

# This document is discoverable and free to researchers across the globe due to the work of AgEcon Search. 

## Help ensure our sustainability. Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

Papers downloaded from AgEcon Search may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.

# Modeling Regulated Open Access Resource Use 

By<br>Frances Reed Homans

B.A. (Pomona College) 1983
M.S. (University of California) 1991

DISSERTATION
Submitted in partial satisfaction of the requirements for the degree of DOCTOR OF PHILOSOPHY
in

## Agricultural Economics

in the
GRADUATE DIVISION
of the
UNIVERSITY OF CALIFORNIA
DAVIS

Approved:


Committee in Charge 1993

## FRANCES REED HOMANS

Frances Reed Homans

December 1993

# Modeling Regulated Open Access Resource Use 


#### Abstract

This thesis develops a new model of regulated open access resource exploitation. The basic H.S. Gordon model of open access rent dissipation is extended in two ways, by adding a model of regulatory behavior and by adding a market sector. The regulatory model assumes that regulators are goal oriented, choosing target harvest levels according to a safe stock concept. These harvest quotas are implemented by setting season lengths, conditioned on the industry fishing capacity. The industry, in turn, enters according to rents, conditioned on season length regulations. Harvest levels, fishing capacity, season length, and biomass are determined jointly and dynamically. Joint dynamics depend upon speeds of adjustment and parameters of the system and may be complex, including asymptotic and oscillatory patterns.

The market sector model endogenizes exvessel prices, by modeling wholesale inventory dissipation behavior with an adjustment cost framework. A dynamic model of optimal inventory dissipation within each marketing period is coupled with a dynamic model of optimal carryover between marketing periods. This forms the basis for characterizing exvessel prices, which emerge as a derived demand for additions to inventory based on the optimal


dissipation model. Exvessel prices are linked to the industry/regulator and wholesale market sectors via total harvest and season length regulations, which determine the marketing period length and optimal dissipation/carryover plans.

The parameters of both the industry/regulator and the market model are estimated for the North Pacific Halibut fishery. Simulations compare predictions from the basic Gordon model with those from the new model. Predictions from the modified model are significantly different from the Gordon model. In particular, the role of the regulatory sector is critical, producing higher biomass levels than under pure open access. These larger capacity levels must be stifled, resulting in very short seasons such as those currently in the halibut fishery. The introduction of the marketing sector clarifies complicated interrelationships between rents, regulations, and revenues. Recent changes associated with new fisheries programs that have been largely unanticipated by economists using the old Gordon paradigm are better explained by the new model.

## Acknowledgements

Many, many thanks to Jim Wilen. I was one of his more challenging students, I think-but he took all of me on, and worked to shape an economist out of me. I would have a very different life story if not for all of his watchful tending. I can't thank him enough for all his gifts of guidance, wisdom, and care.

Thanks to my committee. Thanks to Michael Caputo for his emphatic criticism, effusive praise, and energetic friendship (who can talk about kitties like Michael can?). Thanks to Art Havenner for his great teaching and constant confidence in me.

Mary McNally has been a true friend through the struggles, and also a fine social companion. Constance, Yu Lan, Big John, Ken, Ray, Cathy D., Marca, and Janis are a part of the fabric of my life for good. Peter, of course, too. This time, he didn't send in for prayers for my thesis, but offered support, beers, arbo runs, and a floor to sleep on.

Thanks to Rich Sexton for early encouragement, and thanks to Richard Howitt for stepping in with interest and expertise at just the right time. Thanks to Barbara Huffine for her sweet self and Kathy Edgington for calm helpfulness. Thanks to Brett for some last minute advice.

Thanks to my family. Thanks Mom-thanks for all the support, financial and otherwise. Thanks for making a home of good ideas, good stories, and high expectations.

## Contents

Table of Contents ..... v
List of Figures ..... viii
List of Tables ..... x
1 Introduction, Objectives, and Outline ..... 1
1.1 Introduction ..... 1
1.2 The Gordon Model ..... 3
1.3 Omissions and Refinements of the Gordon Model ..... 6
1.3.1 Rent Dissipation and Biology ..... 6
1.3.2 Rent Dissipation and Revenues ..... 9
1.3.3 Rent Dissipation and Regulations ..... 11
1.4 Thesis Objectives and Outline ..... 12
2 The Pacific Halibut Fishery ..... 17
2.1 Introduction ..... 17
2.2 Biology ..... 18
2.3 Fishing Technology ..... 19
2.4 History of the Fishery and Regulatory Structure ..... 19
3 Modeling Industry/Regulator Behavior ..... 30
3.1 Introduction ..... 30
3.2 Fishermen's Behavior ..... 31
3.3 The Entry Function and Industry Behavior ..... 34
3.4 Regulator Behavior ..... 40
3.5 Equilibrium in a Regulated Open Access Fishery ..... 43
3.6 Comparative Statics ..... 45
3.7 A Dynamic Model of the Regulated Open Access Fishery ..... 48
4 Estimation of the Industry/Regulator Model ..... 56
4.1 Introduction ..... 56
4.2 Empirical Estimation Issues ..... 57
4.3 Data ..... 59
4.3.1 Biomass Data $(X(t))$ ..... 61
4.3.2 Fishing Capacity $(E(t))$ ..... 62
4.3.3 Harvest $(H(t))$ and Exvessel Prices $(P(t))$ ..... 63
4.3.4 Quotas $(Q(t))$ and Season Lengths $(T(t))$ ..... 63
4.4 Parameter Estimates: Static Model ..... 64
4.5 Parameter Estimates: Dynamic Models ..... 70
4.6 Estimating Regulatory Behavior ..... 77
4.7 Simulations ..... 86
5 Modeling Inventory Dissipation and Carryover ..... 94
5.1 Introduction ..... 94
5.2 Optimal Inventory Behavior with Semiperishable Commodities ..... 97
5.3 Inventory Behavior With Parametric Carryover ..... 99
5.4 A Linear Quadratic Adjustment Cost Model of Inventory Behavior ..... 105
5.5 Industry Equilibrium With Inventory Holding ..... 116
5.6 Carryover ..... 123
5.7 An Industry Model of Carryover ..... 139
6 Empirical Estimation of Exvessel Prices ..... 145
6.1 Introduction ..... 145
6.2 Estimation Issues in Models of Dynamic Decisions ..... 147
6.2.1 Expectations ..... 149
6.3 Modeling the Exvessel Demand Curve ..... 163
6.4 Discussion ..... 175
7 Rents, Regulations, and Revenues: Modeling Regulated Open Access Resource Use ..... 181
7.1 Introduction and Overview ..... 181
7.2 The Quota/Biomass Link ..... 184
7.3 Simulating Regulated Open Access Use ..... 189
7.3.1 The Static Gordon Model ..... 191
7.3.2 The Gordon Model with Biological Dynamics ..... 192
7.3.3 A Model of a Regulated Open Access Fishery ..... 194
7.3.4 Regulated Open Access with a Marketing Sector ..... 201
8 Concluding Remarks ..... 209
Bibliography ..... 218

## List of Figures

1.1 The Gordon Model ..... 4
1.2 The Smith Model ..... 10
1.3 Thesis Outline ..... 14
2.1 Biomass ..... 22
2.2 Quota ..... 23
2.3 Capacity ..... 24
2.4 Season Length ..... 25
3.1 Production Function ..... 33
3.2 Entry Function ..... 35
3.3 Marginal Revenue and Cost Functions when Capacity is at its Maximum ..... 38
3.4 Marginal Conditions at an Alternative Capacity Level ..... 39
3.5 One Capacity Level with Two Rent Dissipating Season Lengths ..... 39
3.6 Regulatory Function ..... 42
3.7 Equilibrium ..... 44
3.8 Nonexistence of Equilibrium ..... 48
3.9 The Phase Plane ..... 51
3.10 Characterization of the Dynamics ..... 53
3.11 Alternative Paths to Equilibrium ..... 55
4.1 Map of Regulatory Areas 2 and 3 ..... 60
4.2 Simulation with Post-1965 Parameters ..... 89
4.3 Simulation with Slow Regulatory Adjustment ..... 90
4.4 Simulation with Slow Regulatory and Industry Adjustment ..... 91
4.5 Simulation with Slow Industry Adjustment and Fast Regulatory Adjustment ..... 92
4.6 Simulation with Fast Industry Adjustment and Slow Regulatory Adjustment ..... 92
5.1 Sales and Stocks in a Marketing Period ..... 109
5.2 Sales and Stocks With a Discount Rate ..... 115
5.3 Sales with Myopic Price Expectations ..... 119
5.4 Overview of Carryover ..... 124
5.5 Shadow Values of Carryover ..... 130
5.6 Carryover with a Price Increase in the Final Period ..... 136
5.7 Carryover with a Price Increase in the Middle Period ..... 137
6.1 A Calendar Year ..... 164
6.2 Exvessel Demand ..... 165
6.3 Wholesale and Exvessel Market Links ..... 178
6.4 Exvessel Price Dependence on Marketing Period Length ..... 180
7.1 Thesis Recap ..... 183
7.2 Quota Rules ..... 190
7.3 Phase Diagram with Biomass Increase ..... 196
7.4 Simulation of Biomass: Market Exogenous ..... 197
7.5 Simulation of Quota and Harvest: Market Exogenous ..... 198
7.6 Simulation of Rents and Revenues: Market Exogenous ..... 198
7.7 Simulation of Capacity and Season Length: Market Exogenous ..... 199
7.8 Simulation of Quota and Harvest in the Full Model ..... 203
7.9 Simulation of Biomass in the Full Model ..... 204
7.10 Simulation of Exvessel and Wholesale Prices in the Full Model ..... 204
7.11 Simulation of Capacity and Season Length in the Full Model ..... 206
7.12 Simulation of Rents and Revenues in the Full Model ..... 206

## List of Tables

3.1 Comparative Statics Results for T and E ..... 47
4.1 Estimates of the Entry Equation for Area 2: Static Model ..... 69
4.2 Estimates of the Entry Equation for Area 3: Static Model ..... 70
4.3 Estimates of the Entry Equation for Area 2: Dynamic Model ..... 73
4.4 Estimates of the Entry Equation for Area 3: Dynamic Model ..... 74
4.5 Estimates of the Entry Equation for Both Areas Modeled Jointly: Dynamic Model ..... 76
4.6 Estimates of the Entry Equation for Both Areas Modeled Jointly and Average Revenues Equal Across Areas: Dynamic Model ..... 78
4.7 Estimates of Regulatory Equation for Area 2: Static Model ..... 80
4.8 Estimates of Regulatory Equation for Area 3: Static Model ..... 80
4.9 Estimates of Regulatory Equation for Area 2: Dynamic Model ..... 83
4.10 Estimates of Regulatory Equation for Area 3: Dynamic Model ..... 83
4.11 Estimates of Regulatory Equation for Area 2, Allowing for Structural Change: Dynamic Model ..... 84
4.12 Estimates of Regulatory Equation for Area 2, Allowing for Structural Change: Dynamic Model ..... 85
4.13 Simulation Parameters ..... 88
5.1 Comparative Dynamics Results for Sales in a Marketing Period ..... 111
5.2 Comparative Statics Results for Carryover between Two Marketing Periods ..... 129
5.3 Comparative Dynamics Results for Carryover with Three Marketing Periods ..... 135
6.1 Exvessel Demand Equations: Carryover Parametric ..... 167
6.2 Exvessel/Wholesale Demand Curves: Structural form with Endogenous Car- ryover ..... 172
6.3 Exvessel Demand Curves: Reduced Form with Endogenous Carryover ..... 174
7.1 Estimated Implicit Quota Rules ..... 187
7.2 Estimated Yield Curve Parameters ..... 188
7.3 Equilibrium Values for Alternative Models ..... 208

## Chapter 1

## Introduction, Objectives, and

## Outline

### 1.1 Introduction

One of the most important and oft-cited publications in the natural resource economics literature is the paper published in 1954 by H. Scott Gordon[39] entitled "The Economic Theory of a Common Property Resource: The Fishery." Gordon's paper made two fundamental contributions. First, Gordon's model of the rent dissipation process laid the foundations for subsequent predictive analysis of common property resource use, in fisheries and a wide variety of other settings. Second, the normative implications of Gordon's paper formed the basis for much of the discussion of the economic rationale for fisheries policy that has appeared in the economics literature over the past forty years.

It is difficult to overestimate just how important and how persistent Gordon's ideas have been in framing the manner in which economists have come to think about fisheries and fisheries policy. Part of the reason for this prominence lies in the elegance and simplicity of
his model, coupled with the fact that it seemed to describe so well many important fisheries in the early fifties. There is compelling reason to believe, however, that Gordon's model does not fit the circumstances of today's fisheries as well as it did when he wrote in 1954. Perhaps most important is the fact that most of today's fisheries are no longer pure open access fisheries. Instead, most have come under the purview of some regulatory body and hence are more properly thought of as regulated open access, or regulated closed access in some cases. Thus, at minimum, the proper paradigm for analyzing modern fisheries ought to account for the role of the regulatory structure and the role that regulatory institutions play in affecting incentives and industry behavior.

In addition, for various reasons, the focus of the Gordon paper was on input use to the almost total exclusion of the marketing or revenue side. This focus on inputs has colored the manner in which economists have viewed the open access problem and consequently on the way in which they have come to recommend solutions. In particular, almost all of the attention by economists to policy design issues over the past several decades has focused on controlling input use. No one has paid attention to the possibility that open access alone, or open access in concert with regulations, might also have an important impact on revenues. This might occur, for example, through regulations on catch volumes or catch size composition, or through restrictions on open seasons, or via impacts on quality induced by the incentives in open access.

This thesis develops and tests a new model of renewable resource exploitation, one based on H. Scott Gordon's influential paper, but modified to account for important features of modern fisheries. In particular, we examine the basic Gordon model under conditions that incorporate a regulatory structure and allow for dynamic interaction between the regulated industry and the regulators. In addition, we incorporate a marketing sector and allow for the feedback effects between regulations, input and output selection, product quality, and the strength of the rent dissipation process. The combination of these two added features
leads to a richer set of predictive conclusions and normative implications, more suited to the analysis of current conditions in fisheries. We apply the model to the important Pacific Halibut fishery.

In the remainder of this chapter we develop the Gordon model and discuss the nature of fisheries regulatory institutions in some detail. Then we outline the thesis objectives and briefly discuss the case study.

### 1.2 The Gordon Model

H. Scott Gordon's paper appeared in the Journal of Political Economy in 1954. Gordon's paper focused on the consequences of common property in general, illustrating those consequences for fisheries in particular. Economists had not paid much attention to either of these topics as of the fifties. Similarly, fisheries science and fisheries policy making were also just emerging at this date. In biology, biologists were beginning to make considerable headway into understanding and studying population dynamics and a well developed set of techniques was emerging in the literature by the time that Gordon's paper appeared. With regard to fisheries policy at the time, most fisheries were essentially unregulated, with a few important exceptions in North America, notably the lobster, salmon, and halibut fisheries.

Gordon's paper was exceptional for the elegance and simplicity of his structure, and for the range and importance of the implications of his predictions. Gordon's model focused on the economic incentives at work in an industry where the resource being utilized is not privately owned. Gordon developed his predictions with a simple value of marginal product formulation of input choice. Suppose that there is a single composite fishing input $E$ (effort) and suppose further that this input has an opportunity cost $w$. Assume that the input may be combined with a fish stock $X$ to produce a harvest level under the condition of


Figure 1.1: The Gordon Model
diminishing marginal production. Assume also that the output price $P$ is constant. Then (see Figure 1.1) a sole owner of the resource would choose an amount $E^{*}$ of the input, bringing the value of marginal physical product ( $V M P P_{E}$ ) into equality with the marginal input cost $w$. This level of input choice would produce a rent and wage bill as indicated by the shaded boxes in the diagram. Gordon contrasted this sole owner case with the case where the fish resource is unowned. Gordon's insight was to suggest that if no one owns the resource, inputs will flow in beyond the level $E^{* *}$ because each unit initially will be able to earn more than its opportunity cost by an amount reflecting a share of the rents. The entry of inputs will proceed until the level of inputs $E_{0}$ is reached, at which point the earnings of each input in the fishery are driven down to the level attainable outside of the fishery.

The Gordon theory describing the manner in which inputs can be expected to flow into a common property exploitation setting is simple and elegant. The process is driven by
incentives operating through a dynamic mechanism, in the sense that inputs will continue to be drawn in as long as rents exist in the production process. Because of the absence of enforceable property rights to the fish stock, the normal resource owner's function of guiding input use is absent also and inputs are overapplied. The process is predicted to equilibrate when the value of average physical product ( $V A P P_{E}$ ) equals the opportunity cost of inputs.

Gordon was particularly interested in the normative implications of this rent dissipation process. The most important normative conclusion is that inputs beyond $E^{*}$ create no social value and, in fact, lead to the waste of the potential surplus represented by the resource rents. Thus even though employment of variable inputs may be high, input proportions are inefficient and if these inputs have other socially productive uses, overall social welfare will be lower than it might be. Gordon also believed, following the important work of Scott[81], that proper conservation of resources meant wise stewardship that maximizes social values rather than simple biological criteria. Although there were only a handful of fisheries subject to active fisheries policy in the fifties, Gordon used these to emphasize the problem with policies focusing only on goals such as maximizing sustainable harvests. As Gordon wrote (in 1955):

Neglect of the cost side of the question has had the result that certain conservation programmes are regarded as successful by biologists when from the economic point of view they are palpable failures... (the halibut case) has been hailed many times as the outstanding case of successful fisheries conservation policy, yet I feel quite certain that it must go down in the economic annals as one of the clearest cases of failure.[40, p.69]

In summary, the paper by H. S. Gordon lays out the consequences of common property and open access institutions of resource exploitation. Gordon's paper is notable for its simplicity and for the wide range of positive and normative conclusions. It is certainly one of the few, if not the, fundamental contributions to the paradigm that economists have
come to adopt when thinking about fisheries and fisheries policy making.

### 1.3 Omissions and Refinements of the Gordon Model

The Gordon model abstracted from many dimensions that might have been included, probably in order to focus on what he believed most important, namely the economic consequences of common property. One obvious component that is only treated peripherally is the connection between common property incentives, input levels, and the health of the biological stocks. A second component that is assumed away is the marketing side of the rent dissipation process. A final important component is the regulatory structure and the role it plays in the rent dissipation process.

### 1.3.1 Rent Dissipation and Biology

Gordon developed his model of a fishery without much explicit consideration of the nature of fish biology and particularly the dynamics of harvests and stock growth. Presumably, we would expect some connection between input levels, the harvest level, and the population dynamics of the harvested species. Gordon did append some simple stock/flow relationships to his basic model at the end of the paper, but it is clear that he didn't regard this part of the story as really important. In other writings, in fact, he expressed a view that other biologists had held for some time, which was that environmental factors were the most important determinants of biological population sizes and that man's role as a predator was generally insignificant[84, p. 135]. While this may be true for some species, it is certainly not universally the case. Hence the lack of connection between the economic and biological systems is an important omission.

This connection was, in fact, developed in some detail in another important paper
published by Vernon Smith in 1968[82]. Smith's model is notable also for its elegance and for the range of implications and testable hypotheses developed. Smith began with a simple description of a biological system in differential equation form, following the early developments by Lotka[56] and Volterra[85]. Let $X(t)$ be the stock or population size of some species and assume that the species grows naturally according to some function $F(X(t))$. $F(X(t))$ incorporates the role of density dependent factors which operate in most biological systems. For example, when species density is low, we generally observe rapid growth as natality rates are high and mortality and predation rates are low. As the population begins to impinge on its resource base, however, we also observe mortality increasing and growth dropping as the species approaches some level supported by the environmental carrying capacity. A representative form for $F(X(t))$ thus might be a quadratic, so the species grows naturally by:

$$
\begin{equation*}
\dot{X}=F(X(t))=a * X(t)-b * X(t)^{2} . \tag{1.3.1}
\end{equation*}
$$

The differential equation representation of the growth process in Equation (1.3.1) can be solved to yield an S-shaped population growth path as a function of time which asymptotically approaches the steady state population level $\bar{X}=\frac{a}{6}$. At that point births are just matched by deaths each period and the population level remains constant. Smith adds a simple representation of a harvesting industry to the above mechanism by incorporating a process similar to that postulated by Gordon. In particular, assume that the harvest level is some function of the amount of effort $E(t)$ and the density of the stock $X(t)$ so that we have $H=H(E(t), X(t))$. Then the above biological relationship can be modified to incorporate the harvesting by writing:

$$
\begin{equation*}
\dot{X}=F(X(t))-H(E(t), X(t)) . \tag{1.3.2}
\end{equation*}
$$

Now the species will either grow or decline according to whether the harvest level in any period is less or greater than the natural biological growth at that stock level.

To close the model, Smith adds another differential equation describing the manner in which we might expect the harvesting industry to expand or contract. In particular, assume that the harvest can be sold at some price $P$, and that total industry costs are $C(E(t), X(t))$. Then a reasonable assumption about entry and exit behavior is that effort enters or exits according to something like:

$$
\begin{equation*}
\dot{E}=\delta *(P * H(E(t), X(t))-C(E(t), X(t))) . \tag{1.3.3}
\end{equation*}
$$

Here $\delta$ is a reaction parameter which describes how quickly the industry responds to rents. Note also that entry occurs when rents are positive as Gordon hypothesized and effort exits when rents are negative. An equilibrium occurs when total revenues equal total costs, or when the value of average product equals average costs as in the Gordon model.

Equations (1.3.2) and (1.3.3) form a simultaneous dynamic system in which effort is drawn in in response to rents, leading to a harvest level which either causes the population level to increase or decrease. Smith's model is actually quite general and capable of incorporating production externalities, intertemporal externalities, different price assumptions, etc. For purposes of exposition, it is convenient to focus on a simple version of his model in which costs are a proportion of effort, price is fixed, and the production function is linear in both the effort level and the stock level. ${ }^{1}$ Then with a quadratic biological growth function, the phase diagram in Figure 1.2 depicts some of the dynamic properties of the system.

Figure 1.2 shows the combined dynamics of the industry and biology. Note first that an equilibrium exists at the intersection of the two isoclines which individually describe

[^0]The isoclines are then defined by:

$$
\begin{aligned}
& \dot{X}=0 \quad \rightarrow X=\frac{a-q E}{b} \\
& \dot{E}=0 \quad \rightarrow X=\frac{c}{P q} .
\end{aligned}
$$

points of rest for either the industry or the biology. This bioeconomic equilibrium is defined as a stock and effort level for which the industry has dissipated all rents and is in economic equilibrium, and the corresponding level of effort supports a harvest level which also holds the species at some steady state level. The second thing to note is that various approaches to this equilibrium are possible. The system's stability properties can be examined by linearizing around the equilibrium point and examining the eigenvalues. Among other things, the nature of behavior during the approach to equilibrium depends on the strength of the response parameter $\delta$. When response to rents is sluggish ( $\delta$ low), the industry will gradually and asymptotically approach the equilibrium but when response is fast, there will be a tendency to overshoot. This is seen by the oscillatory approach which at first overshoots, then undershoots, and so on.

The Smith model thus refines the Gordon model in two important ways. First, it adds the connection between effort, harvest, and the species biology which Gordon considered only partially. Second, it makes explicit the nature of industry entry/exit dynamics and connects these with the biological dynamics. The result is a combined dynamic model which embeds the rent dissipation predictions of the Gordon model as the equilibrium, but which also adds richness by describing what might happen in the transition to the equilibrium.

### 1.3.2 Rent Dissipation and Revenues

A second dimension of the common property resource exploitation mechanism that Gordon chose to ignore is the role of the market or revenue side of the rent equation. Gordon focused almost exclusively on the role of inputs and costs and assumed that prices were given. While this is no doubt defensible as a first level of abstraction for many fisheries, in others it may not be. Revenues may be affected by the process of common property resource use in several ways. For example, if input entry drives the stock to a low level, harvests


Figure 1.2: The Smith Model
may also fall and if there is some price flexibility, this will impact the entry process. Or, if input choices such as mesh size affect fish sizes landed, we might expect common property exploitation to result in smaller and smaller fish being landed, perhaps reducing average exvessel prices. Another example (mentioned by Gordon) might arise if nearby grounds are overexploited, reducing sizes and harvests over what they might be under sole ownership with a more spatially rationalized pattern of exploitation.

In these and other cases, the common property exploitation process may affect revenues, which may in turn feed back into the common property entry process. Of particular relevance is the possibility that exploitation not only draws in excess inputs, but it also reduces output price below what might be earned under optimal management. If this is the case, then ignoring the revenue side could have two important consequences. First, any analysis focusing on inputs alone would miss a whole component of rent losses, namely those suffered on the marketing side. Second, with lower actual prices being received by industry entrants, the rent dissipation process would be mitigated somewhat over what we would expect without a revenue effect.

### 1.3.3 Rent Dissipation and Regulations

A third aspect of common property rent dissipation that Gordon did not address is the role of regulations on the process. This omission is not surprising since at the time of Gordon's paper, very few fisheries were in fact regulated. Since his work, however, virtually all important fisheries have come under some form of regulation. Movement towards active fisheries management began in the sixties after the collapse of several important fisheries. Additional impetus was provided by the Law of the Sea convention which led, in 1976, to the establishment of exclusive economic zones (EEZ's) off the coast of all coastal nations. These set up the legal infrastructure for coastal nations to control fishing and other resource
exploitation out to a 200 mile limit. This important institutional change effectively converted many of the common property fisheries in coastal waters into fisheries that could, in principle, be managed rationally as sole ownership fisheries. In practice, the 200 mile extension (and the enabling legislation in the U.S.-the Magnuson Act) led to the development of new regulatory systems, the operation of which were based on biological, if not always economic, criteria.

The omission of the role of the regulatory structure is thus potentially serious if one is concerned with modeling contemporary fisheries. There are virtually no completely open access fisheries, save a few in high seas such as tuna fisheries. The remainder are properly considered either regulated open access, or regulated restricted access. Regulated open access fisheries are essentially open to participation by any citizen, but participants are subject to a variety of (generally) biologically based restrictions. Typical restrictions include closed areas, closed seasons, terminal gear restrictions (e.g., minimum mesh sizes, net dimension regulations, prohibited gear regulations) and landings restrictions (fish size limits, trip limits, restrictions on catching female fish). Regulated restricted access fisheries typically utilize some of the same biologically motivated restrictions and in addition employ some form of limited access mechanism. Typical limited entry programs have limited participation with general licenses to participate, with specific licenses to utilize units of gear (e.g., lobster trap licenses), with licenses to use particular sizes and types of vessels, and recently with individual quota license programs (IQs or if transferable, ITQs) which allow a participant to land a certain quantity dictated by quota held.

### 1.4 Thesis Objectives and Outline

As discussed above, the Gordon model, elegant as it is, omits several components that are important to the increasingly complicated task of analyzing modern fisheries. The
demands on fisheries management for ex ante analysis of policy options have escalated since the extension of jurisdiction and passage of the Magnuson Act in 1976. Today, virtually any changes in management policy must be justified with cost/benefit and impact analyses before being implemented. Often these changes are incremental, such as a season or area closure change, but increasingly managers are also considering wholesale changes to radically different systems that would operate under completely new incentive systems. For example, many fisheries that have been conducted under regulated open access, with season length and gear restrictions, are now being considered candidates for adoptions of ITQs.

Whether changes being analyzed are incremental or quantum, most studies that are done are hampered by an overly simplified view of the rent dissipation process inherited from the Gordon paradigm. For example, a typical study of an impending season length reduction (to counter a falling resource stock) might assume that product prices would remain unchanged, and that inputs would simply be saved as fishermen cut back effort. Little account would be taken of the possibility that fishermen might intensify effort over the remaining shorter season, and that this might lead to the need for another round of season adjustments, and so on, or that shorter seasons would cause more fish to be stored, leading to lower prices and a less intense rent dissipation process.

As the demands for assessment of fisheries management changes continues to intensify, it is important that accurate analysis is made of both the status quo and any impending options in order to generate responsible policy. As discussed above, the status quo for most fisheries includes a regulatory structure at minimum, and potentially important links with the marketing and biological sectors that ought to be included generally. While policy analysts almost always include the biological links, no one has recognized the importance of modeling regulatory behavior and the marketing links. The main objective of the thesis is to develop a more complete conceptual model and demonstrate its application in a case study of the Pacific Halibut fishery.


Figure 1.3: Thesis Outline

Figure 1.3 outlines the structure of the model developed in this thesis. Outlined in bold borders in the lower left is the component of the system as developed by Gordon. In particular, Gordon showed how industry behavior, driven by common property incentives to enter when rents exist, would lead to a particular level of input choice in the fishery. Gordon took exvessel prices as given. Vernon Smith added the connection between biomass and the harvest level and made the Gordon model explicitly dynamic in his bioeconomic model. This thesis adds a regulatory sector and a marketing sector. The regulatory sector is assumed to be motivated by regulatory goals and behavior, which lead to instrument choice. The instrument choices (e.g., season length) interact with industry choice in a simultaneous way to generate a regulated level of effort and corresponding harvest level. The harvest level feeds back via the biological mechanisms generating biomass changes, and these in turn affect regulatory behavior. In addition to affecting raw product quantity, instrument choices may have other effects on the product quality (by intensifying the race for fish, or by affecting size distribution, etc.). They may also affect the structure of the wholesale market and indirectly affect prices. In the case chosen here, the primary instrument is season length, which affects how much of the harvest must be frozen and held in inventory, which in turn affects the wholesale derived demand price. Thus the exvessel price, which is generated as a derived demand from the marketing sector, is affected by regulations, and in turn affects the rent dissipation process, which affects the strength of the regulations needed and so on. The model developed here accounts for both the simultaneous nature of these interactions and also the dynamic nature of the processes.

The remaining chapters are ordered as follows. First, in Chapter Two we present a brief outline and history of the case study to be analyzed. Chapter Three then develops the model of a regulated open access fishery, incorporating both behavioral motivations of an industry subject to regulations and those of a regulatory sector charged with manag-
ing the fishery. The model in Chapter Three abstracts from the marketing sector link by holding exvessel prices as given. Chapter Four then estimates the model developed in the Chapter Three using a variety of specifications from the simple to the sophisticated. Chapter Five begins the task of adding the marketing sector by modeling wholesalers' inventory dissipation decisions under various assumptions about season length, perishability, carryover possibilities, etc. Chapter Six discusses econometric issues associated with modeling dynamic inventory decisions and then estimates a simplified version in order to simulate the case study chosen here. Chapter Seven then draws together the two components-the regulatory and marketing sectors-into a unified structure. Simulation results are presented and a range of conclusions are drawn from the conceptual and empirical model. Chapter Eight concludes.

## Chapter 2

## The Pacific Halibut Fishery

### 2.1 Introduction

This chapter describes the history and institutional details of the fishery selected as the case study for this thesis, the Pacific Halibut fishery. The halibut fishery is a valuable and important West Coast fishery. Its institutional structure has followed the classic pattern of evolving from pure open access to regulated open access to regulated restricted access. It is especially appropriate for this study for two reasons. First, the halibut fishery has a long history of regulation. Hence there is a substantial amount of data including a wide range of variables with which to test various hypotheses. Second, despite the long history of the fishery, many aspects of the structure, including fishing technology, have remained relatively unchanged. Both of these features make the task of model estimation more straightforward.

### 2.2 Biology

Halibut are relatively large flat groundfish that inhabit both the East and West Coasts. ${ }^{1}$ The West Coast species (Hippoglossus stenolopis) extends from California to the Bering Sea with the largest concentrations off both Alaska and British Columbia. Halibut are long-lived and can grow to sizes of several hundred pounds. Sexual maturity is reached at about eight years for males and about twelve years for females.

Biologists believe that there are not distinct stocks of halibut and that there is commingling among virtually all regions along the Alaska and British Columbia Coasts. Tagging studies have shown some very long movements by individual fish but most are recovered close to where they were tagged. There appears to be some seasonal movement to feeding grounds in shallow coastal waters during the summer and back to deeper spawning areas off the Continental Shelf in the winter. Biologists have identified some areas where a substantial amount of spawning seems to take place but spawning is generally diffuse across many areas.

Because of the longevity of halibut, an unexploited population is widely based across many different age classes from young to old. Hence the population as a whole exhibits slow dynamics. Exploited populations of halibut exhibit a similarly wide age distribution which means that the fishery will simultaneously draw from several age classes. Thus, in contrast to other species with dominant year classes and faster inherent dynamics, the catch per unit of effort may not change quickly. It is also the case that an unselective fishery will take from both mature and immature age classes. Hence the impact of high levels of harvests may occur several years after the fact.

[^1]
### 2.3 Fishing Technology

Halibut are caught either in directed fisheries specifically targeting the species, or as incidental catch in other fisheries. Northwest Indians including Tlingit, Haida, and Kwakiutl caught significant quantities of halibut with baited hardwood hooks shaped like a "V" with a bone barb on one side. Once a commercial fishery established itself, directed halibut fishing also adopted a method of hook-based long-line fishing that has endured since the turn of the century. The basic method of fishing for halibut utilizes a unit of gear called a "skate." A standardized skate consists of a longline that is 1800 feet long, with lighter lines attached at regular intervals upon which are attached baited hooks. Most skates are rigged with hooks spaced 18 feet apart, up from 12 feet during the early history of the fishery. Hooks were originally J-shaped but the industry has switched to more efficient circle-shaped hooks more recently.

Skates are fished by ganging several skates together with anchors and buoys attached at the ends. Baited skates are fed over a chute at the stern of a vessel and left at depths between 90 and 900 feet. Skates are left for a "skate soak" that averages 12 hours and then retrieved with a power winch. If conditions and time permit, skates are rebaited and left for subsequent soaks after the catch has been removed. In recent years another innovation utilizing snap-on hooks has been used, particularly on smaller boats with small crews. Snap-on gear involves less clutter and storage space and can be arranged with variable spacing if desired.

### 2.4 History of the Fishery and Regulatory Structure

Commercial fishing began in 1888 when three schooners displaced from the New England halibut fishery started fishing off the Washington Coast. The Pacific fishery had
been largely untapped except by Native Indians and the commercial fishery grew rapidly. Important factors in this growth were the virgin condition of the stocks, the ready supply of vessels that converted from other fisheries (including the fur seal industry), and the growing markets both on the East and West coasts. Market expansion was aided by the completion of the trans-continental railroad in the U.S. in 1869 and the Canadian system in 1885.

Early fishing for halibut was conducted by sailing schooners equipped with about ten small dories. The schooner would sail up the coast, drop anchor, and dispatch the small two or three man dories. The dories would row out and drop the skates and then return to retrieve them. A significant boost to productivity occurred with the conversion to motor powered vessels, first steam and then gasoline and diesel in the early part of the century. Typical vessels ranged from 50 to 80 feet and 25 to 60 tons. With the development of other fisheries off the West Coast, many halibut vessels were also capable of fishing other gear types including trawls and seines. More recent innovations including radar and loran and other electronics as well as refrigeration and power gear handling have also increased fishing power.

As the commercial fishery developed steadily from the 1880s, the early boom period conducted over a virgin fishery soon witnessed declining catches and rising costs. For example, catch per skate soak dropped from a reported 1000 pounds in 1890 to 271 pounds in 1910 and 84 by 1920. Recognition of the declining catches led to two early attempts to establish U.S./Canadian treaties in 1908 and 1919. The later attempt grew out of the post World War I spirit of cooperation and the establishment of the International Fisheries Commission formulated to supervise Fraser River Salmon management. Both of these attempts failed, however, and it wasn't until 1923 that the Convention for the Preservation of the Halibut Fishery of the North Pacific Ocean was signed. This Convention established one of the first international agencies charged with managing a transboundary resource.

The Convention of 1923 laid a modest start for the regulatory system which was to follow. Specifically, the Convention established the International Fisheries Commission (later the International Pacific Halibut Commission) to conduct scientific research and recommend management measures. The commission in turn immediately established a fisheries closure during the three month winter spawning period, formulated rules for the surrender of incidentally caught halibut, and established research plans to examine the halibut life history and to recommend further management measures. Importantly, the Convention also was given enforcement muscle by prohibiting non-treaty boats from landing in Canadian or U.S. ports.

During the period between the Convention of 1923 and the end of the decade, the halibut fishery continued to slide. Catch per skate soak fell to an all time low of 35 pounds in 1930. In that year, another halibut convention was held to assess the scientific knowledge gained and to recommend more stringent measures to deal with the continued decline in stocks. The 1930 Halibut Convention followed scientific recommendations and embarked upon a stock rebuilding program. Two weeks were added to the closure period, certain nursery areas were protected from fishing, and directed catch gear was mandated to be long line only. In addition, the Pacific was divided up into separate regulatory zones. The larger ones were Area 2 (off British Columbia and the Alaskan panhandle) and Area 3 (north of the Alaskan panhandle). Area 1 encompassed the areas off the lower Pacific States. Each of these areas was to be managed by establishing harvest quotas designed to reduce catch until stocks were sufficiently built up to levels that would sustain higher catches. The quotas were met by monitoring harvest levels and using season closures to ensure that targets were not grossly exceeded.

During the period between 1930 and 1960, some dramatic changes occurred in the halibut fishery as a result of the regulatory structure established. First, biomass began a slow recovery to levels similar to those at the turn of the century. As Figure 2.1 shows,


Figure 2.1: Biomass
biomass in regulatory area 2 grew from about 64 million pounds to about 135 million pounds. Area 3 recovered in a similar fashion from about 115 to about 225 million pounds. Although there is a disagreement over whether all of this was due to the harvest regulations, there is little question that the scientific recommendations were instrumental in the stock recovery.

Between 1935 and 1960, and as a result of the stock buildups, harvest quotas were gradually relaxed (see Figure 2.2). Catch per skate soak rose from 25 pounds in 1930 to 60 pounds in 1940 to around 1940 in 1956. But as catch per skate and fishing profitability rose, H. Scott Gordon's scenario began to unfold as new entrants appeared. In 1930 there


Figure 2.2: Quota


Figure 2.3: Capacity
were 459 vessels participating in the halibut fishery and by 1951 this number had risen to 820. Thus as a result of the very success of the biologically based conservation program, rents appeared and began to attract larger amounts of potential effort (see Figure 2.3). This increase in fishing capacity was not allowed to exert itself on the stocks, however, and season lengths were gradually reduced in order to ensure that harvest targets were met. Season length reductions continued until the entire quota for Area 2 was caught in one month in the mid 1950s (see Figure 2.4).

Partly as a result of these developments, some important changes were implemented when the Halibut Convention re-met in 1953. The most important changes divided


Figure 2.4: Season Length
the Pacific up into even finer areas and made provision to establish quotas in each of these in order to manage both harvests and effort distribution more finely. Concern was expressed, particularly as seasons were shortened, that some areas were being over-utilized and others under-utilized. Thus new means were set up to use time and area closures to redistribute effort pressure. Opening dates were rotated, individual small areas were opened up after the main season closed, and early openings were established in some areas to encourage fishermen to exploit those stocks. In addition, the fishing industry itself established a voluntary layover program in order to lengthen the season. The layover program was enforced by fishing unions and required vessels to remain in port a certain period after each landing. This effectively allowed the nominal season to increase to avoid product gluts and excessively long work periods. After dropping to lows in the mid 1950s, season lengths gradually increased again until they reached four months in the early 1960s.

The decade of the sixties witnessed a dramatic collapse of the halibut stocks in a short period of 10 years which effectively lost all that had been gained during the previous 25 years. As Figure 2.1 shows, Area 2 stocks dropped from 131 million pounds in 1962 to 63 million pounds in 1972 and Area 3 stocks similarly fell precipitously from 228 to 76 in the same period. There are various theories about why and how this happened. One is simply that environmental factors might have switched from favorable during the buildup phase to unfavorable during the post sixties phase. Another is that the collapse was purposeful and a reaction to encroachment by foreign trawlers that were beginning to take large amounts of halibut during this period. Former Halibut Commissioner Bell, in his book [6, p.210] states "it would have been naive to believe that imposing greater and greater reductions on the domestic halibut fleet would have benefited any group other than foreign fishing." Still another theory is that regulators erred in pushing the system too hard. Again, a quote from former Commissioner Bell is revealing. He states
... by the late 50 s most segments of the population appeared to have reached
levels of their MSYs under the then prevailing environmental conditions. While such levels had been estimated by both empirical and model studies, they were only estimates, and as with all statistical data they possessed an upper and lower limit of confidence. To test the upper limits, the permitted removals were raised by the Commission[6, p.210].

By far the explanation favored by most fisheries scientists is one developed by Skud[79]. That theory maintains that the data gathering system contained the seeds of its own undoing. In particular, from 1930 onward, fishermen were required to keep extensive log books recording, among other things, fishing location and catch per skate soak. These were used to assess the biomass and in turn to set catch quotas. During the late 50s fishermen switched from skates with 12 foot spacing to skates with 18 foot spacing. Scientists assumed that catching efficiency per hook would remain constant but in fact, it increased. Thus, estimates of biomass using the new skate soaks attributed relatively higher catches to a healthy biomass when in fact they should have been attributed to more effective gear. The result was a seriously overestimated biomass and correspondingly overestimated sustainable harvests.

Whatever the real reason, in the 1960s the stocks collapsed in all areas, driving out participants in a mirror image to the build up and rent generation period of the 40 s and 50s. As the stocks collapsed, regulators clamped down on allowable catch. By the late 1970s it was apparent that this biologically based conservation strategy pursued by the Halibut Commission had worked again and the fishery went through another phase of prosperity and buildup. Again, the increases in fishing capacity were met with severe seasonal restrictions.

In 1978, the extension of jurisdiction by both Canada and the U.S. changed the institutional setting for halibut management in a significant way. The extension of jurisdiction essentially ended the cooperative joint management that had been practiced under the Halibut Conventions and began a new era of effectively nationalized fisheries, although still under Halibut Commission management. Since Area 3 had not been heavily fished by

Canadians, the change did not impact Alaskan regulations significantly. In Area 2, however, a mixed fleet operated and catch was split between the two nations. The distribution issue was resolved by prohibiting Canadian fishing in U.S. waters and vice versa. From 1979 on, each nation went its separate way in conducting fishery management within the umbrella of the IPHC. Canada adopted a limited entry program in 1979 which froze the fleet at slightly over 435 vessels. The U.S. resisted adopting limited entry and continued with the previous structure.

In the 1980s the halibut fishery reached an extreme conclusion of this long period of interaction between an industry responding á la Gordon to rents and a regulatory structure attempting to rein in the capacity and protect the stocks from overharvesting. The regulatory authority was still limited to using the same quota setting/season restriction tools to prevent stock depletion. Yet, even with the short seasons, the fishery was profitable enough to encourage substantial entry. In response to overwhelming entry, the regulatory authority has had to place severe restrictions on fishing days. During the 1990s the entire halibut catch in U.S. waters was caught during seasons of 1-2 days. Conditions during these periods were understandably frenzied, with thousands of vessels vying to catch, load, and transport as much as possible before the short period closed. Fishermen worked continually with little sleep, vessels were dangerously overloaded and sometimes sank, and fish were delivered in poor condition, only to pile up as processors tried to handle a season's production in a couple of days.

