Import Demand Elasticities Based on Quantity Data: Theory, Evidence and Implications for the Gains from Trade

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Import Demand Elasticities Based on Quantity Data: Theory, Evidence and Implications for the Gains from Trade *

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Abstract

Correct estimates of import demand elasticities are essential for measuring the gains from trade and predicting the impact of trade policies. We show that estimates of import demand elasticities hinge critically on whether they are derived using trade quantities or trade values, and this difference is due to properties of the estimators. Using partial identification methods, we show theoretically that the upper bound on the set of plausible estimates is lower when using traded quantities, compared to the standard approach using trade values. Our proposed method using traded quantities leads to smaller point estimates of the import demand elasticities for many goods and imply larger gains from trade compared to estimates based on trade values.

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1 Introduction

Correct estimates of the elasticity of import demand are crucial in order to accurately estimate the gains from trade, predict the impact of trade policies and impute the size of trade costs from data on international trade flows. These elasticities are typically estimated from import trade data and have been estimated for many importing countries and products. The lower are these estimates, the greater the benefits of international trade and economic integration.

Estimates of the elasticity of import demand are traditionally performed using trade value data and price data constructed by dividing trade values by trade quantities, which are known as "trade unit values". An alternative approach is to estimate import demand elasticities using data on traded quantities instead of trade values. We present evidence here suggesting that import demand elasticity estimates based on trade values tend to be higher compared to using traded quantity data. In order to expose these differences between using traded quantities and trade values, we apply the method of partial identification of demand and supply elasticities developed by Leamer (1981). This method permits the estimation of the upper and lower bound on the set of possible estimates for the elasticity of import demand, and underpins the point estimates derived by Feenstra (1994).

In this partial identification framework, we show that the estimates of the upper bound using trade value data are more biased away from zero compared to using trade quantity data. Using data on U.S. imports for the years 1993-2006, we then show that using trade quantities instead of trade values yields estimates of import demand elasticity upper bounds that are smaller than traditional estimates. Since the lower bounds are identical using both approaches, this implies that the range of plausible estimates is much smaller when using traded quantities compared to the standard approach using trade values. Furthermore, approximately one third of the point estimates based on Feenstra (1994) are implausibly high, since they lie above the Leamer (1981) upper bounds.

The international economics literature has avoided using import quantity data when estimating import demand elasticities and authors typically claim that measurement error in the quantity data is at issue. The literature often cites Kemp (1962), who was the first to warn of the bias caused by measurement errors when estimating import demand elasticities. In Kemp's case, however, the bias was caused by measurement error in the price indices, which led to error in the constructed quantity indices that were calculated from trade value and price index data. However, in the context of contemporary international trade data, the raw data reports the value of trade and its quantity (in weight or units), so Kemp's critique does not necessarily apply. We derive the asymptotic bias of our estimators for the upper and lower bounds in the presence of measurement error in both trade quantities and trade values, and show that our original theoretical results are not overturned unless measurement error is sufficiently more severe in the quantity data than the value data.

Using detailed import data for the U.S., we estimate upper and lower bounds on the import demand elasticity using the three possible combinations of trade value, trade quantity and trade unit value data. The pattern of the upper and lower bounds in each approach matches our theoretical predictions for the asymptotic bias. We also derive point estimates using traded quantity data instead of trade value data in the estimation, modifying Feenstra's (1994) methodology. We find that the point estimates based on quantity data are lower on average than the corresponding point estimates using trade value data.

Our results have important implications not only for the estimates of import demand elasticities in particular goods, but also for the implied gains from international trade and economic integration. Our results thus contribute to a recent literature that has attempted to quantity the gains from trade for different countries and time periods that are predicted by workhorse models of international trade. Using the framework developed by Arkolakis et al. (2012), we show that our alternative approach to estimating import demand elasticities implies that the gains from economic integration are larger for many goods compared to previous studies.

Point estimates of import demand elasticities using Feenstra's (1994) methodology have been extensively employed in the field of international economics, and have been used to estimate the impact of new imported product varieties on the price index (Feenstra, 1994), measure the gains from increased variety due to imports (Broda and Weinstein, 2006), impute trade costs from trade flow data (Jacks et al., 2008, 2011; Chen and Novy, 2011; Novy, 2013) , and also to calibrate countless applied models of international trade. The vast majority of this literature estimates this elasticity using trade value data. The Feenstra (1994) approach is commonly used to calculate the trade elasticity, and is conceptually distinct from estimates of the trade elasticity using the Ricardian models of international trade, such as Simonovska and Waugh (2014) and Caliendo and Parro (2015).

While we test and motivate our analysis in the context of international trade, our results are generalizable to any estimation of demand elasticities where price data must be constructed from quantity and value data, and the econometrician must select the most appropriate model. For example, household survey data on expenditures and quantities is used to estimate price elasticities (Deaton, 1987, 1990). Unit values are also prevalent in firm-level datasets, and are used to estimate price elasticities for unit labor costs (Carlsson and Skans, 2012) and electricity unit values (Davis et al., 2013).

The rest of the study proceeds as follows. In section 2 we present the theory behind the partial identification of the import demand elasticities and derive the asymptotic bias associated with the upper and lower bound estimators. Section 3 describes our data and empirical methodology. In section 4 we present the results estimating the upper bounds, lower bounds and point estimates of the import demand elasticity using U.S. import data. Given our new estimates, we quantify the impact of these new estimates on the welfare gains from trade in Section 5. Section 6 concludes.

