Ex Ante Risks in Fed Cattle Production: A New View from Copulas

Yifei Zhang, North Carolina State University, yzhan242@ncsu.edu
Barry Goodwin, North Carolina State University, bkgoodwi@ncsu.edu

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Yifei Zhang*
Barry Goodwin†

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1. Introduction

Cattle producers are exposed to a high level of risk due to the variability of net returns to cattle feeding. Commercial cattle production can be divided into three phases: the production phase of weaned feeder calves, the back-grounding phase of production of feeder-lot ready calves, and the finishing phase in which the cattle are fattened for slaughter. In this paper, we focus on the finishing phase of the cattle production. At this phase, cattle are sent to a drylot on full feed with grains to fatten for slaughter. Many risk factors arise during this stage since feeder cattle producers need to make ex-ante decisions such as the time that cattle are place on feed and cost of feed inputs. The feeding period usually takes three to five months. During this time, uncertainties of the profitability of each pen can come from both the price and yield of cattle. For example, accidental death or injure can happen during the feeding period, also, cattle and corn market prices are changing constantly. In addition to the consideration of the performance of each pen, we account for the increasing demand for higher, more consistent quality beef in nowadays beef market to analyze risks arose from the adoption of a grid pricing mechanism. This pricing mechanism discovers individual animal carcass quality based of a value-based marketing system. A common grid includes five yield grades and five quality grades. Three discount indicators are also included in the analysis. In

*North Carolina State University, yzhan242@ncsu.edu
†North Carolina State University, bkgoodwi@ncsu.edu

general, this study models three aspects of ex-ante risks in fed cattle: the production risk arising from the performance of the cattle, the quality risk introduced by the grid pricing system, and the price risk from volatile cattle and corn prices.

In Unites States (US), cattle producers can buy revenue protection contracts to hedge against ex ante risks, to manage the variability in prices and therefore profits. Insurance such as Livestock Risk Protection (LRP) is designed to insure against declining market prices, and the Livestock Gross Margin (LGM) plan is designed to insure against the loss of gross margin. These insurance contracts are available in many coverage levels, and choosing the optimal contract requires a full understanding of the risks during the cattle production. Not only for cattle producers, addressing risks is also important for insurance companies. The US federal insurance program involves an interesting feature under the Livestock Price Reinsurance Agreement (LPRA), which allows private insurance providers to share risks with the federal government and receive subsided reinsurance. According to the definition by the Risk Management Agency (RMA) of USDA, the LPRA is a "cooperative financial assistance agreement between the Federal Crop Insurance Corporation (FCIC) and an insurance company." Under the LPRA terms, the federal government provides reinsurance on qualified private insurance company, known as Approved Insurance Providers (AIPs), to offer an opportunity for private insurance providers to transfer part of the risk burdens to the federal government. However, the choice of specific portion of the portfolio to cede to the federal government is in hand of AIPs. This, in turn, required concrete measurements of the portfolio risks. Our analysis offers appropriate measurement of livestock insurance portfolio risks for pricing insurance with various coverage levels.

One complication of live animal ventures is the fact that production efficiency can be characterized by a set of dependent variables that are usually highly dependent. Belasco, Ghosh, and Goodwin [4] evaluate the production risks faced by cattle producers using dry matter feed conversion (DMFC), average daily gain (ADG), the mortality rate (MORT), and veterinary costs per head (VCPH). Since these four measures are highly correlated, it would be beneficial to use a multivariate framework to capture the information that these variables brings. We follow after their approach and consider these four variables to evaluate the performance of each pen. In additional to these four performance variables, we have added an additional 13 dependent variables to the multivariate structure to account for unobserved information brought by the grid pricing mechanism. These 13 variables include five yield grades, five quality grades, and three discount factors. A major obstacle we face here when try to address the correlations between all 17 variables is the curse of dimensionality. To model the unknown dependence structure between such high dimension is computationally challenging.