In 1991, Canada adopted a radical change in regulatory structure; they instituted an ITQ program. The ITQ program is radical because it changes the individual incentives away from those described by Gordon and towards incentives more compatible with rational exploitation. ITQs give each fisherman a share of the total allowable catch, to be landed whenever they wish. Thus instead of needing to rush out and engage in a frenzied fishery before their neighbors do, each fisherman can plan to catch, handle, and market in a way that
maximizes profit per unit quota. The impact of this program has been dramatic already. In the first year of operation, Canadian fishermen received about $40 \%$ higher exvessel prices from supplying higher quality fish during the off-glut period produced by the regulated open access Alaska fishery (Fishermen's News [31]). The second year produced similar price premiums. These profit gains are not going unnoticed by U.S. fishermen although there is a considerable amount of inertia against any change. Currently the U.S. system is in the process of making a similar change towards a rights based system similar to Canada's. It is premature to expect that the system will be adopted soon; current stipulations of the Magnuson Act preclude fisheries from taxing themselves for important functions like enforcement and monitoring. Thus the U.S. system may be hung up for several years before adopting anything like the Canadian system.

## Chapter 3

## Modeling Industry/Regulator

## Behavior

### 3.1 Introduction

In this chapter we develop a model of a system of industry/regulator interaction. As discussed above, one important difference between fisheries today and those typical during the time that Gordon's paper appeared is that most contemporary fisheries are regulated. One of the main impacts of a regulatory structure is that it should affect rents and hence the process of rent dissipation. In addition, however, regulations are purposeful in the sense that they are guided by regulatory goals. These goals in turn depend on the status of both the industry and biology and hence we would expect that industry and regulatory behavior to be endogenous. In the next sections we outline a model that views regulation setting and industry response as the outcome of a jointly determined process. We first present a static representation of the process and then move to a more general dynamic representation which nests the static model.

### 3.2 Fishermen's Behavior

H.S. Gordon's model of rent dissipation is a useful point of departure for considering industry behavior. We assume that fishermen behave as Gordon suggested, that is, they enter in response to rents and entry proceeds until effort is earning its opportunity cost. The form of the model presented here adds some concreteness to his formulation by adopting production and cost functional forms which make explicit the connection between behavior and the fishery stock, the regulatory instrument, and relevant parameters.

Rents will be assumed to be simply the difference between industry revenues and industry costs, defined over a given fishing season. Revenues are defined as total seasonal harvest multiplied by an exvessel price $P$ per pound. In this chapter we take $P$ to be given in order to focus on the manner in which adding a regulatory sector modifies the basic Gordon model. In later chapters we endogenize $P$ by adding a marketing sector. Assume that there is an instantaneous harvest rate function defined by:

$$
\begin{equation*}
h(t)=q E X(t) . \tag{3.2.1}
\end{equation*}
$$

This is the most common assumption used in fisheries biology. Here $h$ is the harvest rate, $q$ is the catchability parameter, $E$ is a measure of fishing capacity or power, and $X$ is the biomass in period $t$. We will assume that the industry commits an amount of effort $E$ so that $E$ can be assumed fixed over any given season.

To determine how much will be harvested over a season, assume that the level of biomass at the beginning of any given season is $X_{0}$. Assume also that biomass declines over the fishing season according to:

$$
\begin{equation*}
\dot{X}(t)=-q E X(t) . \tag{3.2.2}
\end{equation*}
$$

We ignore natural mortality in the equation determining the evolution of biomass during the season for analytical convenience. For the fishery we look at, natural mortality is low relative
to fishing mortality during the season. In addition we account for natural mortality between seasons by assuming that the beginning biomass adjusts for between-season mortality and other environmental factors. Equation (3.2.2) is a differential equation describing withinseason biomass that can be solved over a season beginning at time 0 and ending at time $T$. Solving the differential equation gives

$$
\begin{equation*}
X(t)=X_{0} e^{-q E t} \tag{3.2.3}
\end{equation*}
$$

for the biomass level at any date $t$. At the end of the season, ending biomass is given by $X(T)=X_{0} e^{-q E T}$. Total cumulative harvest over the season of length $T$ is thus:

$$
\begin{equation*}
H(T)=X_{0}-X(T)=X_{0}\left(1-e^{-q E T}\right) \tag{3.2.4}
\end{equation*}
$$

This aggregate industry production function for a given season is depicted in Figure 3.1 as a function of total effort $E T$. As the product of daily fishing capacity $E$ and season length $T$ approaches infinity, production approaches the entire initial biomass $X_{0}$. In addition, the marginal product of effort is positive and diminishing. With the fixed amount of fishing capacity throughout the season, the harvest rate declines as the season progresses. This is due to the stock effect: harvesting reduces the stock and since the harvest rate is stock dependent (equation 3.2.1) the harvest rate falls as the biomass is reduced. ${ }^{1}$

With respect to costs, we assume simple linear costs related to both the level of capacity and to the amount of variable effort expended over the season. Consider first capacity costs. We assume a cost $f$ per season per unit of capacity must be incurred to participate in the fishery. In the case study examined here, capacity will be measured in skates, although in other fisheries capacity might be measured in other variables, such as standardized vessels. These costs may thus be assumed to be outfitting, repair, and preparation costs associated with the gear, opportunity cost of the investment, and implicit

[^2]

## ET

Figure 3.1: Production Function
rent associated with other inputs. We also assume that there are variable costs associated with the (assumed constant) rate of input use over the season. Thus the setting and retrieving of a skate requires fuel and bait costs and crew wages. Assume that these costs are $v$ per skate soak per unit time.

With both cost and revenue formulations described above we can write total industry rents anticipated for a season of length $T$ as:

$$
\begin{equation*}
\Pi=\int_{0}^{T}[P h(t)-v E] d t-f E . \tag{3.2.5}
\end{equation*}
$$

We ignore discounting for analytical convenience. Note that the above expression has total variable profits depending on season length under the integral and total fixed costs depending upon capacity outside the integral. Substituting the harvest function from equation
(3.2.4) into the revenue function and integrating leads to an expression for seasonal rents:

$$
\begin{equation*}
\text { Rents }=\left[P X_{0}\left(1-e^{-q E T}\right)\right]-[v E T+f E] . \tag{3.2.6}
\end{equation*}
$$

Total revenues are given by the left hand term and total costs are given by the right hand term. Setting rents equal to zero yields an implicit equation for $E$ as a function of $T, P$, $X_{0}, v, f$, and $q$. This gives the rent dissipating level of capacity identified by Gordon and Smith as the equilibrium level of effort expected in an open access fishery.

### 3.3 The Entry Function and Industry Behavior

The implicit equation $J(E, T)=0$ derived by setting rents equal to zero describes industry behavior associated with effort levels that dissipate rents. The addition of a regulatory instrument, the season length $T$, adds only minor complexity. The forms appropriate for sensible production and cost functions create some analytical complexities, however, because it is not possible to explicitly isolate the rent dissipating capacity $E$ as a function of the other variables and parameters. Hence it is necessary to characterize the entry function $E=E\left(T ; X_{0}, P, c, q, f\right)$ via indirect methods. In what follows we discuss the shape of $E$ heuristically and then show the properties of the implicit function more formally.

A key feature of this model is the assumption that fishermen commit a given amount of effort for the season and must pay a fixed cost in order to outfit for the fishery. As it turns out, this assumption partially bounds the range of equilibria possible and helps determine the shape of the rent dissipating $E$ function over the relevant range. Figure 3.2 depicts the shape of the function which is identified by the implicit relationship described above. The function generally describes a monotonic relationship between $E$ and $T$. That is, as the season length gets longer, the amount of capacity that will dissipate rents is larger. Note that there is a minimum season length below which no effort will be attracted. In


Figure 3.2: Entry Function
addition, it will be shown that there is a maximum season length associated with the largest level of capa : $y$, beyond which the notion of rent dissipating equilibrium breaks down.

Consider first the general shape of the relationship between $E$ and $T$ given by the implicit function $J\left(E, T ; X_{0}, p, v, q, f\right)=0$. We can examine the first derivative of the rent dissipating level of capacity as a function of $T$ through the implicit function theorem. If $J(E, T)=0$ is the rent dissipating equilibrium condition, then the derivative is:

$$
\begin{equation*}
\frac{d E}{d T}=-\frac{J_{T}}{J_{E}}=\frac{E\left[v-P q X_{0} e^{-q E T}\right]}{T\left[P_{q} X_{0} e^{-q E T}-v\right]-f} . \tag{3.3.1}
\end{equation*}
$$

Note that this derivative is positive in the relevant range because the numerator and denominator are both negative. This can be verified because we know that $J_{E}$ is the net value of marginal physical product of an extra unit of capacity. But we know that the net value
of average product is zero (from the rent dissipating condition) and since the net value of marginal product is less than the average product, the denominator must be negative. The numerator is the negative of the marginal value of $E T$. In the relevant range, the marginal value of $E T$ is positive. Note also that this derivative is zero where the term in brackets $\left[P q X_{0} e^{-q E T}-v\right]$ is zero. This defines a maximum of the $E$ function because the second derivative can be shown to be:

$$
\frac{d^{2} E}{d T^{2}}=\frac{E\left(P q X_{0} e^{-q E T}-v\right)^{2}-E^{2}\left(P q^{2} T X_{0} e^{-q E T}\right)}{\left[T\left(P q X_{0} e^{-q E T}-v\right)-f\right]^{2}}
$$

which is negative since the first term in the numerator drops out. Thus the rent dissipating level $E$ has a peak. As discussed below, levels of $E$ that are larger than this peak are not consistent with an equilibrium as we are defining it.

What about levels of $E$ less than those defined by the maximum? Note first that there is a non-zero minimum season length necessary to induce a positive level of effort to enter as long as fixed costs are positive. This can be understood as follows. First define variable profits $\Pi(E, T)$ to be those associated with a variable season length. Variable profits will be revenues less variable costs $v E T$, or:

$$
\begin{equation*}
\Pi=P X_{0}\left(1-e^{-q E T}\right)-v E T . \tag{3.3.2}
\end{equation*}
$$

The marginal variable profit associated with another unit of capacity evaluated where $E$ equals zero is:

$$
\begin{equation*}
\left.\frac{d \Pi}{d E}\right|_{E=0}=P q T X_{0}-v T . \tag{3.3.3}
\end{equation*}
$$

Hence marginal variable profits will compensate for the fixed cost of entry only if the season length is large enough to make marginal variable profits (3.3.3) greater than the cost of entry $f$ per unit of $E$, that is:

$$
T\left(P X_{0} q-v\right)-f>0 \text { iff } T>\frac{f}{P X_{0} q-v} .
$$

The intuition behind this is that as the fixed cost of entry increases, a longer season length is required to generate the variable profits to compensate for the fixed costs. Note that these marginal profits defined at $E=0$ are also a function of the price, the initial biomass, and cost and production parameters so that a higher initial marginal profit (3.3.3) will allow the minimum season length to be shorter. Note also that a necessary condition for any effort to be profitable is that the first units contribute some positive marginal profits, or:

$$
\left.\frac{d \Pi}{d E T}\right|_{E T=0}=P q X_{0} e^{-q E T}-\left.v\right|_{E T=0}=P q X_{0}-v>0 .
$$

This states that the marginal profit from harvesting the first unit of stock $X_{0}$ with variable effort $E T$ must be positive.

Thus there is some minimum length season which will induce some positive entry of capacity. As the season is lengthened the rent dissipating level increases monotonically until the season length reaches some maximum $T_{\text {max }}$. What about seasons longer than $T_{\max }$ ? As it turns out, these are not compatible with a rent dissipating equilibrium that sustains commitments of effort as we have assumed. This can be seen by examining Figure 3.3. This figure depicts the marginal value and cost of an amount of total seasonal effort $E T$. Since the variable profit function (3.3.2) is concave in $E T$, the marginal value of additional seasonal effort declines. Suppose that costs of variable effort $v$ must be incurred. Then an amount of seasonal effort $\hat{E} \hat{T}$ yields maximum variable profits. These are depicted as a function of $E T$ equal to the shaded area. Note that it would be most profitable for the industry to select a low level of $E$ combined with a long season length $T$ to produce $\hat{E} \hat{T}$ in order to minimize capacity costs. However, in open access, incentives will be for capacity to grow until the total payment for fixed capacity costs equals variable profits. Thus in an open access equilibrium, for any season length $T$, the shaded area will be dissipated by fixed costs $f E$.

Now consider two candidate season lengths, one shorter and the other longer than


Figure 3.3: Marginal Revenue and Cost Functions when Capacity is at its Maximum
$T_{\max }$, and a given level of capacity $E_{0}$. Figures 3.4 and 3.5 describe positions of potential equilibrium. Consider $E_{0} T_{1}$ first. With a season length $T_{1}$, variable profit indicated by the shaded area would be generated and dissipated by an amount of fixed capacity costs $f E_{0}$. Note that at this point, the marginal value to the industry of another day of season is positive. Other things equal, on the last day of the season, fishermen would want to fish another day and push total effort towards the point where the marginal value of $E T$ was equal to its marginal cost. The extra variable profits that could be earned are shown by the area shaded in + 's.

Consider another candidate equilibrium where the season length is $T_{2}$ and the relationship $P q X_{0} e^{-q E T}-v$ is negative. This candidate season length has been drawn so that all profits made by expanding the season length to $E T$ are just balanced by losses made by fishing beyond the point where marginal revenues exceed marginal costs. In Figure 3.4 the area shaded by -'s is equal to that shaded by +'s. Clearly, if given the choice, it would not be in the interests of fishermen to fish over the periods of the season where the marginal


Figure 3.4: Marginal Conditions at an Alternative Capacity Level


Figure 3.5: One Capacity Level with Two Rent Dissipating Season Lengths
value of more days is negative. We can conceive of a situation where they are coerced into fishing these extra days but this is unreasonable as an equilibrium assumption.

In summary, the portion of the rent dissipating capacity equation which is beyond and to the right of the peak implies behavior that is unsustainable as an equilibrium. In particular, while positions to the left of the peak involve equilibria where the industry would like the season length expanded, positions to the right imply that the marginal value of the last days of the season are negative. We would thus expect that the industry would not voluntarily choose to fish on those days. This is intuitive because the condition describing the derivative of the capacity equation has the term $v-P_{q} X_{0} e^{-q E T}$ in the numerator. Note that the second part of this is basically $P q X_{T}$, but this is precisely the value of the marginal harvest rate on the last day of the season. Admitting potential equilibria where the whole term is negative thus implies that the value of the marginal units of effort in taking the last units of harvest are negative.

### 3.4 Regulator Behavior

Although economists have not paid much attention to analyzing how agencies operate, there are several alternatives that might be maintained as working hypotheses. One that economists have explored is the notion of rent seeking (Bhagwati[ $[9, B u c h a n a n$ et. al.[11] and Rowley et. al.[73]). Under a rent seeking model, constituents are assumed to lobby the regulators for actions that generate rents. Various mechanisms are assumed whereby regulators know this and are assumed to act in ways that generate shares of the rent. A similar idea is that of regulatory capture whereby it is assumed that the regulatees "capture" the regulators through political or voting processes and manipulate outcomes (Karpoff[54]). A third theory that economists have paid some attention to is a sort of welfare maximizing theory, whereby regulators are assumed to be acting as if they are maximizing
a social welfare function (Alston and Carter[2], Coggins[17] De Gorter et.al.[25], Rausser and Zusman[70]). In empirical examples, some have estimated the welfare weights that are consistent with observed behavior (Fulton and Karp[34]).

In this thesis, we assume a simplified goal structure for the regulatory body which emphasizes the biological orientation of most real world fisheries regulatory bodies. In particular, we assume a hierarchical decision process that focuses, first, on selecting a targeted harvest quota that ensures stock safety. For example, during the early phases of the halibut regulatory history, quotas were set purposely below biological yield in order to bring the stocks up to higher levels. During the period following the 1953 convention, explicit attempts were made to hold the stock at a level as close as scientists could determine was yielding maximum sustainable yield (MSY). In any case, we assume that a stock assessment is made before each season to determine the stock health. Then a quota $Q$ is set according to some explicit or implicit criteria. Thus, we assume at this stage that both $Q$ and $X$ are given each period.

In the second stage of the regulatory process, we assume that regulatory instruments are chosen to achieve the quota target. Regulators are assumed to be knowledgeable about the industry's aggregate production function, about the size of the stock, and about the potential capacity of the industry. For the case study examined here, we assume that the primary instrument used by regulators is control of season length $T$. Suppose that biomass $X_{0}$ and the target $Q$ are given. Then, regulators are assumed to set the season length to ensure that aggregate harvest equals the quota. Using 3.2.4, we have:

$$
\begin{equation*}
Q=X_{0}\left(1-e^{-q E T}\right) \tag{3.4.1}
\end{equation*}
$$

which can be solved for the season length $T$ as a function of effort, biomass, the quota, and the catchability coefficient. In particular, the regulators are assumed to behave by choosing


Season Length:T
Figure 3.6: Regulatory Function
$T$ so that:

$$
\begin{equation*}
T=\frac{1}{q E} \ln \left[\frac{X_{0}}{X_{0}-Q}\right] . \tag{3.4.2}
\end{equation*}
$$

Graphically, this equation is a rectangular hyperbola (see Figure 3.6) since the product of capacity $E$ and season length $T$ is set to equal a constant determined by the biomass and quota. This simple form is in part due to the production function which is a function of the product of capacity and season length ET. It is assumed that regulators are simply concerned about controlling total effort, which may be composed of a high level of capacity exerted over a short season, or a low level of capacity exerted over a long season. Note that the position of the hyperbola depends upon both the biomass and quota (and catchability coefficient). Other things equal, as the quota increases, the hyperbola shifts out since a larger amount of total effort $E T$ will be needed to catch it. On the other hand, a larger biomass with the same quota will shift the hyperbola inward, because the biomass
plays a positive productive role in the harvest process.

### 3.5 Equilibrium in a Regulated Open Access Fishery

The preceding sections have developed two sides of a theory of interaction between a regulated industry and the regulators. The industry makes a capacity "choice" which results in a level of capacity which dissipates rent for any given season length. Regulators, in contrast, are assumed to select a season length, given the capacity choice by the industry and conditional on a biomass and target quota. It remains to discuss the nature of equilibrium under various possible mechanisms.

The simplest assumption to make about an equilibrating mechanism is to draw the analogy from economic models of markets and assume some sort of an auctioneer. For example, assume that the auctioneer posits a potential season length of $T_{0}$. Then we could assume that the industry announces a rent dissipating capacity decision to "supply" $E\left(T_{0}\right)$ and that the regulators announce a corresponding desired or "demanded" capacity level $E\left(T_{0}\right)$ capacity level consistent with their regulatory goals. Assume that the auctioneer follows the rule: adjust $T_{0}$ in proportion to the "excess demand." Then if $T_{0}$ is lower than the equilibrium so that excess demand for season days is positive, the trial season length will be increased to $T_{0}+\delta$ and so on. After repeated trial announcements, the process would reach an equilibrium where the industry's capacity choice contingent on season length exactly matches the choice assumed by regulators when they make their season length choice. Thus the "market" would clear as in Figure 3.7, where an equilibrium is reached for both sets of participants.

There are, of course, alternatives to this analog to the supply/demand mechanism. If agents do not know what the equilibrium decision of the other party will be, they might use proxies or estimators. For example, regulators may not know what capacity the industry will


Figure 3.7: Equilibrium
use and may predict this year's capacity by using past capacity in a forecasting mechanism. On the other side, the industry may not believe announce season lengths and may use information about how announced and actual season lengths diverged in the past to estimate actual season lengths. These alternatives may lead either or both agents to operate off their respective schedules so that (for example) the industry may be making rents and/or the catch may diverge from the quota.

Another alternative story is that there is sluggishness in behavior so that the industry and/or the regulators do not adjust instantly to the levels as defined by the above models of individual behavior. For example, the industry may tend towards rent dissipation but if capacity can only be changed slowly, dissipation may not occur instantly. This is the assumption essentially used by Vernon Smith. Regulators, on the other hand, may be able to change season length rapidly but may decide not to in order to avoid disrupting the industry too much. Obviously there are many potential assumptions that might be made about the nature of the equilibrating process in the system and the implied dynamics.

We take an encompassing approach here by developing a dynamic model that nests the instantaneous static equilibrium model as a special case. This partial adjustment model is presented at the end of this chapter, following a discussion of the comparative statics of the instantaneous Walrasian model.

### 3.6 Comparative Statics

Under the assumption that the system of industry/regulator behavior clears perfectly and instantly, we can examine some of the properties of the equilibrium under various parameter changes. The equilibrium values of capacity and season length are determined as those values that satisfy both the zero rent condition (equation (3.2.6)) and the regulators' equation (3.4.2). In equilibrium, we can solve to determine a reduced form equation for
both the rent dissipating capacity level and the season length. These are:

$$
\begin{equation*}
E_{0}=\frac{P Q}{f}-\frac{v}{q f} \ln \left[\frac{X_{0}}{X_{0}-Q}\right] \tag{3.6.1}
\end{equation*}
$$

and

$$
\begin{equation*}
T_{0}=\frac{f \ln \left[\frac{X_{0}}{X_{0}-Q}\right]}{P q Q-v \ln \left[\frac{X_{0}}{X_{0}-Q}\right]} . \tag{3.6.2}
\end{equation*}
$$

Comparative statics properties of the model can be deduced from these. The parameters $P, v$, and $f$ shift the schedule that determines capacity alone. An increase in price induces more capacity for each season length while increases in costs prompt less capacity for each level of $T$. To offset capacity changes, regulators adjust season length. Thus, changes in prices and costs shift the entry equation, which induces a movement along the season length determination schedule. A quota change induces an outward shift of the regulator schedule so that more total effort can be allowed. If the entry schedule remains fixed, the quota increase will thus produce an equilibrium with higher capacity and a longer season length. Changes in the stock and in the catchability coefficient shift both schedules. Increases in catchability and biomass shift the season length schedule inward, resulting in less equilibrium effort, other things equal. But these same increases also shift the capacity equation upward. The net result is a reduction in equilibrium season length and a capacity increase. These comparative statics properties are summarized in Table 3.1. Analytic results are derived from the equilibrium values above and the signs are presented to the right of each derivative.

It should be noted that these properties of equilibria are valid for interior equilibria. As the discussion of industry behavior suggested, the set of viable rent dissipating equilibria are contained within minimum and maximum season lengths. The configuration of parameters may be such that no seasonal regulation is necessary. For example, with a very high allowed quota, the rectangular hyperbola describing the regulator equilibrium may be shifted out far enough so that it doesn't intersect the peak of the industry equi-

|  | $d E_{0}$ | sign | $d T_{0}$ | sign |
| :---: | :---: | :---: | :---: | :---: |
| $d P$ | $\frac{Q}{f}$ | + | $\frac{-T_{0} Q}{f E_{0}}$ | - |
| $d f$ | $\frac{-E_{0} T_{0}}{v}$ | - | $\frac{T_{0}^{2}}{f}$ | $+$ |
| $d f$ | $\frac{-E_{0}}{f}$ | - | $\frac{T_{0}}{f}$ | + |
| $d Q$ | $\frac{P q\left(X_{0}-Q\right)-v}{q f\left(X_{0}-Q\right)}$ | + | $\frac{P_{q f}\left(\frac{Q}{X_{0}-Q}-\ln \left[\frac{X_{0}}{X_{0}-Q}\right]\right)}{\left[P q Q-v \ln \left[\frac{X_{0}}{X_{0}-Q}\right]\right]^{2}}$ | $t$ |
| $d q$ | $\frac{v \ln \frac{x_{0}}{\left(X_{0}-Q\right)}}{q^{2} f}$ | $+$ | $\frac{-T_{0}\left(v T_{0}+f\right)}{q f}$ | - |
| $d X$ | $\frac{v Q}{q f X_{0}\left(X_{0}-Q\right)}$ | + | $\frac{-P q Q^{2} f}{X_{0}\left(X_{0}-Q\right)\left[P q Q-v \ln \left[\frac{x_{0}}{x_{0}-Q}\right]\right]^{2}}$ | - |

Table 3.1: Comparative Statics Results for T and E


Figure 3.8: Nonexistence of Equilibrium
librium curve (see Figure 3.8). In this case there is no need for a season length regulation because the unregulated choices of $E_{0}$ and $T_{0}$ by the industry do not lead to a harvest as large as the quota. This situation is more likely with lower prices, higher costs, and a lower biomass level.

### 3.7 A Dynamic Model of the Regulated Open Access Fishery

As we discussed in the section on equilibrium assumptions, there are several reasons why this system of interaction might not reach equilibrium in each period. If it does not adjust instantly, we can model a representative process with variants of the sluggish adjustment or partial adjustment models that have been successfully utilized in supply
response and investment theory. Partial adjustment models essentially allow for endogenous variables to adjust partially in any given period to their long run values. Often, no specific mechanism is proposed as a reason for the sluggishness but rather it is simply assumed as a maintained hypothesis. ${ }^{2}$ The adjustment parameter dictates whether adjustment will be fast or slow. In a simultaneous system such as the one developed here, if partial adjustment is assumed on the part of both participants, a rich variety of outcomes is possible. These are discussed below.

The simplest way to develop a dynamic version of the static models discussed above is to assume a simple proportional adjustment process. Recall that we have developed two models of individual behavior, one for the industry and one for the regulatory agency. The industry model is based on the assumption that a level of capacity $E$ can be identified which just dissipates available rents at any given season length. Although we did not (and cannot given our functional forms) derive an explicit expression for this equation, we can define the implicit relationship $E=j\left(T ; P, X_{0}, q, v, f\right)$ that yields the rent dissipating level of capacity. Similarly, we have an (explicit) expression for the season length that regulators would choose, conditioned on the industry's capacity choice, namely $T=g\left(E ; X_{0}, Q, q\right)$.

The partial adjustment model makes the assumption that variables move towards their equilibrium levels with some sluggishness, where the sluggishness is captured by adjustment parameters. Specifically, let $\gamma$ and $\delta$ be the adjustment parameters. Then the

[^3]dynamic model may be specified as:
\[

$$
\begin{align*}
& \dot{T}=\gamma\left[g\left(E ; X_{0}, Q, q\right)-T\right] \quad=G(E, T)  \tag{3.7.1}\\
& \dot{E}=\delta\left[j\left(T ; P, X_{0}, q, v, f\right)-E\right]=J(E, T)
\end{align*}
$$
\]

If the season length is below the level associated with the target, then $\dot{T}$ will be positive and $T$ will increase at a speed determined in part by $\gamma$. Similarly, if $E$ is larger than the rent dissipating level, it will tend to fall, at a speed associated with $\delta$.

In order to analyze the motion of this system, we can first plot the isoclines that identify pairs of $E$ and $T$ with leave each part of the system in equilibrium. These are, of course, simply the behavioral equations that depict the static model as shown in Figure 3.9. These show that the level of capacity is in equilibrium when the capacity level is the rent dissipating level identified in the model of the industry. Similarly, the regulators will be in equilibrium when the season length is equal to the one associated with the target quota. The whole system is in equilibrium at $E_{0}$ and $T_{0}$ when both equations are satisfied.

Now, what if the system is not in equilibrium? What does the approach path look like? How is it different depending upon where we start the process? How does it depend on parameters, including the important speed of adjustment parameters? These and other questions about the nature of the dynamic paths can be analyzed by examining the stability of the system. Since this is a non-linear system, we can analyze (local) stability around $E_{0}, T_{0}$ by linearizing and evaluating the eigenvalues of the linearized system. In particular, consider the system in (3.7.1) above evaluated at its equilibrium point. A Taylor's approximation gives:

$$
A=\left.\left[\begin{array}{cc}
G_{T} & G_{E}  \tag{3.7.2}\\
J_{T} & J_{E}
\end{array}\right]\right|_{E_{0}, T_{0}}=\left.\left[\begin{array}{cc}
-\gamma & \gamma g_{E} \\
\delta j_{T} & -\delta
\end{array}\right]\right|_{E_{0}, T_{0}}
$$

where $A$ is the coefficient matrix of first partial derivatives of the system in 3.7.1 above, evaluated at the equilibrium. In the system analyzed here the coefficient matrix is fairly


Figure 3.9: The Phase Plane
simple since $g_{T}=j_{E}=0$. The eigenvalues are found by taking the determinant of the matrix $A-r I$ and setting it equal to zero, or:

$$
\operatorname{det}(A-r I)=\left|\begin{array}{cc}
-\gamma-r & \gamma g_{E}  \tag{3.7.3}\\
\delta j_{T} & -\delta-r
\end{array}\right|=(-\gamma-r)(-\delta-r)-\delta \gamma g_{E} j_{T}=0
$$

This yelds a quadratic equation whose roots satisfy:

$$
\begin{equation*}
r_{1}, r_{2}=\frac{1}{2}\left[-(\gamma+\delta) \pm \sqrt{(\gamma-\delta)^{2}+4 \delta \gamma g_{E} j_{T}}\right] \tag{3.7.4}
\end{equation*}
$$

The eigenvalues or roots of this equation determine the qualitative nature of the dynamic system when it is in the neighborhood of the equilibrium. Specifically, the system will be dynamically stable and converge directly to the equilibrium from any initial conditions if the eigenvalues are distinct, real, and negative. In this case, the equilibrium can be characterized as a stable node. If the roots are imaginary with a negative real part, the equilibrium will be approached in an oscillatory path. The equilibrium would then be
a stable focus. If the roots are real and positive the equilibrium will be an unstable node and the system will diverge directly from the equilibrium. If the roots are imaginary with positive real parts, the equilibrium will be an unstable focus, and the system will diverge with an oscillatory pattern.

As can be seen from the quadratic equation, the critical factor determining the nature of the approach path is the sign of the discriminant. If the term under the square root sign is negative, the approach path will be oscillatory. The oscillations will be convergent because the real part is the (negative) sum of the adjustment parameters. The sign of the discriminant depends upon the size of the second term relative to the first. The second term contains the product $g_{E} j_{T}$ which is the ratio of slopes of the two equilibrium conditions evaluated the equilibrium. Since $g_{E}$ is always negative and $j_{T}$ is positive, this ratio is negative. If the absolute value of this second term in the discriminant is large, the system will have imaginary roots and, with a negative real part, will result in the equilibrium being a stable focus. Then, the system will exhibit convergent oscillations towards the equilibrium. If the second term is small so that the discriminant is positive, both roots will be negative and real and the equilibrium will be a stable node. Then, the system will converge directly from any point in the phase space.

Thus the critical determinants of the nature of the approach to equilibrium are the respective sizes of the adjustment parameters and the slopes of the two isoclines which, in turn, depend on system parameters. The specific relationship between these can be further examined as follows. First define the ratio of the two adjustment parameters to be $R$. Then $\gamma=R * \delta$. Next define the absolute value of the ratio of slopes of the isoclines (i.e., the slope of the industry equation divided by the slope of the regulator equation) to be $Z$. Then the eigenvalues of the system can be redefined as:

$$
r_{1}, r_{2}=\frac{1}{2}\left\{-(1+R) \delta \pm \sqrt{(R-1)^{2} \delta^{2}+4 R \delta^{2} g_{E} j_{T}}\right\}
$$



Figure 3.10: Characterization of the Dynamics

$$
=\frac{1}{2}\left\{-(1+R) \delta \pm \delta \sqrt{(R-1)^{2}-4 R Z}\right\} .
$$

which is now expressed in terms of the two ratios. From this we can note that there is a critical value of $Z$ which just makes the discriminant zero for any value of $R$. This critical value is $Z^{*}=(R / 4)-(1 / 2)+(1 / 4 R)$. The function $Z^{*}(R)$ describing the critical value which separates the cases qualitatively is upward convex and upward sloping for $R$ greater than one and downward sloping and convex for $R$ less than one as shown in Figure 3.10. Now consider any given level of $R$ and alternative levels of $Z$. If $Z$ is greater than $Z^{*}$ the whole term under the square root will be negative and thus the approach path will be oscillatory. Similarly if $Z$ is less than $Z^{*}$ the approach path will be stable and convergent as shown in the figure.

Now we are ready to summarize how various combinations of circumstances will come together to determine the qualitative nature of the equilibrium between the industry
and the regulators. What happens, for example, if the industry reacts quickly to rents and the regulators react slowly to their targets? Or what happens as the equilibrium approaches the minimum and maximum season lengths? Or what if both parties adjust equally fast, or slow? First, note that whatever the value of the ratio of slopes at the equilibrium the closer the adjustment speeds are to each other the more likely that an oscillatory approach will result. Hence if one group reacts slowly and the other quickly, these differing adjustment speeds act as a stabilizing factor. Second, suppose that adjustment speeds are approximately, but not exactly, equal. Then as the absolute value of the ratio of the slopes gets larger, the system will be more likely to oscillate rather than converge directly. This occurs, for example, when the slope of the entry equation is steep and the slope of the regulatory equation is flat. But when is the absolute value of the ratio of slopes large? As it turns out, we can express this ratio in terms of parameters to arrive at the expression:

$$
Z=\frac{\left(X_{0}-Q\right)-(v / P q)}{\left(X_{0}-Q\right)-Q\left[\ln \left(X_{0} /\left(X_{0}-Q\right)\right)\right]^{-1}}
$$

This ratio will be large when the quota is small or when the price and/or catchability coefficient is large. The ratio will also be larger as the variable cost coefficient is smaller and it is ambiguously related to the biomass.

In summary, this chapter has developed a new theory of open access behavior that combines behavioral models of both an industry subject to regulations and a regulatory body which sets regulations in a manner conditioned on the industry's behavior and on specific goals. In the first part of the chapter we discussed a static version of the model and described comparative statics of the equilibrium properties. In the last section, we developed a more general model which explicitly incorporates dynamics and which nests the static model. The more general model is capable of describing a wide range of behavior, the qualitative properties of which depend on the structural model parameters as well as adjustment speeds. As Figure 3.11 summarizes, the approach to equilibrium can be either direct or oscillatory. The specific behavior that we might expect depends upon the


Figure 3.11: Alternative Paths to Equilibrium
system. If, for example, the industry and regulators are relatively "matched" with respect to adjustment speeds, the system will tend to exhibit oscillations. On the other hand, with one party relatively slow, the system will tend to converge asymptotically. In addition, the nature of the approach will depend in a relatively complicated way on the location of the equilibrium, and specifically on the slopes of the respective decision functions.

## Chapter 4

## Estimation of the

## Industry/Regulator Model

### 4.1 Introduction

In this chapter we estimate the model of industry/regulator interaction developed in Chapter Three. As outlined in Chapter Three, the conceptual underpinning for our model of industry behavior is the hypothesis that entry of fishing capacity is governed by a tendency to enter until rents are dissipated. Similarly, the behavioral hypothesis for modeling regulatory behavior is that regulators have goals in terms of targeted catch, biomass levels, etc., and policy instruments are chosen to meet those goals. An important issue for empirical specification concerns the equilibrium properties of the interactions between the two groups of agents. Should it be assumed that equilibrium is reached each period, or should some sluggishness be allowed? We leave this as an issue to be decided empirically, by nesting the static model in a more general dynamic model. In the next section, we review the complete system to be modeled and briefly discuss some of the econometric issues it
poses. Then we discuss the data used to estimate the model parameters. Following this, we discuss the parameter estimates under the assumption of instantaneous equilibrium and a more general model that allows us to test hypotheses about sluggish adjustment. In the final section we present some simulations of the paths of the endogenous variables generated with the dynamic model.

### 4.2 Empirical Estimation Issues

In the previous chapter we modeled the behavior of the industry and regulators as a simultaneous system. Regulators are assumed to choose the season length $T$, based on the fishing capacity $E$ of the industry, the targeted catch or quota $Q$, and the beginning of the season biomass $X$. The industry is assumed to choose capacity depending upon the season length chosen by the regulators, the biomass $X$, and the exvessel price $P$. In the dynamic specification of the system, contemporaneous values of season length and capacity depend also on past values. The actual harvest taken $H$ depends upon total actual effort $E T$, which is the daily capacity $E$ multiplied by the total number of days $T$ that fishing is allowed. The basic system to estimate can thus be written as follows:

$$
\begin{aligned}
E_{t} & =j\left(T_{t}, E_{t-1}, P_{t}, X_{0_{t}} ; \phi\right) \\
T_{t} & =g\left(E_{t}, T_{t-1}, Q_{t}, X_{0_{t}} ; \phi\right) \\
H_{t} & =h\left(E_{t}, T_{t} ; \phi\right),
\end{aligned}
$$

where $\phi$ is a vector of parameters, including cost and production parameters and adjustment speed parameters. In this system, we assume that $E, T$, and $H$ are endogenous variables and the exvessel price $P$, the biomass $X$, and the quota $Q$ are predetermined.

There are several econometric issues to be considered in estimating parameters of the above model. First we need to choose functional forms and consider the origin and
form of the econometric error term. The choice of functional form will affect the estimation procedure, including whether linear or nonlinear methods must be used. Second, we need to account for the simultaneity of capacity and season length choices since there is unavoidable correlation of the contemporaneous error term with explanatory variables. Finally, we need to consider the covariance among equations and determine whether imposing cross equation restrictions and taking advantage of the covariance structure to increase efficiency is worth potential misspecification problems.

There are essentially two options to consider when faced with simultaneous equations: maximum likelihood and instrumental variables. Maximum likelihood estimation is attractive in that it yields all of the desirable asymptotic properties, including asymptotic efficiency. The desirability of maximum likelihood depends, however, on a correct specification of the distribution of the error term (e.g., multivariate normal). In addition, estimation can become quite cumbersome if nonlinearities are present.

The second possibility is instrumental variables estimation. There are several features of instrumental variables estimation that make it attractive, even though instrumental variables estimators are not efficient. First, consistency is preserved, and this property does not depend on an accurate specification of the distribution of the error term. Second, the technique for estimating nonlinear systems has been well developed, ${ }^{1}$ although the success of instrumental variables estimation depends on the choice of instruments. In linear systems, the instruments that yield the best estimates are, in principle, easy to determine, namely all of the predetermined variables in the system. In nonlinear systems, in contrast, the instruments that yield the most efficient estimates depend on the true parameter vector which is, of course, unknown. Hansen and Singleton suggest using any predetermined variable as an instrument including any lagged endogenous variable.

[^4]
### 4.3 Data

The halibut case was chosen as a case study in part because of the unusually lengthy and high quality data base that has been collected and published. The International Pacific Halibut Commission has consistently compiled an extensive data series on critical variables as well as numerous reports discussing management approaches and goals; scientific findings from studies of the fishery, and annual reports. Much of the important information comes from logbooks that fishermen are required to fill out and report at the point of landing. Logbooks report the number of units of gear fished, where fished, how long each unit is soaked, and the catch per gear haul, including size breakdowns. These data are gathered by the IPHC, compiled and standardized, and utilized for stock assessment, catch estimation, and effort measurement.

In the results reported in this chapter, we estimate equation systems over the period 1935-1978 for two regulatory areas that comprise the bulk (over 90\%) of total North Pacific halibut landings. The two regulatory areas encompass most of Area 2 and Area 3 (see map in Figure 4.1). These areas have been treated as separate regulatory regions since the halibut regulatory program began in the 1930s. Thus each region has had its own biomass estimates, effort level compilations, catch quotas, and season lengths. Estimation over the two regions allows a consistency check of the parameter estimates for the models since there are effectively two individual time series.

From 1935 until 1966, Area 2 consisted of the waters off of the Canadian coast of British Columbia and the Alaskan waters up to Cape Spencer in Southeast Alaska. In 1967, Area 2 was merged with Area 1, which had been the waters off of the Continental U.S. In 1981, Area 2 was split up into three distinct subregions (2A, 2B, and 2C) which effectively split Area 2 by national boundaries. The estimates reported here treat Area 2 and the former Area 1 as a single aggregate by summing biomass, catch, and effort for both


Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
regions. The season lengths and quotas we use are for the regulatory Area 2. Since the catch in Area 1 was negligible (less than $5 \%$ of Area 2's catch), and the area was fished without a quota imposed, this approximation seems permissible.

Area 3 has typically encompassed waters north of Cape Spencer off Alaska but excluding waters in the western Aleutians and in the Bering Sea. Area 3 was divided into subareas in 1952. This fact and the fact that the western boundary of Area 3 has changed from time to time presents a problem for the compilation of a consistent data series because season lengths have not always been the same across the finer subdivisions where geographic boundaries were changing. We compiled a pseudo "Area 3 " data series by summing biomass, catch, and effort over an area whose size remained fixed. The season length and quota we use covered the larger part of the geographical area. As it turns out, this approximation is not of great quantitative significance since the areas where changes in boundaries and different seasons occurred have contributed only a small part of the larger area catches.

The estimation period chosen for this study stops in 1978. In 1978 several changes in management were initiated which made adding more recent years problematic. First, the jurisdiction extension by both the U.S. and Canada led to a separation of Area 2 waters into separate Canadian and U.S. waters. This in turn led to different management practices over the newly formed areas (essentially Areas 2B and 2C). Thus the data series, including quotas and season lengths, split in Area 2 after 1978. In addition, Canada initiated a limited entry program in 1979. We might expect some structural change as a result, adding new problems in return for more degrees of freedom.

### 4.3.1 Biomass Data $(X(t))$

Estimates of biomass have been computed on a yearly basis and updated periodically since the program began in the late 1920s. The series we use is from Quinn, et.al.,[69]
and was published in 1985. These data were estimated using data from logbook entries over the entire halibut program history. The logbook data reveals the density of different year classes via the catch per unit of standardized gear. Each year's catch by size is then used to estimate parameters of an age/size structured biological model by maximum likelihood methods. Population biologists regard this study to be the best source of biomass estimates for the Pacific Halibut.

In the econometric estimates discussed here, biomass enters the regulator's season length choice equation explicitly and the industry entry equation implicitly. Thus one issue is whether the series we use is a good representation of that actually used by regulators over the whole period and by potential entrants, when assessing the profitability of participating. We assume that it is an unbiased representation. We also assume that biomass estimates refer to beginning of the season estimates $X_{0}$ rather than end of season estimates $X_{T}$. This does not seem to make much difference to estimates; the estimating structure could be appropriately modified if the alternative interpretation were more accurate.

### 4.3.2 Fishing Capacity $(E(t))$

Our measure of capacity is also derived ultimately from logbook data compiled by the IPHC. These logs are used to calculate the total number of standard skates (units of longline gear, as discussed in Chapter 2) fished in each region. A standard skate is an 1800 foot long groundline with 100 hooks attached at 18 foot intervals set in the water for 12 hours. Fishermen report actual skate soak times and gear configurations and these are converted into standardized units.

In order to estimate the parameters of the behavioral model of both industry and regulator choice, we need a relatively simple and consistent measure of potential fishing capacity that approximates both the measure utilized by managers and a measure of capac-
ity reflected in actual technology. This is because we are estimating actual season length choices by regulators and cost parameters for the industry, which are influenced by actual gear and capacity configurations. Our measure of fishing capacity is simply total standardized skate soaks fished as published by the IPHC, divided by season length. This measure assumes that effort intensity does not vary over the sample and that each unit of standardized capacity has costs proportional by their conversion factors to actual costs. Thus, for example, if a fisherman utilizes 900 foot skates instead of 1800 foot skates, we assume that the cost of both setting and retrieving a single skate and the cost of outfitting that skate will be half that of a standardized skate.

