An Adaptive Model of Perishable Inventory Dissipation in a Nonstationary Price Environment

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The paper develops an adaptive model of perishable commodity dissipation based on the individual's price expectations and risk perception. A two-step, state-space procedure for modeling nonstationary time series is presented. The method combines an impulse response model for estimating deterministic components with an innovations model for the remaining stationary stochastic noise. Combined parameters are used to generate forecasts and to derive a measure of risk in a nonstationary price environment. Defined as the variance (covariance) of out-of-sample forecast error, the measure of risk is the difference between the historical estimate of the stationary noise auto-covariance and the variance (covariance) of out-of-sample forecasts. The optimal marketing strategy for a hypothetical salmon processor who sells to Japanese wholesalers is developed to illustrate the model. The solution is obtained using quadratic programming algorithm.

Agricultural production is often characterized by a short and concentrated harvest period, followed by a longer marketing period during which the stored product is sold from inventories. The same situation is frequently observable in many fisheries where there exists a biologically determined period when fish are available in high concentrations (e.g. salmon runs) or in fisheries where harvesting season has been shortened to avoid exceeding target harvest levels [Wessells and Wilen (1993)]. In these types of economic activities, agents must decide whether to rush the commodity to the market immediately after harvesting/processing or store it and speculate on the price increase over the storage period.

Many Alaskan salmon processors export the bulk of their inventory within few months after harvest. In this paper we design an alternative marketing strategy where inventory depletion is based on the processor's price expectations and risk perception. The commodity under consideration is frozen sockeye salmon and the economic agent is a hypothetical processor who buys fish from fishermen during harvesting season and, after processing and freezing, stores it for future sales. Because 96 percent (average 1986–1990) of the US fresh/frozen sockeye salmon is exported to Japan (average 1986–1990) of the US fresh/frozen sockeye salmon is exported to Japan [Knapp (1992)], the processor's marketing strategy is oriented towards the Japanese wholesale market.

Based on the adaptive hedging model of Vukina and Anderson (1993), the task is to develop an adaptive dynamic model of perishable inventory dissipation. The model assumes the agent depleting his/her inventory over time with no carryover of perishable fish into the next season in a way that will maximize expected utility of terminal profit. The subjective expectations about future prices and the individual perception of risk are approximated by the price forecasts and the variance-covariance of forecast errors. Forecasts and mean squared forecast errors are generated by the state-space method for modeling nonstationary time series with nonstationarities assumed deterministic in nature [see: Vukina and Anderson (1994)]. The model is adaptive in the sense that the marketing

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strategy can be adjusted each time period when new information becomes available.

Inventory Dissipation Model

The model simulates the decision process of a hypothetical processor who buys an exogenously determined quantity of raw fish. In a given isolated region of Alaska, even a small processor would experience exogenous supply conditions determined by the regional fishery management constraints and stochastic salmon run size. After processing and freezing, the purchased quantity of raw fish yields the processed quantity Q_0^* that commands the known market price $\bar{p}_0$. Price $\bar{p}_0$ can be interpreted as the opportunity cost of tied-up working capital, i.e. the price that would have been obtained had the product been ready for sale in period 0. The model assumes unlimited cold storage capacity at the nonstochastic rental price (unit cost of storage). The problem becomes how to optimally allocate the given quantity Q_0^* throughout the remaining periods of the marketing season, with no stock carryover into the next season, so that the expected utility of terminal profit is maximized. Future prices $p_1, \ldots, p_T$ at which the commodity (risky asset) will be sold are unknown stochastic variables. A hypothetical processor is a price taker, because his strategy cannot influence the Japanese wholesale market price. Assuming an increasing, strictly concave, twice differentiable Von Neumann-Morgenstern utility function U with terminal profit Π_T as the sole argument and the information set Z_0 (i.e. the history of prices up to the data point p_0), the period 0 decision problem is:

$$
\begin{aligned}
&\underset{q_k}{Max}\ J_0 = E[U(\Pi_T)|Z_0] \\
&\text{s.t.} \\
&\Pi_T = (1+r)^T(-\bar{p}_0 Q_0^*) + \sum_{k=1}^{T}(1+r)^{T-k}(p_k - kc)q_k \\
&\sum_{k=1}^{T} q_k = Q_0^* \\
&q_k \geq 0; \quad \forall\, k, \quad k = 1, 2, \ldots, T
\end{aligned}
\tag{1}
$$

where $q_1, \ldots, q_T$ are decision variables denoting monthly quantities of perishable commodity to be sold, c is the unit cost of storage that increases with time k, and r is the one-period risk-free interest rate.