2 Partially Identifying Import Demand Elasticities

We begin by theoretically deriving the difference in asymptotic bias when estimating import demand elasticities using quantity data or value data. The import demand elasticity for a good can be naively estimated by regressing traded quantities on prices:

$$\ln x_{ct} = -\beta \ln p_{ct} + \varepsilon_{ct}, \tag{1}$$

where x_{ct} is the quantity demanded from country c in year t, and p_{ct} is its corresponding price.[1] However, estimating (1) by OLS will lead to biased and inconsistent estimates of β if the errors are correlated with prices, i.e., $E(\varepsilon_{ct} \ln p_{ct}) > 0$. This positive covariance arises if ε_{ct} contains demand shocks — a positive demand shock raises both quantity and price. An IV approach is one potential solution, but the absence of good instruments

[1]Note that we express the elasticity of substitution as a positive value. For simplicity and without loss of generality, we omit the constant in the regression equation and assume all variables have mean zero.

in this context has lead to alternative approaches in the literature.

The challenge of estimating import demand and supply elasticities in the absence of good instruments has a long tradition in economics. The study of an under-identified supply and demand system was pioneered by Working (1927), who showed that under certain conditions the data trace out the demand curve if the supply curve is more variable than the demand curve. Leamer (1981) shows that in a demand–supply system with zero covariance between the residuals, the set of possible maximum likelihood estimates is defined by a hyperbola. Leamer (1981) also shows that if the demand elasticity assumed to be negative and the demand elasticity assumed to be positive, then the set of maximum likelihood estimates for one elasticity is the interval between the direct least-squares estimate (regressing quantities on prices) and the reverse least-squares estimates (regressing prices on quantities). Leamer (1981) shows that (1) defines either the upper or lower bound on the true estimate of the demand elasticity, and the reverse least square estimate will define the other bound. In what follows, we follow Leamer's (1981) partial identification approach to estimating an upper and lower bound for the elasticity of import demand.

2.1 Quantity–Price Approach (Leamer, 1981)

The main principle of partial identification is to estimate an interval in which the true parameter lies. The main objective is to establish that the upper bound is above the true parameter, and the lower bound is below the true parameter. For these bounds to be informative, it is also relevant that the estimated interval is as narrow as possible, while at the same time ensuring that the bounds bracket the true parameter of interest.

We now derive the asymptotic bias of the estimators for the least squares and reverse least squares regressions of import quantities on import prices. The demand equation is given by (1), and the supply equation is given by:

$$\ln x_{ct} = \gamma \ln p_{ct} + \eta_{ct}, \tag{2}$$

which yields the following reduced form:

$$\ln x_{ct} = \frac{\gamma}{\gamma+\beta}\varepsilon_{ct} + \frac{\beta}{\gamma+\beta}\eta_{ct},$$
$$\ln p_{ct} = \frac{1}{\gamma+\beta}\varepsilon_{ct} - \frac{1}{\gamma+\beta}\eta_{ct}.$$

The probability limit of the OLS estimate of β using (1) is

$$\texttt{plim}\ \hat{\beta} = -\frac{E(\ln x_{ct}\ln p_{ct})}{E\left((\ln p_{ct})^2\right)} = \frac{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 + (\gamma-\beta)\sigma_{\varepsilon\eta}}{\sigma_\varepsilon^2 + \sigma_\eta^2 - 2\sigma_{\varepsilon\eta}},$$

where $\sigma_{\varepsilon\eta} = E(\varepsilon_{ct}\eta_{ct})$. Now, consider the reverse regression of $\ln p_{ct}$ on $\ln x_{ct}$. The probability limit of the OLS estimator is

$$\texttt{plim}\ \hat{\beta}^R = -\frac{E(\ln x_{ct}\ln p_{ct})}{E\left((\ln x_{ct})^2\right)} = \frac{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 + (\gamma-\beta)\sigma_{\varepsilon\eta}}{\gamma^2\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2 + (\beta+\gamma)\sigma_{\varepsilon\eta}}.$$

Assume the supply and demand shocks are uncorrelated, i.e. $\sigma_{\varepsilon\eta} = 0$. This yields the following probability limits for the least squares and reverse least squares estimates:[2]:

$$\texttt{plim}\ \hat{\beta} = \beta - (\gamma+\beta)\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_\eta^2} \leq \beta, \tag{3}$$

$$\frac{1}{\texttt{plim}\ \hat{\beta}^R} = \beta + (\gamma+\beta)\frac{\gamma\sigma_\varepsilon^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2} \gtreqless \beta \tag{4}$$

It is clear from (3) that the least squares estimate, which captures the lower bound, brackets the true β from below. With an additional parametric assumption on the sign of the denominator in (4), we obtain the Leamer (1981) result that the least squares and reverse least squares estimates constitute the upper and lower bound on β:

$$0 \leq \texttt{plim}\ \hat{\beta} \leq \beta \leq \frac{1}{\texttt{plim}\ \hat{\beta}^R} \Leftrightarrow \beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 > 0. \tag{5}$$

[2]Leamer (1981) shows that the hyperbola of the maximum likelihood estimates is given by $\hat{\gamma}^2\left(\hat{\beta}s_p^2 - s_{px}\right) + \hat{\beta}^2\left(-\hat{\gamma}s_p^2 + s_{px}\right) = \left(\hat{\beta}-\hat{\gamma}\right)s_x^2$, where s_p^2 and s_x^2 are the sample variances and s_{px} is the sample covariance. Assuming a non–negative supply elasticity, the upper bound for the demand elasticity is found by imposing $\hat{\gamma} = 0$, which yields $\hat{\beta} = \frac{s_x^2}{s_{px}}$, the inverse of the least squares estimate of p on x.