### 4.3.3 Harvest $(H(t))$ and Exvessel Prices $(P(t))$

We use published data for both total harvest in both Areas 2 and 3 . These sum up catches of both Canadian and U.S. fishermen and are measured in dressed weight (eviscerated and head off). Similarly, exvessel prices are the weighted average prices reported for the whole North Pacific halibut fishery by the IPHC. These essentially sum up total exvessel values derived from fish delivery tickets and divide by the total harvest level over the season. We deflate nominal exvessel prices by a wholesale price index with base year 1982 to create a series of real prices.

### 4.3.4 Quotas $(Q(t))$ and Season Lengths $(T(t))$

Quotas for each of the areas are published in annual reports of the IPHC. This is somewhat unique in fisheries management; often quotas are set and known by the regulatory agency and not published or recorded. It should also be pointed out that the quotas are targets used by the regulatory agency to choose the season length instrument. The actual catch $H(t)$ may be and usually is different from the target catch $Q(t)$.

Season lengths are derived from annual reports of the IPHC and from a summary in Skud[80]. These are expressed in days of season length and are essentially continuous seasons. Another reason for truncating the estimation at 1978 is that the subsequent years witnessed split seasons. Hence instead of having continuous seasons, a total season was split into several short openings. In the last few years in Alaska, for example, there have been three one day openings in June, July, and September, following a single two day opening in May. We would expect the industry behavior associated with these types of regulations to be different from what was observed between 1935 and 1978 when seasons ranged continuously from 25 to 250 days.

### 4.4 Parameter Estimates: Static Model

The first model examined is the simplest representation mirroring the first model discussed in the previous chapter. Critical to the estimation is the assumption of instantaneous and complete equilibrium. The process is assumed to work analogously to the Walrasian tatonnement mechanism in that both the industry and the regulators make choices based on expectations about each other's behavior that are realized each period. We refer to this as a "static" model because the equilibrium is simply a sequence of static equilibria. This model was estimated by treating the industry capacity choice equation and the regulator choice equation separately. We make the judgment that possible contamination of estimates from misspecification is more of a problem than the potential efficiency losses from ignoring covariances. Since each contains endogenous variables, we use an instrumental variables method, two stage least squares, to purge the effects of contemporaneous correlation between the error terms and explanatory variables.

We are interested in estimating three parameters of the system, namely the catchability coefficient $q$, and two cost coefficients $f$ and $v$, one each for fixed and variable costs.

The two cost coefficients are embedded in the industry behavioral equation. The industry behavioral equation essentially assumes that capacity enters each period in a manner that completely dissipates rents as in:

$$
\begin{equation*}
\text { Rents }=\left[P X_{0}\left(1-e^{-q E T}\right)\right]-[v E T+f E] . \tag{4.4.1}
\end{equation*}
$$

Equation (4.4.1) presents a specification problem because there is no closed form for the equilibrium level of capacity. As discussed earlier, this occurs because of the functional forms of the production and cost functions. In particular, as long as we wish to retain the desirable features of the exponential production function and the simple linear cost function, it is not possible to solve for and isolate the level of rent dissipating capacity $E_{0}$ as a function of the other variables.

One way of addressing this problem is to give up trying to retain the exact functional form and instead approximate the production function with a form that allows solving for $E$. For example, a second order Taylor's series approximation can be written as:

$$
\begin{equation*}
H_{t}=f\left(E_{t}, T_{t}\right) \simeq q X_{0 t} E_{t} T_{t}\left[1-\frac{q E_{t} T_{t}}{2}\right] \tag{4.4.2}
\end{equation*}
$$

and when inserted into the rent dissipation equation, one can solve for the resulting approximation to the rent dissipating level $E_{0}$, namely:

$$
\begin{equation*}
E_{0} \simeq \frac{2}{q}\left[\frac{1}{T_{t}}-\frac{v}{q X_{0 t}}-\frac{f}{q X_{0 t} T_{t}}\right] . \tag{4.4.3}
\end{equation*}
$$

This approximation has desirable properties for estimation since the resulting equation is linear in the inverses of $T, X_{0}$, and $T X_{0}$, respectively. The second order Taylor's series approximation is not a particularly good approximation, however, because the production function is an asymptotic functional form. A better approximation can be obtained using:

$$
\begin{equation*}
H_{t} \simeq \frac{E_{t} T_{t} X_{0 t}}{a X_{0 t}+E_{t} T_{t}} . \tag{4.4.4}
\end{equation*}
$$

This asymptotes at a catch of $X_{0}$ as $E T$ approaches $\infty$ as does the production function in 3.2.4. When plugged into the rent dissipating equation, this gives rise to another equation
for $E$, namely:

$$
\begin{equation*}
E_{0 t} \simeq \frac{X_{0 t} T_{t}(1-f a)-v a X_{0 t}}{f T_{t}^{2}+v T_{t}} \tag{4.4.5}
\end{equation*}
$$

Unfortunately, this equation, while a more accurate approximation, ends up nonlinear in both the variables and parameters and hence becomes more difficult to estimate.

Given the undesirable outcomes of trying to use approximations, another approach to specifying the rent dissipating equation that retains the specific functional form is to simply estimate the zero rent equation using a nonlinear estimation routine. That is, one could define the equation as:

$$
\begin{equation*}
0=\left[P X_{0 t}\left(1-e^{-q E_{\mathrm{t}} T_{t}}\right)\right]-\left[v E_{t} T_{t}+f E_{t}\right]+\epsilon_{t} \tag{4.4.6}
\end{equation*}
$$

and estimate $q, f$, and $v$ nonlinearly. The only apparently feasible assumption about the error structure is one which appends an additive error. One must also account for the endogeneity of $E$ and $T$, for example, by using nonlinear instrumental variables. We were successful in using this specification, particularly when appending the catch equation and estimating both simultaneously. A problem is that this system is extremely sensitive to starting values (particularly starting values for the catchability coefficient). This problem, in fact, led us to a third approach which we adopt for the discussion to follow.

The third approach simply estimates a linear rent dissipation equation which implicitly incorporates the rent dissipating capacity $E$. The equation is:

$$
\begin{equation*}
0=T R_{t}-v_{0} E T_{t}-f E_{t}+\epsilon_{t} \tag{4.4.7}
\end{equation*}
$$

which substitutes total revenues for the specific, but nonlinear functional form for revenues used in equation (4.4.6) above. This equation is an advantageous specification because it can be estimated as a linear system. The simplest expression regresses total revenues against the two variables explaining total costs, or total seasonal effort $E T$ and total capacity $E$. Simple linear estimation techniques can be used to recover estimates of the two cost
coefficients $v$ and $f$ and $q$ can be recovered by estimating a separate catch or total revenue equation. These can be estimated separately using two stage least squares or as a system using three stage least squares. Alternatively, 4.4.7 may be rearranged in various ways. Two possibilities are to: (i) divide through by price and estimate a normalized form, and (ii) isolate $E$ and estimate the resulting ratio form model using nonlinear methods. Finally, equation (4.4.7) can be estimated in implicit form instead of isolating total revenues or effort or any other variables on the left hand side. The procedure is relatively straightforward. First define a variable called $Z E R O=0$. Then using any linear or nonlinear package (we use SHAZAM 7.0 [78]) to regress $Z E R O$ on the rent equation. This effectively assumes an additive error, which is about the only error specification possible in any case.

We settled on the last specification after trying and comparing all of the others suggested above. We found that parameter estimates were relatively stable across all specifications and the implicit equation with total revenues embedded generally turns out to be the easiest specification to estimate econometrically. Its linearity in variables is the most attractive feature, particularly in light of our findings that the fully nonlinear specification with the catchability coefficient embedded explicitly in revenues is especially sensitive to starting values. ${ }^{2}$

Two simple modifications of the basic model are presented here. First, we generalized the variable cost coefficient by adding a time trend to account for the possibilities of cost increases associated with the rent dissipation process. Although fishing technology has not changed appreciably, we would anticipate some cost increases associated with inefficiencies induced by the race to catch fish. Thus we define variable costs to be $v_{0}+v_{1} * t$ rather than a simple constant as in our theoretical discussion. In addition, since our estimation

[^5]period spans the disruptive World War II period, we modified the fixed cost coefficient by including a dummy variable for the years 1943-1945 when the war was active in the Pacific. This dummy variable is multiplied by the capacity variable to represent an implicit change in opportunity costs. Total fixed costs thus become $f+f_{w} * D W A R$ where $D W A R$ equals one in 1943-1945.

The equation we estimate for both Areas 2 and 3 is:

$$
\begin{equation*}
0=T R_{t}-v_{0} E T_{t}-v_{1} t * E T_{t}-f E_{t}-f_{w} D W A R_{t} * E_{t}+\epsilon_{t} . \tag{4.4.8}
\end{equation*}
$$

Both capacity and season length are endogenous variables (as is total revenues) and hence we estimate this equation using instrumental variables. The instruments we use are all of the predetermined variables in the system, including biomass, the quota, price, the war dummy, and the regulatory variable that determines season length $\left(\ln \left(X_{0} /\left(X_{0}-Q\right)\right)\right)$. In this formulation, this is a two stage least squares (2SLS) estimation. The error is assumed to be additive, and composed of random errors in expected revenues as well as a random error associated with the industry's "choice" of $E_{0}$.

The results of this estimation of the simple static model are presented in Tables 4.1 and 4.2 for Areas 2 and 3 respectively. The models for both regions fit well and produce coefficients that are generally significant and of the correct signs. With respect to magnitudes, the parameter estimates are also reasonable. ${ }^{3}$ We would expect costs to be of the same relative magnitudes with Area 3's remoteness perhaps generating higher costs estimates. The variable costs per skate soak, evaluated in 1978, are approximately $\$ 80$ in Area 2 and $\$ 71$ in Area 3 while fixed costs are approximately $\$ 768$ per unit of gear in Area 2 and almost $\$ 5,000$ per unit in Area 3. These are within reasonable ranges. The War also had the expected effect of increasing the implicit opportunity cost of participating in the

[^6]| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $v_{02}$ | $.0291^{* *}$ | 5.17 |
| $v_{12}$ | $.0012^{* *}$ | 6.00 |
| $f_{2}$ | $.7679^{* *}$ | 2.34 |
| $f_{w 2}$ | $1.5941^{* *}$ | 4.01 |
| D.W. | 2.22 |  |

Table 4.1: Estimates of the Entry Equation for Area 2: Static Model

Halibut fishery. The implicit opportunity cost added by the hazards of the war amount to an extra $\$ 1,600$ per unit of gear in Area 2 and a significant $\$ 14,500$ in Area 3. Management reports suggest that fishermen were warned against fishing in the Bering Sea during the War due to dangers presented by Japanese submarines.

In summary, these are satisfactory results for the first cut at estimating the parameters of the behavioral model of joint industry/regulator interaction. The coefficients all pass a reality check in terms of signs and relative magnitudes. The models also track the data well over the span of 43 years, although with some evidence of autocorrelation in Area 3. This suggests a couple of possibilities, including problems with variables used (e.g.) whether biomass estimates generated in the 1985 study are good proxies for what the industry anticipated over the period), or perhaps some issues related to dynamics. We thus now turn to estimates of the dynamic specification that nests the static models presented

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $v_{03}$ | .0071 | .282 |
| $v_{13}$ | $.0015^{* *}$ | 2.94 |
| $f_{3}$ | $4.9888^{* *}$ | 2.48 |
| $f_{w 3}$ | $14.511^{* *}$ | 4.10 |
| D.W. | 1.08 |  |
| $*:$ significant at $10 \%$ level <br> **: significant at $5 \%$ level |  |  |

Table 4.2: Estimates of the Entry Equation for Area 3: Static Model
here.

### 4.5 Parameter Estimates: Dynamic Models

In Chapter Three a simple dynamic model was developed based on a sluggish adjustment modification of the basic static model. The essential idea for that generalization is that effort and season length may be expected to adjust towards the levels specified by the static models but that they might not reach the respective levels in each period. The dynamic model in Chapter Three captured sluggish adjustment possibilities with two adjustment speed parameters and demonstrated the implications of various assumptions about adjustment speeds. For example, slow adjustment speeds and divergent relative speeds (industry vis à vis regulators) lead to slow asymptotic approaches to the joint equilibrium,
whereas faster reaction speeds and similar adjustment speeds may produce oscillatory approaches. In this section, we estimate a more general model of both industry and regulatory behavior which incorporates the above static model as a special case.

There are various ways one might specify a dynamic model of industry behavior that captures both sluggishness and the tendency towards rent dissipation. Again, we are hindered by the inability to solve explicitly for the rent dissipating level $E_{0}$ in the dynamic specification. Approximations are possible but we choose to follow a procedure similar to that discussed in the previous section in which we estimate the parameters of an implicit equation using total revenues as an explanatory variable. The basic model estimated here assumes that total industry costs adjusts sluggishly to total revenues, or:

$$
\begin{equation*}
T C_{t}-T C_{t-1}=(1-\theta)\left[T R_{t}-T C_{t-1}\right] . \tag{4.5.1}
\end{equation*}
$$

The adjustment factor $1-\theta$ measures how quickly the gap is made up between total costs and total revenues. If $\theta=0$ then total costs adjust instantly to total contemporaneous revenues and this is effectively the static model discussed above. As $\theta$ approaches one, adjustment is less complete and more time is taken to dissipate rents.

Lying behind total costs adjusting to total revenues is the notion that capacity $E$ is entering and driving up total costs for any given season length. Hence we express total industry costs as a function of capacity as in the static formulation. Dividing the equation through by the adjustment factor leads to the estimating equation:

$$
\begin{align*}
0= & T R_{t}-\frac{v_{0} E T_{t}}{1-\theta}+\frac{\theta v_{0} E T_{t-1}}{1-\theta}-\frac{v_{1} t * E T_{t}}{1-\theta}+\frac{\theta v_{t} t E T_{t-1}}{1-\theta}-  \tag{4.5.2}\\
& \frac{f E_{t}}{1-\theta}+\frac{\theta f E_{t-1}}{1-\theta}-\frac{f w D W A R_{t} E_{t}}{1-\theta}+\frac{\theta f_{w} D W A R_{t} E E_{t-1}}{1-\theta}+\epsilon_{t} .
\end{align*}
$$

This equation can be estimated using linear methods but the estimated parameters would be composites rather than individual parameters. The option is thus to use a nonlinear instrumental variables procedure. We use the same instruments as in the static estimation,
adding lagged total revenues. We estimate this equation for each region, using variables and instruments from that region only.

The cost and adjustment coefficients from these estimations are presented in Tables 4.3 and 4.4. Again the results are very reasonable and significant. For Area 2 the variable cost estimates are similar to those estimated in the static model ( $\$ 77.90$ per skate soak), although the intercept is less significant. Fixed costs are also in the same range, with the implied non-Wartime fixed outfitting costs estimated at $\$ 1200$ per skate compared with $\$ 767$ in the static model. For Area 2, the adjustment parameter $\theta$ is .6450 and highly significant. A simple asymptotic t -test shows that it is significantly different from one and significantly different from zero. Hence we have evidence that rents are not completely dissipated instantly in each period.

Area 3 parameter estimates are reasonable and comparable to the static results presented also. Variable costs are close to those for Area 2 (\$80.57) and fixed costs are also closer than in the static model estimates. Implied non-Wartime fixed costs are $\$ 3,193$ which is two and a half times that for Area 2. The adjustment coefficient is also highly significant, different from one and zero, and indicative of a faster speed of adjustment in Area 3 than in Area 2. (Recall that low values of the adjustment parameter mean faster speeds of adjustment.)

As a next step, we modeled industry behavior jointly over the two regions. Since entry was open over the period examined and since fishermen were free to choose either area, we might expect some covariance between the two behavioral equations. We incorporated this possibility by estimating a system of two equations (4.5.2), using nonlinear three stage least squares. The instrument list contained all the variables used in the two stage least squares estimations from both areas. Table 4.5 reports the dynamic model with Areas 2 and 3 modeled jointly. As expected, there is some increase in efficiency, with smaller variances

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $v_{02}$ | $.0177^{*}$ | 1.63 |
| $v_{12}$ | $.0014^{* *}$ | 4.59 |
| $f_{2}$ | $1.2121^{* *}$ | 1.81 |
| $f_{w 2}$ | $1.4595^{* *}$ | 2.49 |
| $\theta_{2}$ | $.6450^{* *}$ | 7.00 |

Table 4.3: Estimates of the Entry Equation for Area 2: Dynamic Model

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $v_{03}$ | .03327** | 2.04 |
| $v_{13}$ | .0011** | 2.63 |
| $f_{3}$ | 3.1930** | 2.61 |
| $f_{w 3}$ | $11.633^{* *}$ | 4.13 |
| $\theta_{3}$ | . $4519^{* *}$ | 4.16 |
| D.W. | 1.81 |  |
| *: signifrant at $10 \%$ level <br> **: sig:.. : 7 level |  |  |

Table 4.4: Estimates of the Entw Fquation for Area 3: Dynamic Model
for most parameter estimates. In general the parameter estimates are similar to two stage least squares estimates although the joint model produces parameter estimates that are more alike across the two regions. Variable cost estimates are $\$ 84$ and $\$ 87$ per skate soak for Areas 2 and 3 respectively and fixed cost estimates are $\$ 1,600$ and $\$ 1,700$. Adjustment coefficient estimates are slightly larger for Area 2 and slightly smaller for Area 3, bringing them closer together in magnitude.

A final set of estimations allowed for the possibility of covariance between regions and, in addition, imposed a between-region equilibrium condition. With sluggish behavior one is allowing for less than complete rent dissipation at any point in time. This could arise, for example, because capacity constraints on fishing capital preclude rapid adjustment in response to rents. A question which arises, however, is would we expect fishing capacity in the fleet as a whole to distribute itself across the regions to equalize returns? Since fishermen could choose either region, we might expect some such process, even when rents in total are not completely and instantly dissipated.

One way to test this is to examine returns per skate, or

$$
\begin{equation*}
\frac{T R_{t}-v_{0} E T_{t}-v_{1} t E T_{t}}{E_{t}} \tag{4.5.3}
\end{equation*}
$$

If returns per skate are equalized across regions 2 and 3 , we would have:

$$
\frac{T R_{2 t}-v_{02} E T_{2 t}-v_{12} t E T_{2 t}}{E_{2 t}}=\frac{T R_{3 t}-v_{03} E T_{3 t}-v_{13} t E T_{3 t}}{E_{3 t}}
$$

which can be imposed as an extra condition to be estimated jointly with the two dynamic adjustment equations. The extra condition imposed on the system can be written as:

$$
\begin{equation*}
0=\left[\left(T R_{2 t}-v_{02} E T_{2 t}-v_{12} t E T_{2 t}\right) E_{3 t}\right]-\left[\left(T R_{3 t}-v_{03} E T_{3 t}-v_{13} t E T_{3 t}\right) E_{2 t}\right]+w_{t} . \tag{4.5.4}
\end{equation*}
$$

Before running the equal returns formulation, we examined average revenues per skate over the period 1935-1978. This is only a proxy of equation (4.5.3) above since it does not include

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $v_{02}$ | .0192** | 2.65 |
| $v_{12}$ | .0015** | 6.10 |
| $f_{2}$ | .9931** | 2.88 |
| $f_{w 2}$ | $1.6097^{* *}$ | 3.49 |
| $\theta_{2}$ | .558** | 5.57 |
| D.W. | 1.86 |  |
| $v_{03}$ | .0569** | 4.79 |
| $v_{13}$ | . $0007^{* *}$ | 2.15 |
| $f_{3}$ | 1.7327** | 1.93 |
| $f_{w 3}$ | 9.82** | 4.14 |
| $\theta_{3}$ | .4728** | 5.52 |
| D.W. | 2.18 |  |
| *: significant at $10 \%$ level <br> **: significant at $5 \%$ level |  |  |

Table 4.5: Estimates of the Entry Equation for Both Areas Modeled Jointly: Dynamic Model
average costs, which are to be estimated. Nevertheless average revenues were suggestive of a pattern of convergence over the period. During the 1930s and 1940s, average revenues in Area 3 were considerably higher than those for Area 2. This is consistent with the fact that Area 3 is considerably more remote and subject to inclement weather than Area 2. By the late 1940s this divergence between average revenues had disappeared. It is likely that fleet expansion and development of other fisheries in Alaska helped open up Area 3 to more fishing pressure and hence eroded the earlier quasi-rents that seemed to exist.

We ran several sets of joint models including the new cross-region equation. One tested for a significant difference over the whole sample between returns per skate soak. We found a significant positive difference favoring Area 3 , but this was obviously influenced by the early period that showed up in the data plots. Table 4.6 reports a set of estimates from a three stage least squares regression run over the later part (1949-1978) of the sample only, when plots showed a convergence of average revenues. An initial run of this regression over the truncated sample testing for a positive difference revealed no significant difference in estimated average rents per skate. Again, parameter estimates are reasonable and significant. Estimated variable costs are $\$ 74$ and $\$ 84$ for Areas 2 and 3 and fixed costs are close to equal across the two regions.

### 4.6 Estimating Regulatory Behavior

The estimates reported above recover cost parameters and speeds of adjustment of the industry as revealed in their behavior over the sample period. In this section we discuss similar results for the regulatory sector. The basis for these estimates are the models discussed in Chapter Three which postulate that regulators choose season length to achieve targeted catches, given the level of biomass and the industry's level of fishing capacity. We report parameter estimates as above, beginning with a static version of the

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $v_{02}$ | $-.0286^{* *}$ | $-1.69$ |
| $v_{12}$ | .0024** | 5.28 |
| $f_{2}$ | 2.2183** | 6.10 |
| $\theta_{2}$ | . $4741{ }^{* *}$ | 4.24 |
| D.W. | 2.25 |  |
| $v_{03}$ | .0323** | 6.10 |
| $v_{13}$ | .0012*** | 5.34 |
| $f_{3}$ | 2.2841** | 5.34 |
| $\theta_{3}$ | .4072** | 5.34 |
| D.W. | 2.18 |  |
| ${ }^{*}:$ significant at $10 \%$ level  <br>   <br>  $=$ : <br> significant at $5 \%$ level |  |  |

Table 4.6: Estimates of the Entry Equation for Both Areas Modeled Jointly and Average Revenues Equal Across Areas: Dynamic Model
model and progressing to dynamic versions.
As presented in Chapter Three, regulators are assumed to choose season length $T$ according to:

$$
\begin{equation*}
T_{t}=\frac{1}{q E_{t}} \ln \frac{X_{0}}{X_{0}-Q}+\eta_{t} \tag{4.6.1}
\end{equation*}
$$

This falls out of the functional form assumed for the industry harvest function and the assumption that $T$ is chosen to achieve the published quota $Q(t)$. We assume that the season length is set following the specific rule so that regulators are in their equilibrium at each point in time except for random errors. The single parameter to be estimated is the catchability coefficient $q$ and it is important to note that this is the coefficient assumed by the regulators to be correct. We compare estimates derived from the regulatory behavior to those arising from actual catch behavior below.

This static regulatory model was first run for the two areas, treating each as independent equations and estimated using nonlinear two stage least squares to account for the endogeneity of $E$. Instruments used included biomass, quota, price, and the regulatory variable $\ln \left(X_{0} /\left(X_{0}-Q\right)\right)$. Tables 4.7 and 4.8 report estimated catchability coefficients. These both are reasonable, with very small variances. The error plots reveal significant autocorrelation however, suggesting, among other possibilities, that the dynamics of behavior might be important for the regulatory structure also.

For comparison, we estimated the actual aggregate harvest functions jointly with the regulator equations for each area. We performed simple hypothesis tests (for each region) that determined that there is no significant difference between the coefficients estimated from assumed regulatory behavior in setting season lengths and coefficients from actual aggregate industry harvest levels. This is not particularly important here, but in simulations reported later, we distinguish between catch targeted by regulators and actual catch realized by the industry. These tests allow us to assume that the catchability coefficients are the

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $q_{2}$ | $.0012^{* *}$ | 21.65 |
| D.W. | .84 |  |
| : significant at 5\% level |  |  |

Table 4.7: Estimates of Regulatory Equation for Area 2: Static Model

| Coefficient | Estimate | asymptotic t-ratio |  |
| :---: | :---: | :---: | :---: |
| $q_{3}$ | $.0010{ }^{* *}$ | 26.82 |  |
| $D . W$. | .39 |  |  |
| : significant at $5 \%$ level |  |  |  |

Table 4.8: Estimates of Regulatory Equation for Area 3: Static Model
same.

The more interesting tests of regulatory behavior examine a model based on sluggish adjustment similar to that used in the industry behavior equations. In particular, we assume that seasons are set according to:

$$
\begin{equation*}
T_{t}-T_{t-1}=(1-\mu)\left[\frac{1}{q E_{t}} \ln \left[\frac{X_{0 T}}{X_{0 t}-Q_{t}}\right]-T_{t-1}\right]+\eta_{t} \tag{4.6.2}
\end{equation*}
$$

In this formulation, $\mu$ measures the adjustment speed of regulators. When $\mu$ is small or close to zero, regulators adjust season length quickly to the level governed by the quota rule and industry capacity. When $\mu$ is large, the response speed of regulators is assumed slower, with only partial adjustment towards the quota goal.

This formulation is inherently ad hoc as was the dynamic industry model, but interestingly, one can derive the above form as an exact optimal rule for a slightly more general regulator problem. Suppose that regulators are concerned about two goals in setting season length. One is staying close to their quota goals to ensure stock safety and the other is not changing the season length too radically from season to season. The latter could fall out of the belief that fishermen need consistent and predictable regulatory rules to plan effectively. Assume that these goals can be represented by a utility loss function that is quadratic in deviations from the quota rule season and in changes in season length, or

$$
\begin{equation*}
\underset{E}{\min } L=\frac{1}{2}\left\{m\left[T_{t}-\left[\frac{1}{q E_{t}} \ln \left(\frac{X_{0 t}}{X_{0 t}-Q_{t}}\right)\right]\right]^{2}+n\left[T_{t}-T_{t-1}\right]^{2}\right\} . \tag{4.6.3}
\end{equation*}
$$

Here $m$ and $n$ measure the relative weights placed on each respective goal. This function can be minimized by choosing a season length according to:

$$
\begin{equation*}
T_{t}-T_{t-1}=\left(1-\frac{n}{m+n}\right)\left[\frac{1}{q E_{t}} \ln \left[\frac{X_{0 T}}{X_{0 t}-Q_{t}}\right]-T_{t-1}\right] \tag{4.6.4}
\end{equation*}
$$

Note that this is exactly the sluggish adjustment model presented in equation (4.6.2) above but with an interpretation based on a two goal minimization framework. In particular, note
that as the weight $m$ placed on remaining close to the quota goal rises relative to the weight $n$ place on season stability, $1-\mu$ rises, implying faster adjustment speeds. In contrast, if $n$ is large relative to $m$, season lengths will be adjusted towards the ones appropriate to the quota target, but at a slower rate. Hence the adjustment speed parameter can be thought of as reflecting the relative weights that regulators place on the two goals of coming close to the quota and instrument stability for the industry's benefit.

The two dynamic equations representing regulatory behavior for Areas 2 and 3 were estimated separately using nonlinear two stage least squares. The form shown above in equation (4.6.2) was used, appending an additive error and using for instruments variables including biomass, quota, price, the regulatory variable, and lagged season length. Tables 4.9 and 4.10 report results over the whole sample from 1935-1978. These reveal virtually identical behavior between the two regions and relatively sluggish adjustment. Adjustment coefficients in the range of .7 suggest that regulators set season length pursuant to their quota goals but only achieve the targets gradually, coming within $30 \%$ of the target each period. Another way to interpret this is that the relative weights placed on achieving the season based quota goals and achieving instrument stability are $30 \%$ and $70 \%$ respectively.

These results are somewhat surprising and led us to further investigate the behavior of the regulatory authorities over the entire period. We ran the dynamic model over various different sample periods and also ran some diagnostic tests á la Harvey[46]. One of the diagnostic checks included a recursive estimation over the sample and Chow tests of parameter change. Although these are strictly appropriate when conditions dictating OLS hold, they appeared suggestive of a significant change in adjustment speed behavior in the early 1960s. We then estimated a new set of regressions that allowed for a discrete shift in the adjustment speed from 1965 forward.

Tables 4.11 and 4.12 report results of the nonlinear two stage least squares spec-

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $q_{2}$ | $.0014^{\times \times}$ | 5.98 |
| $\mu_{2}$ | $.7360^{* *}$ | 5.57 |
| $D . W$. | 2.05 |  |
| *: significant at $10 \%$ <br> ※*: lignificant at $5 \%$ level |  |  |

Table 4.9: Estimates of Regulatory Equation for Area 2: Dynamic Model

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $q_{3}$ | $.0012^{* *}$ | 8.62 |
| $\mu_{3}$ | $.7322^{* *}$ | 8.76 |
| $D . W$. | 1.79 |  |
| : <br> *: significant at $10 \%$ level <br> *: significant at $5 \%$ level |  |  |

Table 4.10: Estimates of Regulatory Equation for Area 3: Dynamic Model

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $q_{2}$ | $.0015^{* *}$ | 10.02 |
| $\mu_{2}$ | $1.028^{* *}$ | 6.92 |
| $\mu 65_{2}$ | $-.6424^{* *}$ | -2.41 |
| $D . W$. | 2.56 |  |

Table 4.11: Estimates of Regulatory Equation for Area 2, Allowing for Structural Change: Dynamic Model
ification testing for structural change in the regulators' behavior. What these show are increases in adjustment speeds from 1965 onward. The increase in reaction speed is substantial and significant at the $5 \%$ level in Area 2 and less significant but in the same direction in Area 3. The post 1965 adjustment coefficients are 0.386 and 0.527 for Areas 2 and 3 respectively. Thus the adjustment coefficients averaging about .7 over the whole sample in the above reported results appear to mask a different scenario, namely one with very sluggish adjustment before the sixties and relatively quick adjustment after.

These results raise the question, what might have happened to alter the manner in which regulators operated and, in particular, why would they have changed their "utility" weights to emphasize quota targets rather than instrument stability? An examination of the history of the fishery, supported by regulatory documents and data, suggests a plausible answer. As discussed in Chapter Two, the early sixties is precisely when the halibut stocks

| Coefficient | Estimate | asymptotic t-ratio |
| :---: | :---: | :---: |
| $q_{3}$ | $.0012^{* *}$ | 11.72 |
| $\mu_{3}$ | $.8061^{* *}$ | 10.21 |
| $\mu 65_{3}$ | $-.2788^{*}$ | -1.47 |
| $D . W$. | 1.756 |  |

Table 4.12: Estimates of Regulatory Equation for Area 2, Allowing for Structural Change: Dynamic Model
first began to show evidence of the catastrophic collapse that erased all of the gains that had been won during the buildup phase starting in the 1930s. Biomass peaked in Area 2 in 1955 and in Area 3 in 1961 and thereafter dropped precipitously to lows reached in 1974, 13 years later. Annual reports and scientific background papers suggest that regulators took this fishery collapse very seriously and sought measures of regaining control almost immediately. Quotas were cut and seasons were reduced, but more importantly, it appears that regulators changed behavior so that seasons began to "track" those consistent with quota goals more precisely. This meant more variable seasons when industry capacity varied which is easily seen in Figure 2.4 in Chapter Two. Although other reasons for these results are plausible (including simply spurious factors or omitted variables), we find these explanations compelling.

### 4.7 Simulations

In this section we simulate paths of the endogenous variables from the dynamic model of industry and regulator interaction. We do this at this stage for two reasons. First, we wish to investigate the stability of the system to ensure that simulations generate reasonable results under the parameters estimated. Second, we would like to examine how the model that ignores the marketing impacts of regulation performs qualitatively in order to be able to interpret results from the more complicated model incorporating an inventory and exvessel price determination mechanism. Thus these simulations can be looked at as a model of the first part of the generalization of the Gordon model discussed in the introduction, namely the addition of an endogenous regulatory sector. In the next two chapters we will be discussing the second part of the generalization, or how the marketing sector is impacted by regulations and how this feeds back into the regulator/industry model. In Chapter Eight, we draw together the complete model in a more comprehensive simulation exercise.

The parameter estimates discussed above are quite consistent across various specifications. As a general summary, industry cost coefficients are in accord with what one might expect in terms of magnitude and are also relatively similar across the two regions. All parameters have correct signs and low standard errors and each equation fits well with reasonable error patterns. The speed of adjustment parameters for the industry are stable while those for the regulators appear to have undergone structural change in the sixties. Thus among other interesting questions, one might examine how various speeds of regulatory response affect predicted dynamic paths.

To investigate these issues, we simulated several scenarios using parameters from

Area 2 estimations. The basic simulation model can be written in implicit form as:

$$
\begin{aligned}
& E_{t}=j\left(E_{t-1}, T_{t} ; X_{0}, P, v, f, \theta\right) \\
& T_{t}=g\left(T_{t-1}, E_{t} ; X_{0}, Q, q, \mu\right) .
\end{aligned}
$$

Parameters used are given in Table 4.13. This system is not easy to simulate because it is dynamic, nonlinear, and simultaneous. For any given set of starting values, the two nonlinear equations above must be solved simultaneously to generate values for $E_{t}$ and $T_{t}$. Then these are used as starting values for another simultaneous solution for $E_{t+1}$ and $T_{t+1}$ and so on. We use the General Algebraic Modeling Simulation (GAMS[10]) package to solve for a complete set of dynamic paths for each specification. This is done by appending errors to each equation above and finding the complete simultaneous solution to the 2 N equations, where $N$ is the simulation period length, which minimizes the sum of squared errors. The package thus searches over the parameter space for $N$ values of $E$ and $T$ that drive errors within $\epsilon$ of zero. Simulation runs of about 50 time periods converged completely and relatively quickly on a DEC Dec Station 5000-125 computer. Those of 100 periods took considerably longer. We thus generated some simulation runs by splicing shorter runs, using ending values of the first as starting values for the next and so on.

We present the below results using various adjustment speed parameters on simulated phase diagrams. For the system simulated here, the simulated long run equilibrium is ( $\bar{E}=4.846, \bar{T}=43.58$ ). This compares with sample mean values of $E=4.359$ and $T=94.35$ over the estimation period. At this equilibrium point, the ratio of the slopes of the isoclines is .4255 . Qualitative properties of the approach to the equilibrium depend upon the adjustment speeds of both the industry and regulators as discussed in Chapter Three. Figure 4.2 below shows simulated dynamic approach paths using parameters appropriate for the post 1965 period where regulators were responding relatively quickly. As can be seen, the combined dynamics of industry and regulatory behavior generate an oscillatory path around the long run equilibrium with over- and under-shooting, particularly by the

| Parameter | Value |
| :---: | :---: |
| $v$ | 0.0835 |
| $f$ | .99 |
| $q$ | .0015 |
| $X_{0}$ | 91 |
| $Q$ | 24.71 |
| $\theta$ | .9079 |
| $\mu$ | .37 |
| $E_{-1}$ | 14 |
| $T_{-1}$ | 8.6 |

Table 4.13: Simulation Parameters


Figure 4.2: Simulation with Post-1965 Parameters
industry adjusting towards the rent dissipation equilibrium.

As discussed above in the section on regulatory behavior estimates, regulators increased their reaction speed, or the weight placed on being close to the quota determined season length, after 1965. A question that arises is thus, what behavior would have prevailed over the earlier period? Figure 4.3 shows the simulated dynamics with much slower regulator dynamics ( $\mu=.9$ ), assuming the same (estimated) industry adjustment speed ( $\theta=.558$ ). Note that an oscillatory path is still the result, although with a slightly different character. In particular, the over- and under-shooting is more pronounced and the regulators are not as close to their isocline. This reflects the obvious fact that if the decision rule places heavy weight on instrument stability, instrument accuracy (in the sense of season lengths chosen so that the target is met) will be sacrificed. This also means that targeted and actual catches will diverge during the approach to equilibrium as the dynamics swing wide and away from the $E T$ combination coincident with the quota.


Figure 4.3: Simulation with Slow Regulatory Adjustment

What would happen if both the industry an regulators reacted very slowly? Figure 4.4 shows simulated dynamics, assuming that $\mu$ and $\theta$ are both .9 . In this case, the results are qualitatively similar to the case above. During the first pass towards equilibrium, the industry does not overshoot to the degree it does when entry dynamics are faster, and hence the season length does go beyond its maximum from the case above. This case ultimately takes longer to arrive in the neighborhood of the equilibrium, of course.

Lastly, we simulated two cases where the relative adjustment speeds diverge substantially so that the approach path is more likely to he convergent rather than oscillatory. In Figure 4.5 below, we show the case with slow industry dynamics and fast regulatory dynamics $(\theta=.9, \mu=.1)$. In this case, the approach path hugs the regulatory isocline so that the quota target is adhered to alinost exactly, while the industry slowly exits towards the long run rent dissipation level. During this long adjustment period, the industry would be earning losses, although this result depends upon our arbitrary initial conditions which


Figure 4.4: Simulation with Slow Regulatory and Industry Adjustment
set $E_{0}$ at a level too high compared with the long run equilibrium. The equilibrium is approached directly in this case and hence is classified as a proper stable node.

At the other extreme, if the industry reacts very quickly to the appearance of rents and the regulators react slowly $(\theta=.1, \mu=.9)$, a dynamic path like that in Figure 4.6 will result. This one involves season lengths that are far off those that would be chosen to meet the targeted quota and hence actual and targeted catches diverge widely. On the other hand, the industry quickly approaches the region of its isocline, indicating rents close to zero as the system approaches the long run equilibrium. This approach path passes once through the industry isocline and hence the equilibrium is characterized as an improper node.

In sum, the parameter estimates discussed in this chapter generate reasonable dynamic simulations which are not only in accord with historically observed values but which also conform to the theory and intuition developed in Chapter Three. In the next


Figure 4.5: Simulation with Slow Industry Adjustment and Fast Regulatory Adjustment


Figure 4.6: Simulation with Fast Industry Adjustment and Slow Regulatory Adjustment
two chapters, we relax the assumption that the exvessel prices are exogenous by developing a model of the market and its relationship with the industry/regulator dynamics discussed up to this point. Then, in Chapter Seven we bring all components together to discuss properties and simulation results from the full model.

## Chapter 5

## Modeling Inventory Dissipation

## and Carryover

### 5.1 Introduction

As discussed in the introductory chapter, the objectives of this thesis are to explore two important facets of contemporary fisheries that Gordon's model of exploitation ignored, namely the role of the regulatory structure and the role of the market. In the previous two chapters, we developed and estimated a new model of a regulated open access industry, holding exvessel prices constant. The workings of the market were held exogenous for analytical convenience and to highlight the implications of adding regulations to the basic Gordon model. In the next two chapters, we close the structure by developing a model of the market for a fishery product that is harvested in a regulated setting. Again, for analytical convenience, we hold constant the rest of the system by assuming that effort, the harvest level, the biomass, and the fishing season length are given. Then the marketing problem reduces to the determination of an exvessel price, which in turn depends upon the
workings of the wholesale sector.

To foreshadow the modeling structure in the next two chapters, we note that in the fishery we are examining (as well as many others), the impact of the regulatory structure mainly affects the market by impacting raw product quality and by affecting the distribution of final product types. As seasons have shortened in the halibut fishery, raw product quality has been reduced simply by the frenzy associated with the race to catch fish. In 1992, for example, it was reported that in the two day Alaska opening, over one third of the landed fish had never been iced and over half were delivered without being gutted. Inevitably, it has also become common for long queues to develop at processing plants so that recently caught fish sit for hours before first handling. After delivery, further processing has also become bottlenecked as massive amounts of fish delivered cause peak load problems.

With a compressed fishing season, there is a further impact in addition to simply reduced quality raw product and that is that a large proportion of the fish must be converted to processed products rather than sold fresh. With long seasons, it is possible to deliver more fish to the generally more lucrative fresh markets but with seasons of five or even two days, virtually all fish must be processed, stored, and marketed over the remaining marketing period. Processing can take many forms including canning, smoking, pickling, and freezing. In the fishery we examine here, the primary form of processing for the marketing period is freezing. Raw product is first cut into slabs and/or fillets, then flash frozen at sub-zero temperatures, and then stored at close to freezing until marketed. At the point of marketing, the frozen product is further processed by trimming, packaging, and sometimes breading and cooking and so on.

The importance of this is to highlight that factors originating in the interaction between regulators and the industry also have an impact on the market and vice versa.

Critical to understanding these links is the notion that exvessel prices (which drive the entry/exit/regulatory process) are themselves determined as a derived demand. In particular, when most of the product is frozen or otherwise processed and held in inventory, the prices that the fishing sector receives will reflect the willingness to pay of inventory holders for additions to a stock of inventory. This willingness to pay, in turn, is forward looking because it depends upon a dynamic supply plan and on an assessment of market conditions over the upcoming marketing period. Thus a proper modeling of exvessel prices in this setting must begin with a model of inventory holders' dynamic plans and then derive the corresponding willingness to pay for additions to stocks held.

In the remainder of this chapter we develop an increasingly complicated model of optimal inventory behavior. We begin first with a stylized model of optimal inventory dissipation plan for a single firm operating in a single marketing period. ${ }^{1}$ Then we generalize to an industry level model which incorporates expectations and explores the nature of the dynamic equilibrium path of prices and quantities. We then move to a multiple period model in which periodic infusions of new additions to inventory arise and in which carryover between marketing periods is possible. We derive a number of qualitative conclusions about optimal inventory behavior under various assumptions. In Chapter 6 we draw from these conceptual foundations in order to specify an exvessel price equation that reflects the wholesale market behavior modeled here.

[^7]
### 5.2 Optimal Inventory Behavior with Semiperishable Commodities

The incorporation of inventories into empirical models of both agricultural and industrial sectors of the economy has always proven problematic. This is significant because inventory levels often seem to be important factors influencing the levels of farm-gate prices and wholesale orders. The difficulties in modeling inventory behavior stem from the durability of inventories. Durability means that inventories may be subject to speculative pressures as well as more tangible marketing and production oriented motives. This in turn suggests that modeling inventory behavior must account for the forward looking dynamic nature of the plans of decision makers.

The nature of the dynamic plan of inventory holders depends upon the characteristics of the product being stored and of the production process. For industrial goods, it is generally the case that production is continuous and that inventories are held to reduce costs of stockouts. There are many models of optimal inventory holding in both the operations research and economics literature. The simplest of these arrive at optimal levels of inventories while more sophisticated models derive closed loop policies which are functions of certain state variables at each point in time.

There is also a significant body of literature treating inventories and storage of agricultural commodities. Most of this has been developed to deal with the non-continuous nature of agricultural production which necessitates storing the year's production and marketing during the post harvest period. Early work in agriculture examined relationships between storage of grain crops and prices, particularly futures and spot prices (Working[90]) during the marketing period. More recent work has embodied dynamic optimization frameworks to account for the forward looking nature of the problem. Most of this work has embodied the implicit assumption that the commodity is infinitely or at least very durable.

