Chance Constrained Programming Models for Risk-Based Economic and Policy Analysis of Soil Conservation

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The random nature of soil loss under alternative land-use practices should be an important consideration of soil conservation planning and analysis under risk. Chance constrained programming models can provide information on the trade-offs among pre-determined tolerance levels of soil loss, probability levels of satisfying the tolerance levels, and economic profits or losses resulting from soil conservation to soil conservation policy makers. When using chance constrained programming models, the distribution of factors being constrained must be evaluated. If random variables follow a log-normal distribution, the normality assumption, which is generally used in the chance constrained programming models, can bias the results.

Many economic and policy analyses of soil conservation [Wade and Heady; Walker and Timmons; Kramer, McSweeny, and Stavros] as well as soil conservation planning have been conducted based on the long-run average soil loss through employing the University Soil Loss Equation (USLE) [Wischmeier and Smith]. For instance, the state and federal agricultural Best Management Practice (BMP) cost-share programs in Virginia require that participants have land that is either rated in the top one-third of highly erodible land or have a water quality index category rating of 5 or greater [Virginia Department of Conservation and Recreation]. The erodibility ranking and water quality index are determined by the USLE used in conjunction with the Virginia Geographic Information System [Shanholtz et al.]. The tolerance of soil loss in a particular area is, thus, implicitly determined by the weighted erodibility ranking and water quality index of land in the area and available cost-share funds for that area.

While the long-run average amount of soil loss is of a basic concern from a policy perspective, so, too, is the probability distribution of annual soil loss. It is known that rainfall related soil loss under field conditions is stochastic. The amount of soil loss depends on many factors including topography, soil characteristics, rainfall rates and amounts, cover, and management [Knisel]. The amount of soil loss estimated by the deterministic form of the USLE represents the long-run average of a particular crop system in a given area. Since the time period that individual farmers participate in a given soil conservation program is generally shorter than the long-run, the effectiveness of a conservation program is influenced by the probability distribution of soil loss. From the government's perspective, trade-offs among the desire to decrease the tolerance level of soil loss, increasing the probability of achieving a certain soil loss tolerance level, and expanding the amount of land with BMPs with limited cost-sharing funds may exist. From the farmers' perspective, increasing the probability of satisfying tolerance levels requires the adoption of more restrictive conservation practices, which may discourage their participation in soil conservation programs. Therefore, information on the relationship between changes in tolerance levels of soil loss, probability levels, and their economic impacts are useful for risk-based economic and policy analysis of soil conservation.

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The objectives of this research are to: (1) discuss the implications of using chance constrained programming models for risk analysis; (2) develop a chance constrained programming model with a log-normal distribution for soil loss; and (3) compare the results of this model with two approximate approaches: a normal distribution of soil loss, and a linearized normal distribution of soil loss.

Chance Constrained Programming Models and Decision Making under Risk

Soil conservation planning consists of decision making under risk because the results of conservation practices are, to a great degree, affected by the weather related random variables. Typically, decision making under risk should involve the costs and benefits of recourse; that is, the costs and benefits of information from observed random variables [Hogan, Morris, and Thompson]. For example, theoretically, the cost of the probability of failure to achieve soil loss tolerances, including on-site and off-site costs of soil loss, should be explicitly included in risk-based economic and policy analysis of soil conservation.

Stochastic programming with recourse (SPR), which directly couples the cost of recourse to the objective function, has been recognized as a proper method to address decisions under risk [Hogan, Morris, and Thompson]. However, three limits to the application of SPR to soil conservation analysis exist. First, measurement of the cost of recourse is difficult in that tons of soil loss cannot easily be measured in the same units as a monetary objective function, thus, a non-commensurable case exists. Little research has been done on the application of SPR to non-commensurable cases. Second,the concept of recourse itself is inapplicable to current soil conservation policies given that soil conservation programs are voluntary in the United States. For example, when the government uses cost-share payments to stimulate farmers to adopt soil conserving best management practices, the participants are subject to inspections for program compliance to get government payment. Third, there is a lack of sufficient monitoring to assess the performance of individual farms' conservation activities. Using an expected utility function as the objective function may be an alternative for application of SPR to soil erosion [Keeney and Raiffa]. However, difficulties associated with the development of a satisfactory utility function may be significant.