The benchmark model used for livestock risk calculation assumes a normal distribution of the portfolios, yet such model does not capture characteristics such as tail dependence and asymmetric risks of the portfolio's underlying joint likelihood functions. To account for these characteristics, we propose to use copula models to provide flexibility when estimating the joint likelihood functions for a portfolio with multiple random variables that are possibly correlated. A copula is a multivariate distribution function for which the marginal probability distribution of each variable is uniform on the interval [0, 1]. Copula models are capable of modeling each marginal distribution independently, and therefore provide great flexibility. They also allow for tail dependence and asymmetric relationships while accommodating the high dimensionality. We fit the copula models using a two-stage maximum likelihood approach, also referred to as the Inference Functions for Margins (IFM) [14]. Multiple copula models are fitted, including the Student-t Copula, Vine Copulas, and Non-parametric Copulas. We use Gaussian Copula as a benchmark.

Another complexity when modeling these variables is the problem of censoring. Close to half of the variables are left censored at zero. We fit the censored data to a classic Tobit model to address the censoring before fitting the copula model. We then conduct simulation studies that evaluate the premiums/discounts that would be received under grid pricing, construct hypothetical insurance policies to calculate loss ratio and premium rates, and conduct analysis on profit variability when accounting for all three risks.

The rest of the paper is organized as the following: section 2 discusses some basic knowledge of grid pricing system, livestock insurance program, and copula models; section 3 introduces the methodology; section 4 provides information on the dataset; section 5 presents the simulation study and results; and the last section concludes.

2.1. Grid Pricing

In recent 40 years, the U.S. slaughter cattle industry is experiencing a market driven transformation from pen-level transactions to sales based on individual animal quality characteristics. This value-based marketing is adopted through the adoption of grid pricing. Compared to the traditional live animal pricing system, grid pricing values cattle on an individual animal basis. Prices are established based on the quality and yield grades of beef, after the animals have been slaughtered. The general consensus of adopting this pricing mechanism is in hopes to motive beef producers to improve carcass quality and consistency over time.

There are three common ways to price fed cattle in the market: by live weight, by dressed weight, and based on a grid price. With live weight pricing and dressed weight pricing, there is usually one average price established for the entire lot. The advantage of having an average price is the flexibility since prices are negotiated after transaction costs are established. The down side, however, is that they do not provide appropriate price signals to cattle producers [18]. The grid pricing system, on the other hand, does not have this concern since the price is discovered based on a single animal. Most grids consist a based price with premiums and discounts for specific quality and yield grades. As a result, higher quality beef is rewarded with a premium, where lower quality beef is penalized with a discount. Grid pricing places a greater emphasis on the quality and consistency of beef, leading producers to improve the quality and produce more market desirable types of cattle.

The idea of using grid pricing for the fed cattle market was first introduced in the 1990s. Value Based Marketing Taskforce 1990 (VBMTF) stated the inconsistent quality issues in the beef market, which leads to an innovation in the pricing mechanism. Stated in the 2016 National Beef Quality Audit (NBQA) conducted by Beef Quality Assurance, consumer satisfaction was ranked the second priority by all marketing sectors except for packers, and a greater share of companies are ”willing to pay a premium for guaranteed quality attributes”. In addition, the audit points out that to many survey respondents, the consistent offering in size is more important than increase in size. A large carcass makes further process harder in ways to meet consumer specific thickness and weights. As a result, about 66 percent of further processors are willing to pay a premium in exchange for a guaranteed weight and size [7].

Following pricing on a grid encourages producers to improve the quality of beef, which is what consumer demands. However, following such pricing system can bring more risk factors into the production process, therefore, introduce more variability in profit. Fausti et. al. [9] compare the risk on a producer's decision to sell cattle on a grid relative to sell cattle on a pen averaged price. They find evidence that the risk premium associated with grid pricing shocks can affect producer's market decisions. Although following a grid pricing system can promote better beef quality, the penalty for marketing lower quality beef maybe larger.

The grid pricing mechanisms can vary by firms and by packing locations, yet they share common features [10]. Table 1 demonstrates an example grid pricing mechanism. This example grid consists five quality grades and five yield grades to reflect animal's carcass attributes. Quality grade choice and yield grade 3 indicate the base price, which reflects the value of a standardized carcass.