The main problem investigated is then how much of a crop to carry over between crop years under stochastic production. Early work of this type included a dynamic programming based approach by Gustafson[41] and more recent theoretical work has been published by Helmberger and Weaver[47], Scheinkman and Schechtman[76], and Gardner[36]. Perhaps the most comprehensive treatment of the agricultural storage and carryover problem is that by Williams and Wright[89].

Most of the literature on inventory behavior in agriculture deals with relatively durable commodities such as grains. Less attention has been paid to products that are semiperishable. Exceptions include some work on fruit crops such as apples (Ben David and Tomek[7]), blueberries (Hoelper and Marra[51]), and potatoes (Glauber and Miranda[60]). For the most part, empirical specifications in these examples are ad hoc, although sensible intuitively. For these and for the product investigated in this thesis, perishability plays a critical role in specification of the problem. In particular, perishable products may not even last long enough to engender a carryover problem. If this is the case, then the essential decision problem of inventory holders will be how fast to dissipate a given initial infusion of inventories over a marketing period.

In the model developed here, in fact, we need a flexible structure capable of dealing with a variable marketing period and hence variable need for carryover. This is the case because the model of regulator/industry interaction developed in previous chapters essentially makes season length endogenous. Hence if exogenous factors are such that the harvest season length turns out to be long, the marketing period will be short and hence carryover will be likely, if any product is stored at all. On the other hand, with a short harvesting season, the corresponding marketing period will be long and inventory holding will be likely. But if the marketing period is very long relative to the product's durability, less carryover will be likely. In the next section, we begin by developing a simple model of optimal inventory dissipation in a single marketing period, assuming that planned carryover
is given parametrically. In later sections, we allow carryover to be endogenously determined and influenced by harvest production period length.

### 5.3 Inventory Behavior With Parametric Carryover

The simplest way to begin is to examine a model in which it is assumed that the level of carryover is given (and possibly zero). Let the marketing period run from period 0 to period $\tau$ and assume that there is some given initial inventory stock level $S(0)$ to be dissipated. The initial level of inventory $S(0)$ will be assumed to consist of an "infusion" of new product $I_{0}$ that comes from the current harvest together with some carryout $C_{0}$ from the previous marketing period. We will use a labeling convention where variables that are taken as parametric at a given time are subscripted and those that are choice variables or state variables will have time in parentheses. We assume that inventory holders take wholesale prices and costs as given and attempt to maximize discounted profits from holding and selling inventory. The dynamic problem is thus:

$$
\begin{align*}
& \max _{q} \int_{0}^{\tau} e^{-r t}[\Pi(q(t), S(t), t)] d t  \tag{5.3.1}\\
& \text { subject to } \quad \begin{aligned}
\dot{S} & =-q \\
S(0) & =C_{0}+I_{0} \\
S(\tau) & =C_{\tau},
\end{aligned}
\end{align*}
$$

where $q(t)$ is sales, $S(t)$ is the stock level, $r$ is the discount factor, 0 and $r$ are the starting and closing dates of the marketing period, and $t$ indexes time. The inventory holder starts with a given stock level, $C_{0}+I_{0}$, and plans to have $C_{\tau}$ at time $\tau$.

We can solve this problem with a variety of dynamic optimization techniques, and we choose to use the Pontyragin conditions of optimal control theory. The Pontryagin conditions are necessary conditions that a path of chosen variables must satisfy for the path
to be optimal. The solution method is to form a current value Hamiltonian function and derive the necessary conditions from the Hamiltonian. For this problem, the current value Hamiltonian and necessary conditions for an interior optimum ${ }^{2}$ are:

$$
\begin{align*}
H(S(t), q(t), \lambda(t), t) & \equiv \Pi(S(t), q(t), t)-\lambda(t) q(t)  \tag{5.3.2}\\
\partial H / \partial q & =0  \tag{5.3.3}\\
-\partial H / \partial S & =\dot{\lambda}-r \lambda  \tag{5.3.4}\\
\partial H / \partial \lambda & =\dot{S} \tag{5.3.5}
\end{align*}
$$

In addition to these conditions, the endpoint conditions must be satisfied: the optimal solution must start with the given initial stock $C_{0}+I_{0}$ and end with the target stock $C_{\tau}$ at the end of the marketing period (time $\tau$ ).

We can characterize the solution by carrying out the differentiation indicated and interpreting the necessary conditions. The first condition (equation(5.3.3)) is restated as follows (where subscripts indicate partial derivatives):

$$
\begin{array}{cl}
H_{q}=0 \Rightarrow & \Pi_{q}-\lambda=0  \tag{5.3.6}\\
\text { or } & \Pi_{q}=\lambda .
\end{array}
$$

This condition states that sales must be chosen so that instantaneous marginal profits are equal to the current shadow value of the stock at every moment of the planning horizon. If a unit is sold, it is no longer part of the stock, so the instantaneous marginal return from selling it $\left(\Pi_{q}\right)$ must compensate for the foregone loss in its value as a unit of stock $(\lambda)$.

The second equation (equation(5.3.4)) is the costate equation, and it describes the evolution of $\lambda$, the current value of a marginal unit of the stock along the optimal inventory

[^8]dissipation path:
\[

$$
\begin{array}{cc}
-H_{S}=\dot{\lambda}-r \lambda \Rightarrow & -\Pi_{S}=\dot{\lambda}-r \lambda .  \tag{5.3.7}\\
\text { or } & \dot{\lambda}=r \lambda-\Pi_{S} .
\end{array}
$$
\]

This condition states that the current marginal value of the stock must grow to cover the cost of holding on to the stock. If the level of the stock has no effect on instantaneous profits (i.e., if $\Pi_{S}=0$ ), then the cost of holding the stock is solely an opportunity cost. The rate of change of $\lambda$ along an optimal inventory path thus will be the discount rate $(\dot{\lambda} / \lambda=r)$. In this case a marginal unit left in inventory must grow in value by a rate reflecting the opportunity cost of holding capital in inventory instead of alternative investments. However, it is plausible to expect that profits are affected by the stock level. In an inventory problem, the effect should be negative because it costs more to have more stock on hand due to the need for more storage space, cooling capacity, and so on. Thus, if $\Pi_{S}<0$, the cost of holding the stock will consist of more than simply the opportunity cost of capital tied up in inventory, and the marginal value of the stock must grow more quickly than the discount rate $\left(\dot{\lambda} / \lambda=r-\Pi_{S} / \lambda\right)$. With $\Pi_{S}<0$, this extra amount reflects the marginal storage costs associated with holding inventory.

Integrating equation (5.3.7) from some date $v$ in the plan yields:

$$
\begin{equation*}
\lambda(v)=\lambda(\tau) e^{-r(\tau-v)}+\int_{v}^{\tau}\left[e^{-r(t-v)} \Pi_{S}(t)\right] d t \tag{5.3.8}
\end{equation*}
$$

This shows that the marginal value of unit of inventory at any time (v) should be equal to the terminal marginal value $(\lambda(\tau))$ discounted to time $v$ less the integral of the discounted marginal cost of holding the stock $\left(\Pi_{S}\right)$. The particular terminal marginal value is as yet unknown but it can solved for using the fixed endpoint conditions. Still, we have the economic insight afforded by this condition. The path of $\lambda$ is determined by holding costs and the opportunity cost, anchored by the shadow value in the terminal period which in turn depends on the endpoint conditions. ${ }^{3}$

[^9]Thus, we have two optimality conditions: one static and the other dynamic. The optimal inventory dissipation plan should reflect, at each date $v$, a tradeoff between selling a marginal unit and leaving it in inventory (5.3.6). If it is left in inventory, its marginal value at some date $v$ should reflect its opportunity cost (what it might earn if sold at some future date $\tau$ ) less the discounted holding costs of carrying it to that date (equation (5.3.8)).

Finally, the third condition returns the equation of motion. The evolution of the inventory stock is determined by the quantity of sales:

$$
\begin{equation*}
H_{\lambda}=\dot{S} \Rightarrow-q=\dot{S} \tag{5.3.9}
\end{equation*}
$$

The integral of this expression from the beginning of the period to the end introduces the endpoint conditions:

$$
\begin{align*}
& \int_{0}^{\tau}[\dot{S}] d t=-\int_{0}^{\tau}[q(t)] d t, \text { or } \\
& S(0)-S(\tau)=\int_{0}^{\tau}[q(t)] d t, \text { or } \\
& C_{0}+I_{0}-C_{\tau}=\int_{0}^{\tau}[q(t)] d t \tag{5.3.10}
\end{align*}
$$

The complete solution to this problem is thus embodied in three equations, (5.3.6), (5.3.7), and (5.3.9), together with endpoint conditions. These three equations can be reduced to a system of two first order differential equations by using the instantaneous marginal profit of sales equation (5.3.6) in the equation of motion (5.3.9). For example, if integration using the endpoint conditions. For example, the differential equations can be specified to be:

$$
\begin{array}{rc}
\dot{\lambda}-r \lambda=-\Pi_{s} & \rightarrow \int\left[\frac{d}{d t} \lambda(t) e^{-r t}\right] e^{r t} d t=\int-\Pi_{s}(t) d t \\
\dot{S}=-q & \rightarrow \int\left[\frac{d}{d t} S(t)\right] d t=\int-q(t) d t .
\end{array}
$$

So that the solution can by solving the following system for the constants of integration ( $c_{1}$ and $c_{2}$ ) using the endpoint conditions ( $S(0)=C_{0}+I_{0}$ and $S(\tau)=C_{\tau}$ )

$$
\begin{array}{r}
\lambda(v)=\int-\Pi_{S}(t) e^{-r t} d t+c_{1} e^{-r t} \\
S(v)=\int-q(t) d t+c_{2} .
\end{array}
$$

Both solution methods yield the same result, but we find the solution method in the text to be more intuitive both here and in the carryover problem in subsequent sections.
$\Pi_{q q} \neq 0$, then by the Implicit Function Theorem, we can solve (5.3.6) for $q$ as a function of the stock, the shadow price, and time, say $q=\phi(S, \lambda, t)$. Inserting $q=\phi(S, \lambda, t)$ in the equation of motion (5.3.9) yields the system:

$$
\begin{align*}
& \dot{S}=-\phi(S, \lambda, t) \\
& \dot{\lambda}=r \lambda-\partial \Pi(S, \phi(S, \lambda, t), t) / \partial S . \tag{5.3.11}
\end{align*}
$$

Alternatively, the instantaneous marginal profit of sales equation (5.3.6) can be differentiated with respect to time, substituting out $\lambda$ using the costate equation (5.3.7), to get:

$$
\begin{align*}
& \dot{q}=f(S, q, t) \\
& \dot{S}=-q . \tag{5.3.12}
\end{align*}
$$

Either system of first order differential equations can be solved with the endpoint conditions $S(0)=C_{0}+I_{0}$ and $S(\tau)=C_{\tau}$ to yield optimal trajectories for the optimal inventory level $S^{*}\left(t ; C_{0}+I_{0}, C_{\tau}, \tau\right)$ or the optimal sales path $q^{*}\left(t ; C_{0}+I_{0}, C_{\tau}, \tau\right)$.

What does the optimal sales profile look like and what determines its qualitative properties generally? For this general model, various possibilities emerge and can be examined as follows. First, we can take the derivative with respect to time of equation (5.3.6):

$$
\Pi_{q}(q(t), S(t), t)=\lambda(t)
$$

Differentiating both sides with respect to time gives:

$$
\begin{equation*}
\Pi_{q q} \dot{q}+\Pi_{q} \dot{S} \dot{S}+\Pi_{q t}=\dot{\lambda} \tag{5.3.13}
\end{equation*}
$$

The time derivative of the shadow value can be substituted out using $\dot{\lambda}=r \lambda-\Pi_{S}$ and $\Pi_{q}=\lambda$ to get:

$$
\Pi_{q q} \dot{q}+\Pi_{q} s \dot{S}+\Pi_{q t}=r \Pi_{q}-\Pi_{S} .
$$

If we assume that the second derivative of the profit function with respect to sales $\left(\Pi_{q q}\right)$ is nonzero, we can rearrange this equation to obtain the time derivative of sales:

$$
\begin{equation*}
\dot{q}=\frac{r \Pi_{q}-\Pi_{S}-\Pi_{q S} \dot{S}-\Pi_{q t}}{\Pi_{q q}} \tag{5.3.14}
\end{equation*}
$$

This gives the rate of change of the optimal sales path. Although the sign of this equation is indeterminate without further specification of the profit function's characteristics, under reasonable assumptions, (5.3.14) is likely to be negative, so optimal sales are likely to decline. We can see this as follows. First, profits should be increasing in sales, so $\Pi_{q}>0$. In addition, storage costs should be positive or zero, but not negative, implying that profits depend negatively or not at all on $S$, so that $\Pi_{S} \leq 0$. Thus, the first term in the numerator is positive and the second term is nonnegative. The cross effects of stock on marginal sales profits $\left(\Pi_{q} S\right.$ ) are likely to be either positive or zero. $\Pi_{q} S$ might be positive if, for example, larger stocks imply lower access costs or if inventory holders sell higher quality stocks first. Finally, it is reasonable to assume that $\Pi$ is concave in $q$. Concavity in $q$ implies $\Pi_{q q} \leq 0$. We have to add the additional requirement that $\Pi_{q q} \neq 0$ to obtain a solution. Thus, the first three terms in the numerator are likely to be positive and the denominator negative. If $\Pi_{q t}$ is negative or zero, $\dot{q}$ is unequivocally negative, and optimal sales decline over time.

If $\Pi_{q t}$ is positive and large it may offset the first three terms and make it optimal to tilt sales towards the future by selling less early and more at the end of the period. $\Pi_{q t}$ will be positive if marginal revenues grow with time which might occur, for example, if prices rise over the marketing period. If this effect is significant enough, it may pay to plan optimal sales so that sales rise. It may even be optimal to postpone sales completely until the later part of the marketing period.

In the relatively general form discussed above, the inventory decision problem is thus capable of predicting a variety of dissipation patterns, depending upon the nature of coats and the anticipated pattern of wholesale prices over the marketing period. In principle, it is possible to numerically solve for the optimal sales path and other variables of interest for almost any general system. In practice, it will be desirable to impose some structure, particularly to carry out empirical work which will necessitate explicit specification of optimal decision variables. In what follows, we explore some exact solutions to
inventory dissipation problems with more explicit structure.

### 5.4 A Linear Quadratic Adjustment Cost Model of Inventory Behavior

A convenient structure for both theoretical and empirical models of dynamic decision making is one in which profits and the state equation are assumed to be either linear or quadratic or some combination. There is a rich literature on these types of problems and they all benefit from the restrictions on form which allow explicit dynamic solutions to be derived. This falls out of the fact that linear quadratic forms themselves produce Pontryagin conditions which are linear in the state and costate variables and hence subject to well known solution methods for simultaneous linear differential equations.

In this section we develop and characterize some explicit solutions for a specification of the inventory problem that is similar to models of investment in the so-called adjustment cost literature. This is a new approach which takes advantage of some of the theoretical insights derived in applications to investment problems. In addition, while limited in generality, these models nevertheless admit a wealth of outcomes with considerable qualitative and quantitative range suitable to econometric estimation. This feature, combined with the possibility of deriving explicit dynamic solutions, makes the framework attractive for our dual purposes of exploring both conceptual and empirical relationships in the marketing sector.

Assume that the profit function is linear/quadratic so that instantaneous sales revenues are a quadratic function of sales. This parallels the adjustment cost literature which assumes that attempting higher rates of investment increases marginal investment costs; here attempting to move larger amounts of inventory reduces marginal profits per unit
sold. This is a reasonable assumption: marginal profits could fall for a variety of reasons including production and shipping bottlenecks, the need to discount to increase sales, etc. We also make the simple assumption that holding costs are linear in the existing inventory level. Assume in this first model that wholesale prices are expected to be constant over the marketing period. Then the inventory decision problem is:

$$
\begin{align*}
& \max _{q} \int_{0}^{\tau}\left[P q-\frac{c}{2} q(t)^{2}-h S(t)\right] d t  \tag{5.4.1}\\
& \text { subject to } \quad \begin{aligned}
\dot{S} & =-q \\
S(0) & =C_{0}+I_{0} \\
S(\tau) & =C_{\tau}
\end{aligned}
\end{align*}
$$

where $q$ (sales) and $S$ (inventory stock) are as defined before, $c$ is the adjustment cost parameter, $h$ is the holding cost parameter, and $P$ is the wholesale price. We ignore discounting in this simple model for analytical convenience. The sales rate, $q$, is under the control of the inventory holder. As in the general model considered earlier, a path of sales is chosen to maximize net revenues, which depend upon both the rate of sales and the level of the stock. The Hamiltonian and necessary conditions for an interior solution to this problem are:

$$
\begin{align*}
H & \equiv P q-\frac{c}{2} q^{2}-h S-\lambda q  \tag{5.4.2}\\
H_{q} & =P-c q-\lambda=0  \tag{5.4.3}\\
-H_{S} & =\dot{\lambda}=h  \tag{5.4.4}\\
H_{\lambda} & =\dot{S}=-q \tag{5.4.5}
\end{align*}
$$

Instantaneous marginal profits, $\Pi_{q}$, are the difference between constant marginal revenue, $P$, and marginal sales cost, cq. Marginal profits are set equal to the marginal shadow value of the stock, $\lambda$. Because there is no discount rate, there is no opportunity cost to holding inventory and so the cost of holding the stock is only $h$, the storage cost.

Equations (5.4.3), (5.4.4), and (5.4.5) can be solved for a path of sales that maximizes net revenues. First, we integrate equation (5.4.4) over the time period $t$ to $\tau$ to yield $\lambda(\tau)-\lambda(t)=h(\tau-t)$. Then, we substitute $\lambda(t)$ into equation (5.4.3) to solve for sales in terms of the (as yet unknown) terminal shadow value, $\lambda(\tau)^{4}$ :

$$
\begin{gather*}
q(s)=\frac{P-\lambda(s)}{c}  \tag{5.4.6}\\
q(s)=\frac{P+h(\tau-s)-\lambda(\tau)}{c} . \tag{5.4.7}
\end{gather*}
$$

Equation (5.4.7) is an expression for the optimal sales path in terms of known parameters of the problem and the shadow value of the stock at the terminal time, $\lambda(\tau)$. To solve for $\lambda(\tau)$, we substitute the sales equation (5.4.7) into the stock constraint (5.3.10), carry out the integration, and solve the resulting equation for $\lambda(\tau)$. This yields:

$$
\begin{equation*}
\lambda(\tau)=P+\frac{c\left(C_{\tau}-\left(C_{0}+I_{0}\right)\right)}{\tau}+\frac{h \tau}{2} . \tag{5.4.8}
\end{equation*}
$$

Finally, we substitute $\lambda(\tau)$ into the sales equation (5.4.7), and obtain the solution:

$$
\begin{equation*}
q^{*}\left(t ; C_{0}+I_{0}, C_{\tau}, \tau, h, c\right)=\frac{\left(C_{0}+I_{0}-C_{\tau}\right)}{\tau}+\frac{h}{2 c} \tau-\frac{h}{c} t . \tag{5.4.9}
\end{equation*}
$$

${ }^{4}$ Alternatively, we can solve:

$$
\begin{array}{cc}
\dot{\lambda}=h & -\lambda(t)=h t+c_{1} \\
\dot{S}=-q & \rightarrow S(t)=-\int q(t) d t+c_{2}
\end{array}
$$

using the endpoint conditions. We know that $q(t)=\frac{P-\lambda(t)}{c}$, so

$$
\begin{array}{r}
S(t)=\int\left[\frac{-P+c_{1}+h t}{c}\right] d t+c_{2} \\
S(0)=C_{0}+I_{0}, S(\tau)=C_{\tau} \\
c_{2}=S_{0}, c_{1}=\frac{c}{\tau}\left[\frac{-h \tau^{2}}{2 c}+\frac{P \tau}{c}+\left(C_{\tau}-C_{0}-I_{0}\right)\right] .
\end{array}
$$

Substituting these constants of integration into $S(t)$ yields:

$$
S(t)=\frac{h}{2 c}\left(t^{2}-t \tau\right)+\left(C_{0}+I_{0}\right)+\frac{\left(C_{\tau}-C_{0}-I_{0}\right) t}{\tau}
$$

as the solution for $S(t)$ and

$$
-\dot{S} \equiv q(t)=\frac{C_{0}+I_{0}-C_{\tau}}{\tau}+\frac{h \tau}{2 c}-\frac{h t}{c}
$$

as the solution for optimal sales.

If storage costs are zero, only the first term remains, so sales are simply a constant fraction of the stock to be dissipated. That fraction is basically the total marketable stock (beginning stock less planned carryover) divided by the number of days in the marketing period. Note, though, that the presence of storage costs shifts sales more toward the present. This is intuitive because more current sales allow the inventory holder to avoid payment of storage costs. In contrast, adjustment costs increase with the rate of sales, so adjustment costs motivate the inventory holder to flatten out sales. Therefore, the degree of concentration of sales towards the beginning of the horizon depends on the relative magnitudes of storage and adjustment costs, $h$ and $c$. Note, however, that with the combination of positive sales and adjustment costs, sales are tilted towards the early part of the sales period. Note also that, for this problem, the output price does not enter into the inventory holder's decisions since, without discounting, the revenue that inventory holders receive does not depend upon the timing of sales. Figure 5.1 contains a diagram of the optimal path of sales and of the stock of inventory. The slope of the sales equation is $-h / c$ : a higher storage cost and a lower adjustment cost steepens the sales function, while a lower storage cost and a higher adjustment cost flattens it. Note also that the optimal inventory level is convex over time, declining from the initial level $C_{0}+I_{0}$ to the planned carryover $C_{\tau}$ by the end of the marketing period.

We can carry out the comparative dynamics of this problem more formally, and the results are presented in Table 5.1. The comparative dynamics show that changing the parameters $h$ and $c$ tilts the sales profile around the midpoint, at time $\frac{\tau}{2}$, where the stock level is $\frac{C_{0}+I_{0}-C_{r}}{\tau}$. The interpretation of these results are discussed above: firms want to avoid storage costs by selling earlier, but the existence of adjustment costs motivate the firm to even out sales over time. Since the sales path is linear and symmetric around its midpoint, a change in the cost ratio rotates the path. Thus whether sales at a particular date rise or fall due to cost changes depends upon whether the date is before or after the


Figure 5.1: Sales and Stocks in a Marketing Period
midpoint. In contrast, changes in the beginning and ending stock shift the entire sales profile. A higher initial stock ( $C_{0}+I_{0}$ ) increases sales at every time because there is more stock to dissipate by the end of the horizon. Similarly, a higher planned ending stock lowers sales at every date since more must remain by the end.

The solution above is written in open loop form, i.e., the problem is solved once over the whole horizon, $[0, \tau]$. This yields the terminal shadow price (5.4.8) expressed in terms of beginning and ending stocks ( $C_{0}+I_{0}$ and $C_{\tau}$ ) and the length of the whole marketing period $(\tau)$. The solution equation defining the optimal sales path (5.4.9) is then simply a function of the date $t \in[0, \tau]$ with all other factors taken as given.

It is also possible to express the solution in closed loop form, i.e., where the optimal sales at any date is expressed as a function of state variables at that date. The procedure is as above, except that the problem is solved over some interval $[t, \tau], t \geq 0$. In this case, the terminal shadow value (5.4.8) is a function of the stock $S_{t}$ at that date, and the remaining time in the horizon, $(\tau-t)$. The problem is:

$$
\begin{align*}
\max _{q} \int_{t}^{\tau}\left[P q-\frac{c}{2} q(s)^{2}\right. & -h S(s)] d s  \tag{5.4.10}\\
\text { subject to } \quad \dot{S} & =-q \\
S(t) & =S_{t} \\
S(\tau) & =C_{\tau} .
\end{align*}
$$

The necessary conditions are the same as before, but the terminal shadow value is expressed in terms of the stock at date $t$ :

$$
\begin{equation*}
\lambda(\tau)=P+\frac{c\left(C_{\tau}-S_{t}\right)}{\tau-t}+\frac{h(\tau-t)}{2} \tag{5.4.11}
\end{equation*}
$$

We substitute this expression for $\lambda(\tau)$ into equation (5.4.7) to solve for sales at any date $s$ between $t$ and $\tau$ :

$$
\begin{equation*}
q^{\prime \prime}\left(s ; t, S_{t}, \tau, C_{\tau}, h, c\right)=\frac{S_{t}-C_{\tau}}{\tau-t}+\frac{h(\tau+t)}{2 c}-\frac{h s}{c} . \tag{5.4.12}
\end{equation*}
$$

| Comparative Dynamic | Expression | sign |
| :---: | :---: | :---: |
|  |  | $>0$ if $t<\frac{\tau}{2}$ |
| $\frac{\partial q^{*}}{\partial h}$ | $\frac{1}{c}\left[\frac{\tau}{2}-t\right]$ | $<0$ if $t=\frac{\tau}{2}$ |
| $<0$ if $t>\frac{\tau}{2}$ |  |  |$|$

Table 5.1: Comparative Dynamics Results for Sales in a Marketing Period

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Then, we evaluate at $s=t$ to find sales at the initial date $t$ :

$$
\begin{equation*}
q^{*}\left(t ; t, S_{\mathfrak{t}}, \tau, C_{\tau}, h, c\right)=\frac{\left(S_{t}-C_{\tau}\right)}{(\tau-t)}+\frac{h}{2 c}(\tau-t) . \tag{5.4.13}
\end{equation*}
$$

Note that the feedback solution is expressed in terms of two state variables, namely the current stock, $S(t)$, and the remaining time, $(\tau-t)$ in the marketing period.. In a deterministic problem, the open loop and closed loop representations give the same solution for a given problem. The optimal sales solution at every "beginning" date $t$ leads to an inherited stock $S_{t+\epsilon}$ at time $t+\epsilon$ so that the optimal sales at time $t+\epsilon$ that is the same as the open loop solution for $s=t+\epsilon$. We will find it convenient to work only with the closed loop solution concept in what follows. ${ }^{5}$

Now we examine how the introduction of discounting modifies the results of our basic model of inventory dissipation. We would expect, of course, that inventory holders would choose a different sales path in the presence of a discount rate. If inventory holders discount future earnings at a rate $r>0$, the problem becomes:

$$
\begin{align*}
& \underset{q}{\max } \int_{t}^{\tau} e^{-r(s-t)}\left[P q(s)-\frac{c}{2} q(s)^{2}-h S(s)\right] d s  \tag{5.4.14}\\
& \text { subject to } \quad \dot{S}=-q \\
& S(t)=S_{\mathrm{t}} \\
& S(\tau)=C_{\tau} .
\end{align*}
$$

We find the solution for this problem by following the same steps as before. The Hamiltonian and necessary conditions are:

$$
\begin{align*}
H & \equiv P q-\frac{c}{2} q^{2}-h S-\lambda q  \tag{5.4.15}\\
H_{q} & =0=P-c q-\lambda \tag{5.4.16}
\end{align*}
$$

[^10]\[

$$
\begin{align*}
-H_{S} & =\dot{\lambda}-r \lambda=h  \tag{5.4.17}\\
H_{\lambda} & =\dot{S}=-q \tag{5.4.18}
\end{align*}
$$
\]

We solve for $\lambda(s)$ in terms of $\lambda(\tau)$ by integrating the costate equation (5.4.17):

$$
\begin{equation*}
\lambda(s)=\lambda(\tau) \epsilon^{-r(\tau-s)}+\frac{h}{r}\left[e^{-r(\tau-s)}-1\right] . \tag{5.4.19}
\end{equation*}
$$

Then, we integrate the stock constraint to solve for $\lambda(\tau)$ :

$$
\begin{equation*}
\lambda(\tau)=\frac{r c\left(C_{\tau}-S_{t}\right)}{1-e^{-r(\tau-t)}}+\frac{\operatorname{Pr}(\tau-t)}{1-e^{-r(\tau-t)}}+\frac{h}{r}\left[\frac{r(\tau-t)}{1-e^{-r(\tau-t)}}-1\right] . \tag{5.4.20}
\end{equation*}
$$

Finally, we find $\lambda(t)$ in terms of $\lambda(\tau)$ and substitute it into the sales equation (equation (5.4.6)) and evaluate at time $t$, to obtain:

$$
\begin{equation*}
q(t)=\left(S_{t}-C_{\tau}\right)\left[\frac{r}{e^{r(\tau-t)}-1}\right]+\frac{h}{c}\left[\frac{1}{r}-\frac{(\tau-t)}{e^{r(\tau-t)}-1}\right]+\frac{P}{c}\left[1-\frac{r(\tau-t)}{e^{\tau(\tau-t)}-1}\right] . \tag{5.4.21}
\end{equation*}
$$

The limit of this expression as $r$ approaches zero is the equation for sales without a discount rate, equation (5.4.13). This is not obvious from inspection, but multiple applications of L'Hôpital's Rule can be used to show the equivalence.

When inventory holders discount future profits, they sell more at the beginning and less at the end than they do in the absence of a discount rate. Inventory holders would rather have current profits than distant profits under discounting, so that if sales do in fact yield positive profits, inventory holders will shift sales towards the present. The output price also becomes relevant to the inventory holder's decisions when a discount rate is introduced, since discounted revenue is no longer invariant to the timing of sales.

Note that the rate at which that sales decline is directly related to the rate at which $\lambda$ grows: $\dot{q}=-\dot{\lambda} / c$. Since $\lambda$ is growing faster with a discount rate than without one in order to cover the opportunity cost, the sales rate slows more quickly with a discount rate than without. We can calculate the rate of change of sales explicitly from the general equation for $\dot{q}$, equation (5.3.14). Instantaneous marginal profits, $\mathrm{H}_{q}$, are equal to $P-c q$ and
the second derivative of the profit function with respect to sales, $\Pi_{q q}$, is $-c$. The marginal profit of holding stock, $\Pi_{S}$, is $-h$. Using this information, we find that without a discount rate $\dot{q}=-h / c$ and with a discount rate, $\dot{q}=-[r(P-c q)+h] / c$. Since marginal revenue $(P-c q=\lambda)$ is positive, the rate of change of sales is more negative with a discount rate than without one. From this expression for $\dot{q}$, we see that a higher output price, storage cost parameter, and discount rate encourage the inventory holder to sell more quickly, while a higher adjustment cost parameter mitigates those effects.

The same amount of stock is sold in either case, and sales decline more quickly with a discount rate than without. These facts together imply that inventory holders sell more stock at the beginning of the period and less at the end when they discount future earnings as intuition would suggest. To see this, note that the slope of the sales function is everywhere steeper with a discount rate. Also, the area under the sales function is the total quantity of stock sold. The implication is that the sales path with a discount rate must cross the no discount rate sales path from above. Figure 5.2 displays the path of sales and stock in the presence of a discount rate. The sales path is steeper, and the stock declines more quickly than without a discount rate.

In summary, the fundamental result that emerges in these two cases is that marginal profits have to rise over time to cover the costs incurred by waiting, here interest charges and storage costs. In the absence of increasing output prices, this function is fulfilled by changes in adjustment costs over the dissipation plan. With higher initial sales, adjustment costs are large but as sales become slower, adjustment costs are reduced. This reduction of marginal cost acts to increase marginal profits over time. It remains to discuss what happens in markets where adjustment costs are unlikely to play such a prevalent role, or where output prices do, in fact, change over time. In the next section, we endogenize prices by assuming a market demand curve and equilibrium behavior on the part of an industry of competitive inventory holders.


Figure 5.2: Sales and Stocks With a Discount Rate

### 5.5 Industry Equilibrium With Inventory Holding

The previous section lays out a relatively rich structure for examining the properties of the path of optimal inventory dissipation that a firm might choose under the conditions specified for profits and costs. An important simplifying assumption is that wholesale prices remain constant over the marketing period. As we showed in the first model with a more general structure, whether profits are shifting over time is important to the determination of the tilt of inventory dissipation. If wholesale prices were allowed to vary with time, of course, profits would shift about and hence we would expect the optimal path to be more complicated.

It is simple in principle to modify the above model of the firm to allow for time varying prices. For example, we could simply assume that the firm expects some price path $P(t)$ which it takes as given, and we could correspondingly modify either the zero discount or positive discount case. This would result in a non-autonomous control problem so that the optimal sales path would depend explicitly on time in more complicated ways than just through the discount factor in equation (5.4.21). This is demonstrated next.

It does not make logical sense, however, to simply posit any arbitrary price path as an equilibrium path, particularly when one moves to a description of the industry. This is obvious because if, for example, every firm expected prices to rise dramatically in the future, then every firm would postpone sales, producing a glut in the future and a shortage in the present. This is clearly not in general a sustainable equilibrium. The sensible way to close the model thus is to posit an industry demand curve for the product sold out of inventory. Then an equilibrium will be sustainable if: (i) each firm taking a price path $P(t)$ as given, makes a present value maximizing inventory path which (ii) results in an aggregate supply path $Q(t)$ which not only (iii) exhausts the total supply to be sold but also (iv) produces an equilibrium price path which is exactly what was expected by all
firms in the first place. This is the basis for the following discussion of a model of optimal inventory decisions arising from an industry of competitive dynamic decision makers. ${ }^{6}$

Assume that $n$ identical firms make sales plans conditioned on an arbitrary path of prices, $P(s)$. Planned optimal supplies out of inventory depend on these price paths, which are taken as parametric to the firm. For simplicity, consider the problem without a discount rate. Individual inventory holders solve the following problem:

$$
\begin{align*}
\max _{q} \int_{t}^{\tau}\left[P(s) q(s)-\frac{c}{2} q(s)^{2}-h S(s)\right] d s  \tag{5.5.1}\\
\text { subject to } \begin{aligned}
\dot{S} & =-q \\
S(t) & =S_{t} \\
S(\tau) & =C_{\tau}
\end{aligned} .
\end{align*}
$$

The difference between this problem and (5.4.10) is that prices are a function of time. The difference in the solution comes in the expression for $\lambda(\tau) . \lambda(\tau)$ now depends on the path of prices, instead of a constant price:

$$
\begin{equation*}
\lambda(\tau)=\frac{c\left(C_{\tau}-S_{t}\right)}{\tau-t}+\frac{h(\tau-t)}{2}+\frac{1}{\tau-t} \int_{t}^{\tau} P(s) d s \tag{5.5.2}
\end{equation*}
$$

The solution for sales in period $t$ is therefore:

$$
\begin{equation*}
q^{*}(t)=\frac{S_{t}-C_{\tau}}{\tau-t}+\frac{h(\tau-t)}{2 c}+\frac{1}{c}\left[P(t)-\frac{1}{\tau-t} \int_{t}^{\tau} P(s) d s\right] . \tag{5.5.3}
\end{equation*}
$$

Note that in comparison with either equation (5.4.13) or equation (5.4.21), individual inventory holder supply decisions are shown here to be a function of the path of future prices as well as other variables. With the assumption that all inventory holders have the same expectations of future prices, we can sum the individual decisions to arrive at a market supply function:

$$
\begin{equation*}
Q(t) \equiv n q(t)=\frac{n\left(S_{t}-C_{\tau}\right)}{\tau-t}+\frac{n h(\tau-t)}{2 c}+\frac{n}{c}\left[P(t)-\frac{1}{\tau-t} \int_{t}^{\tau} P(s) d s\right] . \tag{5.5.4}
\end{equation*}
$$

[^11]Industry supply depends on the total stock of inventory existing in the industry, storage and adjustment costs, and the time left until the end of the marketing period. Also, supply depends critically on the path of future prices. The formulation of price expectations thus will determine the form and location of the supply function. We will examine two plausible expectation formation mechanisms and explore their implications for the industry supply path.

## Myopic Expectations

First, suppose that firms are myopic and so expect that prices will remain fixed at the current level. Then, the third term in equation (5.5.4) vanishes, and industry supply is simply:

$$
\begin{equation*}
Q(t) \equiv n q(t)=\frac{n\left(S_{t}-C_{\tau}\right)}{\tau-t}+\frac{n h(\tau-t)}{2 c} \tag{5.5.5}
\end{equation*}
$$

In this case, industry supply is the same as the sum of the individual decisions derived without a discount rate and with constant prices, i.e., it is equation (5.4.13) times $n$. Note that since supply does not depend on price, the supply curve is vertical, and the level of supply is established as a function solely of available stock, time left in the marketing period, and cost parameters. The time derivative of market supply is $-n h / c .^{7}$
${ }^{7}$ This follows from the derivative of equation (5.5.5) with respect to $t$ :

$$
\dot{Q} \equiv n \dot{q}=\frac{n \dot{S}}{\tau-t}+\frac{n\left(S_{t}-C_{\tau}\right)}{(\tau-t)^{2}}-\frac{n h}{2 c} .
$$

Now, the derivative of the stock at date $t$ is exactly the optimal sales at date $t, q^{*}(t)$ :

$$
\dot{S}=-q^{*}(t)=-\frac{S_{t}-C_{\tau}}{\tau-t}-\frac{h(\tau-t)}{2 c} .
$$

So, substituting this $\dot{S}$ into the $\dot{Q}$ equation, we get:

$$
\begin{aligned}
n \dot{q} & =-\frac{n\left(S_{t}-C_{\tau}\right)}{(\tau-t)^{2}}-\frac{n h(\tau-t)}{2 c(\tau-t)}+\frac{n\left(S_{t}-C_{\tau}\right)}{(\tau-t)^{2}}-\frac{n h}{2 c} \\
& =-\frac{n h}{c} .
\end{aligned}
$$

Note that this is precisely the same as the time derivative of sales derived in an open loop framework. It is the same because we have assumed that the stock changes optimally.


Figure 5.3: Sales with Myopic Price Expectations

The market clearing current price is determined by the intersection of demand and the vertical market supply curve. Now, suppose that demand is a linear function of price, or $Q(s)=\alpha-\beta P(s)$. In inverse demand form, $P(s)=\alpha / \beta-Q(s) / \beta$, or $P(s)=A-B Q(s)$. We can take the time derivative of price to see the actual price path implied by the behavior associated with myopic price expectations. Since $\dot{Q}=-n h / c, \dot{P}=n h / c \beta$. Thus, even though individual firms assume that prices will remain fixed, or that $\dot{P}=0$, prices will actually rise at a positive rate. The only case where prices will not rise is if demand is infinitely elastic, or $\beta=\infty$. Then, the expected and actual price paths will coincide because firms will be correct in their utilization of a myopic expectations framework.

We illustrate this in Figure 5.3. The vertical supply function is implied by myopic expectations. Sales will shrink over time (from $Q(t)$ to $Q(v)$ ), which will give rise to increasing prices (from $P(t)$ to $P(v)$ ). If the demand curve is flat, prices will not rise.

## Perfect Foresight

At the other end of the spectrum, firms might forecast prices correctly. For this to be true, the series of optimal individual supply decisions must aggregate to a series of market supply functions which, in concert with demand, will generate the set of prices that firms expect. This is the concept of rational competitive market equilibrium that is used in the rational expectations literature. We derive the equilibrium sequence of prices by imposing the condition that market demand always equals market supply.

The inverse demand form for the market demand function is $P(s)=A-B Q(s)$. Under perfect foresight, we can express the integral of future prices in equation (5.5.4) as a function of future quantities sold:

$$
\begin{aligned}
\int_{t}^{\tau} P(s) d s & =\int_{t}^{\tau}[A-B Q(s)] d s \\
& =\int_{t}^{\tau} A d s-B \int_{t}^{\tau} Q(s) d s \\
& =A(\tau-t)-B \int_{t}^{\tau} Q(s) d s
\end{aligned}
$$

We can then restrict the sequence of equilibrium future prices to be those that clear the market by substituting in the demand function for the prices. Write current supply as a function of the current price and the integral of future sales:

$$
\begin{align*}
Q(t) \equiv n q(t)= & \frac{n\left(S_{t}-C_{\tau}\right)}{\tau-t}+\frac{n h(\tau-t)}{2 c}+  \tag{5.5.6}\\
& \frac{n}{c}\left[P(t)-\frac{1}{\tau-t}\left(A(\tau-t)-B \int_{t}^{\tau} Q(s) d s\right)\right] .
\end{align*}
$$

But, the integral of future sales is amount of stock left to be sold in the industry $\left(\int_{t}^{\tau} Q(s) d s \equiv\right.$ $n \int_{t}^{\tau} q(s) d s=n\left(S_{t}-C_{\tau}\right)$ ), so that the equation (5.5.6) can be expressed in terms of the current stock:

$$
\begin{equation*}
Q(t)=\frac{n\left(S_{t}-C_{\tau}\right)}{\tau-t}+\frac{n h(\tau-t)}{2 c}+\frac{n}{c}\left[P(t)-A+\frac{B}{\tau-t} n\left(S_{t}-C_{\tau}\right)\right] . \tag{5.5.7}
\end{equation*}
$$

Now, we can express current supply in price dependent form,

$$
P(t)=A-\frac{(n B+c)\left(S_{t}-C_{\tau}\right)}{\tau-t}-\frac{h(\tau-t)}{2}+\frac{c Q(t)}{n},
$$

and current demand can be equated to current supply to yield the current market equilibrium quantity ${ }^{8}$ :

$$
\begin{align*}
A-B Q(t) & =A-\frac{(n B+c)\left(S_{t}-C_{\tau}\right)}{\tau-t}-\frac{h(\tau-t)}{2}+\frac{c Q(t)}{n}  \tag{5.5.9}\\
& \Rightarrow Q(t)=\frac{n\left(S_{t}-C_{\tau}\right)}{\tau-t}+\frac{n h(\tau-t)}{2(n B+c)} \tag{5.5.10}
\end{align*}
$$

Now, we take the derivative of the equilibrium market quantity with respect to time to find the rate of change of sales as well as the rate at which prices rise.

$$
\begin{equation*}
\dot{Q}=\frac{n \dot{S}_{t}}{\tau-t}+\frac{n\left(S_{t}-C_{\tau}\right)}{(\tau-t)^{2}}-\frac{n h}{2(n B+c)} . \tag{5.5.11}
\end{equation*}
$$

Recall that, by definition, $n \dot{S}_{t}=-Q(t)$, so substituting the definition for $Q(t)$ found in equation (5.5.10), we find:

$$
\begin{aligned}
& \dot{Q}=-\frac{n\left(S_{t}-C_{\tau}\right)}{(\tau-t)^{2}}-\frac{n h}{2(n B+c)}+\frac{n\left(S_{t}-C_{\tau}\right)}{(\tau-t)^{2}}-\frac{n h}{2(n B+c)} \text {, or } \\
& \dot{Q}=\frac{-n h}{n B+c}=\frac{-n h}{c+n / \beta} .
\end{aligned}
$$

Therefore, since the demand function is $P(s)=A-B Q(s)$ and $\dot{P}=-B \dot{Q}$, the derivative of price with respect to time is:

$$
\begin{equation*}
\dot{P}=\frac{n h B}{n B+c}=\frac{n h}{n+c \beta} . \tag{5.5.12}
\end{equation*}
$$

Note that in comparison with the myopic expectations case, prices rise more slowly if firms perfectly anticipate prices and if the demand function is not perfectly elastic (if $\beta>0$ ). Since firms know that prices will rise, they delay sales to take advantage of higher prices

[^12]later. This phenomenon acts to tilt current sales towards the future and drive the initial price above what it would be under myopic expectations. Individual firms' sales decisions, in the aggregate, cause their expectations to be borne out. Any other expected price path will cause the individual firms decisions to give rise to a price path that diverges from what they expect.