Assuming a constant absolute risk aversion (CARA) utility function, a normally distributed terminal profit, and jointly normally distributed Π_T and p_k, maximizing $E[U(\Pi_T)]$ then is equivalent to maximizing the mean-variance objective function J $= E(\Pi_T) - (\lambda/2)\,Var(\Pi_T)$, where $\lambda = -U''(\Pi_T)/U'(\Pi_T) = -E[U''(\Pi_T)]/E[U'(\Pi_T)]$ represents the Pratt-Arrow measure of absolute risk aversion. In matrix notation the problem can be specified as follows:

$$
\begin{aligned}
&\underset{\mathbf{q}}{Max}\ J_0 = E(\boldsymbol{\pi})'\mathbf{q} - \frac{\lambda}{2}\mathbf{q}'\,\Omega\,\mathbf{q} \\
&\text{s.t.} \\
&\mathbf{iq} = Q_0^* \\
&\mathbf{q} \geq 0
\end{aligned}
\tag{2}
$$

where Ω is a $(T \times T)$ symmetric matrix of the intertemporal variance-covariance structure of the decision maker's subjective probability distribution of net returns from sales:

$$
\Omega = \begin{pmatrix}
R^{2T-2}\sigma^2_{p_1} & R^{2T-3}\sigma_{p_1p_2} & R^{2T-4}\sigma_{p_1p_3} & \cdots & R^{T-1}\sigma_{p_1p_T} \\
R^{2T-3}\sigma_{p_2p_1} & R^{2T-4}\sigma^2_{p_2} & R^{2T-5}\sigma_{p_2p_3} & \cdots & R^{T-2}\sigma_{p_2p_T} \\
R^{2T-4}\sigma_{p_3p_1} & R^{2T-5}\sigma_{p_3p_2} & R^{2T-6}\sigma^2_{p_3} & \cdots & R^{T-3}\sigma_{p_3p_T} \\
\cdots & \cdots & \vdots & \cdots & \vdots \\
R^{T-1}\sigma_{p_Tp_1} & R^{T-2}\sigma_{p_Tp_2} & R^{T-3}\sigma_{p_Tp_3} & \cdots & \sigma^2_{p_T}
\end{pmatrix},
\tag{3}
$$

$E(\boldsymbol{\pi})$ is a $(T \times 1)$ vector of expected prices net of storage costs:

$$E(\boldsymbol{\pi}) = \begin{matrix} R^{T-1}(\hat{p}_1 - c) \\ R^{T-2}(\hat{p}_2 - 2c) \\ R^{T-3}(\hat{p}_3 - 3c) \\ \vdots \\ \hat{p}_T - Tc \end{matrix}, \tag{4}$$

$\mathbf{q}$ is a $(T \times 1)$ nonnegative vector of monthly sales, $R = (1 + r)$ is a compound factor, and $\mathbf{i}$ is an $(1 \times T)$ vector of ones. Expectations about the future wholesale prices are approximated with price forecasts $\hat{p}_k = E(p_k|Z_0)$, and the subjective perception of risk is measured by variances $(\sigma_p^2|Z_0)$ and covariances $(\sigma_{pp}|Z_0)$ of forecast errors, all based on information set Z_0.

The solution to the problem in (2) can be obtained by quadratic programming. The vector of optimal quantities $\mathbf{q}|Z_0$ determines the marketing strategy for the entire planning horizon based on information available in period 0. However, the model allows for the adjustment of the original marketing policy each time period, after new information becomes available. Therefore, based on the initial estimates, only the first period optimal quantity $q_1^*|Z_0$ is actually sold. The remaining periods quantities $(q_2, \ldots, q_T)|Z_0$ serve as auxiliary decision variables enabling derivation of the optimal inter-temporal solution $q_1^*|Z_0$.

In the next period, the processor has new data and the existing marketing strategy can be revised. Expectations and risk perceptions are now based on information Z_1, which has been updated with new data point p_1. To determine optimal inventory depletion for remaining inventory $(Q_0^* - q_1^*|Z_1)$, the decision problem can be formulated as:

$$\underset{q_2, \ldots, q_T}{Max} \; J_1 = E\,[U(\Pi_T)|Z_1]$$

$$s.t.$$

$$\begin{aligned} \Pi_T = {} & (1 + r)^T(-\bar{p}_0 Q_0^*) \\ & + (1 + r)^{T-1}\,(\bar{p}_1 - c)\,q_1^* \\ & + \sum_{k=2}^{T} (1 + r)^{T-k}\,(p_k - kc)\,q_k \\ & \sum_{k=2}^{T} q_k = Q_0^* - q_1^* \\ q_k \geq 0; \quad & \forall\, k, \quad k = 2, 3, \ldots, T. \end{aligned} \tag{5}$$

The remaining stochastic variables are $p_2, \ldots, p_T$. With the one-period time advancement, $\bar{p}_1$ has become a known constant and all expectations are based on information Z_1. The solution to the optimal marketing policy $(q_2, \ldots, q_T)|Z_1$ can be obtained by solving the quadratic programming problem like the one in (2), with the first row and first column in matrix Ω in (3) and the first row in vector $E(\boldsymbol{\pi})$ in (4) deleted. The $(T - 1)$ vector of optimal quantities $\mathbf{q}|Z_1$ determines the marketing strategy for the rest of the planning horizon based on information available in period 1, but only the nearest period sales of $q_2^*|Z_1$ are actually executed.

To adjust the marketing strategy to new market signals (prices), the same routine is repeated $(T - 1)$ times. The final decision the processor has to make is $q_{T-1}^*|Z_{T-2}$, because the last period sales q_T^* are automatically determined by satisfying the no-carryover constraint.

Price Forecasting With State-Space Models of Nonstationary Time Series

To simulate how inventory dissipation decisions are formulated, the measures of subjective price expectations and uncertainty surrounding these expectations are needed. We use the successively updated out-of-sample price forecasts to approximate price expectations and the variance (covariance) matrices of forecast errors to measure risk. Out-of-sample forecasts are obtained by the state-space method for modeling nonstationary time series. The individual processor's perceived risk is measured by the variance (covariance) of the out-of-sample forecast error (mean squared out-of-sample forecast error).

