strategy to improve productivity and maintain soil fertility in sub-Saharan Africa should focus on a combination of both inorganic and organic fertilisers for maximum complementary benefits (Mafongoya *et al.* 2007). However, research evidence shows low adoption of integrated soil fertility management practices, which includes legume cultivation (Giller *et al.* 2009). A better understanding of the value of these legume systems is needed to develop more effective economic incentives that would facilitate the adoption of best management practices and reward soil conservation efforts.

Using the directional input distance function approach (DIDF), this study appraised the value of symbiotic nitrogen and estimated the technical efficiency of legume-based cropping systems (LBCS) in Malawi. Our results reveal two major findings. First, the results show that smallholder farmers exhibit substantial production inefficiency, with a mean directional inefficiency value of 0.52. By addressing this production inefficiency, an average farm could reduce each of the four inputs by 52 per cent while output remains constant. Second, the average shadow price for symbiotic nitrogen (LBSF-N) is higher than the observed market price for commercial nitrogen fertiliser. The shadow price values of symbiotic nitrogen (and LBCS) reflect only productivity benefits achieved without applying any unit of chemical fertilisers. The total value of LBCS could be greater if other environmental services and socio-economic benefits, such as their value as a disease break and for reducing soil erosion, are accounted for.

In the interest of maintaining a productive stock of soil capital, market-based mechanisms could be used to help enhance legume production and promotion of conservation agriculture. However, given the prevailing market prices, our estimated elasticities of input substitution demonstrate that a complete substitution to organic fertilisers can be detrimental to farm productivity. This result infers that complete conversion to organic production will require some compensation to farmers, for example through higher output prices, for such a substitution to be profitable. Therefore, one of the possible policy options to incentivise farmers' investments in conservation agriculture is to facilitate price premiums for commodities produced from sustainably managed farms, such as LBCS. For example, recent studies have shown that some farmers in other African countries such as Ghana, Kenya, and Uganda are benefiting from price premiums received as a result of participating in certification programs targeting low input sustainable agricultural production systems (Bolwig *et al*. 2009; Kleemann and Abdulai 2013; Ayuya *et al*. 2015). Thus, price premiums can provide incentives to farmers to invest in legume-based conservation agriculture as part of the integrated soil fertility management strategies.

Organic soil amendments build long-term soil fertility benefits that are usually heavily discounted by land users, who mostly seek to maximise their present farm benefits. As a result, there is less investment in conservation practices by the land users, partly because produce from such low input sustainable agricultural systems are considered as nondifferentiated products. Thus, the prevailing commodity prices fail to incentivise conservation agriculture. We contend that price premiums could be a better alternative and a more cost-effective policy instrument for promoting LBCS than the subsidies or public-support programs that are currently being used to promote legume production in Malawi and other African countries. We therefore recommend that future research should investigate the potential demand for produce from sustainably managed farms, and also mechanisms through which farmers can be integrated into existing or emerging regional and export markets for such products. Specifically, policymakers could focus on creating and promoting an enabling environment that allows the potential benefits of LBCS to be fully exploited by: (i) promoting knowledge about soil and other benefits of the integrated cropping systems; (ii) supporting more sustainable production processes (e.g. through certification and labelling requirements); and (iii) channelling support from current input subsidies to the support of extension and market development activities.

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## Supporting Information

Additional Supporting Information may be found in the online version of this article:

**Appendix S1.** Computation of symbiotic nitrogen using the harvest index approach

**Table S1.** Estimates of SNF for the sample grain-legume cropping systems

**Table S2.** Coefficient estimates for the quadratic crop-response model

**Table S3.** Estimated parameters for the quadratic directional distance function