In summary, the previous three sections explore a conceptual model of inventory holding behavior appropriate for analyzing an industry that is marketing a semiperishable commodity. The modeling approach utilizes some simplifying assumptions which are important to clarifying how various parameters affect the optimal sales and inventory paths. ${ }^{9}$ First, a general model is examined with minimal explicit structure to show how the shape of the optimal dissipation path depends upon the respective roles of sales, the inventory level, and exogenous time varying factors in affecting instantaneous profits. Then, an explicit model similar to optimal investment models in the adjustment cost literature is utilized to derive exact dynamic solution paths. Several variants are developed, beginning with the simplest which assumes fixed wholesale prices and no discounting. Simple closed and open loop forms for the optimal sales paths and inventory levels suitable for estimation can be developed. Two generalizations add discounting and time varying prices. The last generalization is used as the basis to discuss industry equilibrium in a setting of identical competitive firms. It is demonstrated that the nature of the equilibrium depends upon the wholesale market demand function and on the expectations mechanisms utilized by individual firms. A rational expectations solution can be developed (again suitable for estimation), which describes industry equilibrium time paths for sales, the inventory level, and the time path of wholesale prices.

[^13]
### 5.6 Carryover

The models developed above all share a common characteristic: the assumption that carryover and carryin are parametric. In markets where the commodity is perishable relative to the length of the marketing period, there may be no carryover and hence these models could be directly used, simply by setting $C_{0}$ and $C_{\tau}$ equal to zero. If the marketing period is short and/or the product is more durable, carryover will be likely positive and endogenous rather than parametric. In this section we generalize our model of optimal inventory dissipation by including optimal carryover as part of the basic decision problem. As done above, we move to the general formulation by first exploring a series of simplified problems and their implications.

A diagram of the general problem as well as possible optimal sales paths is presented in Figure 5.4 to introduce notation and to set the stage for the discussions that follow. For simplicity in this diagram assume that the length of the production period is negligible so that time can be expressed in multiples of $\tau$, defined as the length of the marketing period. The first marketing period starts at date 0 and ends at date $\tau$, the second starts at $\tau$ and continues to $2 \tau$, and so on. At harvest time, inventories are augmented by an infusion of stock ( $I$ ). The amount of stock ( $S$ ) at the beginning of any period $(S(i \tau)$ ) is the sum of the infusion $\left(I_{i \tau}\right)$ and the quantity carried over from the previous period ( $C(i \tau)$ ). Sales ( $q$ ) are simply sales out of the stock of inventory, and so as the commodity is sold, the quantity of stock on hand is depleted. The path of the stock is depicted as $S^{\prime \prime}(t)$, where $t$ is the time index.

There are several sales paths shown here. In one, shown in marketing period 1 , there is no carryover at the end of period 1 into period 2 , and sales take place continuously until the end of the marketing period (until $2 \tau$ ). In another, period 2 , there is no carryover from period 2 to period 3 and the inventory holder chooses to sell out before the end of the


Figure 5.4: Overview of Carryover
period (at time $\bar{t}$ ). A more typical case is shown in period 0 , where there are sales over the entire marketing period and some stock is carried over.

Optimal control theory is particularly suitable to generalizing to include carryover through the use of transversality (or endpoint) conditions. In particular, the optimal level of carryover can be determined by allowing the terminal value of the inventory stock to be a choice variable, similar to what might be done, for example, in a capital problem with a scrap value.

Recall that the transversality condition for a scrap value problem is that the terminal shadow value of the chosen ending stock is equal to its marginal scrap value. If the terminal shadow value is always greater than the marginal scrap value, the optimal choice would be to carry no stock beyond into the last period. The terminal shadow value of a unit of carryover to the current marketing period (period 0 ) is $\lambda_{0}(\tau)$, or its foregone value in the period 0 marketing plan. On the other hand, the marginal scrap value would be
the marginal value of the stock in the subsequent period, $\lambda_{1}(\tau)$. The value of the stock in the subsequent period will depend on the economic conditions that are expected to unfold then; the optimal sales plan in the next period, given the output price and the level of stock infusion, will imply various values for additions to the starting level of the stock. Once we derive the optimal sales plan of the next period, we can infer the shadow value of an extra unit of carryover (or carryin) stock.

Equivalently, we can think of a (shadow) market for carryover stock. The supply function is the marginal cost schedule of carryout, or the marginal cost (to the current period) of not selling the stock in the current period. This marginal cost is its foregone marginal value, or $\lambda_{0}(\tau)$. The demand function is the marginal benefit of carryin to the next period, or $\lambda_{1}(\tau)$. In equilibrium, the optimal carryover from one period to the next will be such that the marginal cost to the period sending carryout will just be balanced by the marginal benefit of the period receiving carryin.

To illustrate the principles involved, consider the simple problem where there are only two marketing intervals over which to plan and wholesale prices are constant, but perhaps different, within each marketing interval. The inventory holder must decide in the first marketing period how much of the initial stock to allocate to the second interval. This choice of the carryover stock will then influence sales decisions in the next or second marketing period.

The objective function is the same as the one in the first model, (5.4.1), except that the condition that the firm must reach a given terminal stock at the end of the current marketing period is replaced by the transversality condition for a free ending stock. Prices may differ between the two periods. The problem for the first marketing period is:

$$
\begin{equation*}
\max _{q} \int_{t}^{\tau}\left[P_{0} q(s)-\frac{c}{2} q(s)^{2}-h S(s)\right] d s \tag{5.6.1}
\end{equation*}
$$

$$
\begin{array}{ll}
\text { subject to } & \dot{S}=-q \\
& S(t)=S_{t} \\
& C(\tau) \geq 0
\end{array}
$$

The transversality conditions imply:

$$
\begin{gathered}
{\left[\lambda_{0}(\tau)-\lambda_{1}(\tau)\right] C(\tau)=0} \\
\lambda_{0}(\tau)-\lambda_{1}(\tau) \geq 0, C(\tau) \geq 0
\end{gathered}
$$

To solve and characterize the solution to this problem, first integrate the optimal sales equation and set it equal to the stock constraint to find $\lambda_{0}(\tau)$ :

$$
\begin{align*}
S_{t}-C(\tau) & =\int_{t}^{\tau} q(s) d s=\int_{t}^{\tau} \frac{1}{c}\left[P_{0}-\lambda_{0}(s)\right] d s  \tag{5.6.2}\\
& =\int_{t}^{\tau} \frac{1}{c}\left[P_{0}-\lambda_{0}(\tau)+h(\tau-s)\right] d s \\
\Rightarrow \lambda_{0}(\tau) & =P_{0}+\frac{h(\tau-t)}{2}+\frac{c\left[C(\tau)-S_{t}\right]}{\tau-t} \tag{5.6.3}
\end{align*}
$$

This is the marginal cost to the first marketing period's program, (evaluated at some date $t$ ) of the last unit carried out $C(\tau)$.

To solve for the optimal carryover we need to next find the marginal value of the carryover into the following period, $\lambda_{1}(\tau)$. We can solve for $\lambda_{1}(\tau)$ with the second period's problem, which is:

$$
\begin{gather*}
\underset{q}{\max } \int_{\tau}^{2 \tau}\left[P_{1} q(s)-\frac{c}{2} q(s)^{2}-h S(s)\right] d s  \tag{5.6.4}\\
\text { subject to } \dot{S}=-q \\
\\
C(2 \tau)=C_{2 \tau} \\
\\
S(\tau)=C_{\tau}+I_{\tau}
\end{gather*}
$$

From the perspective of the second period, first assume that this carryin stock, $C_{\tau}$, is given. Then we can solve for the shadow value of extra units of carryin stock. We solve for the
initial shadow value by integrating $\dot{\lambda}$ from the beginning of the second marketing period to an arbitrary time in the second period (from $\tau$ to $s$ ):

$$
\lambda(s)-\lambda_{1}(\tau)=h(s-\tau) .
$$

We integrate the state equation and set it equal to the second marketing period stock constraint:

$$
\begin{align*}
C_{\tau}+I_{\tau}-C_{2 \tau} & =\int_{\tau}^{2 \tau} q(s) d s=\int_{\tau}^{2 \tau} \frac{1}{c}\left[P_{1}-\lambda(s)\right] d s  \tag{5.6.5}\\
& =\int_{\tau}^{2 \tau} \frac{1}{c}\left[P_{1}-\left(\lambda_{1}(\tau)+h(s-\tau)\right)\right] d s \\
\lambda_{1}(\tau) & =P_{1}-\frac{h \tau}{2}+\frac{c\left[C_{2 \tau}-C_{\tau}-I_{\tau}\right]}{\tau} . \tag{5.6.6}
\end{align*}
$$

This is the value (from the perspective of date $\tau$ ) of an extra unit of beginning stock (or carryin), assuming that the subsequent sales plan is optimized. The determination of optimal carryover is completed by noting that $C(\tau)$ must be chosen so that the marginal cost (to the first marketing period) of having to carry out stock is equal to the marginal value of this carryin to the second period. $C(\tau)$ may thus be determined by setting equation (5.6.3) equal to equation ( $\overline{5} .6 .6$ ) and retrieving the corresponding carryout. This leaves us with:

$$
\begin{equation*}
C(\tau)=\frac{P_{1}-P_{0}}{c}\left[\frac{(\tau-t) \tau}{2 \tau-t}\right]-\frac{h \tau(\tau-t)}{2 c}+\frac{\left(C_{2 \tau}-I_{\tau}\right)(\tau-t)+S_{t} \tau}{2 \tau-t} \tag{5.6.7}
\end{equation*}
$$

This is a closed loop form (since it depends upon $S_{t}$ and $\tau-t$ ) and we can simplify the expression for carryover if we view the decision from the perspective of the beginning of the first period. We evaluate this expression at $t=0$ so that the carryover stock is

$$
\begin{equation*}
C(\tau)=\frac{\left(P_{1}-P_{0}\right) \tau}{2 c}-\frac{h \tau^{2}}{2 c}+\frac{C_{2 \tau}-I_{\tau}+\left(I_{0}+C_{0}\right)}{2} . \tag{5.6.8}
\end{equation*}
$$

Note that the optimal carryover depends in an important way on the price differential between the two marketing periods. The larger the positive difference between future and
current prices, the larger will be the optimal carryover. In addition, carryover depends positively on the beginning inventory ( $I_{0}+C_{0}$ ) and planned end of period stock ( $C_{2 \tau}$ ) and negatively on the expected infusion between marketing periods. These and remaining comparative statics derivatives are reported in Table 5.2.

We can summarize the intuition underlying these determinants of carryover by examining the shadow market for carryover in Figure 5.5. The above determinants act through either the marginal cost or marginal benefit curves (or both) to determine carryover. For example, an increase in the first marketing period wholesale price shifts the marginal cost curve upwards, reducing carryover. Similarly, an increase in holding costs shifts both the marginal cost of carryout upwards and the marginal benefit of carryin downwards, reinforcing effects which reduce carryover. A change in the adjustment cost parameter affects both the intercepts and slopes of both equations, resulting in an ambiguous overall impact.

The above solution for the optimal carryover thus embodies the factors that we might expect to determine carryover and in a manner that accords with intuition. This solution may then be embedded back into the problem associated with maximizing the profits from the sales flow during the marketing period. This is done by substituting the solution to the carryover problem (equation (5.6.7)) into the equation determining the optimal end of period shadow price (equation (5.6.3)). Then, we recall that sales are equal to $q(t)=\left(P_{0}-\lambda(t)\right) / c$, and the current shadow value is equal to $\lambda(t)=\lambda\left(\tau_{0}\right)+h\left(t-\tau_{0}\right)$. Therefore, sales can be written $q(t)=\left(P_{0}-\lambda\left(\tau_{0}\right)+h\left(\tau_{0}-t\right)\right) / c$. Performing the substitution of $\lambda\left(\tau_{0}\right)$ into this equation yields:

$$
\begin{equation*}
q(t)=\frac{\left(P_{0}-P_{1}\right) \tau}{c(2 \tau-t)}+\frac{S_{t}+I_{\tau}-C_{2 \tau}}{2 \tau-t}+\frac{h(2 \tau-t)}{2 c} . \tag{5.6.9}
\end{equation*}
$$

Note that this can be compared with the feedback form (5.4.13) for the simpler problem with carryover parametric. That problem (with no discounting) showed the optimal sales

|  |  | sign |
| :---: | :---: | :---: |
| $\frac{\partial C^{*}(\tau)}{\partial P_{1}}$ | $\frac{\tau}{2 c}$ | + |
| $\frac{\partial C^{*}(\tau)}{\partial P_{0}}$ | $-\frac{\tau}{2 c}$ | - |
| $\frac{\partial C^{*}(\tau)}{\partial h}$ | $-\frac{\tau^{2}}{2 c}$ | - |
| $\frac{\partial C^{*}(\tau)}{\partial c}$ | $\frac{\left(P_{0}-P_{1}\right) \tau+h \tau^{2}}{2 c^{2}}$ | $=0$ if $P_{1}=P_{0}+h \tau$ |
| $\frac{\partial C^{*}(\tau)}{\partial P_{1}}$ | $\frac{\tau}{2 c}$ | +0 if $P_{1}>P_{0}+h \tau$ |
| $\frac{\partial C^{*}(\tau)}{\partial I_{\tau}}$ | $-\frac{1}{2}$ | - |
| $\frac{\partial C^{*}(\tau)}{\partial I_{0}}, \frac{\partial C^{*}(\tau)}{\partial C_{0}}, \frac{\partial C^{*}(\tau)}{\partial C_{2 \tau}}$ | $\frac{1}{2}$ | + |

Table 5.2: Comparative Statics Results for Carryover between Two Marketing Periods


Carryover Stock at time $\tau$ : from period 0 to 1

Figure 5.5: Shadow Values of Carryover
path to be a simple variant of one that divides current stock by the remaining time, modified by a term involving the ratio of holding cost to adjustment cost and time remaining in the marketing period. The absence of discounting and the assumed constant prices held revenues constant regardless of the sales path and hence the price level did not appear in the sales solution. This problem, in contrast, reveals explicitly that current sales patterns depend upon conditions expected in future marketing periods such as future wholesale prices, the infusion expected due to harvest between periods, and the future targeted carryout. In particular, if prices are unequal between marketing periods, the optimal sales profile in the first period will be tilted to produce more (or less) carryover, depending on relative prices.

Figure 5.5 also illustrates conditions that might lead to a corner solution with zero carryover. Generally, if the marginal cost of carryout is everywhere above the marginal benefit of carryin, the optimal carryover from the first marketing period to the second will be zero. This might occur, for example, if the wholesale price in the first period is higher than that in the second period and if the current stock were low. In this case inventory holders would want to market all of their available stock in the current period since the opportunity costs would be too high to warrant carryover. If parameters are configured such that the optimal choice is to carry no stock into the second period, the solution for sales is:

$$
\begin{equation*}
q(t)=\frac{S_{t}}{\tau-t}+\frac{h}{2 c}(\tau-t) \tag{5.6.10}
\end{equation*}
$$

It should be noted that current sales still depend upon future conditions, though implicitly through the transversality condition which dictates a corner solution.

Finally, it should be pointed out that the comparative statics properties may depend in interesting ways on when a prospective change occurs in the marketing period. Table 5.2 comparative statics are computed from the perspective of the beginning of the first marketing period. However, we might also imagine an unforeseen change in a parameter that occurs somewhere in the middle of the first marketing period. For example, suppose that
the expected level of infusion that is expected from the upcoming harvest is revised. If more infusion is expected, we would anticipate less carryover and consequently an acceleration of sales in the current marketing period. However, with less time to adjust, there should be less responsiveness. This can be seen by using equation (5.6.7) which shows the functional dependence of the carryover decision on the inherited stock and on time $t$. Assuming an interior solution, the derivative of the carryover with respect to the infusion is:

$$
\begin{equation*}
\frac{d C(\tau)}{d I_{\tau}}=\frac{-(\tau-t)}{2 \tau-t} \tag{5.6.11}
\end{equation*}
$$

As this shows, carryover is negatively related to the infusion. Interestingly, the magnitude of the response depends upon the timing of the parametric change. If the change in expected infusion occurs at the beginning of the marketing period, that is, when $t=0$, the change in carryover will be $-1 / 2$ per unit infusion change. The closer the time is to the end of the marketing period, the smaller the response will be. For example, as $t$ approaches the end of the marketing period, there will be no revision of carryover.

It is straightforward to extend this model to multiple periods and examine carryover in a long horizon setting with multiple infusions and multiple marketing period prices. It is easiest to see how to proceed by considering how results change with the addition of a third period and then generalizing to many periods by induction. Thus suppose that there are three marketing periods denoted 0,1 , and 2 . The transversality condition that pins down the amount of carryover from period 0 to period 1 is that $\lambda_{0}(\tau)=\lambda_{1}(\tau)$, or the marginal cost of carryout to period 0 equals the marginal benefit of carryin to period 1. The marginal benefit of a unit of carryin stock to period 1 depends on the carryover from period 1 to period $2(C(2 \tau)$ ), which we assume is endogenous in this three period case. Therefore, the optimal carryover from period 1 to 2 must be embedded into the expression for marginal benefit $\left(\lambda_{1}(\tau)\right)$ for it to correctly reflect the marginal benefit of carryover from period 0 to 1 . Similarly, the marginal cost of carrying a unit of stock out of period $1\left(M C(1)=\lambda_{1}(2 \tau)\right)$ depends on the carryin to that period, $(C(\tau))$. The optimal solution
for $C(\tau)$ can be embedded into marginal cost of carryout in period $1, \lambda_{1}(2 \tau)$, to correctly reflect the effect on period 0 of carryover from period 1 to 2 .

It is straightforward to solve for the shadow values in terms of any date $t$ in the initial marketing period, but for ease of presentation, we will solve them from the perspective of the beginning of the horizon, time 0 . First, we solve for the carryover between periods 1 and 2 in terms of the conditions in those periods. Then, we will embed that solution into the marginal benefit equation for a unit of stock at the beginning of period 1 . Second, we solve for the carryover between periods 0 and 1 in terms of the conditions in periods 0 and 1. Then, we will embed that solution into the marginal cost equation for a unit of stock carried out at the end of period 1 . We will then obtain the complete optimal solution for both carryover decisions.

For the carryover between periods 1 and 2 , the marginal condition is that marginal $\operatorname{cost}\left(\lambda_{1}(2 \tau)\right)$ is equal to marginal benefit $\left(\lambda_{2}(2 \tau)\right)$, or:

$$
P_{1}+\frac{h \tau}{2}+\frac{c\left[C(2 \tau)-\left(C_{\tau}+I_{\tau}\right)\right]}{\tau}=P_{2}-\frac{h \tau}{2}+\frac{c\left[C_{3 \tau}-I_{2 \tau}-C(2 \tau)\right]}{\tau} .
$$

From this equality, carryover is expressed in terms of the conditions in periods 1 and 2:

$$
C(2 \tau)=\frac{\left(P_{2}-P_{1}\right) \tau}{2 c}-\frac{h \tau^{2}}{2 c}+\frac{C_{3 \tau}-I_{2 \tau}+\left(I_{\tau}+C_{\tau}\right)}{2} .
$$

The marginal benefit of carryin to period 1 is:

$$
\begin{equation*}
\lambda_{1}(\tau)=P_{1}-\frac{h \tau}{2}+\frac{c\left[C_{2 \tau}-I_{\tau}-C(\tau)\right]}{\tau} \tag{5.6.12}
\end{equation*}
$$

and substituting in the optimal $C(2 \tau)$ for $C_{2 \tau}$ gives:

$$
\begin{equation*}
M B(1)=\lambda_{1}(\tau)=\frac{P_{1}+P_{2}}{2}-h \tau+\frac{c\left[C_{3 \tau}-I_{2 \tau}-I_{\tau}-C(\tau)\right]}{2 \tau} \tag{5.6.13}
\end{equation*}
$$

Now we equate this modified marginal benefit to the marginal cost of carryout from period 0 , that is $\lambda_{0}(\tau)=\lambda_{1}(\tau)$, hence

$$
M C(0)=\lambda_{0}(\tau)=P_{0}+\frac{h \tau}{2}+\frac{c\left[C(\tau)-\left(I_{0}+C_{0}\right)\right]}{\tau}
$$

$$
=\frac{P_{1}+P_{2}}{2}-h \tau+\frac{c\left[C_{3 \tau}-I_{2 \tau}-I_{\tau}-C(\tau)\right]}{2 \tau}=\lambda_{1}(\tau)=M B(1) .
$$

The optimal carryover from period 0 to 1 is found by solving this last equation for $C(\tau)$ :

$$
\begin{equation*}
C(\tau)=\frac{\left(-2 P_{0}+P_{1}+P_{2}\right) \tau}{3 c}-\frac{h \tau^{2}}{c}+\frac{2\left(I_{0}+C_{0}\right)-I_{\tau}-I_{2 \tau}+C_{3 \tau}}{3} . \tag{5.6.14}
\end{equation*}
$$

Now we can find the marginal cost to period 1 of carryout. We embed the expression for carryover from period 0 and 1 in terms of conditions in those periods into the marginal cost of carryout (from period 1) equation:

$$
\begin{equation*}
M C(1)=\lambda_{1}(2 \tau)=\frac{P_{1}+P_{2}}{2}+h \tau+\frac{c\left[C(2 \tau)-I_{\tau}-\left(I_{0}+C_{0}\right)\right.}{2 \tau} \tag{5.6.15}
\end{equation*}
$$

which, when equated to the marginal benefit of carryin to period 2

$$
\begin{equation*}
M B(2)=\lambda_{2}(2 \tau)=P_{2}-\frac{h \tau}{2}+\frac{c\left[C_{3 \tau}-I_{2 \tau}-C(2 \tau)\right]}{\tau} \tag{5.6.16}
\end{equation*}
$$

yields optimal carryover from periods 1 to 2 in terms of all the exogenous variables:

$$
\begin{equation*}
C(2 \tau)=\frac{\tau\left(-P_{0}-P_{1}+2 P_{2}\right)}{3 c}-\frac{h \tau^{2}}{c}+\frac{\left(I_{0}+C_{0}\right)+I_{\tau}-2 I_{2 \tau}+2 C_{3 \tau}}{3} \tag{5.6.17}
\end{equation*}
$$

Equations (5.6.14) and (5.6.17) thus yield reduced form expressions for the optimal carryover between the two periods. The comparative statics of the three period model are derived from these two reduced form equations. The results are summarized in Table 5.3.

Intuitively, these results are due to shifts in the marginal benefit and cost functions as they were in the two period case. Now, though, the marginal cost and benefit functions depend on parameters in all three periods. For example, we can look at the effects of price changes on optimal carryover using Figures 5.6 and 5.7. As in the case of only two periods, price and stock parameters enter into the intercept terms. Storage and adjustment cost parameters are in both the intercepts and the slopes of the marginal cost and benefit schedules. We show the effect of an increase in the price in the second period in Figure 5.6. When the price in the second period rises, the marginal benefits of carryover rise in

|  | $d C^{*}(\tau)$ | sign | $d C^{*}(2 \tau)$ | sign |
| :---: | :---: | :---: | :---: | :---: |
| $P_{0}$ | $-\frac{2 r}{3 c}$ | - | $-\frac{\tau}{3 c}$ | - |
| $P_{1}$ | $\frac{\tau}{3 c}$ | + | $-\frac{\tau}{3 c}$ | - |
| $P_{2}$ | $\frac{\tau}{3 c}$ | + | $\frac{2 \tau}{3 c}$ | + |
| $I_{0}+C_{0}$ | $\frac{2}{3}$ | + | $\frac{1}{3}$ | + |
| $I_{\tau}$ | $-\frac{1}{3}$ | - | $\frac{1}{3}$ | + |
| $I_{2 \tau}$ | $-\frac{1}{3}$ | - | $-\frac{2}{3}$ | - |
| $C_{3 \tau}$ | $\frac{1}{3}$ | + | $\frac{2}{3}$ | + |
| $h$ | $-\frac{\tau^{2}}{c}$ | - | $-\frac{\tau^{2}}{c}$ | - |

Table 5.3: Comparative Dynamics Results for Carryover with Three Marketing Periods


Figure 5.6: Carryover with a Price Increase in the Final Period
both marketing periods 1 and 2. Intuitively, the marginal benefit of carryin to period 1 rises because the marginal benefit of carryout from period 1 rises. A higher carryin level to period 2 is induced because of the direct effect of higher prices. Since the marginal costs of carryover do not change, both carryovers increase and inventories are shifted out of both periods 0 and 1 into period 2 . One half of the new stock in period 2 comes from period 1 and one half comes from period 0 , as the carryover from period 0 to 1 the amount from 1 to 2 .

We show, in contrast, the case where the price in period 1 rises in Figure 5.7. Then, the marginal benefit of carryin to period 1 rises as does the marginal cost of carrying out of period 1. As a consequence, there is more carryover from period 0 to 1 and a reduction in carryover from period 1 to 2 than before the change. In this case, inventories are shifted into the middle period from the two adjacent periods. Thus, we can see that a change


Figure 5.7: Carryover with a Price Increase in the Middle Period
in a parameter in one period affects sales decisions in distant periods, both forward and backward in time. The effects are also muted with time. It is easy to see this characteristic with Figure 5.6, where carryout from period 0 changes, but not by as much as carryout from period 1. The pervasive but diminishing effects of parameter changes are a general feature of this multiple period model with carryover, which we can demonstrate by moving to an $N$ period model.

The general $N$ period model can be constructed in a matrix representation by reproducing the above results for carryover stocks in terms of conditions in adjacent periods. In general and in any marketing period $v$, carryover is a function of the carryin to the period before and the carryout from the period after:

$$
C(v \tau)=\frac{\tau\left(P_{v}-P_{v-1}\right)}{2 c}-\frac{h \tau^{2}}{2 c}+\frac{C_{(v-1) \tau}+I_{(v-1) \tau}-I_{v \tau}+C_{(v+1) \tau}}{2} .
$$

We construct a matrix from a series of $N$ carryover decisions. Ultimately, the carryover
decision must be conditioned on some given initial stock, $S_{t}$ and a given terminal stock in period $N+1, C_{(N+1) r}$. Here again, we assume that interior solutions are optimal. The carryover decisions in matrix form are:

$$
\begin{align*}
& \underbrace{\left[\begin{array}{l}
C(\tau) \\
C(2 \tau) \\
\vdots \\
C((N-1) \tau) \\
C(N \tau)
\end{array}\right]}_{\mathrm{C}: N \times 1}=\frac{\tau}{c} \underbrace{\left[\begin{array}{rrrrr}
-\frac{(\tau-t)}{2 \tau-t} & \frac{(\tau-t)}{2 \tau-t} & 0 & \cdots & 0 \\
0 & -1 / 2 & 1 / 2 & \cdots & 0 \\
\vdots & & \vdots & & \vdots \\
0 & \cdots & -1 / 2 & 1 / 2 & 0 \\
0 & \cdots & 0 & -1 / 2 & 1 / 2
\end{array}\right]}_{\mathrm{A}: N \times(N+1)} \underbrace{\left[\begin{array}{l}
P_{0} \\
P_{N} \\
P_{N-1} \\
\vdots \\
P_{N-1}
\end{array}\right]}_{\mathrm{P}:(N+1) \times 1}  \tag{5.6.18}\\
& -\frac{h \tau}{c} \underbrace{\left[\begin{array}{l}
(\tau-t) \\
\tau \\
\vdots \\
\tau \\
\tau
\end{array}\right]}_{\mathrm{B}: N \times 1} \underbrace{\left[\begin{array}{rrrrr}
-\frac{(\tau-t)}{2 \tau-t} & 0 & 0 & \cdots & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & \cdots & 0 \\
\vdots & & \vdots & & \vdots \\
& & & & \\
0 & \cdots & \frac{1}{2} & -\frac{1}{2} & 0 \\
0 & \cdots & 0 & \frac{1}{2} & -\frac{1}{2}
\end{array}\right]}_{\mathrm{F}: N \times(N+1)} \underbrace{\left[\begin{array}{l}
I_{\tau} \\
I_{2 \tau} \\
\vdots \\
I_{(N-1) \tau}
\end{array}\right]+}_{\mathrm{IN}:(N+1) \times 1} \\
& \underbrace{\left[\begin{array}{cccccc}
0 & \frac{(\tau-t)}{2 \tau-t} & & 0 & \cdots & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 & \cdots & 0 \\
\vdots & & \vdots & & & \vdots \\
0 & \cdots & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\
0 & \cdots & 0 & 0 & \frac{1}{2} & 0
\end{array}\right]}_{D: N \times N} \underbrace{\left[\begin{array}{l}
C(\tau) \\
C(2 \tau) \\
\vdots \\
C(N-1) r \\
C(N \tau)
\end{array}\right]}_{\mathrm{C}: N \times 1}+\underbrace{\left[\begin{array}{l}
\frac{S_{S} \tau}{2 \tau-t} \\
0 \\
\vdots \\
0 \\
\frac{C_{(N+1) \tau}}{2}
\end{array}\right]}_{\mathrm{E}: N \times 1},
\end{align*}
$$

or, more compactly:

$$
\begin{equation*}
\mathbf{C}=\frac{\tau}{c} \mathbf{A} \cdot \mathbf{P}-\frac{h \tau}{c} \mathbf{B}+\mathbf{F} \cdot \mathbf{I N}+\mathbf{D} \cdot \mathbf{C}+\mathbf{E} . \tag{5.6.19}
\end{equation*}
$$

The solutions for the carryover stocks are found by premultiplying this equation by $(\mathbf{I}-\mathbf{D})^{-1}$, where $I$ is the identity matrix:

$$
\begin{equation*}
\mathbf{C}=\frac{\tau}{c}(\mathbf{I}-\mathbf{D})^{-1} \mathbf{A} \cdot \mathbf{P}-\frac{h \tau}{c}(\mathbf{I}-\mathbf{D})^{-1} \mathbf{B}+(\mathbf{I}-\mathbf{D})^{-1} \mathbf{F} \cdot \mathbf{I N}+(\mathbf{I}-\mathbf{D})^{-1} \mathbf{E} \tag{5.6.20}
\end{equation*}
$$

The elements of the matrix $(\mathbf{I}-\mathbf{D})^{-1}$ are all nonzero, so this system is fully integrated. Each carryover stock in the future depends upon all the exogenous variables in the system.

The above matrix representation of the solution to the $N$ period carryover problem closes the model of optimal inventory dissipation by endogenizing carryover. The full solution to the problem thus consists of a sequence of dynamically optimal plans for all marketing periods, coupled with optimal carryovers between and into each period. It should be apparent that the optimal sales level at any given date in any given marketing period is truly part of a complex dynamic plan, since it not only embodies conditions within the particular marketing period, but also all of those expected to unfold in future marketing periods. Any change in parameters will have ripple effects throughout the entire system. For example, a change in the wholesale price or expected harvest level at some future marketing period $v$ will change the whole solution of carryovers via the matrix equation 5.6.20 which in turn will induce tilts in each of the sale profiles in all adjacent marketing periods. The more distant the expected change, the less will be the impact on current optimal plans; impacts are felt most acutely in marketing periods closest to those expected to experience any change in parameters.

### 5.7 An Industry Model of Carryover

As we did in the single period case, we can also construct an industry supply model with multiple marketing intervals. The solution procedure for the firm's decision parallels the fixed price carryover model, and the computation of the market equilibrium price follows
the market model. We solve for the marginal cost of carryout and the marginal benefit of carryin at time $\tau$, solve for the optimal carryover level, embed the carryover level into the firm's sales decision, and aggregate over firms to arrive at an industry supply function. We can then hypothesize an expectations formulation process to complete the specification of supply.

Consider a two period case first. For a single firm expecting some wholesale price path $P(s)$, the marginal cost of carryout of the first marketing period is:

$$
\lambda_{0}(\tau)=\frac{c\left[C(\tau)-S_{t}\right]}{\tau-t}+\frac{h(\tau-t)}{2}+\frac{1}{\tau-t} \int_{t}^{\tau} P(s) d s
$$

and the marginal benefit of carryin to the next period is:

$$
\lambda_{1}(\tau)=\frac{c\left[C_{2 \tau}-C(\tau)-I_{\tau}\right]}{\tau}-\frac{h \tau}{2}+\frac{1}{\tau} \int_{\tau}^{2 \tau} P(s) d s
$$

The intersection of marginal cost and marginal benefit yields the optimal carryover level, $C(\tau)$ :

$$
C(\tau)=\frac{\tau S_{t}+(\tau-t)\left(C_{2 \tau}-I_{\tau}\right)}{2 \tau-t}-\frac{h \tau(\tau-t)}{2 c}+\frac{1}{c(2 \tau-t)}\left[(\tau-t) \int_{t}^{\tau} P(s) d s-\tau \int_{\tau}^{2 \tau} P(s) d s\right] .
$$

We substitute the optimal carryover level into the terminal shadow value, $\lambda_{0}(\tau)$, and this shadow value into the sales equation to obtain optimal sales:

$$
q(t)=\frac{S_{t}+I_{\tau}-C_{2 \tau}}{2 \tau-t}+\frac{h(2 \tau-t)}{2 c}+\frac{1}{c}\left[P(t)-\frac{1}{2 \tau-t} \int_{t}^{2 \tau} P(s) d s\right] .
$$

The market supply function is the sum of individual sales decisions:

$$
\begin{equation*}
Q(t) \equiv n q(t)=\frac{n\left(S_{t}+I_{\tau}-C_{2 \tau}\right)}{2 \tau-t}+\frac{n h(2 \tau-t)}{2 c}+\frac{n}{c}\left[P(t)-\frac{1}{2 \tau-t} \int_{t}^{2 \tau} P(s) d s\right] . \tag{5.7.1}
\end{equation*}
$$

The assumption about the expectations formation mechanism completes the specification. With myopic expectations, the price term vanishes. Alternatively, if firms have perfect foresight or rational expectations, the integral of future prices can be converted into
a function of stock variables:

$$
\begin{align*}
\int_{t}^{2 \tau} P(s) d s= & \int_{t}^{2 \tau} A(s) d s-B \int_{t}^{2 \tau} Q(s) d s \\
& \int_{t}^{2 \tau} A(s) d s-B\left(C_{2 \tau}-I_{\tau}-S_{t}\right) \tag{5.7.2}
\end{align*}
$$

so that the market supply function is:

$$
Q(t) \equiv n q(t)=\frac{n h(2 \tau-t)}{2 c}+\frac{n P(t)}{c}-\frac{n}{c(2 \tau-t)} \int_{t}^{2 \tau} A(s) d s+\left[\frac{n B+c}{c}\right] \frac{n\left(S_{t}+I_{\tau}-C_{2 \tau}\right)}{2 \tau-t} .
$$

The market equilibrium level of sales is therefore:

$$
\begin{equation*}
Q(t)=\frac{n}{n B+c}\left[A(t)-\frac{1}{2 \tau-t}\left[\int_{t}^{2 \tau} A(s) d s\right]\right]+\frac{n\left(S_{t}+I_{\tau}-C_{2 \tau}\right)}{2 \tau-t}+\frac{n h(2 \tau-t)}{2(n B+c)} . \tag{5.7.3}
\end{equation*}
$$

This is the fully optimal sales profile for the two period case when firms have rational expectations and carryover is determined optimally. The analog under parametric carryover is given in equation (5.5.8).

The matrix representation of the market equilibrium solution with $N$ periods is a straightforward modification of the matrix representation with fixed prices, or equation(5.6.19). The vector of prices multiplied by the length of the marketing period, $\mathbf{P} \cdot \boldsymbol{\tau}$, is replaced by a vector of integrals of the demand shifters. Also, the constant $c$, the adjustment cost parameter, is replaced by $n /(n B+c)$. Defining

$$
\begin{align*}
& \alpha_{i}=\int_{\tau}^{(i+1) \tau} A(s) d s, \forall i \neq 0,  \tag{5.7.4}\\
& \alpha_{0}=\int_{t}^{\tau} A(s) d s
\end{align*}
$$

the vector $\alpha$ is written:

$$
\underbrace{\left[\begin{array}{l}
\alpha_{0}  \tag{5.7.5}\\
\alpha_{1} \\
\vdots \\
\alpha_{N-1} \\
\alpha_{N}
\end{array}\right]}_{\alpha: N \times 1}
$$

and the new expression for carryover is:

$$
\begin{equation*}
\mathbf{C}=\frac{n}{n B+c} \mathbf{A} \cdot \alpha-\frac{n h \tau}{2(n B+c)} \mathbf{B}+\mathbf{F} \cdot \mathbf{I N}+\mathbf{D} \cdot \mathbf{C}+\mathbf{E} \tag{5.7.6}
\end{equation*}
$$

This equation can also be pre-multiplied by $(\mathbf{I}-\mathbf{D})^{-1}$ as was done to derive equation 5.6.20. The resulting equation thus generalizes the $N$ period model of optimal decisions by the firm to a similar formulation for the competitive rational expectations industry. This, in combination with the reduced form expressions derived earlier for the optimal industry level within marketing period paths of prices and sales completes the solution to the general problem. Again, within period sales are forward looking, not only to expected demand shifters within the current marketing period, but also to expected parameters and shifters that will unfold within and between future marketing periods. This is what we would anticipate for a properly specified dynamically optimal plan; decisions made at any point incorporate all information about any future events and conditioning factors expected to arise.

In summary, this chapter has outlined a modeling structure suitable for analyzing inventory behavior for the storage of a semiperishable product. As discussed in the introduction, virtually all of the attention that has been devoted to inventories in either industrial or agricultural setting has been framed in annual, infinite horizon models under the assumption that the commodity is infinitely durable. The principle focus has been on the carryover decision, with little attention to the decisions that must be made within the marketing period, or the manner in which those are connected to the carryover decisions.

The model developed here is thus new in its focus and in its analytical structure. This chapter develops a fully integrated model of both the within-marketing period sales decisions and the between marketing period carryover decisions. Individual decision makers are assumed to make plans that are dynamically optimal, and this leads to several models of the competitive inventory holding firm. The adjustment cost framework, although
widely used in the investment literature, has not been applied to dissipation problems. The advantages of the linear/quadratic framework are apparent in this application; the ability to develop closed form analytical solutions allows exploration of a number of comparative statics and comparative dynamics properties of complicated scenarios.

The conceptual models of the competitive inventory holding firm lead to some interesting and plausible structural specifications for dynamically optimal supply plans. Most of these are presented in closed loop form, which is a robust specification suitable for empirical modeling. In the simplest examples, current sales are simply a linear function of the current stock less planned carryover, divided by the time remaining in the marketing period, plus a term involving the ratio of costs and time remaining. Adding discounting makes the structure nonlinear in parameters and adds a term involving the (assumed constant) wholesale price. Generalizing to allow time varying prices complicates the structure by making current sales a function of expected future prices.

Since not all price paths are admissible equilibrium paths, we also discuss the nature of an industry equilibrium with individually optimizing competitive inventory holders. This leads to reduced form expressions for the equilibrium paths of both wholesale prices and industry sales, under the assumption of an arbitrary demand curve. A critical determinant of the sales path is the expectations mechanism used by industry participants. We compare two: the naive myopic expectations process and the rational expectations process.

The second half of this chapter generalizes further by endogenizing the carryover decision that was assumed parametric in the first part. We move from specific to general by developing models of the firm and industry in models with increasing numbers of carryover decisions to make. The final model developed is a multiperiod industry model with an arbitrary time varying wholesale demand curve, under rational expectations. This model lays out an analytical structure which simultaneously solves both the within marketing
period decision of the industry with the between period carryover decisions over $N$ periods. Some illustrative comparative dynamics are presented, showing how parametric changes in either current or distant markets would ripple throughout the full system, affecting both within and between period optimal decisions. This model is, in principle, suitable for empirical modeling of the structure and expectations formulation process of any industry that is dissipating a semiperishable product over time. Its uses are potentially several, including recovering structural and expectations process parameters, forecasting, identifying whether seasonality is demand or supply induced, and so on.

In the next chapter we discuss some of the empirical issues raised by the theoretical model developed here. Then we draw from the conceptual model to specify an exvessel demand equation that is consistent with the structure here and which can be used to close the model of regulatory/industry interaction. This then completes the task of generalizing the basic Gordon model by adding both a regulatory and marketing sector and endogenizing season length, capacity, and exvessel price.

## Chapter 6

## Empirical Estimation of Exvessel

## Prices

### 6.1 Introduction

In the previous chapter a theoretical deterministic model was developed of the inventory dissipation process. The model structures the factors that we would expect to influence inventory dissipation plans, both within and between marketing periods. The model is fully dynamic, incorporating variables such as future prices (or demand shifters) future infusions from harvesting, and planned optimal future carryovers into the current marketing period supply plan. Some simple closed form solutions are derived, suggestive of functional forms that might be used to estimate parameters of such a system.

In this chapter we discuss some of the econometric issues associated with estimating some of the parameters of the dynamic model developed in Chapter Five. There is an extensive literature already developed dealing with estimating dynamic decision models and we will not review this in great depth. Instead, we will briefly review some issues
that might be confronted in estimating the complete model and suggest ways to deal with them. These issues would arise, for example, if we were primarily interested in estimating structural (cost and demand) parameters and expectations process parameters embedded in revealed decisions by the inventory industry about supply paths and equilibrium prices.

As it turns out, for the purposes set out in the introduction of this thesis, we are less interested in explicitly modeling actual inventory sales and carryover decisions than we are in modeling the dual of these plans, namely the exvessel demand price paid to the fishermen for additions to inventory at the beginning of the marketing period. As discussed earlier, in order to close the expanded version of the Gordon model, we need to incorporate both the impact of regulations and industry behavior on exvessel prices and also the impact of exvessel prices on regulatory and industry behavior.

To be conceptually consistent, we need an exvessel demand curve which is a derived demand and which reflects all of the factors determining equilibria in the wholesale market. This is complicated because, as we showed in the previous chapter, the wholesale market operates ideally as a completely dynamic and integrated system of within and between marketing period markets. As we also showed, however, it is possible to derive conceptual reduced forms for industry wholesale price and sales paths that predict how these integrated within and between period decisions would play out in observed data.

The models developed in Chapter Five are grounded in an optimal control formulation of the inventory dissipation process. As briefly discussed, application of the Pontryagin conditions to the inventory problem yields a set of two differential equations (in the inventory stock $S(t)$ and in the dynamic shadow price $(\lambda(t))$ and a single static equilibrium condition in the control variable $q(t)$ (or sales), the stock variable, and the shadow price. These can be combined in either of two ways. Chapter Five utilizes a combination which generally eliminates the shadow price and reduces the system down to one describing the
optimal time paths of the inventory stock and sales, or ( $S(t), q(t)$ ). Alternatively, one could combine the three equations in a manner that eliminates the sales variable and describes the solution in terms of the shadow price and the optimal inventory path, or $(S(t), \lambda(t))$. As it turns out, this is a more useful conceptualization for thinking about how to derive an exvessel demand curve consistent with a wholesale sector that is dynamically optimizing inventory dissipation. This is because the shadow price is, at any time, the marginal value of an exta unit of inventory. Along the optimal paths, $\lambda$ thus measures what the wholesale sector would be willing to pay for another unit of harvest that could be added to inventories. Thus, rather than focusing on the paths of the inventory stock and sales falling out of the wholesale sector's dynamically optimal plans, it is more convenient to focus on the dual of those quantity decisions, namely the shadow price.