2.2. Livestock Risk Protection (LRP)

The Livestock Risk Protection (LRP) insurance program is a popular insurance product among cattle and swine producers to insure against unexpected declining market prices. Producers can choose a coverage plan ranging from 70 to 100 percent, and choose a insured period starting from three week going into a year. The price that cattle producers receive from LRP is discovered from the cattle futures market market, which should present a strong correlation to the price of settlement when cattle are marketed. If at the end of the insurance period, the actual price is below the coverage price (expected price $\times$ coverage level), producers would receive an indemnity payment of a difference between the coverage price and actual ending price. The calculation of LRP producer premium and indemnity is straightforward using the following formulas, which are provided by USDA AMS. Note that there is a subsidy factor included in the calculation. USDA provides subsidies to motivate the purchase of the insurance. The subsidy factor is usually set at 20 percent.

Producer Premium Calculation

Step 1: Insured Value = Number of Head $\times$ Target Weight $\times$ Coverage Price

Step 2: Total Premium = Insured Value $\times$ Rate

Step 3: Producer Premium = Total Premium - (Total Premium $\times$ Subsidy Factor)

Producer Indemnity Calculation

Indemnity = Number of Head $\times$ Target Weight $\times$ (Coverage Price - Actual Ending Price)

The rate used in calculating total premium is published daily by USDA, different rates corresponds to different coverage levels. In our simulation study, we attempt to calculate the probability of loss and the actuarially-fair premium for hypothetical portfolios while incorporating risks from quality/yield grade, and variations in prices.

2.3. Copula Models

Introduced by Sklar in 1959 [19], copula become popular in defining multivariate distribution functions because of the Sklar's Theorem. The theorem states that every multivariate distribution can be written in terms of marginal distributions and a copula function, where a copula function is a joint distribution function for which the margins are uniform on the interval $[0, 1]$. Multiple copula models are developed to accommodate various dependence structure between two variables, yet things becomes more complicated when dealing with multivariate densities. High dimensional copula models can be applied to various situations due to its flexibility, however, the estimations face the problem of curse of dimensionality, for which the convergence to the true density becomes slower as the dimension increases. Such estimations requires large set of observations as well as decent model efficiency. Vine copula models provide remedy to these problems.

Proposed by Joe [13] and developed by Bedford and Cooke [3] and Kurowicka and Cooke [15], the idea of vine copula is the ability to decompose any complicated multivariate copula density into the product of some bivariate copulas. This decomposition structure is call the regular vine (R-vine). Aas et al. [1] extended this idea and proposed the pair-copula constructions (PCCs), to model any arbitrary multivariate densities using pair copulas. The computational efficiency is largely improved since the structural building blocks are bivariate copulas. In addition, with PCCs, we are able to adopt the rich bivariate copula families to improve the flexibility of multivariate copula models.

In this study, we fit our multivariate data into the following five vine copula models: Gaussian Vine (g-vine), Student-t Vine (t-vine), Regular Vine (r-vine), Canonical Vine (c-vine), and Kernel Vine (k-vine). Out of the five models, k-vine is the only non-parametric model. Kernel estimators generally provide more flexibility than parametric estimators, yet they do not integrated to one. To solve this problem, we use the transformation method by Greenes et al.[11]. The data is first transformed to support on full $\mathbb{R}^2$ domain, then estimate the densities using standard kernel techniques. The following equation presents the transformed kernel estimator,

$$\hat{c_n}^T(u_1, u_2) = \frac{\hat{f}_n(\Phi^{-1}(u_1), \Phi^{-1}(u_2))}{\phi(\Phi^{-1}(u_1))\phi(\Phi^{-1}(u_2))} \tag{1}$$

where Φ is the standard normal CDF and ϕ is its first order derivative.

3. Methodology

The goal of this study is to estimate revenue insurance contracts while accounting for correlations between the dependent variables. With four performance variables, five yield grades, five quality grades, and three discount factors, a system of equations that contains 17 equations is modeled. Belasco, Schoreder, and Goodwin [5] incorporate the grid pricing aspect to estimate the quality risk and profitability of cattle production. We follow their definition of the system of equations, and define the following conditioning on a set of pen-characteristics:

$$\begin{bmatrix} P_i \\ Q_i \\ Y_i \\ D_i \end{bmatrix} = x_{1i} * \beta + \epsilon_i, \tag{2}$$

where P_i denotes the four performance variables which are the average daily gain, the log of dry matter feed conversion (the ratio indicating the amount of feed required per pound of weight gain), the mortality rate, and veterinary costs per head. Q_i is the five quality grades: Prime, CAB, Choice, Select; and Y_i denotes the five yield (Y1-Y5). Lastly, D_i represents the three discount factors: dark cutters, heavy, and light. Each dependent variable is conditioned on the same set of independent variables, x_{1i}, which includes the log of average placement weight, the sex of each pen (steer, heifer, or mixed), the location of each pen (Kansas or Nebraska), and the time of placement (Spring, Summer, Fall, or Winter).