2.2 Value–Price Approach

In international trade data, the price is constructed as the average unit value of each trade flow i.e. $p_{ct} = v_{ct}/x_{ct}$, where v_{ct} is the value of trade. Logging this expression and rearranging yields

$$\ln v_{ct} = \ln p_{ct} + \ln x_{ct}. \tag{6}$$

This simple relationship between trade values, trade quantities and trade unit values in the data implies that β and γ can be estimated using any two of the components from (6) and then transforming the resulting point estimate. For example, one can use (6) to transform (1) and (2) into regression of trade values on trade unit values, yielding the following expressions for demand and supply:

$$\ln v_{ct} = (1 - \beta) \ln p_{ct} + \varepsilon_{ct}, \tag{7}$$

$$\ln v_{ct} = (\gamma + 1) \ln p_{ct} + \eta_{ct}. \tag{8}$$

Feenstra's (1994) point estimates are based on structural equations similar to (7) and (8), which require using constructed trade unit values. The reduced form of this system of equations is given by:

$$\begin{aligned} \ln v_{ct} &= \frac{1+\gamma}{\gamma+\beta}\varepsilon_{ct} + \frac{\beta-1}{\gamma+\beta}\eta_{ct}, \\ \ln p_{ct} &= \frac{1}{\gamma+\beta}\varepsilon_{ct} - \frac{1}{\gamma+\beta}\eta_{ct}. \end{aligned}$$

The probability limit of the lower bound on β using (7) is the ordinary least squares regression of trade values on prices, transformed using (6):

$$1 - \texttt{plim}\ \hat{\beta}^P = 1 - \frac{E(\ln v_{ct} \ln p_{ct})}{E\left((\ln p_{ct})^2\right)} = \beta - (\gamma + \beta)\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \leq \beta. \tag{9}$$

The probability limit of the lower bound in (9) is identical to (3). This stems from the fact that price is on the right hand side when estimating the lower bounds, regardless of whether quantities or values are the dependent variable. We can show, however, that the upper bounds are not identical to the quantity–price approach. The probability limit for the upper bound based on the reverse least squares estimation takes the following form:[3]

$$\texttt{plim} \quad -\frac{1-\hat{\beta}^{R,P}}{\hat{\beta}^{R,P}} = -\frac{1-\frac{E(\ln v_{ct}\ln p_{ct})}{E((\ln v_{ct})^2)}}{\frac{E(\ln v_{ct}\ln p_{ct})}{E((\ln v_{ct})^2)}}$$

$$= \beta + (\beta+\gamma)\frac{(1+\gamma)\sigma_\varepsilon^2}{(\beta-1)\sigma_\eta^2-(1+\gamma)\sigma_\varepsilon^2}. \tag{10}$$

It is clear from the denominator in (10) that the reverse least squares regression will unambiguously not bound the true β from above if $\beta < 1$. However, $\beta > 1$ is a common assumption in the literature and will be satisfied for may imported products. Feenstra (1994) assumes a demand elasticity in excess of unity due to CES preferences, and Scobie and Johnson (1975) argue that the elasticity of demand will be elastic if supplying countries are sufficiently "small" in the sense that there are several suppliers of a similar good to the export market. It is also evident in (10) that the Feenstra upper bound will hold under in some cases in the presence of a downward-sloping supply curve. The numerator of the bias term in (10) is larger than the numerator in equation (4), which suggests that the upper bound is likely larger using trade value data compared to using trade quantity data.[4]

2.3 Discussion and the Impact of Measurement Error

To sum our results so far, we have shown that the Leamer and Feenstra lower bounds are identical and unambiguously bound the true demand elasticity from below. The Leamer and Feenstra upper bounds are not identical, and both bracket the true import demand elasticity from above under certain parameter restrictions. Moreover, the Feenstra upper bound is more likely to be violated when the true import demand elasticity is small.

[3] Inverting equation (7) without the error term yields the transformed reverse least squares estimate $\hat{\beta}^{R,P} = \frac{1}{1-\hat{\beta}^P}$. Rearranging yields $\hat{\beta}^P = -\frac{1-\hat{\beta}^{R,P}}{\hat{\beta}^{R,P}}$.

[4] As suggested by Scobie and Johnson (1975), another way to partially estimate import demand elasticities is to regress $\ln x_{ct}$ on $\ln v_{ct}$ and vice versa, thus avoiding the need to construct price data. We derive the asymptotic bias of the upper and lower bounds using this approach in the Appendix. We find that the quantity–value lower bound is identical to the Leamer upper bound, and that the quantity–value upper bound is identical to the Feenstra upper bound. Since the lower bound is not likely the bracket the true elasticity in this case, estimating import demand elasticities without constructing trade unit values thus leads to implausibly high estimates.

We plot the predicted asymptotic biases of each estimator for various values of the true import demand elasticity in order to illustrate the relationships between the bounds. The results of this exercise are reported in figure 1 where we plot β between 0 and 10, and we hold constant $\gamma = 1$, $\sigma_\varepsilon^2 = 0.5$, and $\sigma_\eta^2 = 1$. It is evident from the figure that the Feenstra upper bound is larger than the Leamer upper bound for most values of β. Figure 1 also illustrates that the Feenstra upper bound is highly unstable at low values of β, and becomes negative when the true import demand elasticity is below a certain threshold. The Leamer approach is thus particularly well–suited to situations where the true import demand elasticity is low.

A related question is whether or not our theoretical results hold in the presence of measurement error. Kemp (1962) was the first to warn of the bias caused by measurement errors using quantity data for the purpose of estimating import demand elasticities. In Kemp's case, the bias was caused by constructing quantity indices from trade value and price index data. In the second paragraph of Kemp (1962), he writes:

> In aggregative studies, however, the quantity variables almost always is constructed by dividing the index of import prices into an index of the total money value of imports. The quantity variable is subject therefore to a measurement error of its own.