Chance constrained programming (CCP) is a possible alternative to SPR for risk-based decision making and it was widely applied in 1960's and 1970's. The CCP model maximizes the objective function subject to constraints that must be maintained at a prescribed level of probability [Charnes and Cooper]. Advocates of CCP argue that CCP is a simple technique to analyze risky decisions with recourse, while opponents of CCP deny that it is a viable alternative to SPR. The debate centers around whether the chosen risk level can capture the complexities of the costs and benefits of recourse[1] [Hogan, Morris, and Thompson].

In spite of this controversy, CCP still can be an alternative model for risk analysis of soil conservation. The CCP model provides information on the trade-offs among the objective function value, tolerance values of the constraint, and the prescribed level of probability, which could be valuable to policy makers and farmers.

A typical CCP model can be expressed as follows:

$$\text{(1)}\qquad \begin{aligned} &\text{maximize } f(c,X) \\ &\text{Subject to: } \Pr[AX \leq b] \geq \alpha \\ &X \geq 0 \end{aligned}$$

where: f(c, X) is the objective function; X is the decision variable vector; A is matrix of technical coefficients; b and c are vectors of coefficients; and α is the prescribed level of probability. Not all of the coefficients in A or b are necessarily random in empirical CCP models. Assuming that the soil loss coefficient is the only random variable in the model, then the soil loss constraint is the only chance constraint in the model. Let the i^{th} row of the A matrix represent the soil loss constraint in this study. The model can be rewritten as:

$$\text{(2)}\qquad \begin{aligned} &\text{maximize} \quad f(c,X) = \sum_j c_j X_j \\ &\text{Subject to: } \Pr\left[\sum_j a_{ij} X_j \leq b_i\right] \geq \alpha_i \\ &\sum a_{kj} X_j \leq b_k \qquad \forall\, k \neq i \\ &X \geq 0 \end{aligned}$$

where: the objective function coefficient, c_j is net returns, X_j is decision variable, crop rotation, $1 - \alpha_i$ represents the acceptable risk of not meeting the soil loss constraint, and the a_{ij} represent soil loss coefficients, which are functions of a random variable, the rainfall and runoff factor, R.

[1] A summary of advocates' and opponents' views of CCP modeling can be found in Hogan, Morris, and Thompson. CCP models are used to explore the trade-offs, but not the cost of recourse in this study.

The USLE can be expressed as follows:

$$a_{ij} = RKLSC_jP_j \quad (3)$$

where: a_{ij} is the tons of soil loss per acre per year associated with the crop rotation j; R is the rainfall and runoff factor; K is the soil erodibility factor; L is the slope-length factor; S is the slope-steepness factor; and C_j and P_j are the cover and management factor and the conservation support practice factor associated with the rotation j, respectively [Wischmeier and Smith]. In the area where the amount of thaw and snowmelt are not significant,[2] the rainfall and runoff factor, R, is equal to Rainfall Erosion Index, EI. R, K, L, S, C_j, and P_j are typically used in a deterministic fashion by using their mean values.

Rainfall related soil loss under field conditions is, however, stochastic. The large number of possible combinations of the levels of R, K, L, S, C_j, and P_j could make accurate measurement of soil loss infeasible. Wischmeier and Smith found that for a particular rainfall area, when the probability of R can be calculated, the USLE can be used for estimation of the probable range of soil loss if other factors are at known levels. That is, if K, L, S, C_j, and P_j can be treated as known parameters for a given area, the distribution of soil loss coefficient, a_{ij}, depends on the random variable, R.