Since more than half of the dependent variables such as mortality rate are left censored at zero, traditional modeling such as OLS estimations are biased and should not be used. For the left censored variables, we propose the use of the standard Tobit model [20]. The uncensored equations are modeled using maximum-likelihood estimation (MLE).

Belasco, Ghosh, and Goodwin [4] find that the residuals in the above equations are likely to be correlated. The correlations usually come from unobserved variables such as weather and genetics contributes. They proposed a multivariate Tobit model to account for the cross-equation correlation, however, this method is limited to having only one censored dependent variables. Instead, we fit the residuals to various vine copulas to control for the possible correlations. To fit the copula models, we first rank transformed each residual vectors into uniform distributions, then the copula models ares fitted. After the joint distribution is defined, we simulate data points from each copula structure to preserve the rank correlation. Lastly, we inversely transformed the simulated data back to their empirical marginal distributions. At this point, the simulated data points can be used for calculations of probability of loss and the actuarially-fair premium rates for various portfolios. More details on the simulation study are specified in section 5.

4. Data

We utilize the data obtained from five feedlots in Kansas and Nebraska who use grid pricing as their main marketing strategies, which includes pen characteristics, production costs, and quality data. Our sample consists a total of 9,681 observations with an average of 129.4 head per pen. When feeder cattle enter the lot, characterises such as in-weight and sex are recorded. In addition, feeding costs, medical charges, and mortality rates are calculated to determine the performance of each pen. At the end of the feeding period, cattle are slaughtered and categorized into five yield grades (y1, y2, y3, y4, y5) and five quality grades (Prime, CAB, Choice, Select, Standard) as a percentage of total carcass weight. The descriptive statistics are reported in Table 2. Notice that close to half of the dependent variables are left censored and few of them are almost fully censored. For example, the degree of censoring of yield grade five is more than 80 percent; in addition, all of the discount factors are censored for more than half of the observations. Figure 1 contains histograms for yield and quality grades, and the discount factors. We can visually see Prime, CAB, Standard, YG1, YG4, YG5, and all of the discount factors are left-censored at zero.

It is important to determine the correlations between quality and yield categories. We estimate the Spearman's rank correlations of the residuals to see the trade-offs producers face between yield and quality grades. The results are presented in Table 3, 4, and 5.

Table 3 presents the correlation coefficient within five yield grades. We observe strong negative correlation between YG1 and all of the other grades, the same applies to YG2. For yield grades below YG3, which is treating as a benchmark, the correlations become positive. This indicates that producers need to choose between targeting high or low yields. The correlation coefficients within quality grades are presented in Table 4. No specific patterns can be observed here except for grade Standard, for which is negatively correlated with all the other quality grades. Lastly, in Table 5, we observe a trade-off between the quality and yield grades, except for CAB, for which no specific pattern is discovered of the trade-offs between quality and yield. For both Prime and Choice, the correlation changes from negative to positive as the yield grade increases. The converse holds for the lower quality grades: for Select and Standard, the correlation changes from positive to negative as the yield grade increases. This implies that in general, producers need to make decision between more desirable yield grades and better quality grades.

5. Simulations and Results

In this section, we consider various simulation studies to estimate the risks brought by adopting a price grid and changes in cattle market price. As mentioned previously, we estimated each marginal density using MLE for the non-censored variables and Tobit model for the censored ones. At this point, we obtained a series of random vectors that are supposed to correlated due to unobserved factor such as weather and genetics. We then rank transformed the residuals to uniform distribution and fitted five vine copula models to characterize the dependence relationship. The goodness-of-fit statistics of each copula model is presented in Table 6. Out of the five models, the kernel vine copula has the lowest Akaike information criteria (AIC) yet highest Bayesian information criteria (BIC), while other models' AIC and BIC are quite consistent. In addition to the goodness-of-fit statistics, we perform a Vuong's [22] likelihood ratio test to compare R-vine and C-vine models. The null hypothesis of this test is that both models are equally close to the true density. A positive test statistic implies that the former model outperforms the latter one, and the converse conclusion holds for negative test statistics. We are interested in making this comparison because C-vine is an extension of the regular vine models with different degree structure. The Vuong test result is presented in Table 6, p-value is included in the parenthesis. The positive and statistically significant test statistic suggests that R-vine has better performance than C-vine. Therefore, we drop the C-vine specification in the rest of the simulation study.