In his derivations, Kemp assumes a measurement error term in the price index data, but not in the money value of imports. Kemp goes on to show that using constructed quantity index data leads to biased and inconsistent estimates of the import demand elasticity, which correspond to our lower bound estimates. In the context of contemporary international trade data, however, the raw data reports the value of trade and its quantity (in weight or units). In the raw 10-digit US import data, for example, there are 47 different units of measuring quantities. Moreover, one cannot rule out that measurement error exists in the contemporary trade value data. Transfer pricing, for example, can lead to measurement error in the trade value data.

The asymptotic bias of the upper and lower bounds of import demand elasticities in the presence of classical measurement error in both trade values and traded quantities are provided in the Appendix. We find that the expression for the asymptotic bias in the presence of measurement error for the least squares estimator (the lower bound) is identical regardless of using trade value or trade quantity data, just as it is in the case without measurement error. The parameter restrictions required for the bounds to hold in the presence of measurement error become slightly more complicated in all approaches since they also hinge on the magnitudes of the error variance and covariance. We therefore study three specific cases of measurement error. In the first case, we assume that the measurement error variance in traded quantities and trade values, and their covariance, are equal in magnitude, which we call the "quantity and value error" case. In the second and third cases, we assume that there is only measurement error in traded quantities only and trade values only. The results of this exercise are illustrated in figures A.1, A.2 and A.3 in the Appendix.

In figure A.1 we plot the true import demand elasticity, and the lower bound with no measurement error, with quantity measurement error, and with value measurement error. It turns out that measurement error in quantities only attenuates the Leamer lower bound, and measurement error in values only attenuates the Feenstra lower bound. However, the attenuation bias is relatively more severe with measurement error in values.

In figure A.2 we plot the true import demand elasticity and the Leamer upper bound with no measurement error, with measurement error of equal magnitude in quantities and values, and with measurement error in quantities only. In figure A.3 we plot the true import demand elasticity and the Feenstra upper bound with no measurement error, with measurement error of equal magnitude in quantities and values, and with measurement error in values only. The results suggest that measurement error of equal magnitude in both quantities and values leads to a very similar outcome to our results without measurement error. Measurement error in quantities only attenuates the Leamer upper bound, while measurement error in values only inflates the Feenstra upper bound.

Overall, our partial identification theoretical results suggest that it is best to estimate import demand elasticities using traded quantities if measurement error is not an issue, or it is similar in magnitude in the quantity and value data. If measurement error is a relatively larger problem in the quantity data then it should be avoided, while if measurement error is relatively larger problem in the value data then it should be avoided. In general, quantity data is particularly well-suited to estimating import demand elasticities for goods with an expected low elasticity. With these theoretical predictions in hand, we now describe the trade data and estimate the bounds on the elasticity of substitution and the point estimates using the Leamer and Feenstra approaches.

3 Data and Empirical Application to U.S. Imports

We now describe how we estimate the upper and lower bounds using the Leamer and Feenstra approaches. We first describe the data, then we provide our estimating equations for the bounds. Finally, we explain how we derive point estimates using the quantity data.

3.1 Data

Our main data source is the U.S. import data available at the Center for International Data, which is based on data from the U.S. Customs Service.[5] The data includes the value of U.S. imports (in USD) and its associated quantity by country of origin at the 10-digit HS level. Following Soderbery (2015), we focus on the years 1993–2006. From the trade values and trade quantities we compute trade unit values. We thus observe the trade value, trade quantity and trade unit values by HS product, partner country and year. We study the U.S. since it is a large importer that imports from many countries, even withing narrowly defined product categories, and allows us to relate our results to those of Feenstra (1994), Broda and Weinstein (2006) and Soderbery (2015).

We perform our estimations at the 8-, 6-, 4-, and 3-digit HS levels, which we achieve by aggregating the data across products. There are 47 different types of quantity units in the data. Since it is crucial to use the same quantity unit for each product, we keep only the trade flows that use the most common quantity unit within before aggregating the data to more coarse product definitions. The units used to measure quantity are very often the same, even within broad product categories. Approximately 5 percent of trade flow observations are dropped when harmonizing the quantity units at the 8-digit HS level. When harmonizing quantity units at the 3-digit HS level, our most aggregated product definition, we drop approximately 20 percent of observations.

As a robustness check we perform our estimations using data from the COMTRADE database, which is administered by the United Nations. We use importer-reported data

[5]See Feenstra et al. (2002) for a detailed description of the U.S. import data. The data can be found at http://cid.econ.ucdavis.edu/usix.html

for U.S. imports at the 6-digit HS level for the years 1991–2015, where both the value of trade (in USD) and the quantity of trade (in kilograms) are reported.

In order to calculate the gains from trade for each imported product, we require data on import penetration ratios for each product, which we take from the U.S. Bureau of Economic Analysis (BEA) 2007 input-output tables, available at the 6-digit level. We collapse the BEA commodity/industry classification to the 4-digit level, then merge it with the Center for International Data U.S. import data at the 4-digit NAICS level.