In the next section, we discuss some general issues that would be confronted when estimating parameters of a dynamic supply system incorporating expectations. We illustrate using our model of the wholesale system of optimal sales, inventory, and equilibrium price paths. Then we focus on developing an exvessel demand curve from information contained in the shadow price equations for the wholesale model. Finally we discuss parameter estimates derived from an econometric model of exvessel prices consistent with the theoretical specification.

### 6.2 Estimation Issues in Models of Dynamic Decisions

There is a substantial literature on estimating models that are derived from the behavior of individuals who make dynamically optimal plans. Many of these empirical models consider the investment decision, where the purchase of a capital good involves intertemporal tradeoffs. The literature in this field has treated expectations in several ways, and we draw from this literature in discussing empirical implementation of the inventory
models developed in Chapter Five.

The basic decision rule for a stochastic empirical model is a simple adaptation of the continuous time rule developed in the previous chapter. Instead of solving a deterministic continuous time present value maximization problem, we solve for the set of contingency plans that maximizes the expected present value of profits in discrete time. We can write the individual inventory holders problem as:

$$
\begin{aligned}
& \underset{S(s)}{\operatorname{maximize}} \mathcal{E}_{t} \sum_{s=t}^{\tau} P_{s} q(s)-\frac{c}{2} q(s)^{2}-h S(s) \\
& \text { subject to } S(s)-S(s-1)=-q(s) \\
& S(t-1)=S_{t-1} \\
& S(\tau)=S_{\tau} .
\end{aligned}
$$

Note that we are considering a within market period model here, with the planned carryout $S_{\tau}$ a parameter. The sole piece of additional notation is due to the uncertain environment: the expectation of the future given information available at time $t$ is denoted $\mathcal{E}_{t}$. The first order conditions are the derivatives with respect to the stock at every time period. We solve the series of first order conditions for the path of stocks that maximizes the expected stream of profits.

First, we substitute the equation of motion into the objective function to get the objective function in terms of stocks. The objective function becomes

$$
\begin{equation*}
\underset{S(s)}{\operatorname{maximize}} \mathcal{E}_{t}\left[\sum_{s=t}^{\tau} P_{s}[S(s-1)-S(s)]-\frac{c}{2}[S(s-1)-S(s)]^{2}-h S(s)\right] \tag{6.2.1}
\end{equation*}
$$

A representative first order condition is:

$$
\partial J / \partial S(v)=\mathcal{E}_{v}\left\{-P_{v}+P_{v+1}+c\left[S_{v-1}-2 S(v)+S(v+1)\right]-h\right\}=0 .
$$

The firm chooses the stock in the current period, knowing the parameters in the current period, and the choice of stock implicitly determines sales. Since this is linear, we can pass
the expectations operator through, where the expectation of any realized random variable is its realization, and the representative first order condition becomes

$$
\begin{equation*}
-P_{v}+\mathcal{E}_{v} P_{v+1}+c\left[S_{v-1}-2 S(v)+\mathcal{E}_{v} S(v+1)\right]-h=0 \tag{6.2.2}
\end{equation*}
$$

The solution for the stock at any date $v$ is therefore

$$
\begin{equation*}
S(v)=\frac{\mathcal{E}_{v} P_{v+1}-P_{v}}{2 c}+\frac{S_{v-1}+\mathcal{E}_{v} S(v+1)}{2}-\frac{h}{2 c} . \tag{6.2.3}
\end{equation*}
$$

We solve for the current stock in terms of the exogenous variables using all of the first order conditions:

$$
\begin{equation*}
S(t)=\frac{S_{\tau}+(\tau-t) S_{t-1}}{\tau-t+1}-\frac{P_{t}}{c}+\frac{\mathcal{E}_{t} \sum_{s=t}^{\tau} P(s)}{c(\tau-t+1)}-\frac{h(\tau-t)}{2 c} \tag{6.2.4}
\end{equation*}
$$

We can also express the solution in terms of sales instead of stock levels by using the equation of motion, $q(s)=S(s-1)-S(s)$ evaluated at $s=t$, to get an expression that is similar to the continuous time version of the sales decision:

$$
\begin{equation*}
q(t)=\frac{S_{t-1}-S_{\tau}}{\tau-t+1}+\frac{h(\tau-t)}{2 c}+\frac{1}{c}\left[P_{t}-\frac{\mathcal{E}_{t} \sum_{s=t}^{\tau} P_{s}}{\tau-t+1}\right] . \tag{6.2.5}
\end{equation*}
$$

As in the deterministic setting, sales are a function of the amount of stock left, cost terms, and prices. In the deterministic case, firms respond to a known path of future prices. Here, firms base their decisions on the path of expected future prices.

### 6.2.1 Expectations

Since the sum of expected future prices in the current cycle is an important determinant of sales, the manner in which firms form expectations is clearly a critical element of the specification. The type of expectations formation process that is assumed on the part of the firms (e.g., myopic or rational) will alter the form of the sales decision. We will discuss the implications of several different expectations assumptions including myopic, extrapolative, and rational.

## Myopic

If firms have myopic expectations, they assume temporarily that current prices will remain the same throughout the marketing cycle. Then, during the next priod, it is assumed that firms update their expectations, solve a new optimization problem, and make the first decision of that new solution. This is a common assumption, both in the theoretical literature on dynamic factor demands and in many empirical applications. For example, Berndt, Fuss, and Waverman [8] employed a myopic expectations assumption in their paper, which was the first empirical piece on investment that was based on the behavior of individual dynamic optimizing firms. Also, Epstein [29] uses static expectations in his development of the dynamic dual approach to modeling investment. Though somewhat unrealistic (in the sense that firms probably know that prices will change) it may be a reasonable approximation of the truth since firms are assumed to observe new prices as they unfold and update their decision.

As in our earlier discussion of the deterministic (undiscounted) case, if prices are assumed to remain constant, the price level does not matter to the firn since the average of future expected prices will be the same as the current price. Firms simply sell a fraction of their remaining stock, modified by storage and adjustment costs, using the decision rule:

$$
\begin{equation*}
q(t)=\frac{S_{t-1}-S_{\tau}}{\tau-t+1}+\frac{h(\tau-t)}{2 c} . \tag{6.2.6}
\end{equation*}
$$

Under these assumptions, supply is perfectly inelastic with respect to price. The appropriate estimation procedure in this case would be ordinary least squares, though one would not be able to identify $h$ from $c$ separately. There would not be a simultaneous equation problem in estimating the supply decision, since the system is recursive, where the remaining stock determines the amount of sales in the supply equation, and sales determine price in the demand equation. Demand can take on any form and a single additive error term can be appended to the supply equation, justified by random optimization errors and random
errors in the data (see Epstein and Yatchew [30], e.g.).
For estimation with aggregate data, the market supply function would be assumed to be the sum of individual supply functions. The econometric model would then be:

$$
\begin{align*}
& \text { (S) } n q(t)=\frac{n\left(S_{t-1}-S_{r}\right)}{\tau-t+1}+\frac{n h(\tau-t)}{2 c}+\omega_{t}  \tag{6.2.7}\\
& \text { (D) } P(t)=g(n q(t), \text { demand variables })+\epsilon_{t}
\end{align*}
$$

Thus, if firms are believed to employ myopic expectations, the estimation of the composite parameter $h / c$ is a simple matter and the econometrician has considerable flexibility in specifying demand.

## Extrapolative

As an alternative expectations process, firms may expect prices to increase at a particular rate from the current price so that the expected future price is a linear function of the current price (e.g., $\mathcal{E}_{t} P_{t+1}=\alpha P_{t}$ ). This is reasonable with a discount rate and a razor's edge model in which prices rise according to the discount factor and storage costs. ${ }^{1}$

If inventory holders expect prices to rise at a constant rate, it is simple to calculate the sum of expected future prices. The form of expectation is:

$$
\mathcal{E}_{t} P_{t+1}=\alpha P_{t},
$$

so that the expectation of a price in the future is

$$
\mathcal{E}_{t} P_{t+s}=\alpha^{s} P_{t}
$$

and the sum of future expected prices is therefore

$$
\mathcal{E}_{t} \sum_{s=t}^{\tau} P_{s}=P_{t} \sum_{s=t}^{\tau} \alpha^{s}=P_{t} \frac{1-\alpha^{\tau+1}}{1-\alpha}
$$

[^14]The decision rule with this expectations mechanism is then

$$
\begin{equation*}
q(t)=\frac{S_{t-1}-S_{\tau}}{\tau-t+1}+\frac{h(\tau-t)}{2 c}+\frac{P_{t}}{c}\left[\frac{\alpha^{\tau+1}-\alpha}{1-\alpha}\right] \tag{6.2.8}
\end{equation*}
$$

In this case, sales are a function of the stock left, the current price, cost parameters, and the expectational parameter $\alpha$.

The appropriate estimation procedure depends upon what we assume about the demand side. If demand is perfectly elastic, we have a triangular system where the location of demand determines the price, and price determines the quantity supplied. Supply in this case can be estimated as a single equation.

If demand is responsive to price, there is a simultaneous equation problem since both quantity and price are endogenous in both equations. One could then employ either a single equation method (e.g., instrumental variables, limited information maximum likelihood) or a systems method (e.g., three stage least squares, full information maximum likelihood) to estimate the parameters of the supply (and demand) equations. The econometric error terms are simply appended onto each equation for estimation. Again, the justification is an appeal to random optimization errors and random errors in the data.

In summary, if extrapolative expectations are assumed, the econometric model with perfectly elastic demand is:

$$
\begin{align*}
& \text { (S) } n q(t)=\frac{n\left(S_{t-1}-S_{\tau}\right)}{\tau-t+1}+\frac{n h(r-t)}{2 c}+\frac{n P_{t}}{c}\left[\frac{\alpha^{r+1}-\alpha}{1-\alpha}\right]+\omega_{t}  \tag{6.2.9}\\
& \text { (D) } \quad P(t)=g \text { (demand variables) }+\epsilon_{t} .
\end{align*}
$$

The econometric model with price responsive demand is:

$$
\begin{align*}
& \text { (S) } n q(t)=\frac{n\left(S_{t-1}-S_{r}\right)}{\tau-t+1}+\frac{n h(\tau-t)}{2 c}+\frac{n P_{t}}{c}\left[\frac{\alpha^{r+1}-\alpha}{1-\alpha}\right]+\omega_{t}  \tag{6.2.10}\\
& \text { (D) } P(t)=g(n q(t), \text { demand variables })+\epsilon_{t} .
\end{align*}
$$

## Rational Expectations

We may also assume that firms have rational expectations, the stochastic analog of perfect foresight. Under this assumption, firms use all of the information available to them to predict the future; in particular, they are assumed to know (or behave as if they know) the econometric model, including the parameters of the relevant stochastic processes and the parameters that guide the decisions of agents in the model. If firms form rational expectations, they do not make forecast errors that are correlated with any information available to them. This follows because if forecast errors were correlated with known information, firms could use that information to improve their forecasts. These underpinnings of rational expectations have been used to develop econometric models that can get at the "deep structural parameters" that guide economic decisions in a dynamic context. ${ }^{2}$

Rational expectations models have been have been widely studied following Muth's[62] original paper introducing the idea. There are essentially three strategies to choose from when estimating a dynamic rational expectations model: the solution method, the Euler equation method, and the forecasting model method. In the solution method, the parameters of the relevant stochastic processes are used to solve for the path of future expected variables in terms of past values. The parameters of the stochastic processes are thus embedded into the decision rule, and the decision rule is estimated along with the stochastic process. ${ }^{3}$ In the Euler equation method, only the first order condition is estimated. The future (unknown) variables in the Euler equation are replaced with their actual values and the ensuing econometric difficulties are treated with an instrumental variables technique.

[^15]In the forecasting model method, a forecasting structure is estimated outside of the structral equation estimation and then forecasts of future variables are substituted in. We will explore these methods in the next sections.

Solution Methods Methods which ultimately derive explicit functions for decision variables depend on a linear/quadratic problem specification and also on whether demand is downward sloping or perfectly elastic. If demand is perfectly elastic, the price can be modeled as following a stochastic process where the firms know the parameters of that process. If the quantity demanded depends on price, though, there is a simultaneous equation problem with estimating current supply and demand because firms have to predict the path of prices that comes from the intersection of supply and demand in the future. We can develop a general model that includes both possibilities. First, we model an inventory dissipation system where demand is responsive to price and then show the restriction that leads to the simpler form where demand is perfectly elastic.

If demand is downward sloping (e.g., $P(t)=A-B n q(t)+\gamma Y(t)$ where $Y$ is a new shifter variable), the concept of rational price expectations must be based on market clearing assumptions. In particular, firms' supply decisions are based on an expected path of future prices, and their optimized supply decisions must act in concert with demand to match those expectations. In this case, the solution is like the perfect foresight solution of the previous chapter: future market clearing is assumed by using the expected market demand function in the sum of expected future prices. The sum of future expected prices is:

$$
\begin{aligned}
\mathcal{E}_{t} \sum_{s=t}^{\tau} P_{s}= & \mathcal{E}_{t} \sum_{s=t}^{\tau} A-B n q(s)+\gamma Y_{s}+\epsilon_{s} \\
& (\tau-t+1) A-B n \mathcal{E}_{t} \sum_{s=t}^{\tau} q(s)+\mathcal{E}_{t} \sum_{s=t}^{\tau}\left[\gamma Y_{s}+\epsilon_{s}\right] .
\end{aligned}
$$

Since the sum of current and future sales is expected to exhaust the current stock (minus
carryover), $\left(\mathcal{E}_{t} \sum_{s=t}^{\tau} q(s)=S_{t-1}-S_{\tau}\right)$ the sales decision is reduced to:

$$
\begin{equation*}
q(t)=\left[\frac{n B+c}{c} \frac{S_{t-1}-S_{\tau}}{\tau-t+1}\right]+\frac{h(\tau-t)}{2 c}+\frac{P_{t}}{c}-\frac{A}{c}-\frac{\mathcal{E}_{t} \sum_{s=t}^{\tau} \gamma Y_{s}+\epsilon_{s}}{c(\tau-t+1)} . \tag{6.2.11}
\end{equation*}
$$

Current individual supply, in inverse form is then:

$$
\begin{equation*}
P(t)=c q(t)-\frac{(n B+c)\left(S_{t-1}-S_{\tau}\right)}{\tau-t+1}-\frac{h(\tau-t)}{2}+A+\frac{\mathcal{E}_{t} \sum_{s=t}^{\tau} \gamma Y_{s}+\epsilon_{s}}{\tau-t+1} \tag{6.2.12}
\end{equation*}
$$

It remains to specify the stochastic process of the demand $\operatorname{shifter}(\mathrm{s}), Y(s)$, and the stochastic shock to demand, $\epsilon_{s}$. For example if we assume that the demand shifter (say income) follows a first order autoregressive process and the demand shock is white noise, then the stochastic assumptions are:

$$
\begin{gathered}
Y_{s}=\rho Y_{s-1}+u_{t}, u \sim N\left(0, \sigma_{u}\right) \\
\epsilon \sim N\left(0, \sigma_{\epsilon}\right)
\end{gathered}
$$

and the supply equation (6.2.12) can be written:

$$
\begin{align*}
P(t)= & c q(t)-\frac{(n B+c)\left(S_{t-1}-S_{\tau}\right)}{\tau-t+1}-\frac{h(\tau-t)}{2}+A+ \\
& \frac{\gamma}{\tau-t+1} Y_{t} \frac{1-\rho^{T-t+1}}{1-\rho}+\frac{\epsilon_{t}}{\tau-t+1} . \tag{6.2.13}
\end{align*}
$$

The appropriate econometric procedure is then to estimate this equation along with the stochastic process for the exogenous demand shifter(s) using maximum likelihood and imposing the cross equation restrictions that the $\rho$ s in each equation are equal:

$$
\begin{align*}
& P(t)=c q(t)-\frac{(n B+c)\left(S_{t-1}-S_{-}\right)}{\tau-t+1}-\frac{h(\tau-t)}{2}+A+\frac{\gamma}{\tau-t+1} Y_{t} \frac{1-\rho^{\tau-t+1}}{1-\rho}+\frac{\epsilon_{t}}{\tau-t+1}  \tag{6.2.14}\\
& Y_{t}=\rho Y_{t-1}+u_{t} .
\end{align*}
$$

Alternative stochastic assumptions would not change the fundamental nature of the estimation.

If demand is perfectly elastic, firms' supply decisions do not affect price and the path of future prices is determined by an independent stochastic process. For example, if prices follow a first order autoregressive process, the firm's forecast of the future would
depend only on the current price (as it did in the previous section where firms are assumed to make extrapolative predictions). In this case, the parameter that determines expectations is assumed to be the same as the one that guides the true stochastic process. We gain efficiency (and precision of identification) by estimating the decision rule along with the stochastic process. The system to estimate would then be:

$$
\begin{align*}
& \text { (S) } n q(t)=\frac{n\left(S_{t-1}-S_{\tau}\right)}{\tau-t+1}+\frac{n h(\tau-t)}{2 c}+\frac{n P_{t}}{c}\left[\frac{\alpha^{\tau+1}-\alpha}{1-\alpha}\right]+\omega_{t}  \tag{6.2.15}\\
& \text { (D) } P_{t}=\alpha P_{t-1}+\epsilon_{t}
\end{align*}
$$

In sum, the above discussion outlines the solution method for rational expectations econometric models of supply and demand under two hypotheses about demand: either perfectly elastic or linear. The perfectly elastic case is nested within the linear model, since the parameter $B$ is the slope of the inverse demand function. Simple analytic solutions are easily computable if the stochastic process guiding the price (for the perfectly elastic case) or the exogenous demand shifters (for the linear demand case) are first order autoregressive.

If prices (or other shifters) follow a second order or higher autoregressive process, the formula becomes complicated. Sargent[75] derives expressions for forecasts of the future when the forecast horizon is infinite with more general processes. He calculates the infinite sum of future expectations expressed in terms of a parameters of the polynomial lag function associated with past exogenous variables. This method does not seem as easy to use in our case since we have a finite sum for each market period. It is straightforward though tedious to calculate individual future period predictions using the Weiner-Kolmogorov formula of least squares projections, however. For example, in the second order case, we have:

$$
\begin{gather*}
\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right) x_{t}=\epsilon_{t} \\
\left(1-\left(\lambda_{1}+\lambda_{2}\right) L+\lambda_{1} \lambda_{2} L^{2}\right) x_{t}=\epsilon_{t}  \tag{6.2.16}\\
x_{t}=\alpha_{1} x_{t-1}+\alpha_{2} x_{t-2}+\epsilon_{t},
\end{gather*}
$$

where $\alpha_{1}=\left(\lambda_{1}+\lambda_{2}\right)$ and $\alpha_{2}=-\lambda_{1} \lambda_{2}$.

The polynomial in the lag operator is ( $1-\alpha_{1} L-\alpha_{2} L^{2}$ ) or $\alpha(L)$. The Weiner Kolmogorov prediction formula is that the prediction $k$ steps ahead of the current date is:

$$
P_{t} x_{t+k}=\left[\frac{1}{\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right) L^{k}}\right]_{+}\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right) x_{t}
$$

where the subscripted + means "ignore negative powers of L ", and $P_{t} x_{t+k}$ means the prediction at date $t$ of $x$ at date $t+k$. For a one step ahead prediction, we have:

$$
P_{t} x_{t+1}=\left[\frac{1}{\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right) L}\right]_{+}\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right) x_{t}
$$

Now, we re-express the denominator and multiply through by $L$ to see which are the negative powers of $L$ to ignore.

$$
\begin{gathered}
\frac{1}{L\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right)}=L^{-1}\left(1+\lambda_{1} L+\lambda_{1}^{2} L^{2}+\lambda_{1}^{3} L^{3}+\ldots\right)\left(1+\lambda_{2} L+\lambda_{2}^{2} L^{2}+\lambda_{2}^{3} L^{3}+\ldots\right)= \\
\left(L^{-1}+\lambda_{1}+\lambda_{1}^{2} L+\lambda_{1}^{3} L^{2}+\ldots\right)\left(1+\lambda_{2} L+\lambda_{2}^{2} L^{2}+\lambda_{2}^{3} L^{3}+\ldots\right)= \\
\lambda_{1}\left(1+\lambda_{1} L+\lambda_{1}^{2} L+\ldots\right)\left(1+\lambda_{2} L+\lambda_{2}^{2} L^{2}+\ldots\right)+L^{-1}\left(1+\lambda_{2} L+\lambda_{2}^{2} L^{2}+\lambda_{2}^{3} L^{3}+\ldots\right)= \\
\frac{\lambda_{1}}{\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right)}+L^{-1}+\frac{\lambda_{2}}{1-\lambda_{2} L} .
\end{gathered}
$$

Now, ignore the negative power of $L$, and rewrite the prediction formula as

$$
P_{t} x_{t+1}=\left(\frac{\lambda_{1}}{\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right)}+\frac{\lambda_{2}}{1-\lambda_{2} L}\right)\left(1-\lambda_{1} L\right)\left(1-\lambda_{2} L\right) x_{t}
$$

and hence the expression for the one step ahead forecast of x is:

$$
P_{t} x_{t+1}=\left(\lambda_{1}+\lambda_{2}-\lambda_{1} \lambda_{2} L\right) x_{t}
$$

or

$$
P_{t} x_{t+1}=\alpha_{1} x_{t}+\alpha_{2} x_{t-1}
$$

which accords with our intuition. The two and three step ahead forecasts are:

$$
\begin{gathered}
P_{t} x_{t+2}=\left(\lambda_{1}^{2}+\lambda_{2}^{2}+\lambda_{1} \lambda_{2}\right) x_{t}-\left(\lambda_{1} \lambda_{2}\right)\left(\lambda_{1}^{2}+\lambda_{2}^{2}\right) x_{t-1} \\
P_{t} x_{t+3}=\left(\lambda_{1}^{3}+\lambda_{2}^{3}+\lambda_{1} \lambda_{2}^{2}+\lambda_{1}^{2} \lambda_{2}\right) x_{t}-\left(\lambda_{1} \lambda_{2}\right)\left(\lambda_{1}^{2}+\lambda_{2}^{2}+\lambda_{1} \lambda_{2}\right) x_{t-1}
\end{gathered}
$$

In principle, these could be computed for each of the remaining periods and summed up in a complicated expression. Estimation using higher order processes in the solution method over a finite horizon is obviously difficult, though.

Another and much simpler option is to assume rational expectations, but produce the forecasts of the exogenous variables (prices, income, substitutes) outside of the decision rule and substitute those predictions into the sum of expected future variables. ${ }^{4}$ This method would permit the use of more sophisticated forecasting techniques (e.g., state space, VAR, etc.). The forecast error would have to be acknowledged and incorporated into the variance/covariance matrix of parameter estimates. This method would require, at every period, using just the information up to that period (though with parameters estimated from the whole series).

Euler Equation Methods Another estimation option, developed by Hansen and Singleton [45] and first implemented by Pindyck and Rotemberg[68], is to estimate the first order condition. ${ }^{5}$ This method has emerged because it has been recognized that, although linear/quadratic problems can be solved explicitly, more general and flexible specifications of the objective function do not lend themselves to an explicit solution. Since many of the important parameters are contained in the first order conditions, an estimation of the first order conditions should yield valuable information. The critical insight of the econometric Euler equation method is that, if the firms have rational expectations, the errors made in implementing the first order conditions are, in fact, forecasting errors which will be uncorrelated with information known at the time. Thus under rational expectations, we can expect that firms will not persistently make the same mistake.

The problem with estimating the first order condition is that it contains unob-

[^16]servable variables, namely the firm's expectation of the stock in the subsequent period and the expectation of the future price (see equation (6.2.2) for example). Furthermore, the firm's expectation of the subsequent stock should be highly correlated with the choice of the current stock and thus the problems of a missing variable and simultaneous equations bias must be overcome before consistent estimators can be found. The solution proposed by Hansen and Singleton is to substitute the realization of the future stock in place of its expectation and to use an instrumental variables technique to eliminate the simultaneity problem. In order for the consistency of the estimator to be proven, the econometric error must be orthogonal to the instruments used. Since expectations are assumed to be rational, any information known to the firm will be uncorrelated with the forecast error. Therefore, any information available to the firm in the current period can be used as instruments in estimation.

The econometric procedure used to obtain the estimates is now fairly standard. ${ }^{6}$ The objective to be minimized is a weighted sum of squared errors, where the error is the forecast error associated with the first order condition. The weighting matrix is made up of the chosen instruments. If $e$ is the error term (independent and identically distributed), and $W$ is a nonstochastic matrix of instruments (with at least as many instruments as parameters), the objective to be minimized is $e^{\prime} W\left(W^{\prime} W\right)^{-1} W^{\prime} e$. Consistency and asymptotic normality of this nonlinear instrumental variables estimator has been proven by Amemiya[3]. Some of the assumptions regarding the error term and instruments have been relaxed by Amemiya[3] and Hansen[42]. Amemiya allows for a system of equations, with errors correlated with each other within the same time period. He terms this estimator nonlinear three stage least squares, since it could be used in a system of equations with a non-diagonal covariance matrix. Hansen generalizes the estimator further, to permit conditional autocorrelation and heteroscedasticity and correlated regressors, and terms his procedure the

[^17]Generalized Method of Moments. Pindyck and Rotemberg note that nonlinear three stage least squares is a special case of the Generalized Method of Moments, and choose to use nonlinear three stage least squares in their estimation. ${ }^{7}$

As an illustration of the mechanics of the Euler equation model, consider the decision to be made in period $t$ for our inventory model. The first order condition for this period is:

$$
\begin{equation*}
\mathcal{E}_{t} P_{t+1}-P_{t}-h+c\left[S(t-1)-2 S(t)+\mathcal{E}_{t} S(t+1)\right]=0 \tag{6.2.17}
\end{equation*}
$$

so that the optimal choice of the current stock, $S(t)$, is

$$
\begin{equation*}
S(t)=\frac{\mathcal{E}_{t} P_{t+1}-P_{t}}{2 c}+\frac{h}{2 c}+\frac{S(t-1)+\mathcal{E}_{t} S(t+2)}{2} \tag{6.2.18}
\end{equation*}
$$

The firm knows the current price and the inherited stock, $S_{t-1}$, but does not know the future value of the stock, $S(t+1)$ or the future price, $P_{t+1}$ and hence it must forecast their values. The Euler equation is assumed to hold exactly, but the econometrician uses a proxy for the expected value of the future stock, $\mathcal{E}_{t} S(t+1)$ as well as the expected future price in the estimated equation. The difference between the expected value (used by the firm) and the actual value (observed by the econometrician) thus plays the role of the econometric error and the error is therefore:

$$
\begin{equation*}
\frac{c}{2}\left[S(t+1)-\mathcal{E}_{t} S(t+1)\right]+\left[P_{t+1}-\mathcal{E}_{t} P_{t+1}\right] \tag{6.2.19}
\end{equation*}
$$

The future stock, $S(t+1)$, is a choice variable. It will be chosen optimally in the subsequent period as a function of $S(t)$, as well as the subsequent stock, $S(t+2)$. We have already solved the Euler equation, so we can see explicitly what it means to substitute the actual $S(t+1)$ for the expected $S(t+1)$. The realized value of the chosen stock in the future depends on the conditions in the future. Expectations will be newly formed, given the

[^18]realizations of what occurs in the future period, $t+1$, so that
$$
S(t+1)=\left[\frac{S_{\tau}-(\tau-t-1) S_{t}}{\tau-t}\right]-\frac{P_{t+1}}{c}+\frac{\mathcal{E}_{t+1} \sum_{s=t+1}^{\tau} P(s)}{c(\tau-t)}-\frac{h(\tau-t-1)}{2 c} .
$$

The expected value of the future stock, however, depends on current conditions:

$$
\mathcal{E}_{t} S(t+1)=\left[\frac{S_{\tau}-(\tau-t-1) S_{t}}{\tau-t}\right]-\frac{\mathcal{E}_{t} P_{t+1}}{c}+\frac{\mathcal{E}_{t} \sum_{s=t+1}^{\tau} P(s)}{c(\tau-t)}-\frac{h(\tau-t-1)}{2 c} .
$$

If price follows a first order autoregressive process, the difference between the expected and actual values can be calculated:

$$
\begin{aligned}
& P_{t+1}=\alpha P_{t}+\epsilon_{t+1} \\
& \mathcal{E}_{t} P_{t+1}=\alpha P_{t} \\
& P_{t+1}-\mathcal{E}_{t} P_{t+1}=\epsilon_{t+1}
\end{aligned}
$$

The difference between the actual future stock and the expected future stock is:

$$
-\frac{P_{t+1}}{c}-\left(-\frac{\mathcal{E}_{t} P_{t+1}}{c}\right)+\frac{\mathcal{E}_{t+1} \sum_{s=t+1}^{\tau} P(s)}{c(\tau-t)}-\frac{\mathcal{E}_{t} \sum_{s=t+1}^{\tau} P(s)}{c(\tau-t)} .
$$

Simplifying,

$$
S(t+1)-\mathcal{E}_{t} S(t+1)=-\frac{\epsilon_{t-1}}{c}+\left[P_{t+1} \frac{1-\alpha^{\tau-t}}{1-\alpha}\right]-\left[P_{t} \frac{\alpha-\alpha^{\tau-t+1}}{1-\alpha}\right]
$$

or

$$
-\frac{\epsilon_{t+1}}{c}+\epsilon_{t+1}\left[\frac{1-\alpha^{\tau-t}}{1-\alpha}\right] .
$$

Finally, the econometric error is

$$
\frac{c}{2}\left[-\frac{\epsilon_{t+1}}{c}+\epsilon_{t+1}\left[\frac{1-\alpha^{\tau-t}}{1-\alpha}\right]\right]+\epsilon_{t+1}
$$

or

$$
\frac{\epsilon_{t+1}}{2}+\frac{c}{2} \epsilon_{t+1}\left[\frac{1-\alpha^{\tau-t}}{1-\alpha}\right] .
$$

In the estimation procedure proposed by Hansen and Singleton, the exact form of the econometric error remains unknown. The econometric error is written as the Euler equation,
with the realized future stock and price substituted for the expected future stock and price. The expectational error is made to be uncorrelated with information known by the firm. We know from this example that the source of the econometric error is the stochastic process of the exogenous variable. Here, the error is heteroscedastic, so that efficient estimation must take the heteroscedasticity into account.

The Euler equation techniques have seen increasing use because dynamic rational expectations models can be estimated with relative ease, and without the stringent functional form requirements of other methods. However, this technique has a few shortcomings. First, there may be a significant information loss from not using the transversality conditions that come in to the solution when the first order conditions are solved. It is assumed that the firm is still using the terminal conditions when it solves for its optimal decision, but the econometrician does not incorporate those terminal conditions into estimation. ${ }^{8}$ Second, there may be randomness which is not attributable to forecast error. For instance, there may be shocks to technology or preferences that are correlated with the instruments. This misspecification can lead to serious misinterpretation of the parameter estimates. ${ }^{9}$ Third, Rotemberg [72] notes that even though the consistency of the estimators does not depend upon the choice of instruments, different choices yield widely different estimates. This feature does not engender confidence in the estimates. Rotemberg suggesis using a series of instruments to determine a band of point estimates.

To summarize, the solution method is both more complicated and more restrictive (it requires a linear/quadratic objective function) than the Euler equation method, but permits the use of information from the transversality condition. The Euler equation method is easier to implement and allows for a more general functional form, but does not ultimately provide information about the expectational process and sacrifices potentially

[^19]valuable information from the transversality condition.

### 6.3 Modeling the Exvessel Demand Curve

As discussed at the beginning of this chapter, the dynamic shadow price from the fully optimized wholesale inventory dissipation model measures at every date the marginal value to the wholesale sector of an additional unit of stock or inventory. In a fishery where some fish are sold into a fresh market and some into a processed market (and perhaps both) during the harvest period, a question arises as to when one should be estimating the exvessel market clearing process. Should the market be modeled at the beginning of the harvest season, or at the end, or throughout the harvest period? ${ }^{10}$ These questions can be considered by examining Figure 6.1 which separates a calendar year into a fishing season of length $T$ days, and a marketing period of length $\tau$ days. In a typical fishery, harvests during most of the early part of the harvest season would be sold into the fresh market. At some date, however, it is possible that inventory holders would begin buying fresh fish to process, hold, and allocate during the subsequent marketing period. The modeling question is thus how should we depict the market clearing mechanism that determines exvessel price in this setting?

In the simplest case that we might confront, the harvest period would be negligible ( $T$ would be small) and all of the product would be devoted to a single product form and wholesale market. In this case, the relevant demand curve could be determined simply by determining an equation for $\lambda$ at the initial date of the marketing period as a function of its determinants and of the size of the expected total harvest ( $H_{0}$ ). These conditions do,

[^20]

Figure 6.1: A Calendar Year
in fact, describe circumstances during recent periods in the halibut fishery when the season length has been reduced to just a few days and when virtually all product is frozen.

To examine what the conceptual model tells us about the derived demand curve for these conditions, recall the simple model where planned carryover is taken as parametric, where the wholesale price is also taken as given over the marketing period, and where we ignore discounting. Then from equation (5.4.8) we can compute an expression for the marginal value of another unit of inventory, looked at from the beginning of the marketing period, namely:

$$
\begin{equation*}
\lambda_{0}=P_{0}-\frac{c}{\tau}\left[S_{0}+I_{0}-C_{\tau}\right]-\frac{1}{2} h \tau \tag{6.3.1}
\end{equation*}
$$

Note that the marginal value is negatively related to the infusion $I_{0}$, which we assume is the total harvest. This simply reflects a diminishing marginal value of additions to inventory and effectively generates a downward sloping demand curve for inventory additions. Note also that the demand curve is negatively related to carryin or holdings at the start of the marketing period $S_{0}$ and to both adjustment and holding costs ( $c$ and $h$ ), and positively related to both the wholesale price $P_{0}$ and the targeted carryout $C_{\tau}$. These can be considered determinants of the intercept of the exvessel demand curve. Figure 6.2 shows how this


Figure 6.2: Exvessel Demand
market would clear at an exvessel price ( $P_{E V}^{*}$ ) that reflects the willingness to pay of the wholesale sector for another unit of inventory.

The above structural equation is a representation of the exvessel demand curve under the simplest possible setting where planned carryover is taken to be parametric. This is unlikely in general, of course, but there are circumstances in which equation (6.3.1) can be estimated using OLS or 2SLS by regressing exvessel prices against wholesale prices, the inventory level on hand at the beginning of the marketing period, the harvest level, and the length of the marketing period. Another case where this simple equation might be used is if there is an independent source for the planned carryover variable, such as a survey of wholesalers and inventory holders. In this case, one could do as in the previous case, adding the planned carryover variable as an additional explanatory variable. A third case where
(6.3.1) might be used directly is if it is believed that wholesalers are myopic in that they expect the carryout to be the same as this year's carryin. In that case, since $\lambda$ is a function of the difference between these two variables, an estimating equation could be derived which regressed exvessel prices against wholesale prices, the harvest level, and the market period length. The error for this equation ought to be heteroscedastic in the marketing period length.

We examined two of these hypothesized cases by collecting data over the period 1959-1978 on halibut wholesale prices, inventories at the beginning and end of the fishing season, harvest levels, and season and hence marketing period lengths. ${ }^{11}$ The data were used to estimate two simple structures based on (6.3.1), one embodying the assumption that wholesalers expect carryover to be zero, and the other that they expect carryover to be what it was this year. Two stage least squares was used to estimate parameters and the results are presented in Table 6.1 below.

As can be seen, the results are consistent with the simple theory developed. Exvessel prices are positively related to wholesale prices $P_{W}$ and negatively related to infusions or harvest ( $I_{0}$ ) and to holding costs. ${ }^{12}$ The coefficient on the wholesale price can be taken to measure some handling and markup costs between the wholesale and exvessel markets. The implied adjustment coefficient is 1.27 cents per pound per million pounds ${ }^{13}$ and the implied holding costs are 2 cents per pound per month. Although the holding cost variable has low significance, it is in accord with what industry wholesalers estimate freezing and storage costs are in the halibut industry. Overall, these models do well in predicting exvessel price, with the dominant explanatory role played by wholesale price. As can be seen, the remainder of the specification does not add too much explanatory power, and one cannot really

[^21]| Independent Variable | Carryover Assumption <br> $C_{\tau}=0$ |  |
| :---: | :---: | :---: |
|  | .7211 <br> $(15.808)$ | .7195 <br> $(15.16)$ |
|  |  | $-.132 * 10^{-4}$ <br> $(1.82)$ |
| $\left(S_{0}+I_{0}\right) / \tau$ | $-.127 * 10^{-4}$ <br> $(2.04)$ | $-.223 * 10^{-1}$ <br> $(0.937)$ |
| $\tau / 2$ | $-.232 * 10^{-1}$ <br> $(0.926)$ |  |
| $D U M_{75}$ | -.312 <br> $(3.13)$ | -.308 <br> $(3.01)$ |
| $R^{2}$ | .9633 | .9611 |
| $D W$ | 2.19 | 2.25 |

Table 6.1: Exvessel Demand Equations: Carryover Parametric
distinguish between the two carryover assumptions on the basis of model performance.
While the above are reasonable estimates, it may not be sensible to assume that carryover is simply given or treated naively as a predetermined variable. In the previous chapter, we developed a fairly elaborate model that treats carryover as part of the optimization decision of inventory holders and hence part of the solution. The question thus arises, how does the more realistic assumption that carryover is endogenous alter the hypothesized structure of the exvessel demand equation? For expositional simplicity, suppose first that there are only two marketing periods (a present and future period), with expected prices $P_{0}$ and $P_{1}$ respectively, and that an infusion of $I_{r}$ is expected between seasons. Then, from the two period model of optimal carryover summarized by equation (5.6.8), the optimal carryover will be:

$$
\begin{equation*}
C(\tau)=\left(\frac{P_{1}-P_{0}}{c}\right) \frac{\tau}{2}-\frac{h \tau^{2}}{2 c}+\frac{S_{0}+I_{0}}{2}+\frac{C_{2 \tau}-I_{\tau}}{2} \tag{6.3.2}
\end{equation*}
$$

When this is inserted into equation (6.3.1) above, the resulting shadow price at the beginning of the first marketing period is:

$$
\begin{equation*}
\lambda_{0}=\frac{1}{2}\left(P_{0}+P_{1}\right)-\frac{c}{\tau}\left[\frac{S_{0}+I_{0}+I_{\tau}-C_{2 \tau}}{2}\right]-h \tau . \tag{6.3.3}
\end{equation*}
$$

In comparison with the derived demand curve computed without carryover determined endogenously, this derived demand curve is different in several respects. First, the impact of wholesale prices now incorporates both present and future marketing period prices (averaged). Second, the impact of infusions is similarly dependent on both present and future infusions, averaged. Finally, the impact of an increase in holding costs on the derived demand curve is doubled in this case, because these costs must be incurred over both periods. Hence, as would be expected with endogenized carryover, derived demand is more forward looking, since optimization must be accomplished over both periods. In this simple two period case, the demand for additions to inventory at the beginning of the first period must account for factors extending into the second period. A more general model incorporating
$N$ periods would similarly result in a derived demand curve in which the current willingness to pay depended upon expected wholesale prices and infusions for all of the $N-1$ future periods. ${ }^{14}$

Several estimation issues are posed by these more realistic formulations, most of which were foreshadowed in the previous section. In contrast to the model where carryover is assumed exogenous, in this case the endogenous and unobserved level of planned carryover in equation (6.3.1) is basically substituted out using (6.3.2) and replaced by its determinants, which are themselves current and future variables. Thus the new problem introduced in equation (6.3.3) is how to deal with the future variables whose expectations appear as explanatory variables. In particular, how should one handle the anticipated wholesale prices and infusions (harvests) for the upcoming period or periods?

In fact, there are various ways to handle expectations of future variables as discussed in the previous section, none completely satisfactory. One possibility, if futures markets existed, would be to use futures prices as proxies for expected future wholesale prices. In some cases, one might even have an analogous proxy for expected infusions, where, for example, regulatory authorities publish upcoming target quota levels. Unfortunately, this is rarely the case with inventories fisheries products and hence were are generally left with more indirect and complicated econometric methods. As discussed in the previous section, one relatively straightforward method is to develop a forecasting model outside of the derived demand curve equation and estimate the future variables from past variables. Then one can plug in the forecasts of future variables at each point, essentially assuming that industry participants have access to the same forecasting model. Another method,

[^22]the so-called direct solution method developed by Hansen and Sargent, is to postulate a general stochastic process for the unknown future variables and then rewrite the exvessel demand curve in terms of the parameters of the stochastic process. In principle one could then estimate both the general derived demand equation and the stochastic process for relevant variables jointly. This procedure is elegant although restricted to linear/quadratic model formulations and computationally burdensome for all but the simplest stochastic structures. Another method is to use the generalized method of moments technique developed by Hansen and Singleton. This allows from more general functional forms but is less useful for forecasting than for recovering structural parameter estimates.

We considered estimating more complicated structural equations of a monthly inventory model but abandoned the idea mainly because of data limitations. While excellent data are available from the IPHC on variables including catch, timing of seasons, and exvessel prices, the quality and quantity of data from the wholesale sector is less reliable. In particular, there are reasonable data on inventories but spotty data on monthly wholesale prices. In the end, this is not of great consequence for our purposes since we are interested in modeling and simulating the industry on an annual basis anyway. Hence it is sufficient for our purposes to derive a yearly exvessel demand curve whose reduced form or structural properties are simply consistent with theory.

For comparison with the above reported estimates of the naive parametric carryover models, we tried several alternatives which reflect the spirit of more forward looking models where carryover is endogenous. In the final analysis, most sophisticated treatments of expectations end up converting the sequence of expected future variables into estimates of these using past lagged variables. Hence some distributed lag formulation arises and the main distinction between alternative approaches involves the degree of complexity embedded regarding error structures and cross equation restrictions.

In Table 6.2 below, we show results from a distributed lag formulation of the structural form for the exvessel demand equation using OLS. We also present results for several structural wholesale demand equations, regressing wholesale price against total disappearances (beginning holdings+harvest-ending holdings), lagged wholesale prices, and marketing period length, using 2SLS.

The forward looking exvessel demand curve, which is a representation of the willingness to pay of inventory holders who are anticipating certain future wholesale prices $P_{W}$ and future infusions $I$, predicts exvessel prices closely and consistently with the inventory model with endogenous carryover developed in Chapter Five. The basic structure is as presented above, with the exception of the addition of lagged wholesale prices and infusions. The coefficients on these two variables and their lags represent composite effects of current and lagged prices and infusions on the expectations of future levels of those variables. We would expect the sum of the effects to be consistent with the inventory model, which they are (wholesale prices affect exvessel prices positively, infusions affect them negatively, in total). The holding cost variable is significant, correctly signed and of the same magnitude as in the naive model estimated above.