We simulated 50,000 data points out of each copula structures for each quality and yield grades, and the three discount factors. The simulated variables are then inversely transformed back to their empirical marginal distributions. The uncensored variables are transformed to a normal distribution, while the specified marginal distribution for the censored variables is logistic.

The quality risks are generally introduced from two aspects. The first aspect is that the quality and yield grades are unknown to producers until the slaughter date, which adds an layer of uncertainty to the production process. The second aspect of quality risks is related to the unknown premiums and discounts at the decision making stage. The stochastic nature of the premiums and discounts might explain the fact that large premiums might not lead to large supply of higher quality cattle. To incorporate premiums and discounts to our simulation study, we obtained weekly premium-discount grid for the past 20 years (2001 - 2021) reported by USDA AMS. This report contains the national premiums and discounts for slaughter cattle with various carcass traits. This dataset provides information on how premiums and discounts change over time due to market conditions. In general, yield premiums and discounts are more stable and predictable than ones associated with quality grades. We estimate the probabilistic characteristics of these premiums and discounts by fitting each one to an ARIMA model. Model statistics are shown in Table 7. The optimal model is selected based on AIC. Each premium and discount price serie is first deseasonalized before fitting the model. We then forecast data for 20 weeks to obtain the expected premiums and discounts for each grade.

In addition to incorporating the quality risk, we also want to consider price risks. Constant changes in cattle prices, as well as feed prices such as corn, increase the uncertainty producers face. Knowing how price plays a role in profit variability can provide insights for creating effective risk management strategies. Corn and cattle prices are likely to be correlated and their dependence structure can be characterized using copula models. We obtained daily corn futures and live cattle futures from 1/1/2009 to 12/31/2018 reported by Chicago Mercantile Exchange (CME) and the price series are fitted to a bivariate copula model after rank transformation. We then simulated 50,000 data points out of the fitted copula models which preserves the rank correlation, and inversely transform the simulated data points to empirical distribution with the assumption that prices are log-normal.

5.1. Simulation Study

Our goal of this simulation study is to calculate the probability of loss and the actuarially-fair premium rates for a hypothetical mixed pen in Kansas with a targeted weight of 1,100 pounds. We define the probability of loss as the amount of times out of the 50,000 simulated data points that the actual ending price is less than the coverage price. The coverage price is calculated by multiplying the expected end value to the coverage level. We set the expected end value to \$122. This is the expected end value for 17-week endorsement contact in Kansas reported by USDA AMS. Four coverage levels are considered in our calculation: 85%, 90%, 95%, 99%. Lastly, we define the actuarially-fair premium rates as the ratio of expected loss and coverage price. Our calculations are based on the following two scenarios:

1. *Consider uncertainties in premiums and discounts.*
2. *Consider uncertainties in both premiums/discounts and price variability.*

In order to calculate the probability of loss and the actuarially-fair premium rates for these two scenarios, we need to define a base price. In real life, this base price is usually provided by meat packers. They can derived this price based on the market reported price or tie it to the futures market price. We assume the base price to be \$123.95, which is the settlement price for October contracts reported by CME at the time of writing this paper.

For the first scenario, we first calculated the premiums and discounts associated with each portfolio. For each simulated portfolios, we multiply the expected value of each premiums/discounts variables obtained from the forecast to the quality and yield grades. Recall that the quality and yield grades, as well as the three discount factors are percentage of carcass weight falls into each grade categories. The base price is then added to the calculated premiums/discounts to obtain the actual ending price. For the second scenario, we replace the base price with the simulated futures price. The results are presented in Table 8. We also included a benchmark model that only compares the simulated futures price to the expected end value.