3.2 Empirical Methodology

In order to compare our results with the literature, we follow Feenstra (1994) by normalizing trade values and trade quantities as a share of total imports. In the case of traded quantities, a country's market share for good g is defined as

$$q_{gct} \equiv \frac{x_{gct}}{\sum_{c \in C_{gt}} x_{gct}} \tag{11}$$

In the case of trade values, a country's expenditure share for good g is defined as

$$s_{gct} \equiv \frac{v_{gct}}{\sum_{c \in C_{gt}} v_{gct}} \tag{12}$$

Normalizing the data with respect to a reference country absorbs the origin–product–year fixed effect, which contains the importer's price index term that would arise in a CES demand framework. Following Feenstra (1994), we also difference the data with respect to a reference country k.

We estimate the lower and upper bounds of the elasticity of import demand for each good at the 3-, 4-, 6- and 8-digit HS level of aggregation, normalizing the variables as described above. Formally, the Leamer lower bound regression for good g is:

$$\Delta^k \ln q_{gct} = -\hat{\beta}_g \Delta^k \ln p_{gct} + \varsigma_{gct}, \tag{13}$$

where

$$\Delta^k \ln q_{gct} \equiv \Delta \ln q_{gct} - \Delta \ln q_{gkt},$$
$$\Delta^k \ln p_{gct} \equiv \Delta \ln p_{gct} - \Delta \ln p_{gkt}$$

The Leamer upper bound regression is:

$$\Delta^k \ln p_{gct} = -\hat{\beta}_g^R \Delta^k \ln q_{gct} + \upsilon_{gct}, \tag{14}$$

The Feenstra lower bound regression is:

$$\Delta^k \ln s_{gct} = -\hat{\beta}_g^P \Delta^k \ln p_{gct} + \xi_{gct}, \tag{15}$$

where

$$\Delta^k \ln s_{gct} \equiv \Delta \ln s_{gct} - \Delta \ln s_{gkt},$$

The Feenstra upper bound regression is:

$$\Delta^k \ln p_{gct} = -\hat{\beta}_g^{R,P} \Delta^k \ln s_{gct} + \zeta_{gct}, \tag{16}$$

We also develop a method for deriving point estimates of import demand elasticities using data on traded quantities instead of trade values, based on Feenstra's (1994) methdology. The structural model's "demand" and "supply" equations are as follows:

$$\Delta^k \ln q_{gct} = -\sigma_g^q \Delta^k \ln p_{gct} + \epsilon_{gct}^k \tag{17}$$

$$\Delta^k \ln p_{gct} = \omega_g^q \Delta^k \ln q_{gct} + \delta_{gct}^k \tag{18}$$

where ϵ_{gct}^k and δ_{gct}^k are unobservable demand and supply shocks, respectively and $\omega \geq 0$ is the inverse supply elasticity. Feenstra (1994) derives equations similar to (17) and (18), but using expenditure shares instead of quantity shares, from a model of CES preferences, using the Armington (1969) assumption of product differentiation by country of origin. [6] Note, however, that σ_g^q in (17) is identical to our estimate of the Leamer lower bound, $\hat{\beta}_g^R$, in equation (13). Moreover, ω_g^q in (18) is the reverse least squares estimate, which is the same as our estimate of the Leamer upper bound with the opposite sign, $\hat{\beta}_g^R$, in equation (14). Thus, the structural equations used to establish point estimates of the elasticity of demand are directly related to the

[6] Harberger (1957) shows that this system yields an elasticity of substitution with more general assumptions on consumer preferences than CES.

estimating equations of the upper and lower bounds.

Feenstra's innovation is to multiply ϵ^k_{gct} and δ^k_{gct} together in order to convert equations (17) and (18) into one estimable equation. Following Feenstra (1994), we assume that ϵ^k_{gct} and δ^k_{gct} are independent. We define $\rho^q_g = \frac{\sigma^q_g \omega^q_g}{1+\sigma^q_g \omega^q_g} \in [0,1)$, scale by $\frac{1}{\sigma^q_g \left(1-\rho^q_g\right)}$ and rearrange to obtain the analogue of Feenstra's (1994) estimating equation:

$$\left(\Delta^k \ln p_{gct}\right)^2 = \theta^q_{1g} \left(\Delta^k \ln q_{gct}\right)^2 + \theta^q_{2g} \left(\Delta^k \ln p_{gct} \Delta^k \ln q_{gct}\right) + u_{gct}, \tag{19}$$

where

$$\begin{aligned} \theta^q_{1g} &= \frac{\rho^q_g}{\left(\sigma^q_g\right)^2 \left(1-\rho^q_g\right)}, \\ \theta^q_{2g} &= \frac{2\rho^q_g - 1}{\sigma^q_g \left(1-\rho^q_g\right)} \end{aligned} \tag{20}$$

and

$$u_{gct} = \frac{\epsilon_{gct}\delta_{gct}}{\sigma^q_g \left(1-\rho^q_g\right)} \tag{21}$$

Feenstra (1994) shows that estimating (19) by 2SLS, where the instruments are dummy variables across the countries $c \neq k$, leads to consistent estimates of θ^q_{1g} and θ^q_{2g}. We implement the most recent refinement of Feenstra's method, by Soderbery (2015), who applies a limited information maximum likelihood (LIML) estimator in order to reduce bias and improve constrained search efficiencies.