Wholesale prices are also predicted relatively well with a very simple model. We present several formulations, including one that simply regresses wholesale price against current harvest, one which also allows for partial demand adjustment with the inclusion of a lagged wholesale price variable, and a third which includes the lagged variable and the marketing period length. Interestingly, longer marketing period lengths seem to have a positive effect on wholesale prices. Since longer marketing period lengths imply shorter harvesting periods, one's first inclination would be to anticipate poorer quality raw product translating into a reduced retail and hence wholesale demand. That longer marketing periods increase wholesale prices is therefore somewhat surprising. Assuming the result is not simply spurious, other explanations may be habit formation effects within the marketing

| Exvess | $\begin{aligned} & \text { Price }\left(P_{t}^{E V}\right) \\ & \text { OLS } \end{aligned}$ | Wholesale Price $\left(P_{t}^{W}\right)$ 2SLS |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Ind Var |  |  |  |  |  |
| $P_{t}^{W}$ | $\begin{gathered} .5883 \\ (7.4677) \end{gathered}$ | $S_{0}+I_{0}-C_{\tau}$ | $\begin{gathered} -.306^{*} 10^{-4} \\ (8.5872) \end{gathered}$ | $\begin{gathered} -.2204^{*} 10^{-4} \\ (3.3897) \end{gathered}$ | $\begin{gathered} -.1832 * 10^{-4} \\ (3.2006) \end{gathered}$ |
| $P_{t-1}^{W}$ | $\begin{gathered} .1737 \\ (2.4311) \end{gathered}$ | $P_{t-1}^{W}$ |  | $\begin{gathered} .3147 \\ (1.8297) \end{gathered}$ | $\begin{gathered} .3494 \\ (2.3708) \end{gathered}$ |
| $I_{t} / \tau$ | $\begin{gathered} .6664^{*} 10^{-4} \\ (3.0445) \end{gathered}$ | $\tau$ |  |  | $\begin{gathered} .1008 \\ (2.639) \end{gathered}$ |
| $I_{t-1} / \tau$ | $\begin{gathered} -.7688^{*} 10^{-4} \\ (3.6613) \end{gathered}$ | CONST | $\begin{gathered} 3.3374 \\ (16.394) \end{gathered}$ | $\begin{gathered} 2.3383 \\ (4.0427) \end{gathered}$ | $\begin{gathered} 1.2968 \\ (2.0519) \end{gathered}$ |
| $\tau / 2$ | $\begin{gathered} -.3091^{*} 10^{-1} \\ (1.4339) \end{gathered}$ |  |  |  |  |
| $D_{75}$ | $\begin{gathered} .3412 \\ (4.0301) \end{gathered}$ |  |  |  |  |
| $R_{a d j}^{2}$ | . 9800 |  | . 8016 | . 8257 | . 8730 |
| D.W. | 2.0229 |  | 1.2200 | 1.6163 | 2.1112 |
| Asymptotic t statistics in parentheses |  |  |  |  |  |

Table 6.2: Exvessel/Wholesale Demand Curves: Structural form with Endogenous Carryover
period, better ability to plan marketing and product rotation, impacts of marketing periods with substitute or complementary products whose prices are not included, etc.

For comparison, we also estimated some simple reduced form exvessel price equations which collapse all impacts into those from two key endogenous variables in our system, namely the catch (infusion) and marketing length variables. Results are presented in Table 6.3. For these estimations, we again used OLS to estimate exvessel prices with contemporaneous and lagged values of harvests and marketing period lengths but without the explicit form and wholesale prices embedded in the structural equations. These equations capture the effects of harvest levels as they impact wholesale prices directly (and negatively). They also capture the impact of harvest levels as they affect exvessel prices directly (and negatively through diminishing marginal value of added inventory) and as they affect expectations of future harvest levels (positively and negatively). We would expect the overall impact of harvest level increases to be negative, which it is as can be seen by the sum of lags impacts. Similarly, marketing period length has a complicated impact, by affecting wholesale prices positively, and by affecting exvessel prices in an ambiguous way depending upon the relative roles of holding and adjustment costs. This ambiguity results because longer marketing periods increase the holding costs that must be paid, but simultaneously allow adjustment costs to be reduced by spreading out sales. As it turns out, for the parameters in this case, the adjustment cost effect outweighs holding cost impacts so that exvessel prices are positively affected by longer marketing periods. This is amplified by the above discussed finding of a similar positive effect in the wholesale market.

In sum, then, we have several alternative formulations of an exvessel demand equation which are consistent with different specifications of the inventory dissipation model developed in Chapter Five. The structural system estimated here contains both a wholesale market demand curve and an exvessel demand curve. It is assumed that wholesale prices are simply markdowns of retail prices and hence a price dependent wholesale demand curve

| Ind Var |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{t}$ | $\begin{gathered} -.1826^{*} 10^{-4} \\ (5.813) \\ \hline \end{gathered}$ | $\begin{gathered} -.1679 * 10^{-4} \\ (4.8492) \\ \hline \end{gathered}$ | $\begin{gathered} -.1303^{*} 10^{-5} \\ (.1565) \\ \hline \end{gathered}$ | $\begin{gathered} .1458^{*} 10^{-5} \\ (.1689) \\ \hline \end{gathered}$ | $\begin{gathered} .1393^{*} 10^{-5} \\ (.1970) \\ \hline \end{gathered}$ |
| $I_{t-1}$ |  |  | $\begin{gathered} -.1811 * 10^{-4} \\ (2.0056) \\ \hline \end{gathered}$ | $\begin{gathered} -.1790^{*} 10^{-4} \\ (1.9102) \\ \hline \end{gathered}$ | $\begin{gathered} -.4500^{*} 10^{-5} \\ (.4666) \\ \hline \end{gathered}$ |
| $I_{t-2}$ |  |  |  |  | $\begin{gathered} .1461^{*} 10^{-4} \\ (1.8984) \\ \hline \end{gathered}$ |
| $\tau_{t}$ | $\begin{gathered} .1023 \\ (2.6606) \\ \hline \end{gathered}$ | $\begin{gathered} .1179 \\ (2.6299) \\ \hline \end{gathered}$ | $\begin{gathered} .9994^{*} 10^{-1} \\ (12.4809) \\ \hline \end{gathered}$ | $\begin{gathered} .9670^{*} 10^{-1} \\ (3.3018) \\ \hline \end{gathered}$ | $\begin{gathered} .1282 \\ (3.3926) \end{gathered}$ |
| $\tau_{t-1}$ |  | $\begin{gathered} .684^{*} 10^{-1} \\ (.3965) \\ \hline \end{gathered}$ |  | $\begin{gathered} .1094^{*} 10^{-1} \\ (.2799) \\ \hline \end{gathered}$ | $\begin{gathered} .2355 \\ (.6652) \\ \hline \end{gathered}$ |
| $\tau_{t-2}$ |  |  |  |  | $\begin{aligned} & \hline-.1263 \\ & (.3693) \\ & \hline \end{aligned}$ |
| CONST | $\begin{gathered} 1.1563 \\ (2.9599) \\ \hline \end{gathered}$ | $\begin{gathered} .8420 \\ (1.6416) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 1.2954 \\ (3.0169) \\ \hline \end{gathered}$ | $\begin{gathered} 1.2245 \\ (2.3954) \\ \hline \end{gathered}$ | $\begin{gathered} .9695 \\ (1.619) \\ \hline \end{gathered}$ |
| $R_{a d j}^{2}$ | . 7382 | . 7303 | . 7882 | . 7732 | . 8473 |
| DW | 1.5701 | 1.5976 | 1.3325 | 1.300 | 1.335 |
| $\begin{gathered} \hline \text { sum of } \\ \text { lags }(I) \end{gathered}$ |  |  | $\begin{gathered} -.1941^{*} 10^{-4} \\ (5.8328) \\ \hline \end{gathered}$ | $\begin{gathered} -.1936^{*} 10^{-4} \\ (5.6141) \end{gathered}$ | $\begin{gathered} -.1771^{*} 10^{-4} \\ (5.2367) \\ \hline \end{gathered}$ |
| $\begin{aligned} & \text { sum of } \\ & \text { lags }(\tau) \end{aligned}$ |  | $\begin{gathered} .1347 \\ (2.4638) \end{gathered}$ |  | $\begin{gathered} .1086 \\ (2.0902) \end{gathered}$ | $\begin{gathered} .1391 \\ (2.2234) \end{gathered}$ |
| Dependent Variable: $P_{t}^{E V}$ |  |  |  |  |  |
| Asymptotic t statistics in parentheses |  |  |  |  |  |

Table 6.3: Exvessel Demand Curves: Reduced Form with Endogenous Carryover
can be estimated as a function of quantity. We also assume that the inventory sector takes prices generated in the wholesale marketing sector as given and then makes within period dissipation decisions according to an optimal plan. Embedded in this optimal plan are decisions about flow over the marketing period that balance holding and adjustment costs and that look forward to future harvest and marketing periods in planning optimal carryover. The results of all of these optimal plans can be summarized in a marginal value of additions to inventory curve, which we take to be the exvessel demand curve at the beginning of the marketing period. These same relationships can be summarized and estimated in a reduced form equation which makes exvessel prices a function of lagged values of harvests and marketing period lengths.

### 6.4 Discussion

For purposes of illustration, consider two of the demand equations whose parameter estimates were discussed above. Consider in particular the structural equation estimates for the exvessel demand curve in Table 6.2, together with the structural equation for wholesale demand which includes both lagged price and season length:

$$
\begin{align*}
& P_{t}^{E V}=.588 P_{t}^{W}+.174 P_{t-1}^{W}+.666 * 10^{-4} \frac{H_{t}}{\tau_{t}}-.769 * 10^{-4} \frac{H_{t-1}}{\tau}-.0309 \frac{\tau}{2}  \tag{6.4.1}\\
& P_{t}^{W}=1.297-.183 * 10^{-4} H_{t}+.101 \tau_{t}+.349 P_{t-1}^{W} .
\end{align*}
$$

Both of these are short run equations whose dynamics will be complicated by the lagged variables for wholesale price lagged infusions (harvests) and marketing period length in the exvessel equation. For simplicity it is convenient to convert these into long run equations by assuming the equilibrium values for prices, quantities, and marketing period length. The long run equations can then be written as:

$$
\begin{align*}
& \bar{P}^{W}=1.992-.281 * 10^{-4} \tilde{H}+.55 \bar{\tau}  \tag{6.4.2}\\
& \bar{P}^{E V}=.762 \bar{P}^{W}-.103 * 10^{-4} \frac{\bar{H}}{\bar{\tau}}-.031 \frac{\bar{\tau}}{2} .
\end{align*}
$$

The long run elasticity of wholesale demand, calculated at the sample means for harvests ( 51.57 million pounds) and marketing period length ( 7.9 months) can be computed as:

$$
\begin{equation*}
1 /\left[\frac{d \bar{P}^{W}}{d \bar{H}} \frac{\bar{H}}{\bar{P} W}\right]=\frac{-1}{.8646}=-1.1566 \tag{6.4.3}
\end{equation*}
$$

Thus at the sample means, wholesale demand is elastic. There is a wide range of harvest levels and marketing periods over the estimation period, however, and the data spans elasticity estimates that are in both the elastic and inelastic ranges.

With regard to the derived demand elasticity for the exvessel demand curve, several points are worth mentioning. First, the derived demand curve can be considered a markdown equation, since exvessel price is proportional to wholesale price, modified by terms that account for both adjustment and holding costs. For example if both of these costs were zero, the inverse exvessel demand curve would be a simple markdown relationship. The effect of adjustment costs is basically to make the exvessel demand curve more inelastic since the slope of the curve is related to the adjustment cost parameters. Similarly, the intercept of the inverse exvessel demand curve is affected by the holding cost parameter. Higher holding costs shift the inverse exvessel demand curve downward by the average holding cost over the whole marketing period.

The price flexibility of exvessel demand with respect to quantity thus has two components. One is a direct component associated with adjustment costs, and another is an indirect component associated with the fact that increases in quantities reduce wholesale price, which in turn reduce exvessel demand. We can compute the direct component as the price flexibility of the exvessel demand curve, holding wholesale price as given and evaluated at the sample means:

$$
\begin{equation*}
\left.\frac{d \bar{P}^{E V}}{d \bar{H}} \frac{\bar{H}}{\bar{P}^{E V}}\right|_{\bar{P} W}=-\left(.103 * 10^{-4} \frac{1}{\bar{\tau}} \frac{\bar{H}}{\bar{P}^{E V}}\right)=-\frac{1}{\bar{\tau}} .5192 \tag{6.4.4}
\end{equation*}
$$

Thus the own price flexibility of the derived demand is small and dependent on the marketing
period length. Correspondingly, the own direct price elasticity of exvessel demand (due to the presence of adjustment costs), is relatively large since it is the inverse of the flexibility.

The full or total exvessel demand elasticity accounts for both the direct effect due to adjustment costs and the indirect effect due to the wholesale price effect. Thus we can write the full price flexibility as:

$$
\begin{equation*}
\frac{d \bar{P}^{E V}}{d \bar{H}} \frac{\bar{H}}{\bar{P}^{E V}}=\left[.762 \frac{d \bar{P}^{W}}{d \bar{H}}-.103 * 10^{-4} \frac{1}{\bar{T}}\right] \frac{\bar{H}}{\bar{P}^{E V}} \tag{6.4.5}
\end{equation*}
$$

This depends upon the marketing period length and we can compute the full long run exvessel demand elasticities evaluated at the sample mean harvest level and various marketing period lengths as:

| $\tau$ | full elasticity |
| :--- | :--- |
| 1 | -.6258 |
| 5 | -.8457 |
| 8 | -.8744 |
| 10 | -.8850 |

Thus evaluated at the sample means, the exvessel demand curve is more inelastic than the wholesale demand curve. This is what we expect from intuition since the full derived demand elasticity transmits the wholesale price effects through to the exvessel demand curve by shifting the intercept. Figure 6.3 show how an increase in harvest levels from $H_{0}$ to $H_{1}$ would first reduce wholesale prices. These reduced wholesale prices would shift the exvessel demand curve downward and the full effect would be that depicted by the intersection of the price/quantity combinations on the two exvessel demand curves. Thus increases in harvests to be marketed have two effects. First, for a given marketing period, more product must flow into the market each month, and hence there is an increase in adjustment cost which is reflected in a movement along the ceteris paribus exvessel demand curve. Second, since more product enters the wholesale market, wholesale prices must fall, reducing exvessel prices by shifting the inverse demand curve.


Figure 6.3: Wholesale and Exvessel Market Links

How does a change in the marketing period affect exvessel demand? In a similar fashion, exvessel demand is affected both through own effects and through effects originating in the wholesale market. As discussed above, we found that longer marketing periods resulted in higher wholesale prices. In the exvessel market, the own impact is ambiguous. because longer marketing periods reduce adjustment costs but increase holding costs. The own impact of a change in marketing period length on (long run) exvessel prices can be computed as:

$$
\begin{equation*}
\left.\frac{d \bar{P} E V}{d \bar{\tau}}\right|_{\bar{P} W}=\frac{.103 * 10^{-4} \bar{H}}{\bar{\tau}^{2}}-\frac{.0309}{2} \geq 0 \tag{6.4.6}
\end{equation*}
$$

and this can be positive, negative, or zero, depending upon the relative sizes of the harvest level and marketing period length. For the parameters of the exvessel demand curve discussed here, it can be seen that the sign of the direct effect of an increase in marketing period length on exvessel price is positive or negative depending upon whether:

$$
\begin{equation*}
.66 * 10^{-3} \bar{H}-\tau^{2} \gtreqless 0 \tag{6.4.7}
\end{equation*}
$$

Figure 6.4 plots the equation separating these two cases. As can be seen, for a given harvest level, an increase in marketing period length will increase exvessel prices, ceteris paribus. when the marketing period is short and decrease them when $\tau$ is large. The intuition is as discussed above: with short marketing periods adjustment costs are large relative to holding costs becanse a large amount of product must be sold over a short period. In these cases. we would expect the marginal value of extra marketing period length to be positive, which it is. and vice versa.


Figure 6.4: Exvessel Price Dependence on Marketing Period Length

## Chapter 7

## Rents, Regulations, and Revenues:

## Modeling Regulated Open Access

## Resource Use

### 7.1 Introduction and Overview

In this chapter we bring together all of the theoretical and empirical components from previous chapters and use them to model regulated open access resource use in our case study. As set out in the introduction, the objective of this thesis is to develop and test a new conceptual model which takes the Gordon model of rent dissipation as a foundation and adds some dimensions which make it more applicable to modern renewable resource exploitation settings. The two components which have been added are depictions of the respective roles of the regulatory sector and the market in the rent dissipation process. Particularly important in the conceptualization of the new model is the role of dynamics, in governing the nature of interaction between the regulators and the industry, in determining
exvessel prices through the mechanisms of inventory dissipation and carryover decisions, and in connecting up harvests with subsequent biomass levels.

To put the whole picture in perspective, we return to a schematic of our system (reproduced and reconfigured slightly as Figure 7.1 below). As we discussed in the introduction, the basic Gordon model depicts open access resource use as a process governed by the pull of rents which draw inputs in until the value of average product is equal to opportunity costs. In his model, several factors were abstracted from purposefully in order to focus on rent dissipation. One of these was the output market, which he simply took as exogenous and implicitly unaffected by the rent dissipation process itself. Another factor Gordon abstracted from was the connection with the biological system and in particular the role of effort determining catch which in turn should govern ecological dynamics. Finally, since most fisheries at the time Gordon wrote were not regulated, Gordon ignored the role the regulations might have in the rent dissipation process.

We have added biomass dynamics, the regulatory sector, and the market sector to our analysis. Chapter Three develops a new theoretical model of regulator/industry interaction which depicts each as goal seeking, and which derives testable hypotheses about joint behavior. Chapter Four empirically tests the models of Chapter Three, estimating parameters of the technology (cost and production function parameters) as well as speeds of adjustment, holding market effects constant. Chapter Five develops a new model of inventory dissipation which depicts the wholesale industry as maximizing the profits from within and between market period sales and carryover decisions. This model casts the forward looking nature of the optimization problem in a structure (an adjustment cost framework) which is particularly illuminating since it allows derivation of closed form representations of optimal decisions. In Chapter Six, we estimate parameters of the inventory decisions. These are then used to construct stock demand curves which determine exvessel prices as a function of outputs from the model of regulator/industry interaction. The full model


Figure 7.1: Thesis Recap
thus determines capacity, season length, harvest, biomass, and prices endogenously in a simultaneous system.

The organization of this chapter is as follows. First, in the next section, we close the biological part of the model by linking up the target harvest or quota (assumed given in Chapters Three and Four) with biomass via a quota rule. Then in the third section we present simulation results from the full model. These are run in a manner which allows us to compare results of our model with various depictions of the Gordon model which ignore facets of the problem that we have included.

### 7.2 The Quota/Biomass Link

Chapters 3 and 4 developed and estimated models based on the simplifying assumption that the harvest quota can be taken as exogenous. Additionally, we did not consider the evolution of the biomass which would naturally occur under any circumstance except the fortuitous one where the quota is correctly chosen at the steady state harvest level. In this section we close the model by postulating and estimating a simple relationship between observed quota levels and biomass.

The basis for this simple extension is the depiction of regulatory behavior already discussed in Chapter 3, namely as a two stage process where, first, a quota level is determined, and then the season length is set to approach that quota, given the capacity choice by the industry. The question this raises is, how is the quota level chosen in the first place? We postulate that the regulatory structure has some notion of a "safe" level of biomass for the species. For example, regulators may wish to keep the biomass close to the level yielding maximum sustainable yield $X_{M S Y}$. When biomass is below that level, some rule must be used to decide how much may be taken. For example, a simple rule which would get the biomass to $X_{M S Y}$ from any arbitrary stock fastest would be: set $Q(t)=0$ if $X(t)$
is less than $X_{M S Y}$ and harvest some maximum level $Q_{M A X}$ if $X(t)$ is greater than $X_{M S Y}$. When the stock equals $X_{M S Y}$, harvest the total yield $H_{M S Y}$ for that level. Alternatively, a simple rule which sets the quota equal to a fraction of $H_{M S Y}$ would achieve the result of getting to $X_{M S Y}$ although more gradually.

Needless to say, one could also choose any other biomass level to key on as the safe level. Another possibility is the so-called $F_{0.1}$ strategy (see Hilborn and Walters[52]), which proscribes a constant exploitation rate designed to leave exploitation at a level slightly less than the one that maximizes yield per recruit. This is actually used in many fisheries, including some in eastern Canada and elsewhere (see Doubleday et. al.[26], Andrew and Butterworth[5]) and its ultimate effect is to leave the biomass at a level larger than the one yielding the largest sustained yield. These and others have been compared under stochastic settings to examine both mean yields and variability by Deriso[24] in 1985.

With this background in mind, we propose to estimate the quota rule that has been implicitly in use as reflected in actual decisions by the IPHC over the 1935-1978 period. That is, we postulate that regulators set a quota according to some function of biomass:

$$
\begin{equation*}
Q(t)=k(X(t)) \tag{7.2.1}
\end{equation*}
$$

Once functional forms for $k$ are proposed, one can use data on $Q(t)$ and biomass $X(t)$ to test alternatives.

Using data already described, we first estimated linear and quadratic functional forms for $k$ by regressing published quota targets against biomass estimates for Areas 2 and 3. As we discovered in our estimates of regulatory behavior, the biomass crash that began to occur in both regions in the 1960s also apparently affected the implicit rules used to set quotas. Recursive residual tests and Chow tests for structural stability were suggestive of a break in behavior which we took to occur here in 1965 also. Hence a second set of regressions were run, using dummy variables as slope and intercept shifters with dummy
values of one during the post-1965 period. Quadratic formulations did not prove to fit better than linear specifications and so Table 7.1 reports using linear results, corrected for first order autocorrelation using the Cochrane-Orcutt procedure. As can be seen, quota policies in Area 2 can be described by a simple rule, the structure of which breaks in 1965. After 1965, a more conservative rule is operative, with a lower intercept and a steeper slope. Similarly in Area 3, prior to 1965 the quota rule can essentially be described as a constant set at 28 million pounds. After 1965, the rule again becomes more conservative, with a lower intercept and steeper slope. These are shown diagrammatically below, superimposed on biological yield curves.

To complete the task of closing the model with a biological system, we also estimated a yield curve, which depicts the growth in halibut biomass as a function of the biomass level and harvest. As Chapter 1 discussed, a popular biological dynamic model is one due to Lotka and Volterra which postulates that stock dependent mortality factors will cause a population to approach its carrying capacity level $X_{M A X}$ in a fashion that is S-shaped as a function of time. This in turn implies that the yield curve can be written as:

$$
\begin{equation*}
\dot{X}=F(X(t))=a * X(t)-b * X(t)^{2}-H(t) \tag{7.2.2}
\end{equation*}
$$

where $a$ and $b$ are parameters and $H$ is he harvest level. With this specification, the carrying capacity $X_{\max }$ is $a / b$, the stock that yields the maximum sustainable yield $X_{\text {MSY }}$ is $a / 2 b$, and the yield $H_{\text {MSY }}$ is $a^{2} / 4 b$. We also estimated these parameters using yield and harvest data for Areas 2 and 3. Biomass less lagged harvest was regressed on lagged biomass and lagged biomass squared, correcting for first order autocorrelation. The results are presented in Table 7.2. These are in accord with results from other studies (see Criddle[19], Criddle and Havenner[20], Deriso[24], and Capalbo[14]).

The quota rules revealed by actual decisions and their properties when combined with yield curve estimates are shown in Figure 7.2. As can be seen, the maximum sustainable

| Area 2 |  |  | Area 3 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | w/o break | with break | w/o break | with break |
| CONST | $\begin{aligned} & 12.329 \\ & (2.990) \end{aligned}$ | $\begin{aligned} & 18.295 \\ & (6.255) \end{aligned}$ | $\begin{aligned} & 16.417 \\ & (2.807) \end{aligned}$ | $\begin{gathered} 27.993 \\ (3.137) \end{gathered}$ |
| CONST* ${ }_{65}$ |  | $\begin{gathered} -12.840 \\ ((2.9431) \end{gathered}$ |  | $\begin{array}{r} -18.659 \\ (1.551) \end{array}$ |
| slope | $\begin{gathered} .8974^{*} 10^{-1} \\ (2.943) \end{gathered}$ | $\begin{gathered} .5972^{*} 10^{-1} \\ (2.291) \end{gathered}$ | $\begin{gathered} .5752^{*} 10^{-1} \\ (1.703) \end{gathered}$ | $\begin{gathered} .3538^{*} 10^{-4} \\ \left(.754^{*} 10^{-3}\right) \end{gathered}$ |
| slope ${ }^{*} \mathrm{D}_{65}$ |  | $\begin{array}{r} .10345 \\ (2.477) \end{array}$ |  | $\begin{gathered} .8746^{*} 10^{-1} \\ (1.307) \end{gathered}$ |
| $\rho$ | $\begin{gathered} .968 \\ (25.27) \end{gathered}$ | $\begin{gathered} .8199 \\ (9.105) \end{gathered}$ | $\begin{gathered} .925 \\ (15.860) \end{gathered}$ | $\begin{gathered} .928 \\ (15.95) \end{gathered}$ |
| $R^{2}$ | . 9405 | . 9418 | . 8318 | . 8360 |
| D.W. | 1.326 | 1.482 | 2.008 | 1.942 |
| Asymptotic t statistics in parentheses |  |  |  |  |

Table 7.1: Estimated Implicit Quota Rules

|  | Area 2 | Area 3 |
| :---: | :---: | :---: |
| $1+\mathrm{a}$ | 1.4171 <br> $(15.043)$ | 1.3227 <br> $(21.066)$ |
| b | $-.1602^{*} 10^{-2}$ <br> $(1.966)$ | $-8782^{*} 10^{-3}$ <br> $(2.623)$ |
| $\rho$ | .895 <br> $(13.026)$ | .865 <br> $(11.184)$ |
| $R^{2}$ | .9914 <br> D.W. <br> 1.894 | 1.257 |
| Asymptotic t statistics in parentheses |  |  |

Table 7.2: Estimated Yield Curve Parameters

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
yield in Area 2 can be harvested with a harvest level of 27 million pounds. The post-65 quota rule implied by actual behavior on the part of regulators has some slope and intersects the yield curve virtually at $X_{M S Y}$. The relevant part of the quota rule function is that below the yield curve; when the stock is below the targeted safe rule, the target is some fraction of the level associated with the sustainable safe level and hence the stock will rise over time. ${ }^{1}$ If the stock is greater than the safe level, a catch larger than the yield is allowed, bringing the stock down to a safe yield over time. Over the period actual biomass has ranged between about 60 and 180 million pounds while the quota has ranged between 11 and 28 million pounds. For Area 3, the estimated quota rule implied by actual behavior intersects beyond $X_{M S Y}$ at 218 million pounds of biomass. Over the 1935-1978 period, biomass in Area 3 has ranged between 82 and 265 million pounds and quota has ranged between 11 and 38 million pounds.

### 7.3 Simulating Regulated Open Access Use

In this section we present simulation results, comparing successive contributions of our modeling exercise to those of the basic Gordon model. To place the results in a real world setting, we assume the perspective of a fisheries analyst in 1977, faced with the task of forecasting what might evolve in Area 2 under continuation of open access. Thus we ask, how would we expect the halibut fishery to unfold from this date (1977) forward, given the

[^23]

Figure 7.2: Quota Rules
following parameters:

| biomass | 53.5 million pounds |
| :--- | :--- |
| fishing capacity | 2.21 thousand skates |
| season length | 73 days |
| wholesale price | $\$ 2.89 / \mathrm{lb}$ |
| exvessel price | $\$ 1.95 / \mathrm{lb}$ |

As a point of reference, we first develop a prediction that falls out of the basic Gordon model. That is, we answer the above question by applying the logic contained in Gordon's 1954 paper, without embellishments. Then we make a first embellishment, that of adding an explicit biological sector, which links up Gordon's instantaneous rent dissipation model with the corresponding harvest level and the results of that harvest on biological dynamics. This is similar to Vernon Smith's model of bioeconomics dynamics, although it is a special case which retains the Gordon assumption of complete and instantaneous rent dissipation in each period. Then we follow with two more embellishments associated with the addition of a regulatory and a market sector.

### 7.3.1 The Static Gordon Model

The basic Gordon model develops a powerful prediction based on a very simple behavioral hypothesis. The essential core of Gordon's model is that entry will proceed until all rents are dissipated, or until average product of the last unit of capacity equals its opportunity costs. Gordon took as given the price and the biomass level. Hence quantification of Gordon's prediction is simply a matter of using production and cost function parameters to solve for the rent dissipating level of $E_{0}$. Suppose that we take our estimated levels of $q$, $f$, and $v(q=.0015, f=.99, v=.0835)$ to be the correct values, and we combine them with the above actual levels for the exvessel price $P_{E V}$ and the biomass level $X$. Then Gordon
would predict a rent dissipating equilibrium where the value of average product equals the opportunity cost or:

$$
\begin{equation*}
\frac{P X_{0}\left[1-e^{-q E T}\right]}{E}=f+v T \tag{7.3.1}
\end{equation*}
$$

What should be assumed about the season length in this case? In the context of our conceptual model, it is sensible to simply assume that the season length will be $T_{\text {max }}$ or the level where fishermen would not choose to fish another day because variable costs are just equal to returns. Under these assumptions, the predictions from the basic Gordon model (7.3.1) would be that the season length and capacity would equilibrate at:

$$
\begin{array}{ll}
E_{0} & =13.83 \text { thousand skates } \\
T_{\max } & =30.28 \text { days } \\
H & =24.95 \text { million pounds. }
\end{array}
$$

Thus the static Gordon model of instantaneous and complete rent dissipation, estimated using representative cost and production function parameters and values for exvessel price and biomass at their 1977 levels, predicts an equilibrium level of capacity of about 14 thousand skates fished over a season length of about a month. That these values were not observed in 1977 is due, of course, to the fact that the halibut fishery was not a pure open access fishery but was in fact regulated. Thus these values can be considered a first estimate of what might happen under abandonment of the regulatory system.

### 7.3.2 The Gordon Model with Biological Dynamics

A problem that arises right away with these static predictions is that the level of capacity $E_{0}=13.83$ fishing over a season length of $T_{\max }=30.28$ will produce an aggregate harvest level which will not be sustainable except under fortuitous circumstances. For example, with the biomass level which was estimated at 53.5 million pounds in 1977, the above predicted values for $E_{0}$ and $T_{\text {max }}$ would result in a harvest level of about 25 million
pounds. But if our yield curve parameters are correct, at a biomass level of 53.5, any harvest level greater than 17.73 will cause the biomass to fall. Hence Gordon's model would miss, from the start, an important part of the dynamics of the real system, and would predict a rent dissipating equilibrium which is really not a sustainable equilibrium.

Consider then, a first extension in which we append to the Gordon model a dynamic model of the biomass/harvest dynamics. Then, if we continue to assume instantaneous rent dissipation, our dynamic Gordon model would be:

$$
\begin{align*}
\frac{P X_{t}\left[1-e^{-q E_{t} T_{t}}\right]}{E_{t}} & =f+v T_{t} \\
X_{t+1} & =(1+a) X_{t}-b X_{t}^{2}-X_{t}\left(1-e^{-q E_{t} T_{t}}\right)  \tag{7.3.2}\\
T_{t} & =T_{\max }=-\frac{1}{q E_{t}} \ln \left[\frac{v}{P q X_{t}}\right] .
\end{align*}
$$

Note that this is no longer a simple static model that can be solved for a single level of capacity and season length. Instead, this is a dynamic model which would evolve from the initial period with a reduced biomass level, followed by another period with the harvest level determined by rent dissipation, and so on.

We simulated this dynamic extension to the Gordon model, using the same parameters and starting values as above. As expected, since initial harvest is too high for a bioeconomic equilibrium using the static Gordon model, the simulation predicts a drop in the biomass from its initial 1977 value. After initial period increases, biomass, the season length, and effort all smoothly and asymptotically fall and end up at values lower than the 1977 initial values with:

$$
\begin{aligned}
\bar{E} & =5.92 \text { thousand skates } \\
\bar{T} & =48 \text { days } \\
\bar{X} & =43.73 \text { million pounds } \\
\bar{H} & =15.176 \text { million pounds. }
\end{aligned}
$$

In comparison with the simplest static Gordon model, then, adding biomass dynamics affects predictions by generating long run equilibrium values for the biomass which are
lower than the level prevailing in 1977 and embedded in the predictions of the simple static Gordon model. The dynamic Gordon model also predicts an equilibrium effort level that is lower, and a season length which is longer than the static model predictions. These results with respect to season length and effort occur because total revenues are affected by the stock size. With biomass dynamics generating reduced stock sizes, the harvest production function shifts down over time, mitigating the open access effect somewhat by reducing rents. This in turn results in a lower long run equilibrium level of effort, harvesting the reduced catch over a longer period than predicted with the static model.

### 7.3.3 A Model of a Regulated Open Access Fishery

With the above two baseline Gordon model predictions, we turn now to the results predicted with the modified model developed in this thesis. The first of the major contributions of this thesis is the modification of the basic Gordon model which involves introducing a regulatory sector. As we have discussed, we assume that the regulatory sector is motivated by goals and follows a two stage process by first setting quota targets according to a quota rule tied to the biomass level, and second, by choosing the instrument level (season length) to achieve the target, conditional on fishing capacity and biomass.

In this section, we simulate the role of the regulatory sector and examine how this modification alters the predictions from the naive models above. To isolate this part of the model development, we hold the exvessel price constant at its 1977 level and ignore the feedback effects that the market generates. Thus these results add the regulatory sector, and simulate the interaction between the industry and regulators as they approach a regulated open access equilibrium from values for biomass and exvessel price in 1977. The model
structure simulated is:

$$
\begin{align*}
P^{E V} X_{t}\left[1-e^{-q E_{t} T_{t}}\right] & =\frac{1}{1-\theta} *\left(f E_{t}+v T_{t} E_{t}\right)-\frac{\theta}{1-\theta} *\left(f E_{t-1}+v T_{t-1} E_{t-1}\right) \\
T_{t} & =\gamma T_{t-1}+\frac{1-\gamma}{q E_{t}} \ln \left[\frac{X_{t}}{X_{t}-Q_{t}}\right] \\
Q_{t} & =5.355+.163 X_{t}  \tag{7.3.3}\\
X_{t+1} & =(1+a) X_{t}-b X_{t}^{2}-X_{t}\left(1-e^{-q E_{t} T_{t}}\right) \\
P^{E V} & =1.95 .
\end{align*}
$$

An important part of the predictions produced by the model of industry/regulator interaction concerns approach paths to equilibrium. As we discussed in Chapter 3, the qualitative nature of the combined dynamics depends importantly on the respective speeds of adjustment to rents and to the quota-based targeted season lengths. We demonstrated in Chapter 4, for example, that (holding prices, quota, and biomass constant) with the adjustment speed coefficients estimated over the post-1965 period, the approach path is oscillatory rather than asymptotic. This is basically the result of the fact that both groups seem to be reacting relatively quickly to both rents and to deviations from long run equilibrium. This is a significant conclusion and one that would not be expected with the use of naive versions of the Gordon model. When biomass dynamics are added to the system, we essentially have a three dimensional phase plane system in ( $E, T, X$ ) space. Hence an important question is how the qualitative characteristics of the industry/regulator sector dynamics are affected by the additional biomass dynamics. In particular, do biomass and quota dynamics smooth or amplify the overshooting tendencies observed when $X$ and $Q$ were held constant?

We ran these simulations, holding the marketing sector constant, and allowing the biomass and quota to recover from their low 1977 values. As will be seen below, the simulation results incorporating regulatory/industry interaction dynamics with biomass dynamics and the quota rule still produce some under and over-shooting in transition to the long run equilibrium. These are altered somewhat in comparison to the results presented in Chapter 4 because biomass dynamics shift both the industry and regulator isoclines.


Figure 7.3: Phase Diagram with Biomass Increase

As biomass rises and approaches the safe level embodied in the quota rule, the harvest production function shifts up and revenues rise, ceteris paribus. This shifts the industry isocline upward. Simultaneously, the regulatory isocline shifts inward because the same catch can be taken with less aggregate effort. The combined effects, as discussed in the comparative statics section in Chapter 3, should be to increase the equilibrium capacity level and reduce season length as biomass rises over time. These anticipated effects are shown in Figure 7.3, which depicts a snapshot of the industry/regulatory system at two levels of biomass and quota. Note that the transition dynamics which are added to the system by biomass and quota growth may dampen the tendency for the industry/regulatory system to overshoot because the equilibrium is shifting northwestward. This dampening effect is partly due to the starting position of the system, however. Since the system starts at 1977 values associated with the southeast quadrant, as the biomass grows and shifts the equilibrium northwestward, the initial overshooting tendency should diminish. On the other hand, if starting values for the capacity and season length placed the system initially in the


Figure 7.4: Simulation of Biomass: Market Exogenous
southwest quadrant of the phase diagram, the tendency to overshoot could be amplified.
These patterns foreshadowed by the theoretical model discussed in Chapter 3 are, in fact, exactly what happens in the simulation runs. As Figure 7.4 shows, the initial biomass is high relative to its long term trajectory ${ }^{2}$, generating relatively large initial harvest levels which overshoot the quota during early periods (see Figure 7.5). These high harvest levels, in turn, generate positive and significant revenues and rent levels during the first few years as can be seen in Figure 7.6. The large rent levels result in accelerated entry of new capacity, exacerbating the already high harvest levels asssociated with high biomass levels. The ultimate impact on the biomass level is to cause a minor collapse as harvests exceed yields for a brief period.

During the next phase of the joint dynamics the system recovers, as a result of internal dynamics associated with rents and entry and as a result of conscious regulatory

[^24]

Figure 7.5: Simulation of Quota and Harvest: Market Exogenous


Figure 7.6: Simulation of Rents and Revenues: Market Exogenous


Figure 7.7: Simulation of Capacity and Season Length: Market Exogenous
system action. First, because of the high initial harvest levels which have exceeded biomass yield, the biomass begins to fall, shifting the industry production function and revenue function downward. This causes rents to actually turn negative, which slows the initial burst of capacity entry. At the same time, regulators severely cut the season length in response to the overshooting of the harvest target and the biomass reduction, amplifying the rent reversal (see Figure 7.7). Both of these forces cause actual harvests to fall below yield, allowing the biomass to return to its long path of buildup towards the safe stock level.

During the long buildup period, the biomass grows, allowing the regulatory authorities to relax the quota and allow harvests to rise. There are two impacts of this. First, the production function shifts up as the biomass grows, allowing, ceteris paribus lower amounts of aggregate effort to take any given level of quota. At the same time, the actual quota is relaxed, requiring ceteris paribus more effort. In this particular case, the biomass effect
outweighs the increasing quota effect so that total effort (capacity times season length) is slowly reduced as the system moves towards the safe stock.

Although total effort falls slowly over time, its composition is dependent on revenues, rents, and regulations. In particular, because biomass growth is shifting up the industry production function, revenues rise over time. This positively impacts rents and results in entry of capacity. In order to contain the growing capacity, regulators match increases with reduced season lengths. Hence over the long run, a larger and larger industry is harvesting over a shorter and shorter season.

In the long run, the system arrives at the equilibrium level defined by:

$$
\begin{aligned}
& E_{0}=40.69 \text { thousand skates } \\
& T_{0}=3.74 \text { days } \\
& X_{0}=133.2 \text { million pounds } \\
& H_{0}=27.14 \text { million pounds. }
\end{aligned}
$$

Besides the above discussed transition dynamics, there are important changes in the predicted long run equilibrium when the regulatory sector is added. First, the regulatory sector guides the system to the safe stock level over the long run, by gradually raising the allowable catch as the biomass grows. This is in contrast to the dynamic open access and unregulated scenario, where biomass falls from its initial level, because there is no institution governing harvest levels. Second, the very success of the rebuilding program causes biomass and allowable catch to rise, increasing revenues and rents and drawing in fishing capacity as Gordon suggested. The main point of departure of our new model with the Gordon model, however, concerns the further role of the regulatory sector in mitigating the potential effects of this larger capacity by shortening the season. Importantly, the increased potential rents associated with the larger biomass and quota draw in an even larger level of potential capacity, which must be stifled by the regulatory sector so that overexploitation doesn't occur. Thus the economic consequences of open access are amplified and waste is
more severe as the larger capacity operates over a very short season. This mirrors, of course, the current situation in the halibut fishery.

### 7.3.4 Regulated Open Access with a Marketing Sector

The second major contribution of this thesis is the development of a marketing sector model which reflects both the impacts of regulations (season length and harvest quota) on price, and also the feedback effects of prices on effort and other endogenous variables. One result that falls out of the above simulation is that biomass and quota growth increase revenues and hence attract an even larger amount of fishing capacity than would occur in pure open access. A question which arises, however, is could these added incentives that arise because of the shifting revenue function be dampened by the market? This might happen, for example, if larger harvests drove the elasticity of exvessel demand into the inelastic range, or if changes in season length shifted exvessel demand downward. In this section we utilize the same model as in section 7.3 .3 above but with the addition of exvessel and wholesale price equations which close the model. The two equations added are:

$$
\begin{align*}
P_{t}^{E V} & =.588 P_{t}^{W}+.174 P_{t-1}^{W}+.666 * 10^{-4} \frac{H_{t}}{\tau_{t}}-.769 * 10^{-4} \frac{H_{t-1}^{\tau}}{\tau}-.0309 \frac{\tau}{2}  \tag{7.3.4}\\
P_{t}^{W} & =1.297-.183 * 10^{-4}\left(S_{0}+H_{t}-C_{\tau}\right)+.101 \tau_{t}+.349 P_{t-1}^{W} .
\end{align*}
$$

Before discussing the simulation results from the full model, it is worthwhile to review the model structure and the conceptual connections between the regulator/industry sector and the market. As the above two equations suggest, the manner in which the complete model is closed is to link up exvessel and wholesale prices to two variables, the harvest level and the marketing period length. Since marketing period length is inversely related to the regulated season length, these two variables are endogenously determined in the regulator/industry section.

We assume that the equilibrium in the marketing sector is determined by a process as follows. First, we (implicitly) assume a retail sector which generates wholesale prices in a markdown fashion from a periodic retail demand function. We aggregate to an annual level by assuming, for example, that there is a single annual wholesale inverse demand function ${ }^{3}$ which depends upon the amount marketed over the whole period (beginning inventories plus harvests less ending inventory), and marketing period length.

The total supply to the retail sector and its temporal pattern depends upon the inventory dissipation behavior of the wholesale sector. Inventory holders plan within marketing period supplies in a manner which reflects the annual wholesale price, adjustment and holding costs, and carryin and carryout plans. Carryin and carryout plans are determined by expectations of future period wholesale prices, harvests, marketing period lengths, as well as adjustment and holding cost parameters. These expectations, in turn, are generated by observations of past values of these variables. Hence the exvessel inverse demand curve is a function of contemporaneous values of wholesale prices, harvests, and marketing period lengths, as well as lagged values. We also assume that the process generating expectations that is embedded in the exvessel demand curve is unchanging over time. Finally, we assume that the difference between carryin and carryout is of a second order in magnitude, so that wholesale prices can be simulated as a function of the harvest level alone. This turns out to be approximately true in the data used to estimate wholesale prices.