Looking at results in Table 8, compare to the benchmark model, we see a large increase in probability of loss and actuarially-fair premium rates when include quality and price risks. More interestingly, the increase in probability of loss and premium rates is larger when only consider uncertainties in premiums/discounts. This is reasonable since premiums and discounts are established relative to the market conditions. Another important factor that affects the two measures is the coverage level. The higher the coverage level, the larger the probability of loss and therefore the higher the premium rates. Also keep in mind that the risks associated with grid pricing is affected by the base price: the higher the base price, the lower the effect of the grid premiums and discounts.

6. Conclusion

In this study, we consider three aspects of risks in fed cattle production: the production risks associated with the performance of each pen, the quality risks introduced by grid pricing, and price risks coming from cattle and corn market prices. These risks can be correlated with each other, and appropriate mathematical representations of these risks can benefit not only the cattle producers, but also private insurance providers. To address the possible dependence structure, we propose to use vine copula models to capture the underlying correlations. Simulated data points are then obtained from the estimated copula models that preserve the rank correlation. We then calculate the probability of loss and actuarially-fair premium rates for various hypothetical portfolios with different coverage levels. We find that the inclusion of quality and price risks greatly increases the probability of loss and premium rates.

Vine copula models utilize pair-copula constructions technique to handle the curse of dimensionality. These models provide great flexibility when characterizing the dependence structure for multivariate variables, and can have a wide range of applications. Vine copulas are beneficial for data exhibits tail dependence and skewness. In this study, however, we do not observe large differences between the copula models. This might be due to the fact that the non-censored variables are normally distributed and the censored variables are fitted to a classic Tobit model. Classic Tobit models are estimated under the assumption that the error terms are normally distributed, moreover, heteroskedastic error terms can bias the coefficient estimates. For future work, we would like address the issue of heteroskedastic nature of error terms in a censored regressions.

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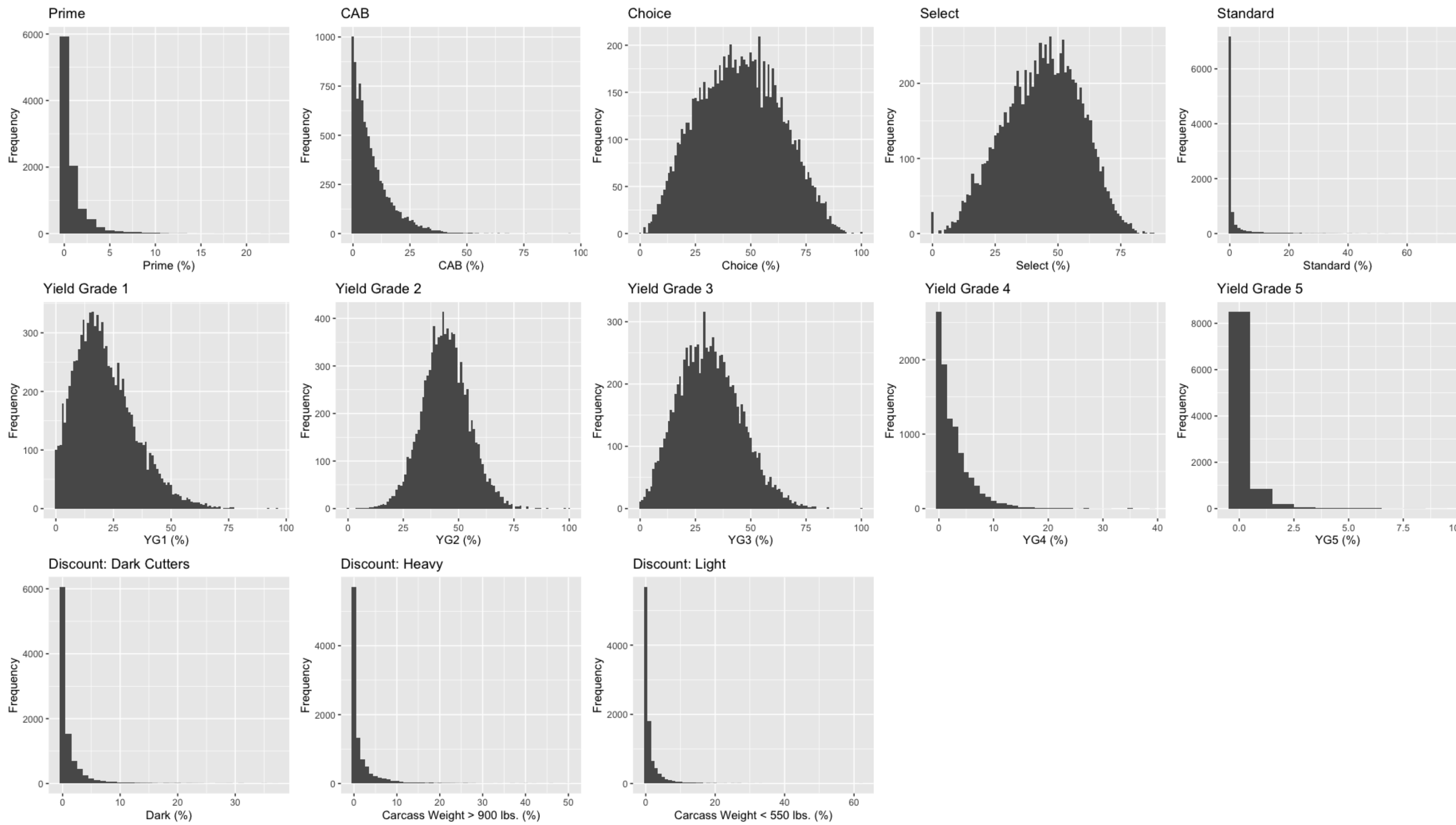