Once we have obtained the estimates of $\hat{\theta}^q_{1g}$ and $\hat{\theta}^q_{2g}$, the values of $\hat{\sigma}^q_g$ and $\hat{\rho}^q_g$ can be solved from the quadratic equations in (20). As long as $\hat{\theta}^q_{1g} > 0$, these equations yield two solutions for $\hat{\sigma}^q_g$, one positive and one negative.[7] We restrict attention to the positive solution. Formally:

[7] This system can also be solved in terms of σ^q_g and ω^q_g, where $\theta^q_{1g} = \omega^q_g / \sigma^q_g$ and $\theta^q_{2g} = \left(\sigma^q_g \omega^q_g - 1\right) / \sigma^q_g$. It is clear here that θ^q_{1g} must be positive so that σ^q_g and ω^q_g are both positive.

$$\hat{\rho}_g^q = \frac{1}{2} + \left(\frac{1}{4} - \frac{1}{4 + \left(\hat{\theta}_{2g}^q\right)^2 / \hat{\theta}_{1g}^q} \right)^{1/2} \quad if\ \theta_{2g}^q > 0, \tag{22}$$

$$\hat{\rho}_g^q = \frac{1}{2} - \left(\frac{1}{4} - \frac{1}{4 + \left(\hat{\theta}_{2g}^q\right)^2 / \hat{\theta}_{1g}^q} \right)^{1/2} \quad if\ \theta_{2g}^q < 0, \tag{23}$$

$$\hat{\sigma}_g^q = \left(\frac{2\hat{\rho}_g^q - 1}{1 - \hat{\rho}_g^q} \right) \frac{1}{\hat{\theta}_{2g}^q} > 0. \tag{24}$$

If θ_{1g}^q is negative, then the solution fails to provide estimates of σ_g^q and ρ_g^q that satisfy the restriction that $\sigma_g^q > 0$ and $0 \leq \rho_g^q < 1$. The restriction on ρ_g^q implies that the supply elasticity must be non-negative, i.e. $\omega_g^q > 0$, which falls directly from Leamer's (1981) inequality constraints. In the event that θ_{1g}^q is negative or there is an imaginary solution, then we apply the constrained search algorithm developed by Soderbery (2015).

4 Results

4.1 Partial identification using quantity data

We first estimate the upper and lower bounds using the trade value – trade unit value specification as given by (15) and (16), which produces the bounds on the set of plausible Feenstra point estimates. We call this set of possible estimates the "Feenstra bounds". The results for each 3-digit HS import product are illustrated in figure 2. The x-axis ranks each HS3 product by its lower bound (least squares) estimate. While all lower bound estimates are positive and lie close to one, the estimates of the upper bound vary widely. For many products with a small lower bound estimate, the corresponding reverse least squares estimate is negative, which agrees with the predicted asymptotic bias. For several products the upper bound is very high. We thus truncate the figure to display estimates between 0 and 30. We also report all "Feenstra point estimates" based on trade values that the Soderbery (2015) procedure yields. The vast majority of the point estimates lie within the bounds given by the estimates of equations (15) and equation (16), with only a few exceptions.

We then estimate the point estimates and the upper and lower bounds using the Leamer trade quantity – trade unit value specification as given by (13) and (14), which we call the "Leamer bounds". We report both the Leamer bounds and the Feenstra bounds, plus the Feenstra point estimates, in figure 3. As predicted by the theory, the Leamer and Feenstra lower bounds are nearly identical, while the Leamer upper bound is far below the Feenstra upper bound. It is also evident that many of the Feenstra point estimates (around one third) lie above the Leamer upper bound. This suggests that many of the elasticity estimates used in the literature are implausably large. Finally, it is evident that the Leamer upper bounds are positive and lie above the lower bounds for all products, including those for which the Feenstra upper bound was negative.

We also check whether our results regarding the difference between the Leamer and Feenstra bounds are sensitive to the level of product aggregation. Imbs and Mejean (2015) show, for example, that estimates of trade elasticities are smaller in aggregate data than at finer levels of aggregation. In figures A.5 and A.6 in the Appendix we illustrate the alternative bounds with the original bounds and point estimates at the HS 4-digit and 6-digit levels respectively. We find that the difference between the Feenstra and Leamer upper bounds persists at finer levels of product aggregation. We also find that many of the Feenstra point estimates lie below the Feenstra and Leamer lower bounds even at finer levels of aggregation.

4.2 Point estimates using quantity data

We now turn to our point estimates of the import demand elasticities using quantity data, and compare them with the point estimates derived from using trade value data, which is the standard approach in the literature. In figure 4 we illustrate the point estimates based on traded quantity data for each 3-digit HS import product, which we call the "Leamer point estimates", as well as the corresponding Feenstra point estimate using trade value data. We also include the Leamer bounds, which allows us to discern how well the point estimates fit withing the set of plausible estimates. Figure 4 illustrates that nearly all our Leamer point estimates lie within the bounds. The figure also illustrates that the Feenstra point estimates tend to be larger on average, especially for those products where the Feenstra point estimate lies above the Leamer bound. We also calculate the Leamer point estimates at the 4-digit and 6-digit levels, and the results are illustrated in figures A.7 and A.8 in the Appendix.

Descriptive statistics of all of the bounds and point estimates at the 3-digit, 4-digit and 6-digit level are provided in Table 1, where we report the number of products, the raw mean and the median. The mean and median of the Leamer upper bounds are always lower than the corresponding measure of the Feenstra upper bounds, regardless of the level of product aggregation. The median is lower than the mean in all cases for the upper bounds, which is driven by a small number of products with relatively high upper bounds. The difference between the mean and the median is especially pronounced for the Feenstra upper bounds. Table 1 also highlights that the Leamer point estimates are lower than the Feenstra point estimates in all cases, for all levels of product aggregation. The raw average and median of the point estimates are very stable across product aggregations.