Based on the insights of linear systems theory, a state-space method for modeling vector-valued time series has been proposed by Aoki (1987) and Aoki (1990). As is typical of most time series procedures, the method assumes that series are stationary stochastic processes. In particular, the weak stationarity assumption requires that mean, variance, and covariances of the series are invariant with respect to displacement in time. Many time series in business and economics are not generated by stationary processes. In this case, accurate modeling of time series requires various transformations to achieve stationarity before the standard methods can be implemented [see: Havenner and Aoki (1988)].

Nonstationarities are categorized as either stochastic or deterministic. Integrated stochastic processes, such as random walks or random walks with drift, exhibit stochastic nonstationarities. Deterministic trends or cycles, dummy variables, or any other nonstationary exogenous (known and observable) variables are the examples of deterministic nonstationarities. The characteristics of the world salmon market suggest the possible presence of deterministic cycles in salmon prices. Cycles may result from the complex interplay of various factors, such as fish population dynamics, seasonal variations in demand, and cyclical nature of overall business activity.

Sockeye salmon (*Oncorhynchus nerka*) are anadromous. The great majority of sockeye are four years old when they return to spawn. Fisheries managers tend to rely on the pattern of these returns for managing the resource. The cycles of return runs have received particular attention because they create severe harvest and marketing problems [Groot and Margolis (1991:95)]. Other factors contributing to the cyclical price behavior include the Japanese traditional holiday demand for sockeye and the availability of various substitute species/products.

The standard approach to incorporate deterministic effects into state-space time-series models requires the stochastic (innovations) model be augmented with regression equation [Dorfman and Havenner (1991)]. The procedure will remove time varying deterministic mean, thus rendering the resulting series stationary. In the alternative approach presented in this paper, deterministic components of the original time series (scalar or vector-valued) are modeled as an impulse response first, and then the resulting residuals are modeled with an innovations model. Out-of-sample forecasts are obtained by summing the forecasts from the impulse response and the innovations models:

$$\text{(6)}\quad \begin{aligned} \hat{y}_{t+k} &= h_{t+k} + \hat{u}_{t+k} \\ &= GF^{k-1}b + CA^{k-1}x_{t+1}, \quad k \geq 1. \end{aligned}$$

where $\hat{y}$ represents the forecast of the original nonstationary time series, h denotes the deterministic nonstationarity, and $\hat{u}$ is the forecast of the stationary stochastic component. If $k = 1$, (6) reduces to a one-step-ahead forecast; $k = 2$ generates $\hat{y}_{t+2}$, and so forth.

In state-space description, the impulse response model (i.e. the response of the linear time-invariant discrete system to the discrete time Kronecker delta function) is specified as:

$$\text{(7)}\quad \begin{aligned} \chi_{t+1|t} &= F\chi_{t|t-1} + b\delta_t \\ h_t &= G\chi_{t|t-1} \end{aligned}$$

where F, b, and G are parameters to be estimated, δ_t is Kronecker's delta function defined as $\delta_t = 1$ for $t = 0$, and $\delta_t = 0$ otherwise, and χ_t is a vector of minimal variables needed to summarize the effect of previous inputs on all future outputs. Observations on m time series $\{y_t\}$ are assumed to be given by the impulse response sequence observed in noise: $y_t = h_t + u_t$, $t = 1, \ldots, T$; where $\{u_t\}$ is a zero mean, mutually independent, Gaussian random variable. The theoretical underpinning of the impulse response model, and the methodology for estimating parameters F, b, and G, are explained in details in Vukina and Anderson (1994).

Using the estimates of F, b, and G, the forecasts of the original time series are generated as the sequence of Markov parameters $h_t = GF^{k-1}b$, and forecasting errors (residuals) are calculated. The obtained errors from the first step are modeled with the innovations model in the second step. The problem requires fitting an appropriate model to T observations on m series of a zero-mean, weakly stationary, vector-valued Gaussian stochastic process with covariance sequence $\Gamma_j = E[u_{t+j} u_t']$. In the state-space format the innovations model consists of state and observation equations:

$$\text{(8)}\quad \begin{aligned} x_{t+1|t} &= Ax_{t|t-1} + Be_t \\ u_t &= Cx_{t|t-1} + e_t \end{aligned}$$

where matrices A, B, and C are the coefficients to be estimated, input $\{e_t\}$ is a white Gaussian process with $E(e_te_t') = \Psi$, and x_t is a vector of unobservable states that are minimal sufficient statistics for history of the process $\{u_t\}$. Subscripts on x refer to the conditional expectation of x in the period of the first subscript given the information set at the time of the second subscript.

The procedure for estimating Markovian model for a stochastic process (i.e. a state-space model driven by white noise) requires obtaining a parametric model for auto-covariance sequence of the process from raw data first, and then obtaining a model for the stochastic process itself. The parameters of the autocovariance model A and C are estimated with the procedure similar to the one used to estimate the Markov sequence parameters from the impulse response model [see: Vukina and Anderson (1994)], and the procedure to obtain an innovations model, i.e. the estimation of the Kalman filter matrix B, is based on Vaccaro and Vukina (1992). Out-of-sample forecasts are generated by $\hat{y}_{t+k} = CA^{k-1} x_{t+1|t}$.