The solution to the CCP model in equation (2) maximizes the objective function subject to total soil loss being less than or equal to the tolerance level of soil loss, b_i, at a probability greater than or equal to a preassigned level, α_i, as well as satisfying other constraints in the model such as labor and land. The double inequality of the soil loss chance constraint can be reformulated as:

$$\sum_j \bar{a}_{ij}x_j + \theta\sqrt{\sum_k\sum_j x_k x_j \sigma_{ikj}} \leq b_i \quad (4)$$

where: $\bar{a}_{ij}$ is the mean value of a_{ij}, σ_{ikj} is the variance-covariance matrix of the a_{ij}'s, and θ is a constant that depends on the distribution of random variable, a_{ij} [Merrill]. While θ can be determined through either Tchebysheff's approximation or assuming the distribution of the random variable, most previous researchers used the latter method of assuming, implicitly or explicitly, that the random variable has a normal distribution since this assumption leads to a linear problem that is readily solvable [Kirby].

[2] A detailed description of the R value for thaw and snowmelt can be found in Wischmeier and Smith. In the rest of this article, thaw and snowmelt will not be considered since they are not important factors affecting soil loss in the study area.

However, Wischmeier and Smith analyzed 20 to 25-year rainfall records from 181 weather stations across the United States, and showed that the annual EI, and therefore also R, tends to follow a log-normal distribution rather than a normal distribution. Applications of CCP to soil conservation have to assume a normal distribution or a new CCP model with a constraint that includes a random variable that follows a log-normal distribution must be developed. In this article, a CCP model with a log-normal distribution is introduced and comparisons among the results of CCP models with a normal distribution, linearized-normal constraint, and log-normal distribution are provided with an empirical CCP model. These three models are discussed below.

Chance Constrained Programming Model with a Normal Distribution

Following Segarra, Kramer, and Taylor and for ease of exposition, it will be assumed that the coefficients a_{ij} are mutually independent. Equation (4) can be rewritten as:

$$\sum_j \bar{a}_{ij}x_j + \theta\sqrt{\sum_j \sigma_{ij}^2 x_j^2} \leq b_i \quad (5)$$

where σ_{ij}^2 is the variance of a_{ij}. When the distribution of random variables is assumed to be normal, equation (5) becomes:

$$\sum_j \bar{a}_{ij}x_j + K_{\alpha_i}\sqrt{\sum_j \sigma_{ij}^2 x_j^2} \leq b_i \quad (6)$$

where: $(\Sigma\sigma_{ij}^2x_{ij}^2)^{1/2}$ is the standard deviation of $\Sigma a_{ij}x_j$, and K_{α_i} is the standardized normal value with an α_i percent probability; that is, the constraint will be met α_i percent of the time. Given left hand side of equation (6) is non-linear, nonlinear programming would have to be used to obtain a solution. Also, it should be noted that the function in equation (6) is neither strictly convex nor concave. Thus, the feasible region for an empirical problem may not be a convex set.

Chance Constrained Programming Model with a Linearized Normal Chance Constraint

Segarra, Kramer, and Taylor demonstrated that the non-linear chance constraint of equation (6) could be linearized, and would result in the following constraint:

$$\sum_j \bar{a}_{ij}x_j + K_{\alpha_i}\sum_j \sigma_{ij}x_j \leq b_i \quad (7)$$

They pointed out that this would be a conservative constraint, as $\Sigma\sigma_{ij}x_j$ in equation (7) is greater than $(\Sigma\sigma_{ij}^2x_j^2)^{1/2}$ in equation (6). The advantage of this approach is that linear programming may be used to solve the problem.