Figures 7.8 through 7.12 show some of the simulation plots from the full (regulator plus market sector) model simulations. These use the production and cost parameters for Area 2 discussed above, adjustment speed parameters estimated for the post 1965 period, and exvessel and wholesale price equations from Table 6.2. As can be seen from Figures 7.8 and 7.9 , the quota rule holds aggregate harvests below the yield for much of the period, allowing the biomass to grow over time towards the safe level as it does in the case discussed

[^25]

Figure 7.8: Simulation of Quota and Harvest in the Full Model
above. The initial period values (relatively high $X$ and $P^{E V}$ ) create startup period dynamics which overshoot the quota by about $30 \%$ in the first few years. This is primarily caused by rents attracting excessive capacity, which is not fully dampened by regulators as discussed above. However, unlike the above case, within a few periods this entry pressure induced by rents is mitigated partially by falling wholesale and exvessel prices (see Figure 7.10) which (with falling biomass and harvests) drop by enough to reduce revenues and rents. This dampens growth of capacity and generates falling harvest levels for a brief period. As the harvest levels fall temporarily, another dynamic reversal occurs as prices recover, and a turnaround follows with a long sustained period of rising harvests and biomass. During the recovery, rising harvest levels induce reductions in both wholesale and exvessel prices. These are caused by and cause changes in the regulatory sector. Price changes are caused by regulations in two ways. First, increased quotas and harvests directly reduce prices. Second, as the fishing season changes, the marketing period also changes, generating shifts


Figure 7.9: Simulation of Biomass in the Full Model


Figure 7.10: Simulation of Exvessel and Wholesale Prices in the Full Model
in both the wholesale and exvessel demand curves. The effect of a lengthening marketing period on the wholesale market is positive, but the effect on the exvessel market depends upon the interplay between adjustment costs, holding costs, and the levels of harvests and the season lengths as shown in Figure 6.4. On the one hand, total and marginal holding costs unambiguously increase with larger harvests and longer marketing periods, giving downward impetus to exvessel prices. On the other hand, during the tarnsition to long run equilibrium, the effects of these changes on total and marginal adjustment costs is ambiguous because a larger quantity is being spread over a longer marketing period. Hence seasonal regulation changes could either mitigate or amplify the inherent open access incentives to enter through their market effects.

In turn, these price changes and their market impacts also cause changes in the regulatory sector. Falling prices should ceteris paribus, reduce the entry pressure and hence allow regulators to relax season lengths. Over the recovery period, however, conditions are not ceteris paribus because of increased biomass and quotas. It appears, in fact, that for the parameters we use to simulate in this case, the combined effects work to exacerbate the regulatory problem because over the approach to equilibrium, capacity continues to rise gradually (see Figure 7.11). This is associated with the rising revenues and positive but small rents (see Figure 7.12). Thus on the whole for this system, the combined effect of rising biomass on production and increased marketing period lengths on price must be outweighing any possible revenue reducing effects associated with the essentially inelastic exvessel demand curve. The implication of this for the regulatory sector is that the increasing growth of capacity must be matched by continual reductions in season length. At the long run equilibrium, the level of capacity is lower than for the case modeled wiithout the marketing sector, indicating that the price dampening effect plays some positive role in easing the regulatory task, although not a significant one.

In summary, the addition of the marketing sector adds complexity to both the


Figure 7.11: Simulation of Capacity and Season Length in the Full Model


Figure 7.12: Simulation of Rents and Revenues in the Full Model
approach paths and to the determination of the characteristics of the equilibrium. During the approach to equilibrium, price changes smooth what would otherwise be more dramatic swings and depending upon initial values, may dampen or increase the possibility of oscillations and over /undershooting. The impact on the equilibrium is complicated, however, and essentially depends upon the interplay between harvest levels, elasticities of demand, and elasticities with respect to marketing period (season length). Table 7.3 below summarizes the four cases considered, from the simple static Gordon model, to the dynamic Gordon model, to the two models we develop that add regulations and the market. As can be seen, moving from the naive versions of the pure open access Gordon model to more realistic depictions of a regulated open access system affects predictions profoundly. In particular, the result of adding a goal driven regulatory institution is both a higher biomass and harvest level. Importantly, however, this ends up exacerbating the social waste associated with open access because there is ultimately more excess capacity operating over a shorter season length than would otherwise be the case. Adding the marketing sector identifies the impacts of these regulations on exvessel prices, and traces the respective roles of price levels on entry and regulations. In general these can be complicated and may mitigate or amplify the basic regulated open access incentives to overcapitalize. In the case modeled here, the combined effects work to reduce rent dissipation pressures, although the ultimate effect on season length is minimal.

|  | $\bar{E}$ | $\bar{T}$ | $\bar{X}$ | $\bar{H}$ | $P_{E V}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Static Gordon | 13.83 | 30.28 | 53.5 | 24.95 | 1.95 |
| Dynamic Gordon | 5.92 | 48 | 43.7 | 15.17 | 1.95 |
| Regulated Open Access | 40.69 | 3.74 | 133.2 | 27.14 | 1.95 |
| Regulated Open Access <br> with Marketing Sector | 32.87 | 4.63 | 133.2 | 27.14 | 1.66 |

Table 7.3: Equilibrium Values for Alternative Models

## Chapter 8

## Concluding Remarks

As pointed out in the introduction, the H.S. Gordon model of the rent dissipation process is one of the most influential papers in the field of natural resource economics. It has not only shaped the profession's general view of how resource exploitation takes place but it has also become the principal paradigm underlying policy design aimed at tackling the problem of open access resource overexploitation. In this chapter we summarize fisheries policy developments since Gordon and discuss the role that his thinking has played in the process of policy evolution. This places the contributions of this thesis in a normative as well as predictive context.

The decade following Gordon's paper witnessed stock collapses in many important fisheries, including salmon, halibut and herring on the West Coast, and cod and various groundfish species in the Atlantic. The response in some cases was for governments to initiate some form of fishing controls similar to those developed for halibut in the thirties. Most regulatory programs evolved with a structure much like that developed in the theory of regulatory behavior presented in Chapter 3, namely quotas (often implicit) establishing targeted harvest levels based on biomass or other criteria, and instruments (including
season lengths, closed areas, gear restrictions) set to achieve the targets. Throughout most of the fifties and sixties, virtually all important fisheries remained open access, not only to domestic fishermen but often also to foreign fleets.

As happened in the halibut program of regulation, where these new regulatory programs proved successful in controlling harvests, they often paradoxically suffered from their very successes, exactly as predicted by our full model. That is, as regulators gained control of harvests and as biomass recovered, rents were generated and new entry occurred, necessitating further tightening of regulations. As early as the sixties, biologists were looking for other regulatory means to harness the effects of the rent dissipation on fishing capacity growth. In the early sixties, fisheries managers in the Australian rock lobster fishery attempted to cap total effort by freezing the number of lobster traps in the industry and allowing fishermen to fish only the numbers of gear they had permits for (Meany[59]). This was one of the first limited entry programs (gear based) and others soon followed, typically restricting participation to those holding one of a limited number of entry permits. ${ }^{1}$ These new programs of fisheries regulation were largely instigated by fisheries biologists (rather than economists) who saw threats to stock health (rather than rent dissipation) as the principal problem associated with open access fisheries (see Adasiak[1]).

Although few fisheries economists were directly involved with designing or implementing these programs, most were quick to embrace these emerging regulatory institutions (Crutchfield [21]). It seems in hindsight that one of the reasons that economists quickly became advocates of limited entry programs is because of the compelling influence of the

[^26]Gordon paradigm. Gordon supplied not only the rationale for regulations (waste of potential resource rents) but also a possible design. In particular, since, as Gordon predicted, we should expect to see rents drawing in excess inputs into open access fisheries, one obvious answer to the problem might be to restrict access.

By the late 1970s, evidence was accumulating that the fisheries regulation problem was not quite so simple (McConnell and Norton[58], Pearse[66]). In many fisheries where limited entry programs were established to control capacity growth, fishermen simply circumvented the program by investing in unrestricted inputs (Anderson[4], Pearse and Wilen[67]). In the British Columbia salmon fishery, for example, regulations were first placed on vessels, restricting the total number in the fleet in 1968. Next, regulators observed fishermen replacing small vessels with bigger ones and hence controls were switched to restrict total tonnage. Then, regulators found fishermen building larger configured vessels of the same tonnage and so additional regulations were placed on dimensions. Finally, once basic vessel dimensions were tied up, regulators found fishermen concentrating in high efficiency gear types and hence subsequent regulations had to limit numbers by gear type (Fraser[33], Campbell[13]). What early regulatory designs failed to incorporate was that fishing technology is not, as Gordon chose to illustrate with his abstraction, a one dimensional metric called "effort." In a resource exploitation setting where technology is flexible, the rent dissipation process will be pervasive and exerted across all dimensions over which fishermen have discretion.

A debate has persisted over many years over the practical significance of this point and over exactly how to interpret the evidence that has been revealed in limited entry programs (see Pearse[64], Townsend[83], and Wilen[88]). Some economists believe that restricting a few key dimensions (tonnage and dimensions and gear configurations) ought to be enough to lock up and sustain most of the potential rents in fisheries (Crutchfield[21]). Evidence supporting this view are the large and persistent transfer prices in many limited
entry programs. ${ }^{2}$ Other economists take the Gordon paradigm to its logical conclusion by arguing that as long as there are any rents in a fishery, there will be a tendency to dissipate them with wasteful investments in redundant inputs, and other means of gaining temporary capacity advantages over their competitors (Wilen[87], Pearse[65], Pearse[66]).

Partly as a result of the inconclusive evidence over the long run ability of limited entry programs to sustain rents, economists began to advocate a radically different program beginning in the late seventies. The new idea was to create property rights by converting aggregate industry regulated quotas into individual transferable quotas or ITQs. ${ }^{3}$ The advantage foreseen for ITQs was that they could attack the problem identified by Gordon directly rather than indirectly. That is, instead of fighting symptoms of forces of rent dissipation by controlling excess inputs, ITQs could attack the cause (lack of proper incentives) by creating rights which encourage efficient behavior. During the eighties and nineties, a debate nearly as rancorous as the preceding one on the ultimate merits of limited entry has surfaced over the potential of ITQs to "solve" once and for all the open access problem identified by Gordon. ${ }^{4}$

Beginning in 1984, Iceland, New Zealand, Australia, and a few other countries began to experiment with ITQs (see Clark et. al.[16] and Wesney[86]). Programs were instituted in New Zealand's offshore groundfish industry and Iceland's offshore cod fishery, followed by others in New Zealand's inshore fisheries and elsewhere. Although judgment has not been universally positive, most economists, managers, and fishermen view these new programs as successful. During the past few years a virtual explosion in new adoptions has taken place. Presently there are over 40 individual quota programs in place, with the most prominent ones in the New Zealand fisheries, Australia's Bluefin tuna fisheries, British

[^27]Columbia's halibut fishery, and the New England surf clam fishery.

An interesting outcome has emerged in virtually all of the new ITQ programs, however, and one that was not expected or even noticed by any economists until recently (Homans and Wilen[52]). The unexpected outcome is that virtually all of the rent gains initially induced by ITQs have emerged not from input and cost reconfigurations but instead through changes on the marketing or revenue side of the ledger. There are several examples, discussed below.

Australian Southern Bluefin Tuna In 1984, the bluefin tuna fishery was converted into an ITQ system by allocating quotas to fishermen based on historical catches. During the regulated open access fishery, the fleet intercepted tuna in their counterclockwise migration in the southwestern seas before they had a chance to reach significant size and maturity. After ITQs were adopted, fishermen fishing off the southeastern seas purchased quota from their southwestern located counterparts and began to fish for larger tuna. These larger (and better handled) tuna earned much higher prices in the Japanese sashimi market and revenues grew substantially. Geen and Nayar[37] report that after just three years of operation, the number of large fish (greater than 15 kg .) increase from 15 to $35 \%$ of the catch. As a result, revenues rose from 988 to 2000 dollars per ton in constant dollars between 1984 and 1987. During this same period, there was some vessel consolidation but Geen and Nayar estimate this to have reduced costs by only $25-30 \%$. While this is significant, it is dwarfed by the over $100 \%$ increase in revenues induced by market side effects.

British Columbia Halibut In 1991, Canadian fishermen adopted an ITQ fishery regulation program side by side with the continued use of a "fishing derby" by U.S. halibut fishermen off Alaska. In 1991, there were reports that over fifty percent of the U.S. catch was landed without ever being iced and about a third of the fish were not even gutted
during the biggest one day opening in the first week in May. The British Columbia fleet, in contrast, chose to hold most of their quotas over to use after the May opening in the U.S. and before the final closing date in November. As a result, significantly higher prices were received by Canadians. Fishermen's News (June 1991) reported that the exvessel prices received during the latter part of May by B.C. fishermen averaged $\$ 1.10$ per pound higher than those received by U.S. counterparts (who received about \$1.70). This suggests a revenue rent loss due to regulated open access of about forty percent of potential. A recent study of the structure of the fleet in B.C. after ITQs by EB Associates[27] concludes that there have been few significant changes in costs and that most of the changes have occurred in the marketing sector. Virtually all of the B.C. halibut is now sold in the much more high valued fresh market almost year round.

New Zealand Inshore Groundfish The pre-ITQ inshore groundfish fishery off New Zealand was primarily a trawl fishery, with catch comprised of many species of various sizes and relatively low quality. Early reports after ITQs were introduced indicated that revenues in some case tripled. This occurred because of major shifts in products marketed, primarily away from mixed batches of small, net marked fish caught in compressed seasons, towards better handled fish targeted with long line gear and selected for size and market characteristics over the whole year. The red snapper fishery is interesting in this regard, switching from a frozen trawl caught product to a long line caught product that is marketed as live fish shipped to Japan. This is particularly interesting because long line fishing is a costlier technology whose introduction was made possible only by the substantially larger revenues earned in the new market. Hence this case is an extreme version of how dramatic the errors might be in adopting a naive Gordon model to anticipate the gains from rationalization. In this case, not only would there be no cost savings from input reduction, but costs would go up and all rent gains would emerge from the revenue side of the picture.

These three examples are not isolated cases but in fact appear to be representative of a general phenomenon emerging out of experience in most recent rationalizations of fisheries via the use of ITQs. What these point to quite clearly is how important a proper paradigm is to accurate anticipation of policy changes. We would argue that the reason fisheries economists have essentially failed to anticipate these important sources of rent gains in modern fisheries is that the dominant paradigm, base on pure open access, static assumptions, and exogenous market conditions, is basically incomplete as a description of most modern fisheries. As we have demonstrated, it is particularly important to account of the fact that modern fisheries are not pure open access but rather regulated open (or restricted) access. What seems to be emerging in all of these cases is that the initial and dominant impact of switching from regulated open access to schemes that encourage rent maximization is that the first and perhaps easiest sources of rent gains come from freeing up the system from the constraints of the old regulatory system. In particular, for fisheries that have been regulated with season and other restrictions whose impacts have distorted the output market, when these are lifted there are almost immediate gains to be made from marketing changes which raise exvessel prices. For example, simple switching from compressed seasons with frozen product to year long seasons with fresh product should almost always results in higher revenues. Or, switching fishing methods to target larger or otherwise better quality fish ought to raise exvessel values. That these changes might be the first to emerge after ITQs has been missed because economists have been bound up in a paradigm which basically ignores the source of these rents (the regulatory sector) and the mode of transmission (the market), focusing instead on anticipated cost savings from reduced inputs. It is likely that cost savings will follow after these schemes mature, of course, but these may turn out to be incidental to the rent gains made on the revenue side.

What does this say about how economists should be thinking about modeling policy impacts under regulated open access conditions? First, as argued throughout this
thesis, it is important to capture the fundamental nature of both the regulatory sector and the marketing sector. As this thesis has demonstrated, this requires some modeling judgment and care. One cannot, for example, simply take regulations as given because they are generally endogenous in the system. This means that the modeler must delve into the specifics of actual policy making and attempt to synthesize the procedures that are being exhibited in decisions. In addition, although there are broad similarities that run across many fisheries, the specific form that regulations take in any one fishery and the impact of those on the market and industry may depend on a process of institutional evolution. The case we have examined has been simple to model because technology has been relatively constant and the primary instrument used has been a single one, season length. Still, even in this case, when the biomass collapsed in the early sixties, regulators changed the nature of their decision making procedures by adopting new quota rules and tighter adherence to targets. Hence modeling may require not only capturing the basic nature of the system, but also looking for and possibly anticipating structural change in regulatory procedures.

In addition, we have demonstrated that the interplay between the industry, the regulatory sector, and the market can be complicated and capable of exhibiting patterns that at first appear counterintuitive. For example, we have shown that the degree of over and under shooting to the long run equilibrium depends upon relative speeds of adjustment. Hence if the industry began to react more sluggishly to rents, we would expect wider swings in capacity and season length, more divergence between actual harvests and targeted quota, and a slower approach to the safe stock level. We have also shown that the market effects of regulations are potentially complicated, particularly because short seasons induce storage which is inherently difficult to model. In our estimations, we have uncovered relationships between harvest levels, inventory costs, and the wholesale market which are consistent with sophisticated dynamic decision making. Still, since revenues depend in complicated ways on price and marketing period elasticities, the implications of policies that simultaneously affect
harvest quantities and quality and season lengths in combination are hard to anticipate even qualitatively without some idea of the values of critical parameters.

In sum, the model developed here is both illuminating pedagogically and potentially useful for policy analysis of a complicated system. We have illustrated its usefulness in an application to regulatory Area 2, and similar results follow for Area 3. Our results show that a considerable amount of potential rent is at stake in this fishery. A quick calculation using estimated cost and production parameters suggests that, even at current prices associated with the predominantly open access fishery off Alaska, the potential rent gains from cost saving alone may be on the order of 30 million dollars per year for Area $2 .{ }^{5}$ It is difficult to forecast how much rents are lost on the revenue side due to regulations which have compressed the season and resulted in most harvest being frozen, because there is only limited experience with new market conditions where halibut is available fresh over the whole year. However, another quick calculation suggests that inventory holding and adjustment costs of 7-8 cents per pound would be saved and in addition, wholesale prices would be higher because of the premium for fresh fish. If the first few year's experience in British Columbia is indicative, exvessel revenues may rise by $60-70 \%$, generating another 30 million dollars per year. These, of course, would emerge almost immediately while the cost savings from reduced and consolidated gear may take several years.

[^28]
## Bibliography

[1] Adasiak, A. "Alaska's Experience With Limited Entry." Journal of the Fisheries Research Board of Canada 36 (1979):770-82.
[2] Alston, Julian, and Colin Carter. "Causes and Consequences of Farm Policy." Contemporary Policy Issues 9 (1991):107-21.
[3] Amemiya, Takeshi. Advanced Econometrics. Cambridge: Harvard University Press, 1985.
[4] Anderson, Lee G. (ed.) Economic Impacts of Extended Fisheries Jurisdiction. Ann Arbor: Ann Arbor Science, 1977.
[5] Andrew, P.A., and D.S. Butterworth. "Is $\mathrm{F}_{0.1}$ an Appropriate Harvesting Strategy for the Cape Hakes." South African Journal of Marine Science 5 (1987):1936-1947.
[6] Bell, F. Heward. The Pacific Halibut. Anchorage: Alaska Northwest Publishing Co., 1981.
[7] Ben David, Shaul, and William Tomek. "Storing and Marketing New York State Apples, Based on Intraseasonal Demand Relationships." Ithaca, 1965.
[8] Berndt, Ernst R., Melvin A. Fuss and L. Waverman. "Dynamic Models of Energy Demand: An Assessment and Comparison." in E. Berndt and B. Field (eds.), Measuring and Modelling Natural Resource Substitution. Cambridge: MIT Press, 1981.
[9] Bhagwati, Jagdish N. "Directly Unproductive Profit Seeking Activities." The Journal of Political Economy 90(1982):988-1002.
[10] Brooke, Anthony, David Kendrick and Alexander Meeraus. GAMS: A Users Guide. Redwood City: The Scientific Press, 1988.
[11] Buchanan, James M., Robert Tollison, and Gordon Tullock (eds.) Toward a Theory of a Rent Seeking Society. College Station: Texas A\&M University Press, 1980.
[12] Cameron, Colin. Economics 240D class notes. UC Davis, 1990.
[13] Campbell, B. A. "License Limitation Regulations: Canada's Experience." Journal of the Fisheries Research Board of Canada 30 (1973): 2070-76.
[14] Capalbo, Susan Bioeconomic Supply and Imperfect Competition: The Case of the North Pacific Halibut Industry. Ph.D. Dissertation, University of California at Davis, 1982.
[15] Christy, F.T., Jr. "Fisherman Quotas: a Tentative Suggestion for Domestic Management." Occasional paper no. 19. Kingston, R.I.: University of Rhode Island, Law of the Sea Institute, 1973.
[16] Clark, Ian, Philip Major, and Nina Mollett. "The Development and Implementation of New Zealand's ITQ Management System." in Philip Neher, Ragnar Arnason, and Nina Mollet (eds.) Rights Based Fishing. Dordrecht: Kluwer Academic Publishers, 1989, pp.117-145.
[17] Coggins, Jay S. "Rationalizing the International Coffee Agreement Virtually." University of Wisconsin-Madison Agricultural Economics Staff Paper Series No. 362, 1993.
[18] Copes, Parzival. "A Critical Review of the Individualized Quota as a Device in Fisheries Management." Land Economics 62(1986):278-91.
[19] Criddle, Keith Modeling Dynamic Nonlinear Systems. Ph.D. Dissertation, University of California at Davis, 1989.
[20] Criddle, Keith, and Arthur Havenner. "Forecasting Halibut Biomass Using Systems Theoretic Time Series." American Journal of Agricultural Economics 71(1989):422431.
[21] Crutchfield, James A. "Economic and Social Implications of the Main Policy Alternatives for Controlling Fishing Effort." Journal of the Fisheries Research Board of Canada 36(1979):742-52.
[22] Crutchfield, James A. "Regulation of the Pacific Halibut Industry." in Scott, A.D., J. Crutchfield, P. Pearse, G. Munro, and M. Tugwell (eds.) The Public Regulation of Commercial Fisheries. Economic Council of Canada, Queens Printer Ottowa, June 1981.
[23] Crutchfield, James A., and Arnold Zellner. "Economic Aspects of the Pacific Halibut Fishery." Washington, D. C.: U.S. Department of the Interior, Fishery Industrial Research, 1962.
[24] Deriso, R. B. "Risk Averse Harvesting Strategies." In Mark Mangel (ed.) Resource Management: Proceedings of the Second Ralf Yorque Workshop, Lecture Notes in Biomathematics No. 61. Berlin: Springer-Verlag, pp. 65-73, 1985.
[25] De Gorter, Harold, D. Nielsen, and Gordon Rausser. "Productive and Predatory Public Policies-Research Expenditures and Production Subsidies in Agriculture." American Journal of Agricultural Economics 74 (1992):27-37.
[26] Doubleday, W.G., D. Rivard, and W.D. McKone. "Estimation of Partial Recruitment and Yield per Recruit for an Otter Trawl Fishery for Deepwater Redfish." North American Journal of Fisheries Management 4(1984):150-31.
[27] EB Associates. "An Evaluation of the British Columbia ITQ Program." Executive Summary, 1993.
[28] Epple, Dennis, and Lars P. Hansen. "An Econometric Framework for Modeling Exhaustible Resource Supply." in J. Ramsey, ed., The Economics of Exploration for Energy Resources. Greenwich: JAI Press, 1981.
[29] Epstein, Larry G. "Duality Theory and Functional Forms for Dynamic Factor Demands." Review of Economic Studies 48(1981)81-95.
[30] Epstein, Larry G., and Adonis J. Yatchew. "The Empirical Determination of Technology and Expectations: A Simplified Procedure." Journal of Econometrics 27(1985)235258.
[31] Fishermen's News June 1991.
[32] Fishery Market News Report, National Marine Fisheries Service, New York Market Statistics Food Fish Market Review and Outlook, December 1977.
[33] Fraser, G.A. "License Limitation in the British Columia Salmon Fishery." Technical report no. PAC/T-77-B. Vancouver: Canada Department of Fisheries and the Environment, Fisheries and Marine Service, 1972.
[34] Fulton, Murray, and Larry Karp. "Estimating the Objective of a Public Firm in a Natural Resource Industry." Journal of Environmental Economics and Management 16 (1989):268-287.
[35] Garber, Peter M. and Robert G. King. "Deep Structural Excavation? A Critique of Euler Equation Methods." National Bureau of Economic Research Technical Working Paper No. 31, 1983.
[36] Gardner, Bruce. The Optimal Stockpiling of Grain. Lexington: Lexington Books, 1979.
[37] Geen, Gerry and Mark Nayar. "Individual Transferable Quotas in the Southern Bluefin Tuna Fishery: An Economic Appraisal." Marine Resource Economics 5 (1988):365-387.
[38] Goodwin, Thomas H., and Steven Sheffrin. "Testing the Rational Expectations Hypothesis in an Agricultural Market." Review of Economics and Statistics 64 (1982):55867.
[39] Gordon, H. Scott. "The Economic Theory of a Common Property Resource: The Fishery." The Journal of Political Economy 62 (April 1954):124-42.
[40] Gordon, H. Scott. "Obstacles to Agreement on Control in the Fishing Industry." In Ralph Turvey and Jack Wisemen, (eds.), The Economics of Fisheries. Rome: FAO, 1957.
[41] Gustafson, Robert L. "Carryover Levels for Grains: A Method for Determining Amounts that are Optimal Under Specified Conditions." USDA Technical Bulletin 1178, Washington D.C., 1958.
[42] Hansen, Lars P. "Large Sample Properties of Generalized Method of Moments Estimators." Econometrica 50(1982):1029-1054.
[43] Hansen, Lars P., and Thomas Sargent. "Formulating and Estimating Dynamic Linear Rational Expectations Models." Journal of Economic Dynamics and Control 2 (1980):7-86.
[44] Hansen, Lars P., and Thomas Sargent. Rational Expectations Econometrics. Boulder: Westview Press, 1991.
[45] Hansen, Lars, and Kenneth Singleton. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." Econometrica 50(1982):1269-1286.
[46] Harvey, A.C. The Econometric Analysis of Time Series. Oxford: Philip Allan, 1981.
[47] Helmberger, Peter and R. Weaver. "Welfare Implications of Commodity Storage under Uncertainty." American Journal of Agricultural Economics 64 (1977):266-70.
[48] Hilborn, Ray. "A Comparison of Harvest Policies for Mixed Stock Fisheries." In Mark Mangel (ed.) Resource Management: Proceedings of the Second Ralf Yorque Workshop, Lecture Notes in Biomathematics No. 61. Berlin: Springer-Verlag, pp. 750-87, 1985.
[49] Hilborn, Ray and Carl Walters. Quantitative Fisheries Stock Assesment: Choice, Dynamics, and Uncertainty. New York:Chapman and Hall, 1992.
[50] Hoag, Stephen H., Richard J. Myhre, Gilbert St-Pierre, and Donald A. McCaughran. "The Pacific Halibut Resource and Fishery in Regulatory Area 2" International Pacific Halibut Commission Scientific Report No. 67. 1983.
[51] Hoelper, Antonia L. and Michele Marra. "Quality Changes and Limited Marketing Season Effects on the Demand for Fresh Blueberries." Northeastern Journal of Agricultural and Resource Economics 20(1991):174-180.
[52] Homans, Frances, and James E. Wilen. "Marketing Losses in Regulated Open Access Fisheries." VIth International Institute for Fisheries Economics and Trade, Paris, 1992.
[53] The International Pacific Halibut Commission. The Pacific Halibut: Biology, Fishery, and Management Technical Report No. 22 (1987).
[54] Karpoff, J.M. "Suboptimal Controls in Common Resource Management: The Case of the Fishery." The Journal of Political Economy 95 (1987):179-94.
[55] Lin, Biing Hwan, Hugh Richards and Joseph Terry. "An Analysis of Exvessel Demand for Pacific Halibut." Marine Resource Economics 4 (1987): 305-314.
[56] Lotka, A. J. Elements of Physical Biology, Williams and Wilkins, 1925.
[57] Lucas, Robert E. "Econometric Policy and Evaluation: a Critique." in K. Brunner and A. Meltzer,(eds.). The Phillips Curve and the Labor Market. Vol. 1 of CarnegieRochester Conferences in Public Policy, a supplementary series to the Journal of Monetary Economics. Amsterdam: North Holland, 1976.
[58] McConnell, K.E., and V. J. Norton. "An Evaluation of Limited Entry and Alternative Approaches to Fishery Management." in Proceedings of a Workshop on Limited Entry. Seattle: University of Washington, 1979.
[59] Meany, T.F. "Limited Entry in the Western Australian Rock Lobster and Prawn Fisheries: An Economic Evaluation." Journal of the Fisheries Research Board of Canada 36 (1979):789-98.
[60] Miranda, Mario and Joseph Glauber. "Intraseasonal Demand for Fall Potatoes Under Rational Expectations." American Journal of Agricultural Economics 75 (1993):104 112.
[61] Moloney, D.G., and P. Pearse. "Quantitative Rights as an Instrument for Regulating Commercial Fisheries." Journal of the Fisheries Research Board of Canada 36 (1979):859-66.
[62] Muth, John. "Rational Expectations and the Theory of Price Movements." Econometrica 29 (1961): 315-35.
[63] North Pacific Fishery Management Council. "Supplemental Analysis of the Individual Fishing Quota Management Alternative For Fixed Gear Sablefish and Halibut Fisheries" (March 1992): 2-6.
[64] Pearse, Peter (ed.) Symposium on Managing Fishing Effort. Journal of the Fisheries Research Board of Canada 36(1979).
[65] Pearse, Peter. "Turning the Tide: A New Policy for Canada's Pacific Fisheries." Final Report. Vancouver: Commission on Pacific Fisheries Policy, 1982.
[66] Pearse, Peter. "Rationalization of Canada's West Coast Salmon Fishery: An Economic Evaluation." In Organisation for Economic Co-operation and Development, Economic Aspects of Fish Production, Paris: OECD, 1972.
[67] Pearse, Peter, and James E. Wilen. "Impact of Canada's Pacific Salmon Fleet Control Program." Journal of the Fisheries Research Board of Canada 36(1979):764-69.
[68] Pindyck, Robert S., and Julio Rotemberg. "Dynamic Factor Demands and the Effects of Energy Price Shocks." American Economic Review 73(1983):1066-1079.
[69] Quinn, Terrance J., Richard Deriso, and Stephen H. Hoag. "Methods of Population Assessment of Pacific Halibut." International Pacific Halibut Commission Scientific Report No. 72. 1985.
[70] Rausser, Gordon C. and Pinhas Zusman. "Public Policy and Constitutional Prescription." American Journal of Agricultural Economics 74 (1992):247-257.
[71] Rosen, Sherwin. "Dynamic Animal Economics." American Journal of Agricultural Economics 69 (1987):547-557.
[72] Rotemberg, Julio. "Interpreting the Statistical Failures of Some Rational Expectations Macroeconomic Models." American Economic Review 74(1984):188-193.
[73] Rowley, Charles, Robert Tollison, and Gordon Tullock (eds.) The Political Economy of Rent Seeking. Boston: Kluwer Academic Publishers, 1988.
[74] Sargent, Thomas J. "Estimation of Dynamic Labor Demand Schedules under Rational Expectations." in Robert E. Lucas, Jr. and Thomas Sargent (eds.) Rational Expectations and Econometric Practice, Vol. 2. Minneapolis: The University of Minnesota Press, 1981.
[75] Sargent, Thomas J. Macroeconomic Theory, 2nd Ed.. Orlando: Academic Press, 1987.
[76] Scheinkman, J., and J. Schectman. "A Simple Competitive Model with Production and Storage." Review of Economic Studies 50(1986):427-441.
[77] Shapiro, Matthew D., "The Dynamic Demand for Capital and Labor." Quarterly Journal of Economics 69(1986):513-542.
[78] SHAZAM User's Reference Manual Version 7.0. New York: McGraw-Hill, 1993.
[79] Skud, Bernard E. "Revised Estimates of Halibut Abundance and the ThompsonBurkenroad Debate." International Pacific Halibut Commission Scientific Report No. 63., 1975.
[80] Skud, Bernard E. "Regulations of the Pacific Halibut Fishery." International Pacific Halibut Commission Technical Report No. 15., 1977.
[81] Scott, Anthony D. "The Fishery: The Objectives of Sole Ownership." Journal of Political Economy 63 (1955):116-124.
[82] Smith, Vernon L. "Economics of Production from Natural Resources." The American Economic Review 58 (June 1958):409-431.
[83] Townsend, Ralph. "Entry Restrictions in the Fishery: A Survey of the Evidence." Land Economics 66(1990):359-378.
[84] Turvey, Ralph and Jack Wisemen, (eds.), The Economics of Fisheries. Rome: FAO, 1957.
[85] Volterra, V. Leçons sur la théorie mathématique de la lutte pour la vie, Gauthier-Villars, 1931.
[86] Wesney, David. "Applied Fisheries Management Plans: Individual Transferable Quotas and Input Controls." in Philip Neher, Ragnar Arnason, and Nina Mollet (eds.) Rights Based Fishing. Dordrecht: Kluwer Academic Publishers, 1989, pp.153-181.
[87] Wilen, James E. "Fisherman Behavior and the Design of Efficient Fisheries Regulation Programs." Journal of the Fisheries Research Board of Canada 36 (1979):855-58.
[88] Wilen, James E. "Limited Entry Licensing: A Retrospective Assessment." Marine Resource Economics 5(1988):313-324.
[89] Williams, Jeffrey and Brian Wright. Storage and Commodity Markets. New York: Cambridge University Press, 1991.
[90] Working, Holbrook. "Theory of the Inverse Carrying Charge in Futures Markets." Journal of Farm Economics 30(1948):1-28.


[^0]:    ${ }^{1}$ In particular assume the cost function $C(E(t), X(t))$ is $c E(t)$, and the harvest function $H(E(t), X(t))$ is $q E(t) X(t)$ Then, with the quadratic growth function (1.3.1), we have the system:

    $$
    \begin{aligned}
    & \dot{X}=a X(t)-b X(t)^{2}-q E(t) X(t) \\
    & \dot{E}=\delta[P q E(t) X(t)-c E(t)] .
    \end{aligned}
    $$

[^1]:    ${ }^{1}$ Much of the material in this and following sections is from a report of the International Pacific Halibut Commission[53] and articles by Crutchfield [22] and Skud[80].

[^2]:    ${ }^{1}$ Note that alternative production function forms used in other settings have undesirable and less realistic properties. For example, a Cobb-Douglas form for the harvest rate or the cumulative harvest would permit production to exceed the total biomass.

[^3]:    ${ }^{2}$ One reason for the lack of specificity is that many different stories have been shown to be indistinguishable empirically. For example, in the literature on rational expectations, it has been shown that optimal forecasting in a MA(1) setting generates the same decision structure as if the decision maker were using adaptive expectations ála Nerlove. The main point is that any sluggishness in the adjustment to equilibrium can be motivated by a number of mechanisms. This includes the possibility that sluggishness is dynamically optimal. For example, the quadratic adjustment cost model of investment under simple myopic expectations assumptions reduces to an equation for the optimal level of the capital stock which is basically a sluggish adjustment model.

[^4]:    ${ }^{1}$ Both Hansen and Singleton [45] and Amemiya [3] have developed nonlinear instrumental variables techniques. These have been further refined and incorporated into the econometrics package SHAZAM [78].

[^5]:    ${ }^{2}$ Our comparisons suggest close agreement between estimates from the model with total revenues and the catch equation estimated separately, and the nonlinear equation embedding the catchability coefficient in the revenue function explicitly. In fact, we first used the linear specification to generate starting values for the nonlinear specification but found such close accord as to make the full nonlinear specification redundant.

[^6]:    ${ }^{3}$ A word about units. Price is in price per pound, catch is in millions of pounds, and effort is in thousands of skate soaks. The implication of this is that the cost coefficients measure thousands of dollars per unit of effort. For example, if $v_{0}$ is .01 , the cost per skate soak is $\$ 10$.

[^7]:    ${ }^{1}$ We define marketing period to be the time between the end of this year's production period and the beginning of next year's production period.

[^8]:    ${ }^{2}$ It is possible that the configuration of parameters might lead to a solution with negative sales. For instance, if prices rise sharply towards the end of the period, it would pay to build up stock at the beginning in order to sell it later. If this is so, we need to employ a control restriction: $q(t) \geq 0, \forall t \in[0, \tau]$. This would introduce substantial complexities, and so we choose to focus on the simpler case where parameters are configured so that negative sales are not optimal.

[^9]:    ${ }^{3}$ An alternative solution method is to solve the two differential equations and find the constants of

[^10]:    ${ }^{5}$ The closed loop solution is particularly appealing for empirical work. With the introduction of stochastics, it doesn't make sense to refer to the start of the program (date 0 ) when conditions have changed by time $t$ due to shocks. Therefore, we will want a solution in terms of the current state: the amount of stock on hand and the time left in the horizon. Though the solutions are exactly the same in the deterministic context, we want to set the stage for empirical models to follow.

[^11]:    ${ }^{6}$ This approach is closely related to the rational expectations literature. Our exposition is carried out in a model that is deterministic in order to first clarify the nature of the equilibrium and its properties under perfect certainty.

[^12]:    ${ }^{8}$ We can also derive the equilibrium quantity of sales if the demand function shifts over time. If the intercept is a function of time, $A(s)$, then the equilibrium quantity is:

    $$
    \begin{equation*}
    Q(t)=\frac{n\left(S_{t}-C_{\tau}\right)}{\tau-t}+\frac{n h(\tau-t)}{2(n B+c)}+\frac{n}{n B+c}\left[A(t)-\frac{1}{\tau-t} \int_{t}^{\tau} A(s) d s\right] \tag{5.5.8}
    \end{equation*}
    $$

    As with the case where prices are exogenous, if the average of future demand intercepts is more than the current intercept, the current equilibrium market quantity is less than otherwise. Firms are motivated to delay sales until demand conditions improve. This is important for empirical work since the role of demand shifters and exogenous shocks can be incorporated into $A(s)$.

[^13]:    ${ }^{9}$ Recall that all models assume that the marketing horizon is fixed and known, that beginning and ending stocks are given parametrically, and a deterministic environment.

[^14]:    ${ }^{1}$ See, for example, the model in Rosen [71].

[^15]:    ${ }^{2}$ Lucas[57] made the case that estimating reduced form equations (e.g., demand curves) was not as useful as estimating the parameters of the root choice functions (e.g., utility functions). Since policy changes affect choices at the structural level, reduced form parameters, which are composites of structural parameters (including expectational parameters), are are not appropriate to use in policy analysis. The "deep structural parameters" are then the parameters of root choice functions such as expected utility and present value of profit functions as well as parameters of expectational processes.
    ${ }^{3}$ Much of the theory was developed by Hansen and Sargent (See [45,44,75]). An application of this method is found in Sargent[74].

[^16]:    ${ }^{4}$ This method was used by Goodwin and Sheffrin[38] in an application to the chicken broiler industry.
    ${ }^{5}$ These articles spawned a substantial literature, both in investment and finance. See, for example, Shapiro[77].

[^17]:    ${ }^{6}$ This section borrows substantially from Cameron [12].

[^18]:    ${ }^{7}$ Hansen and Singleton also provide a test of overidentifying restrictions. In order to estimate the models, one only needs as many instruments as parameters. Additional instruments should also be uncorrelated with the error term, and should not substantially change the parameter estimates.

[^19]:    ${ }^{8}$ This weakness is explained by Pindyck and Rotemberg, who considered the efficiency loss to be acceptable.
    ${ }^{9}$ Analysis of this problem was carried out by Garber and King[35].

[^20]:    ${ }^{10}$ In many fisheries the question is moot because institutions have developed which determine a single price fishermen receive over the whole season. This institutional peculiarity of fisheries has evolved to reduce the uncertainty for both fishermen and processors but the operational significance is that we can consider a market clearing mechanism as taking place at a particular date or instant in time rather than over an interval.

[^21]:    ${ }^{11}$ The price data are New York wholesale price per pound for dressed frozen Pacific Halibut[32].
    ${ }^{12}$ We found that the fit was improved by dummying out 1975, which, for some unknown reason was an outlier.
    ${ }^{13}$ The mean monthly sales rate over the sample is about 5.4 million pounds. Hence the adjustment cost is about 6 cents per pound moved out of inventory, compared with a mean wholesale price of $\$ 1.67$.

[^22]:    ${ }^{14}$ We have developed and illustrated most of the inventory models by ignoring discounting. This seems excusable for pedagogical purposes, particularly because the zero discount case results are so transparent, or when we are mainly dealing with within period marketing period choices. However, as the focus shifts to situations where carryover is a more significant factor in the decision process, it is important to reintroduce discounting, even at the expense of computational clarity. The impact on the structure turns out mainly to be one of messier expressions involving future wholesale price and infusion variables. This is demonstrated in the introduction to Chapter Five for the single period case.

[^23]:    ${ }^{1}$ The safe stock level can be derived by using the quadratic formula to find the solution to the intersection of the two curves.

[^24]:    ${ }^{2}$ The 1977 level exvessel price is also probably high relative to its long term equilibrium as will be shown in the next section.

[^25]:    ${ }^{3}$ This could occur by a mechanism such as forward contracting.

[^26]:    ${ }^{1}$ Perhaps the first limited entry program was that initiated in the British Columbia salmon fishery in 1889. That program gave permits to fish to canneries. It was abandoned in 1892 under pressure from new entrants. In recent times, the next limited entry program was adopted in the South African pilchard and mackerel fishery (1953), the Western Australia Rock Lobster fishery in 1963, followed by another one in the Australian Prawn fishery in 1965. These were followed by the Canadian Maritime Lobster program (1967), the British Columbia salmon program (1968), Wisconsin and Michigan's Great Lakes programs (1968), several in Eastern Canada including herring (1970), and the Bay of Fundy scallops, offshore scallops and lobester, and groundfish fisheries (all in 1973). Alaska and Washington instituted limited entry programs in their salmon fisheries in 1974.

[^27]:    ${ }^{2}$ Alaskan salmon permits (area and gear specific) sell for up to $\$ 400,00$. British Columbian roe herring permits lease for $\$ 100,000$ per year.
    ${ }^{3}$ This idea originated with Christy[15] and was then developed in a fishery setting by Moloney and Pearse[61].
    ${ }^{4}$ See Copes[18] for a critical view.

[^28]:    ${ }^{5}$ This is determined by assuming $X=133.2, Q=27.14, q=.0015, f=.99$, and $P^{E V}=1.66$. These are estimated parameters and simulated equilibrium values for the full regulated open access model including the marketing sector. By contrast, suppose that the season were stretched to a maximum of 270 days, allowing a closure period to protect the biomass during spawning. Then only 562 skates of capacity would be needed to harvest the quota, operating over the longer season. This compares with the predicted regulated open access level of 32,870 skates of capacity. Since the estimated outfitting and opportunity costs are approximately $\$ 100$ per season per skate, the savings would be about $\$ 32$ million.