Measuring Risk

The individual processor's perceived risk is measured by the variance (covariance) of the out-of-sample forecast error (mean squared out-of-sample forecast error). For example, variance of the one-step-ahead forecast error is defined as:

(9) $\Sigma_{11} = E[(y_{t+1} - \hat{y}_{t+1})(y_{t+1} - \hat{y}_{t+1})']$.

Recall that the observation equation for the actual process is:

$$(10) \qquad \begin{aligned} y_{t+1} &= h_{t+1} + u_{t+1} \\ &= h_{t+1} + Cx_{t+1} + e_{t+1} \end{aligned}$$

and the one-step-ahead forecast is generated by:

$$(11) \qquad \begin{aligned} \hat{y}_{t+1} &= h_{t+1} + \hat{u}_{t+1} \\ &= h_{t+1} + Cx_{t+1}. \end{aligned}$$

The observed (actual) process consists of deterministic part h_{t+1}, and the weakly stationary stochastic error u_{t+1}, whose forecasts $\hat{u}_{t+1}$ are generated as a linear combination of unobservable states (Cx_{t+1}). Substituting (10) and (11) into (9) and expanding the product gives:

$$(12) \quad \begin{aligned} \Sigma_{11} = {} & E[(h_{t+1} + u_{t+1})(h_{t+1} + u_{t+1})'] \\ & - 2E[(h_{t+1} + Cx_{t+1} + e_{t+1}) \\ & \quad (h_{t+1} + Cx_{t+1})'] \\ & + E[(h_{t+1} + Cx_{t+1}) \\ & \quad (h_{t+1} + Cx_{t+1})']. \end{aligned}$$

An important feature of weakly stationary stochastic processes is the fact that variances (covariances) of all equidistant lags are identical [e.g. $\Gamma_0 = E(u_t u_t') = E(u_{t+1} u_{t+1}')$]. After replacing u_{t+1} in the remaining cross-products with $(Cx_{t+1} + e_{t+1})$, multiple cancellation of terms in (12) yields:

$$(13) \qquad \begin{aligned} \Sigma_{11} = {} & \Gamma_0 - 2E(Cx_{t+1}x'_{t+1}C') \\ & + E(e_{t+1}x'_{t+1}C') \\ & + E(Cx_{t+1}x'_{t+1}C'). \end{aligned}$$

Observing the definition of the symmetric state covariance matrix $P = E(x_{t+1|t}x_{t+1|t}')$, and invoking the orthogonality of states with current and future innovations gives:

$$(14) \qquad \Sigma_{11} = \Gamma_0 - C P C'$$

where CPC' is the variance of the one-step-ahead forecast defined as:

$$(15) \qquad \begin{aligned} var(\hat{y}_{t+1}) &= E[(\hat{y}_{t+1} - E(\hat{y}_{t+1})) \\ & \quad (\hat{y}_{t+1} - E(\hat{y}_{t+1}))'] \\ &= E\{[h_{t+1} + Cx_{t+1} \\ & \quad - E(h_{t+1} + Cx_{t+1})] \\ & \quad [h_{t+1} + Cx_{t+1} \\ & \quad - E(h_{t+1} + Cx_{t+1})]'\} \\ &= E[Cx_{t+1}x'_{t+1}C'] = CPC', \end{aligned}$$

since $E(x_{t+1}) = 0$, and $E(\hat{y}_{t+1}) = h_{t+1}$.

Similarly, the covariance of the two-step-ahead and the one-step-ahead forecast errors is:

$$(16) \quad \begin{aligned} \Sigma_{21} &= E[(y_{t+1} - \hat{y}_{t+1})(y_{t+2} - \hat{y}_{t+2})'] \\ &= \Gamma_1 - CAPC'. \end{aligned}$$

where $\Gamma_1 = E(u_{t+1}u_t') = E(u_{t+2}u_{t+1}')$ represents the sample estimate of the one-lag stationary noise auto-covariance matrix, and CAPC′ is the covariance of the two-step-ahead and the one-step-ahead forecasts. In case of vector-valued time-series, variances and covariances are (m × m) nonsymmetric (except Γ_0) matrices, and the covariance of the one-step-ahead and the two-step-ahead forecast errors is the transpose of the covariance of the two-step-ahead and the one-step-ahead forecast errors:

$$(17) \qquad \begin{aligned} \Sigma_{12} &= (\Sigma_{21})' \\ &= \Gamma'_1 - CPA'C'. \end{aligned}$$

However, since in this case m = 1, variance-covariance terms are scalars and $\sigma_{21} = \sigma_{12}$.

Consider now the general case of variance (covariance) between the k-step-ahead and the j-step-ahead forecast errors:

(18) $\Sigma_{kj} = E[(y_{t+k} - \hat{y}_{t+k})(y_{t+j} - \hat{y}_{t+j})']$.