Figure 1: Histogram of Percentage Weight of Quality and Yield Grades, and Discount Factors

Table 1: Example of Grid Premiums and Discounts ($/cwt carcass)

		Yield Grades				
Quality Grades		*1*	*2*	*3*	*4*	*5*
Prime		9.90	8.30	6.50	-7.50	-13.50
CAB		5.40	3.80	2.00	-12.00	-18.00
Choice		3.40	1.80	Base	-14.00	-20.00
Select		-5.60	-7.20	-9.00	-23.00	-29.00
Standard		-14.60	-16.20	-18.00	-32.00	-38.00
Dark Cutters	-27.10					
Light (Carcass <550 lbs.)	-20.00					
Heavy (Carcass >900 lbs.)	-14.00					

Table 2: Descriptive Statistics

Varibale	Mean	Median	Std. Dev.	Minimum	Maximum	Censoring Degree (%)
Performance Variables						
ADG	3.42	3.42	0.48	0.47	5.78	0.00
DMCF	6.54	6.59	1.50	4.29	8.82	0.00
MORT	0.90	0.42	0.56	0.00	25.83	47.94
VCPH	7.08	7.07	0.67	0.49	9.52	0.00
Yield Grades (%)						
Y1	21.37	19.40	12.92	0.00	95.80	0.94
Y2	44.16	44.00	10.50	0.00	100.00	0.01
Y3	31.47	30.70	13.92	0.00	100.00	0.10
Y4	2.82	1.70	3.44	0.00	38.80	25.30
Y5	0.18	0.00	0.60	0.00	8.60	85.40
Quality Grades (%)						
Prime	0.84	0.00	1.61	0.00	22.60	57.49
CAB	8.60	6.00	8.68	0.00	95.00	2.46
Choice	45.00	45.00	18.29	0.00	100.00	0.01
Select	43.75	46.60	15.07	0.00	87.80	0.30
Standard	1.81	0.00	5.49	0.00	73.20	72.34
Discount Factors						
Dark Cutters	1.19	0.00	2.72	0.00	37.30	59.80
Heavy	1.72	0.00	3.68	0.00	50.00	56.92
Light	1.33	0.00	2.85	0.00	61.70	55.50
Independent Variables						
Avg. Place Weight (.lb)	758.15	758.00	98.20	371.00	1162.00	
Log(Weight)	6.62	6.63	0.13	5.92	7.06	
Dummy Variables (%)						
Steers	0.53					
Heifers	0.33					
Kansas	0.88					
Spring	0.25					
Fall	0.24					
Winter	0.25					

Table 3: Spearman's Correlation between Yield Grades

	YG1	**YG2**	**YG3**	**YG4**
YG2	-0.012	–		
YG3	-0.758	-0.544	–	
YG4	-0.476	-0.381	0.051	–
YG5	-0.122	-0.103	0.098	0.345

Table 4: Spearman's Correlation between Quality Grades

	Prime	**CAB**	**Choice**	**Select**
CAB	-1.56	–		
Choice	0.286	-0.584	–	
Select	-0.313	0.257	-0.832	–
Standard	-0.354	-0.125	-0.040	-0.071