5 Implications for the Gains from Trade

We now quantify the economic importance of our alternative approach to measuring import demand elasticities for the welfare gains from economic integration. We use the framework developed by Arkolakis et al. (2012), which distills the welfare effect of openness to trade across a wide array of trade models into a simple formula:

$$\hat{W}_j = \hat{\lambda}_{jj}^{1/\epsilon}, \tag{25}$$

where $\hat{W}_j$ is the percentage change in welfare in destination country j, $\hat{\lambda}_{jj}$ equals the percentage change in country j's internal trade (1 minus the import penetration ratio), and ϵ is the elasticity of imports with respect to variable trade costs, also known as the "trade elasticity". In the Armington (1969) model, $\epsilon = 1 - \sigma$, where σ is the import demand elasticity.[8] The formula given in (25) thus highlights that estimates of the import demand elasticity play a central roll in measuring the gains from trade.

We first follow Arkolakis et al. (2012) and calculate the gains from economic intregration for the U.S. in 2000, where the import penetration ratio was seven percent, which implies $\lambda_{jj} = 0.93$. Using the average Leamer point estimate at the 8-digit

[8] In the Melitz (2003) model, $\epsilon = 1 - \sigma - \gamma_j$, where γ_j is the extensive margin elasticity. In the Ricardian model, $\epsilon = 1 - \sigma + \gamma_{jj}^i - \gamma_{ij}^i$, where γ_{jj}^i and γ_{ij}^i denote the extensive margin elasticities.

level from Panel D of Table 1 (2.59), the gains from trade compared to autarky are $1 - 0.93^{1/(1-2.59)} = 4.6$ percent. Using the corresponding 8-digit average Feenstra point estimate (6.75), the gains from trade are $1 - 0.93^{1/(1-6.75)} = 1.3$ percent. The Leamer point estimates suggest gains from trade that are 3.5 times as large as the Feenstra point estimates.

We also calculate the gains from trade on at the industry level, using the 4-digit BEA commodity classification. Ossa (2015), for example, shows that the gains from trade are often higher when calculated at the industry-level and then aggregated. We first calculate the Leamer and Feenstra point estimates at the 4-digit BEA level. These estimations yield 46 BEA commodities for which we have viable Feenstra and Leamer point estimates, and we report these estimates in figure A.10 in the Appendix. We then combine these point estimates with data on the import penetration ratio from the 2007 BEA input-output tables. Plugging these values into (25) for each commodity yields an estimate of the gains from trade for each BEA commoditity. We report the gains from trade across the 10th, 25th, 50th, 75th and 90th percentiles of these commodities in Table 2. The gains from trade are clearly higher when using the Leamer point estimates compared to the Feenstra point estimates across the entire distribution of commodities. The gains from trade for the median commodity is 43.2 percent using the Leamer point estimates, compared to 30.8 percent when using the Feenstra point estimates. This difference in the gains from trade is driven by the fact that the Leamer point estimates of the import demand elasticities are lower than the Feenstra point estimates for most goods. Using traded quantity data instead of trade value data to estimate import demand elasticities thus leads to much higher estimated gains.[9] We also report the gains from trade using the Leamer and Feenstra upper bound elasticity estimates in Table 2. The calculated gains from trade based on these bounds provide a conservative estimate of the gains from trade. The gains from trade for the median commodity is 15.7 percent using the Leamer upper bounds, and only 2.6 using the Feenstra upper bounds.

[9]We find very large gains for a small number of BEA commodities in both the Leamer and Feenstra cases, so that aggregating to the country-level following Ossa (2015) and Costinot and Rodrguez-Clare (2014) provides uninformative results. We thus elect to report industry-level gains across different percentiles instead.

6 Conclusion

Correct estimates of import demand elasticities are essential for measuring the gains from trade and predicting the impact of trade policies. The international economics literature has typically estimated these elasticities using trade value data instead of trade quantities. Using partial identification methods, we show theoretically that the upper bound on the import demand elasticity is more biased upward compared to using traded quantity data. We confirm our theoretical predictions using detailed U.S. import data. We also generate import demand elasticity point estimates based on traded quantity data and compare them with corresponding point estimates using trade value data. Our results suggest that import demand elasticities are lower than previously thought for many goods, which implies that the gains from economic integration have been underestimated in earlier studies.

While we test and motivate our analysis in the context of international trade, our results are generalizable to any estimation of demand elasticities where price data must be constructed from quantity and value data, and the econometrician must select the most appropriate model. Our derivations of the asymptotic bias suggest that using quantity data is superior to value data in cases where measurement error is of similar magnitude in the quantity and value data.

Our results have many implications in international economics that we leave for further research, such as analyzing the impact on the variety gains from trade or the magnitude of trade costs implied by trade flow data. Given that these elasticities are so important for understanding the gains from trade, it is hoped that our study encourages discussion on the pros and cons of using quantity versus value data when estimating demand elasticities.