The observation equation for the actual process (10) can be combined with the state equation from (8) to yield the equation for the process output at time $t+k$ expressed in terms of its deterministic component at time $t+k$, the state vector at time $t+1$, and innovations $e_{t+1}, \ldots, e_{t+k}$:

$$y_{t+k} = h_{t+k} + CA^{k-1}x_{t+1} + \sum_{i=1}^{k-1} CA^{k-i-1}Be_{t+i}$$

$$(19) \qquad + e_{t+k}.$$

Substituting (19) and (6) into (18) yields the general formula for the variance (covariance) of out-of-sample forecast errors:

$$(20) \quad \Sigma_{kj} = \Gamma_{k-j} - CA^{(k-1)}PA'^{(j-1)}C', \quad k,j \geq 1.$$

In the vector-valued time series case $\Sigma_{kj} = \Sigma_{jk}'$, whereas in the scalar case $\Sigma_{kj} = \Sigma_{jk}$.

The results show that the measure of uncertainty defined as the covariance (variance, if $k = j$) of the out-of-sample forecast errors (mean square forecast error) is in fact a difference between the historical (sample) estimate of the lag $k - j$ stationary noise auto-covariance Γ_{k-j}, and the covariance of the k-step-ahead and the j-step-ahead (variance, if $k = j$) out-of-sample forecasts $E\{[\hat{y}_{t+k} - E[\hat{y}_{t+k}][\hat{y}_{t+j} - E(\hat{y}_{t+j})]'\}$. Because the eigenvalues of the transition matrix A lie inside the unit circle (the property of stationarity), increasing the exponent of A in (6) gradually brings $\hat{u}_{t+k} =$

$CA^{k-1}x_{t+1}$ to zero. The speed of decay depends on the magnitude of these eigenvalues. Fast eigenvalues (those close to zero) cause the out-of-sample forecasted residuals to degenerate quickly, while slow eigenvalues (those close to unity) enable forecasts to stretch further out into the future before collapsing to zero.

As a result, the combined forecasts of the actual process revert to the deterministic time series component $h_{t+k} = GF^{k-1}b$, and its variance (covariance) approaches zero. When such a point is reached, the mean square out-of-sample forecast error collapes to the historical stationary noise auto-covariance Γ_{k-j}. The further away the out-of-sample forecasts are from the present, the larger is the mean square forecast error and the lower is the forecast reliability. Maximum risk associated with forecasting is the historical variance of the stationary random noise process. For sufficiently large k, the combined out-of-sample forecast is not better than the deterministic component of the process.

The Toeplitz matrix of stationary noise auto-covariance sample estimates and the Hankel matrix of out-of-sample forecast variances (covariances) can be used to rewrite (20) as:

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \cdots & \Sigma_{1j} \\ \Sigma_{21} & \Sigma_{22} & \cdots & \Sigma_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ \Sigma_{k1} & \Sigma_{k2} & \cdots & \Sigma_{kj} \end{pmatrix} = \begin{pmatrix} \Gamma_0 & \Gamma_1' & \cdots & \Gamma_{j-1}' \\ \Gamma_1 & \Gamma_0 & \cdots & \Gamma_{j-2}' \\ \vdots & \vdots & \vdots & \vdots \\ \Gamma_{k-1} & \Gamma_{k-2} & \cdots & \Gamma_0 \end{pmatrix} - \begin{pmatrix} CPC' & CPA'C' & \cdots & CPA^{j-1\prime}C' \\ CAPC' & CAPA'C' & \cdots & CAPA^{j-1\prime}C' \\ \vdots & \vdots & \vdots & \vdots \\ CA^{k-1}PC' & CA^{k-1}PA'C' & \cdots & CA^{k-1}PA^{j-1\prime}C' \end{pmatrix} \tag{21}$$

which will be used to produce the out-of-sample forecast error variance (covariance) matrix Ω from (3) by multiplying each element of the Σ matrix by the corresponding compound factor.

Empirical Results

Japan's total supply of salmon increased by over 200 percent between 1976 and 1991. In the same period, imports increased by over 4100 percent, accounting for 40 percent of Japan's supply. Although, Japan, is the primary country of destination for the U.S. fresh/frozen salmon exports, the position of the U.S. on the Japanese market is diminishing as aquaculture's share in Japan's salmon imports increases, and as Japan further diversifies its pool of suppliers. At its peak in 1985, the U.S. had nearly 89 percent of total Japanese frozen salmon imports, by 1991 the US share dropped to only 63 percent [Kusakabe (1992)].

The wholesale markets are the core of the distribution system of seafood in Japan. There are 340 producer wholesale markets, and 505 consumer wholesale markets located throughout Japan. The Tokyo Wholesale Central Market (Tsukiji) is the largest fish market in Japan. Marketing channels that bypass wholesale markets are emerging due to improvements in processing technology and transportation, as well as increasing imports. Large supermarket chains that deal directly with importers also reduce the flow through wholesale markets. In general, there are four different outlets for salmon products in Japan: retail market, gift market, restaurants, and industrial use. The majority of imported frozen sockeye is usually processed as salted and sold through the retail market [for details see: Kusakabe (1992)].

In such a highly competitive environment, a hypothetical Alaskan processor perceives the Japanese wholesale prices as exogenous. The prices used in this study are the monthly average frozen sockeye salmon wholesale prices from the three wholesale markets in Tokyo area as reported in the *Tokyo Central Wholesale Market Yearbook*. Prices are expressed in Yen/Kg, and the data set covers the period from January 1978 to July 1992, for the total of 175 observations. The mean of the series is 1295, the standard deviation 234, the minimum value 804, and the maximum value 1883.