Chance Constrained Programming with a Log-normal Distribution

When the random variable such as the rainfall and runoff factor, R, follows a log-normal distribution,

$$(8)\quad \ln(R) \sim N(m_R,\sigma_R^2) \text{ and } R \sim \Lambda(U_R,D_R^2)$$

where: m_R is the mean value of the corresponding normal distribution of ln(R); σ_R^2 is the variance of the corresponding normal distribution of ln(R); U_R is the mean of the log-normal distribution of R; and D_R^2 is the variance of the log-normal distribution of R. Here, m_R and σ_R^2 can be calculated based on historical rainfall records and U_R and D_R^2 can be calculated based on the definition of the log-normal distribution [Crow and Shimizn], as follows:

$$(9)\qquad U_R = e^{\left(\frac{\sigma_R^2}{2}+m_R\right)}$$

and

$$(10)\qquad D_R^2 = e^{(\sigma_R^2+2m_R)}(e^{\sigma_R^2}-1)$$

Since $a_{ij} = R[KLSC_jP_j]$, $a_{ij} \sim \Lambda(k_jU_R, k_j^2D_R^2)$, where: $k_j = KLSC_jP_j$, and

$$(11)\qquad a_{ij}x_j \sim \Lambda(k_jx_jU_R, k_j^2x_j^2D_R^2)$$

Numerical convolution of the log-normal distribution has shown that the sum of such a distribution, $\Sigma a_{ij}x_j$, is a distribution that approximately follows the log-normal law. That is, $\Sigma a_{ij}x_j$ is not identically a log-normal distribution, but an equivalent log-normal distribution can be found that has the same first and second moments as the exact sum distribution [Fenton; Crow and Shimizn; Sarin and Srivastava]. Assuming this equivalent log-normal distribution has a mean value of U and variance of D^2:

$$(12)\qquad \sum_j a_{ij}x_j \sim \Lambda(U,D^2)$$

$$(13)\qquad U = \sum_j k_jx_jU_R, \qquad \text{and:}$$

$$(14)\qquad D^2 = \sum_j k_j^2x_j^2D_R^2, \qquad \text{so:}$$

$$(15)\qquad \ln\left[\sum_j a_{ij}x_j\right] \sim N(m,\sigma^2)$$

where: $m = \ln(U) - (\sigma^2/2)$, and $\sigma^2 = \ln(D^2/U^2 + 1)$. Therefore, $\ln(\Sigma a_{ij}x_j)$ follows a normal distribution with mean value m and variance σ^2, both of which are a function of m_R and σ_R.

Taking the logarithm of both sides of the soil loss constraint, and since $\Sigma a_{ij}x_j$ is a monotonic function, the soil loss constraint can be rewritten as:

$$(16)\qquad \ln\left[\sum_j a_{ij}x_j\right] \leq \ln b_i$$

Since $\ln(\Sigma a_{ij}x_j)$ has a normal distribution, the chance constraint can be expressed as:

$$(17)\qquad m + K_{\alpha_i}\sigma \leq \ln b_i$$

where: m is the mean value of $\ln[\Sigma a_{ij}x_j]$, $m = E[\ln(\Sigma a_{ij}x_j)]$; σ is the standard deviation of $\ln[\Sigma a_{ij}x_j]$; and K_{α_i} is the standardized normal value with a α_i percent probability. When written out in detail, the chance constraint based on the log-normal distribution of R has the following form:

$$(18)\qquad \ln\left[\sum_j k_jx_je^{\left(\frac{\sigma_R^2}{2}+m_R\right)}\right] - \frac{1}{2}\left[\frac{\sum_j k_j^2x_j^2(e^{\sigma_R^2}-1)}{\left(\sum_j k_jx_j\right)^2}+1\right] + K_{\alpha_i}\sqrt{\ln\left[\frac{\sum_j k_j^2x_j^2(e^{\sigma_R^2}-1)}{\left(\sum_j k_jx_j\right)^2}+1\right]} \leq b_i$$

At $\alpha_i = 50$ percent, the third term on the left-hand side of equation (18) is zero because K_{α_i} is zero. Thus, given the skew of the log-normal distribution to the right, the soil loss constraint is less restrictive than might otherwise be anticipated at $\alpha_i = 50$ percent, because with the third term being zero, the first term on the left-hand side has the second term subtracted from it. Noting of this skewness effect is important for interpretation of the results later in this article. In this case as in the case of the normal distribution, non-linear programming would have to be used to obtain a solution and given the nature of equation (18), the feasible region may not be a convex set.