Table 5: Spearman's Correlation between Quality Grades, Yield Grades, and Discounts

	YG1	**YG2**	**YG3**	**YG4**	**YG5**	**Dark**	**Heavy**	**Light**
Prime	-0.221	-0.081	0.199	0.279	0.398	-0.328	-0.303	0.341
CAB	0.277	0.083	-0.265	-0.056	0.010	0.284	0.096	0.003
Choice	-0.473	-0.122	0.468	0.230	0.036	-0.089	0.518	-0.062
Select	0.306	0.164	-0.363	-0.197	-0.044	-0.060	-0.495	-0.00012
Standard	0.199	-0.092	-0.083	-0.246	-0.404	0.324	0.100	-0.113
Dark	0.114	-0.033	-0.058	-0.121	-0.336	–	0.188	-0.239
Heavy	-0.067	-0.052	0.086	0.030	-0.182	0.188	–	-0.455
Light	0.081	0.044	-0.090	-0.026	0.219	-0.239	-0.455	–

Table 6: Goodness of Fit Statistics for Copula Models and Vuong Test Statistic

	Gaussian	**Student-t**	**Kernel**	**R-vine**	**C-vine**
Log-Likelihood	42178.73	47684.65	90397.06	61381.41	55986.52
AIC	-84085.45	-94825.30	-151681.70	-122298.8	-111499.00
BIC	-83109.33	-92873.04	-47205.82	-120633.7	-109798.00
Voung Statistics	32.5				
R-vine vs. C-vine	(0.00)				

Table 7: ARIMA Selection and Forecasts for Premiums and Discount Factors

	In-Sample			Forecast (20 Weeks)	
Grade Variables	Model (P, I, Q)	AIC	Sigma$\hat{2}$	Expected Value	Std. Error
YG1	(0, 1, 5)	1519.16	0.248	3.458	0.044
YG2	(0, 1, 1)	-1167.61	0.019	1.697	0.020
YG4	(5, 1, 0)	5911.23	16.69	-12.972	0.215
YG5	(5, 1, 0)	6518.93	30.34	-19.709	0.266
Prime	(2, 1, 2)	-625.12	0.031	6.887	0.151
CAB	(3, 1, 3)	-3464.22	0.001	3.391	0.130
Select	(0, 1, 3)	8174.75	148.9	-4.918	0.432
Standard	(2, 1, 2)	8850.39	285.6	-17.695	0.520
Dark Cuters	(1, 1, 1)	9110.25	367.2	-28.032	0.198
Heavy	(0, 1, 2)	4777.32	5.74	-9.208	0.001
Light	(0, 1, 1)	6885.00	42.84	-15.327	0.197

Table 8: Probability of Loss and Actuarially-fair Premium Rates

	Gaussian Copula		**Student-t Copula**		**Kernel Copula**		**R-Vine Copula**	
	Pr(Loss)	Rate	Pr(Loss)	Rate	Pr(Loss)	Rate	Pr(Loss)	Rate
Quality Risk Only								
85% Coverage Level	0.503	0.045	0.496	0.044	0.502	0.045	0.499	0.045
90% Coverage Level	0.733	0.077	0.733	0.076	0.730	0.077	0.727	0.077
95% Coverage Level	0.898	0.116	0.902	0.116	0.898	0.116	0.895	0.116
99% Coverage Level	0.964	0.149	0.967	0.149	0.963	0.149	0.960	0.149
Quality and Price Risk								
85% Coverage Level	0.565	0.099	0.563	0.099	0.565	0.100	0.561	0.099
90% Coverage Level	0.673	0.128	0.674	0.128	0.674	0.128	0.673	0.128
95% Coverage Level	0.773	0.160	0.772	0.159	0.770	0.160	0.769	0.159
99% Coverage Level	0.839	0.186	0.837	0.185	0.835	0.186	0.837	0.185
Benchmark	Pr(Loss)	Rate						
85% Coverage Level	0.168	0.016						
90% Coverage Level	0.265	0.027						
95% Coverage Level	0.386	0.042						
99% Coverage Level	0.493	0.058						