Forecasting Results

To implement the presented theoretical model, sequentially updated price forecasts and variance-covariance matrices of forecast errors are required. The results of the initial and final estimation round, together with forecasting statistics for all rounds are presented in Table 1.

Parameters are initially estimated using the first 164 observation, and 11 out-of-sample forecasts are generated by (6), together with the (11×11)

Table 1. Impulse Response/Innovations Model Parameter Estimates and Forecasting Statistics

Parameter Estimates	Impulse Response Model ($\tilde{n}$ = 4)							
	Initial Round: 164 obs.				Final Round: 173 obs.			
F	0.9985	0.0222	−0.0005	−0.0025	0.9986	0.0136	−0.0022	−0.0040
	−0.0004	0.9915	−0.1575	−0.0223	0.0008	0.9877	−0.1508	−0.0371
	−0.0038	0.1406	0.9904	0.0139	−0.0037	0.1397	0.9935	0.0371
	0.0011	−0.0025	−0.0040	0.9907	0.0014	0.0037	−0.0181	1.0047
b'	56481	−6199	−7975	60656	12170	−918	1067	−1700
G	0.1059	−0.1006	−0.0472	−0.0890	0.1029	−0.0878	−0.0444	−0.1695
	Innovation Model (j = 3, $\bar{n}$ = 2)							
A	0.7784	−7.0758			0.8240	−6.5370		
	0.0008	0.3301			0.0009	0.2488		
B'	0.0074	−0.0000			0.0082	−0.0000		
C	81.8351	838.500			74.1744	816.743		

Forecase Statistics	In Sample				Out of Sample	
	Obs.	Var (p_1)	MSE	MAPE	No. of Forecasts	MAPE
	164	53290.1	10434.2	5.90%	11	40.41%
	165	54057.8	10157.0	5.79%	10	37.75%
	166	54551.5	10158.6	5.78%	9	36.03%
	167	55110.2	10221.2	5.83%	8	31.19%
	168	55525.3	10178.0	5.82%	7	27.31%
	169	55333.1	10211.5	5.84%	6	32.47%
	170	55014.1	10231.3	5.87%	5	38.83%
	171	54898.2	10275.8	5.92%	4	32.12%
	172	54581.8	10346.9	5.95%	3	43.28%
	173	54380.9	10378.4	5.99%	2	38.60%

out-of-sample forecast error variance-covariance matrix Σ (21). Next, an additional observation (the 165th) is added to the existing data pool to update system matrices and 10 new out-of-sample forecasts with the new (10 × 10) Σ matrix are generated. The same procedure is repeated 10 times. In the final round estimates are obtained with 173 observations, and only 2 months ahead forecasts are generated. To secure the positive definiteness of Σ, the sample estimates of the stationary noise auto-covariances of different lags are obtained using a biased estimator $\Gamma_k = (1/T)\Sigma_{t=1}^{T-1} u_{t+1}u_t'$ [for details see: Kay, (1988)].

Using the first 164 observations, the estimated order of the impulse response model in (7) is $\tilde{n}$ = 4. The order of the model equals the dimension of the transition matrix F. The eigenvalues of F correspond to the poles of the transfer function. The location of poles allows for the characterization of the response properties of linear systems. For example, when the transfer function has a pair of complex-conjugate poles, given by $1_1 = a + ib$ and $1_2 = a - ib$, they will give rise to the response term $\beta d^k cos[k\theta + \alpha]$ for $k = 0,1,2,\ldots$; where $d = \sqrt{(a^2 + b^2)}$ is the magnitude of the pole, $\theta = \tan^{-1}(b/a)$ is its angle, and β and α are constants. If $\sqrt{(a^2 + b^2)} > 1$, poles are located outside the unit circle (an unstable system) which results in a sinusoidal-like oscillation increasing in magnitude. If poles are located on the unit circle, the response would be a constant sinusoidal oscillation. Finally, if poles are located inside the unit circle (stable system), the response would be sinusoidal oscillation decreasing in amplitude [Cadzow (1973: 260)].

The eigenvalue decomposition of matrix F gives one complex-conjugate pair of eigenvalues modeling the cycle, and two real eigenvalues very close to unity modeling the linear trend (intercept and slope). The complex-conjugate pair, $0.9902 + 0.1488i$ and $0.9902 - 0.1488i$, has the modulus of $d = 1.0013$ (indicating a slightly unstable system) and the angle of $\theta = \pm 0.1492$. The frequency estimate calculated as $\theta/2\pi$ yields 0.0237 cycles per month (0.2849 cycles per year) or equivalently the period of 3.5096 years for one full cycle.

The final round's cycle estimates with 173 points are only slightly different, with the complex-conjugate pair modulus of $d = 1.0001$ and slightly smaller angle of $\theta = \pm 0.1484$. The estimated impulse response model parameters are used to forecast and subtract out the deterministic com-