Empirical Model

In order to compare the results among these three CCP models that employ chance constraints based on a log-normal distribution, a normal distribution, and linearized version of the normal distribution, this analysis uses the data on the farming practices from a study by Segarra, Kramer, and Taylor. Storm data from gauge records of rainfall intensity from Blacksburg, VA, from 1943 to 1971 are employed to calculate the annual rainfall erosion index, EI,[3] and develop the distribution of soil loss coefficient a_{ij}.

In brief, Segarra, Kramer, and Taylor's model consists of an objective function, which maximizes net returns to land, management, and capital on a representative farm in south-central Virginia. Constraints, in addition to the soil loss constraint, include 174 acres of land, monthly labor availability, and a tobacco allotment of 37,800 pounds. The model contains production activities for various crop rotations, selling activities for each crop, and labor hiring activities. The crops considered in the model are: conventional tillage corn (CT); no-till corn (CNO); wheat (W); barley (BA); grass (G); soybeans (S); tobacco (TB); double cropped wheat and soybeans (DWS); and tobacco with a cover crop (TB/c). The sixteen crop rotations incorporated in the model include: (1) two year rotations of: conventional tillage corn—winter wheat (CTW), no-till corn—winter wheat (CNOW), conventional tillage corn—tobacco (CTBA), no-till corn—barley (CNOBA), tobacco—winter wheat (TBW); tobacco—barley (TBBA), conventional tillage corn—double cropped wheat and soybeans (CTDWS), and no-till corn—double cropped wheat and soybeans (CNODWS); (2) three year rotations of: conventional tillage corn—wheat—soybeans (CTWS), no-till corn—wheat—soybeans (CNOWS), conventional tillage corn—barley—soybeans (CTBAS), and no-till corn—barley—soybeans (CNOBAS); (3) four year rotations of: tobacco with a cover crop—tobacco—barley—grass (TB/cTBBAG) and tobacco with a cover crop—tobacco—wheat—grass (TB/cTBWG); and (4) continuous tobacco (TB) and continuous grass (G). Soil loss was constrained to 5, 6, 8, and 10 tons per acre per year. For illustration, four levels of α_i, 50 percent, 80 percent, 90 percent, and 95 percent, were employed.

To find the optimal solution of the normal and log-normal models, different starting values were employed for each scenario since their feasible regions may not be convex sets. First, to start the solution at the boundary of the feasible region, a starting value of 174 acres was assigned to one rotation and zero was assigned to the others. This process was repeated until each of the sixteen rotations had started at 174 acres. More than ten stochastically selected interior starting points were also examined. The results at each starting point for a given soil loss tolerance and probability level were the same, indicating that the optimal solution is unique. All the solutions being the same from different starting points suggests that the normal and log-normal models did, in fact, have a convex feasible region in this empirical analysis.

Results

The results for 50 percent probability with the linearized-normal model and normal model represent the solution to the deterministic linear programming model. Comparing these results to the results with other probability levels indicates the impacts of coupling soil loss probability on the soil conservation decision and incomes of farmers. The acreage of each rotation, resulting from the modeling, is presented in Table 1. Several points from Table 1 are: (1) more restrictive soil loss tolerances lead to the use of less erosive rotations and can lead to the idling of land. For example, with the log-normal model and a probability level of 90 percent, no acreage is assigned to the more erosive rotation, TB, and 26.72 and 67.08 acres of land must be idled in order to meet the six and five tons per acre soil loss levels, respectively, while no land needs to be idled when the soil loss tolerances are ten and eight tons. (2) A higher probability of meeting the chance constraint also has similar effects on land use as more restrictive soil loss tolerances. For example, with the log-normal model, when soil loss tolerance is five tons, the acreage of land that must be idled increases from 34.63 to 88.87 acres when the probability levels increase from 80 to 95 percent. (3) When the soil loss constraint is binding, the log-normal model results in a more restricted land use than a model with the normal distribution. For example, with a probability level of 95 percent and a soil loss tolerance of five tons, the log-normal model idles 88.87 acres of land while 39.75 acres are idled with the normal model. (4) Finally, when the soil loss constraint is binding, the linearized-normal model results in the use of less erosive rotations and the idling of more land at all soil loss tolerance and probability levels than the model with the normal distribution since the linearized-normal chance constraint is more restrictive than the normal distribution chance constraint.