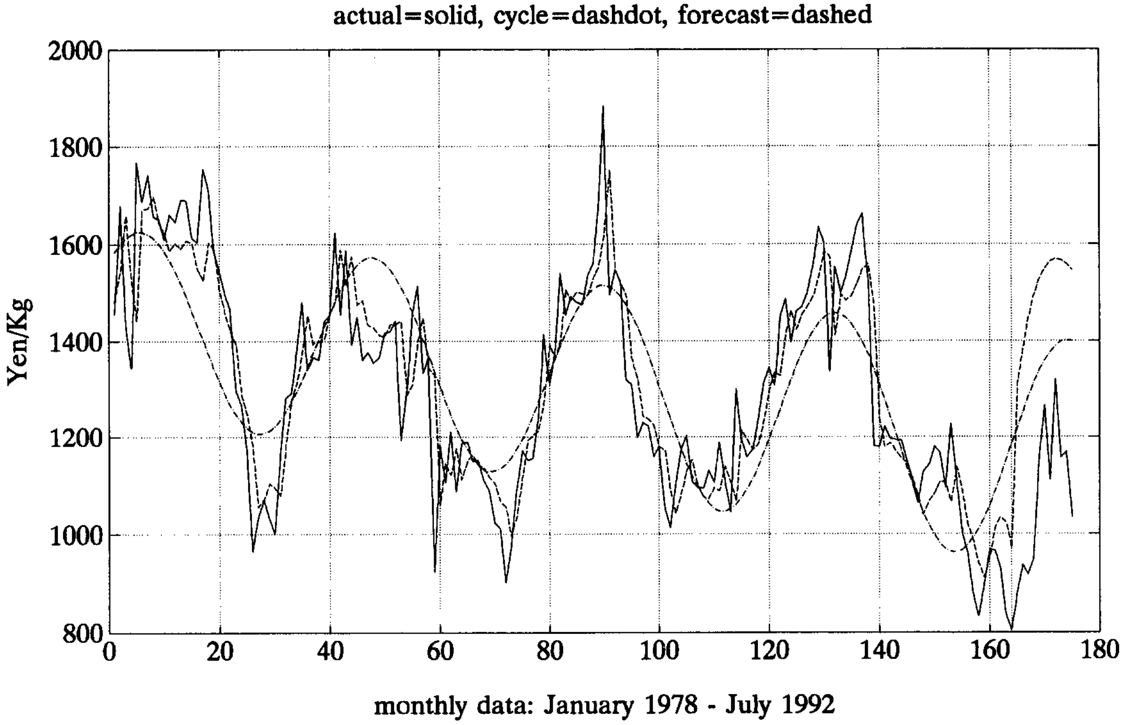

Figure 1. Forecasting with Nonstationary Data: Tokyo Wholesale Frozen Sockeye Salmon Prices (11 periods out-of-sample)

ponent of the original series, rendering the remaining noise stationary.[1]

To estimate innovations model parameters in (8) up to 3rd lag auto-covariances ($j = 3$) of the stationary noise are used and the estimated number of states (the order of the innovations model $\bar{n}$) is found to be 2 in all sequences. The innovations model parameters (A, B and C) are used together with the impulse response model parameters (F, b, and G) to forecast original price series. The combined procedure with 164 sample observations and 11 out-of-sample forecasts is illustrated in Figure 1.

A comparison of the in-sample mean squared forecast error (MSE) with the corresponding zero-lag unconditional data auto-covariance [$Var(p_t)$] indicates that combined models reduce the total sum of squares by a factor of 5. As one would expect, in out-of-sample forecasting, smaller forecast errors result when the forecast distance decreases. Mean absolute percentage error (MAPE) decreases from 40.41% for 11-step-ahead forecasts to 27.31% for the 7-step-ahead. For out-of-sample forecasts closer to the origin (6 and less), the observed tendency seemingly disappears due to large sampling error (small number of forecasts).

Simulation Results

This section illustrates how Alaskan salmon processors could potentially improve the profitability of their operations by gradually depleting inventories over the marketing period. Harvesting and processing occur each summer and inventory accumulation is completed in August. The problem becomes one of optimally allocating the given quantity throughout the remaining 11 months (September–July) of the marketing season. The incentive to liquidate inventory within one year is derived from the fact that frozen salmon will suffer quality deterioration if held for more than one year. For the purposes of this example, initial inventory $Q_0{}^*$ is set at 100 MT of frozen fish and monthly interest rate is assumed to be 0.75% or approximately 9% per year.

Using the obtained sequentially updated out-of-sample forecasts and corresponding variance-covariance matrices of out-of-sample forecast errors, the optimal frozen salmon inventory manage-

[1] To avoid possibility of high frequency cycles being swamped by long swings of the trend, prior detrending of the original data may sometimes be warranted. With detrended data, the order of the model is typically 2, with one conjugate-complex pair of the matrix F eigenvalues modeling the remaining cycle.

ment is calculated for the September 1991–July 1992 period. The process takes 10 iterations to complete and involves repeatedly solving the problem (2) by successively deleting first row and first column of the matrix Ω in (3) and the first element of vector $E(\boldsymbol{\pi})$ in (4). The solution is obtained by a quadratic programming algorithm available in the MATLAB Optimization Toolbox.

Some insight into the nature of the solution can be provided by analyzing a two-period case ignoring the nonnegativity constraints. The final decision the processor has to make is a two-period problem whose interior solution has the following form:

$$
\begin{aligned}
q_{T-1}^{*}|Z_{T-2} &= \frac{(1+r)\,[\hat{p}_{T-1} - (T-1)c] - (\hat{p}_T - Tc)}{\lambda[(1+r)^2\sigma^2_{p_{T-1}} - 2(1+r)\sigma_{p_{T-1}p_T} + \sigma^2_{p_T}]} - \frac{Q_0^{*} - \sum_{k=1}^{T-2} q_k^{*}\,[(1+r)\sigma_{p_{T-1}p_T} - \sigma^2_{p_T}]}{(1+r)^2\sigma^2_{p_{T-1}} - 2(1+r)\sigma_{p_{T-1}p_T} + \sigma^2_{p_T}} \\
q_T^{*} &= Q_0^{*} - \sum_{k=1}^{T-1} q_k^{*}|Z_{k-1}.
\end{aligned}
\tag{22}
$$

where the last period sales $q_T{}^*$ are automatically determined by satisfying the no-carryover constraint. Given a predetermined inventory size, the optimal marketing plan is a function of the future price expectations (forecasts) adjusted for the storage costs and the variances and covariances surrounding price expectations. The subjective expectations (forecasts) of an individual processor may differ from those of other market participants. As the result, in the short run, their decisions and actions are likely to be different. Because the presented model is an adaptive one, which means that false expectations (poor forecasts) are quickly realized (proven wrong), the entire strategy for the remaining (T-k) periods is promptly adjusted to comply with the new market signals.

Marketing decision also depends on the subjective risk parameter λ. For a risk averse individual, λ can be any positive number. However, for sufficiently strong risk aversion, the first term in (22) may become inconsequential and expected returns become unimportant. In this case, the optimal sales volume is determined as a proportion of the existing inventory, and the proportionality factor is determined only by the forecast error variance (covariance) structure. Indeed, optimal inventory dissipation schemes are found to be very similar for all values of λ greater than 0.01.

Table 2 summarizes the results of different simulation scenarios with varying risk aversion parameter. Diminishing risk aversion causes the inventory depletion to slow down, as the inventory

Table 2. Optimal Inventory Dissipation Strategy in September 1991–July 1992 period

Storage Cost: Yen/Kg		$c = 10$			$c = 30$	
Risk Aversion Parameter	$\lambda \geq .01$	$\lambda = .001$	$\lambda = .0001$	$\lambda \geq .01$	$\lambda = .001$	$\lambda = .0001$
Month			Percent			
August	100.00	100.00	100.00	100.00	100.00	100.00
September	39.09	33.51	0.00	39.55	38.17	15.29
October	26.63	24.08	0.00	26.85	26.31	18.70
November	15.22	15.21	4.00	15.34	16.48	21.18
December	8.68	8.72	3.09	8.69	8.77	10.79
January	4.75	5.06	4.85	4.72	4.47	6.95
February	2.81	5.58	31.60	2.72	4.13	20.95
March	1.58	4.62	33.67	1.48	1.68	6.14
April	0.58	0.65	1.34	0.48	0.00	0.00
May	0.42	1.70	14.30	0.15	0.00	0.00
June	0.13	0.23	1.33	0.00	0.00	0.00
July	0.13	0.64	5.82	0.00	0.00	0.00

holder expects the prices to go up during the course of the marketing season. If forecasts (expectations) are correct, postponing the sales until later part of the season should increase profits. This result occurs because the relative weight placed on the expected returns increases with decreasing risk aversion.

The sensitivity of the optimal marketing strategy was also tested by varying the unit cost of storage. As expected, the increase in the unit cost of storage would induce faster inventory depletion in the earlier months of the marketing period. As a result of linear storage cost function specification ($TC_k = kcq_k$; $k = 1,2, \ldots 11$), this tendency is more pronounced with less risk averse individuals.

Conclusions

The prevailing practice of many Alaskan salmon processor is to sell the bulk of their inventory to Japanese buyers within a few months after harvest. This paper designs a marketing strategy based on the processor's individual expectations about future prices and his/her individual perception of risk. To adjust the marketing strategy to new market signals (prices), the model is repeatedly solved with updated price forecasts and variances (covariances) of forecast errors. The solution was obtained using quadratic programming algorithm.

There is ample evidence that salmon prices, as well as many other economic time series, are not generated by stationary stochastic processes. Under assumption that nonstationarities are deterministic in nature, a two-step procedure for estimating state-space model parameters of nonstationary time series is used. The method combines an impulse response model that estimates deterministic components of the time series with an innovations model that models the remaining stationary noise.

The estimated parameters of the combined model are used to derive a measure of risk in a nonstationary price environment. Defined as the mean square out-of-sample forecast error, the measure of uncertainty is in fact a difference between the historical (sample) estimate of the stationary noise auto-covariance and the covariance (variance) of out-of-sample forecasts. The presented optimal frozen sockeye salmon inventory management using scalar time series forecasting illustrates the method. Modeling multiple time series may yield even better forecasting results.

The simulation results illustrate how an Alaskan salmon processor could potentially improve his/her marketing strategy by gradually depleting inventories during the period between two harvesting seasons. The model is general enough to be applicable to other commodities characterized by a short and concentrated harvesting season, followed by a longer marketing period. We treated as exogenous the initial quantity of raw fish purchased by the processor. However, there are examples where this assumption may be overly restrictive, as one can argue that the economic agent simultaneously determines the initial inventory to be acquired and the timing of its release. The presented analytical framework can accommodate this additional complexity by augmenting the existing model with the new decision variable Q_0, and then solve the linear system of $T + 1$ equations with $T + 1$ decision variables ($q_1, q_2, \ldots, q_T$, and Q_0) initially, reducing the dimension of the system and adjusting the strategy as time progresses.

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