The economic consequences of these different

[3] A detailed description of calculation of EI values can be found in Wischmeier and Smith.

Table 1. Distribution of Acreage Among the Rotations for the Three Models by Soil Loss Constraint and Probability Level[a]

α_i (%)	Soil Loss Limit (Tons/Acre)	CNOW	CNOBA	TB/c TBBAG	TB/c TBWG	TB	CTDWS	CNODWS	Idle (G)
				LOG-NORMAL DISTRIBUTION					
	10					18.00	39.95	116.05	
50	8					18.00	39.95	116.05	
	6					18.00	39.95	116.05	
	5				3.28	16.36	0.08	153.55	
	10			18.00				156.00	
80	8			18.00				156.00	
	6			12.28	12.85	5.44		143.44	
	5			17.96	18.04			103.37	34.63
	10					18.00		156.00	
90	8			1.50	1.95	16.27		154.27	
	6			17.97	18.03			111.28	26.72
	5			17.98	18.02			70.92	67.08
	10					18.00		156.00	
95	8			13.44	13.62	4.47	6.28	136.19	
	6	8.23	8.42	17.98	18.02		0.45	73.79	47.11
	5	1.69	1.81	17.99	18.01			45.63	88.87
				NORMAL DISTRIBUTION					
	10					18.00		156.00	
50	8					18.00		156.00	
	6					18.00		156.00	
	5				13.66	11.17		149.17	
	10					18.00		156.00	
80	8					18.00		156.00	
	6			3.39	4.24	14.19		152.87	
	5			17.93	18.07			129.25	8.75
	10					18.00		156.00	
90	8					18.00		156.00	
	6			9.94	10.45	7.80		145.81	
	5			17.96	18.04			110.70	27.30
	10					18.00		156.00	
95	8					18.00		156.00	
	6			15.19	15.53	2.64		140.64	
	5			17.97	18.03			98.25	39.75
				LINEARIZED NORMAL DISTRIBUTION					
	10					18.00		156.00	
50	8					18.00		156.00	
	6					18.00		156.00	
	5				13.66	11.17		149.17	
	10					18.00		156.00	
80	8					18.00		156.00	
	6				17.93	9.03		147.04	
	5				36.00			110.24	27.76
	10					18.00		156.00	
90	8					18.00		156.00	
	6				34.50	0.75		138.75	
	5				36.00			81.26	56.74
	10					18.00		156.00	
95	8					18.00		156.00	
	6				36.00			116.77	21.23
	5				36.00			60.93	77.07

[a] CNO = no-till corn; W = winter wheat; BA = barley; TB/c = tobacco with a cover crop; TB = tobacco; CT = conventional tillage corn; DWS = wheat-soybean double-crop; G = grass.

Table 2. Total Net Income for the Representative Farm as Influenced by Chance Constraint Formulation, Levels of Soil Loss Permitted, and Probability Level

Chance Constraint	Probability (α_i)	Soil and Loss Limit (Tons/Acre)			
		5	6	8	10
Log-Normal	50%	42,944	42,944	42,944	42,944
	80%	29,958	40,425	42,944	42,944
	90%	21,168	32,099	42,598	42,944
	95%	15,089	25,732	40,229	42,944
Normal	50%	41,576	42,944	42,944	42,944
	80%	36,965	42,180	42,944	42,944
	90%	31,943	40,900	42,944	42,944
	95%	28,571	39,865	42,944	42,944
Linearized-Normal	50%	41,576	42,944	42,944	42,944
	80%	31,821	41,149	42,944	42,944
	90%	23,972	39,490	42,944	42,944
	95%	18,467	33,589	42,944	42,944

approaches to constraining soil loss are summarized in Table 2. The objective function value of $42,944 indicates that the soil loss constraint is not binding. Therefore, with a soil loss limit of ten tons per acre, the chance constraint is not binding for any probability level or model. At eight tons of soil loss, the constraint is binding for the log-normal model at a probability of 90 percent and higher. At six tons per acre, the constraint is binding at probability levels greater than 50 percent. Finally, at five tons per acre of soil loss, the constraint is binding at all probability levels for all models, except for the 50 percent probability level with the log-normal model due to the rightward skew of log-normal distribution. Given that the more erosive rotations, such as tobacco, have higher net returns in the study area, the farmer's net returns decreases with lower soil loss tolerance and higher probability levels. Decreases in net returns from these three models, however, are different. The log-normal model results in the most decrease in net returns and the normal model results in the least decrease in net returns when the soil loss constraint is binding. For example, with a 95 percent probability level and a five tons per acre soil loss restriction, the log-normal chance constraint produces an income of $15,089 while with the normal distribution, the income is $13,482 higher, and with the linearized-normal approach, income is $3,378 higher than the log-normal results. The approximations of the log-normal distribution with the normal distribution and linearized chance constraint, therefore, allow more soil loss than the log-normal model at higher probability levels of meeting the lower soil loss tolerance level.

Conclusions

Chance Constrained Programming models provide quantitative trade-offs associated with soil conservation among net returns, tolerance levels of soil loss, and the probability of meeting that tolerance level, which can provide valuable information to farmers and policy-makers. Given the random nature of soil loss, limited cost sharing funds, and non-commensurability between the quantity of soil loss and net returns, selection of land targeted for soil conservation programs and the determination of cost-share payment eligibility could be improved by using information on these trade-offs.

This analysis also demonstrates that when using a CCP model, attention must be given to the distribution of the factors being constrained. While normality is often implicitly, if not explicitly, assumed, it needs to be tested rather than assumed. With chance constrained programming, assuming normality when the distribution is not normal biases the results of the analysis. An *a priori* direction to this bias cannot be assigned because it depends upon the behavior of the actual distribution compared to the normal distribution. In the empirical example of this study, assuming normality underestimates the amount of soil erosion once the chance constraint becomes binding, and thus allows for more adverse environmental impacts than the probability level would suggest. Also, when the constraint is binding, maintaining the normality assumption consistently overestimates farm income compared to the actual log-normal distribution.

To avoid non-linear programming, researchers have often linearized the chance constraint. While the linearized-normal chance constraint produces results closer to the log-normal distribution than the normal distribution does in this study when the soil loss constraint becomes binding, the precision of the linearized approximation can be expected to vary from case to case depending on the actual distribution of the random variable. Given the availability of powerful non-linear programming software such as GAMS[4] [Brooke, Kendrick, and Meeraus], it is possible to model the actual distribution in order to enhance the accuracy of results.

The log-normal model introduced in this article could be empirically employed for soil conservation analysis and policy formulation. However, attention must be given to evaluating whether a local or global optimal solution has been obtained.

Finally, this study provides important information to policy analysts using stochastic programming. In particular, when evaluating policy associated with reducing adverse environmental impacts of agriculture relating to pesticide and nutrient leaching and runoff, it is likely that data associated with these problems will not be normally distributed, because runoff and leaching are driven by rainfall.

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[4] Solutions to the three models were obtained with GAMS version 2.05 [Brooke, Kendrick, and Meeraus] on a 486 personal computer. Computer time necessary to solve each of three models is less than one minute.