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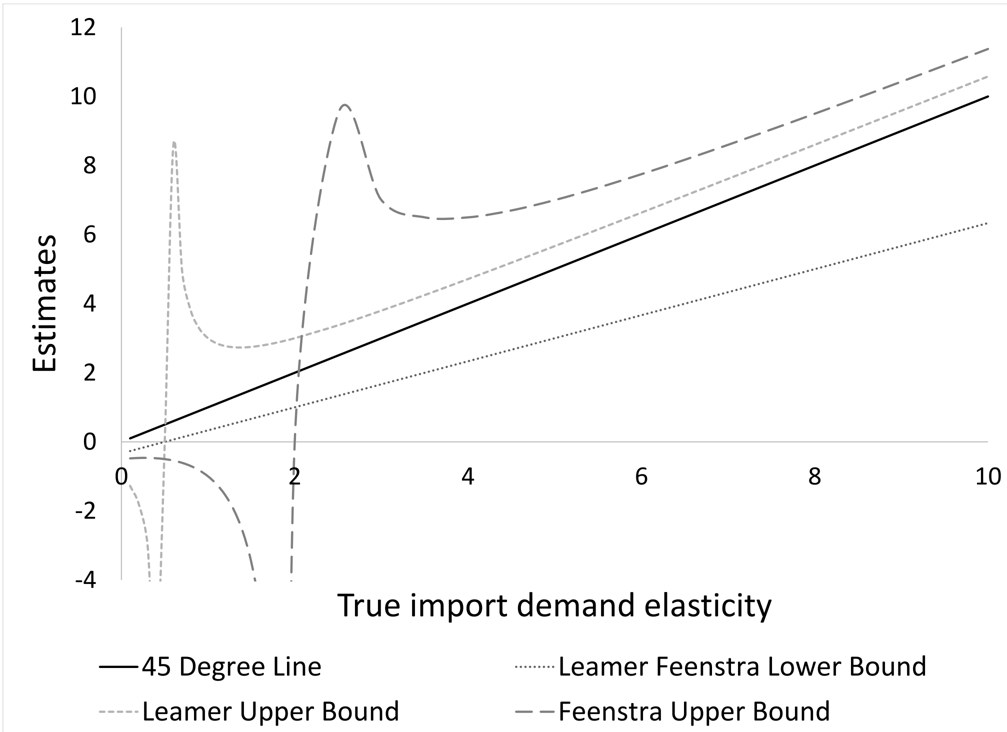

Figure 1: Theoretically predicted upper and lower bounds as function of true import demand elasticity, no measurement error. Source: authors' calculations

Notes: $\gamma = 1, \sigma_{\varepsilon}^2 = 0.5, \sigma_{\eta}^2 = 1$ in all cases.

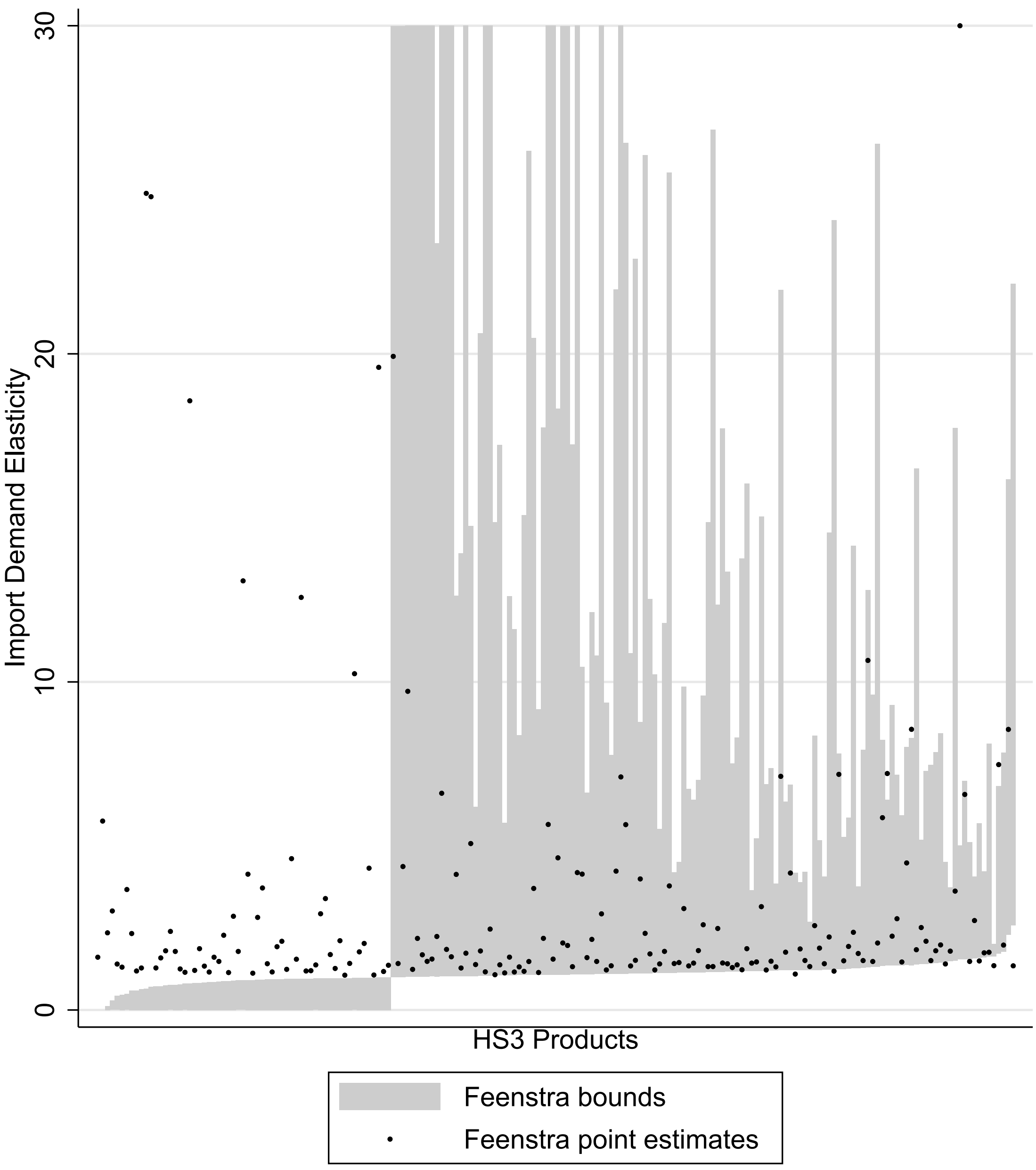

Figure 2: Feenstra bounds and point estimates, by 3-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

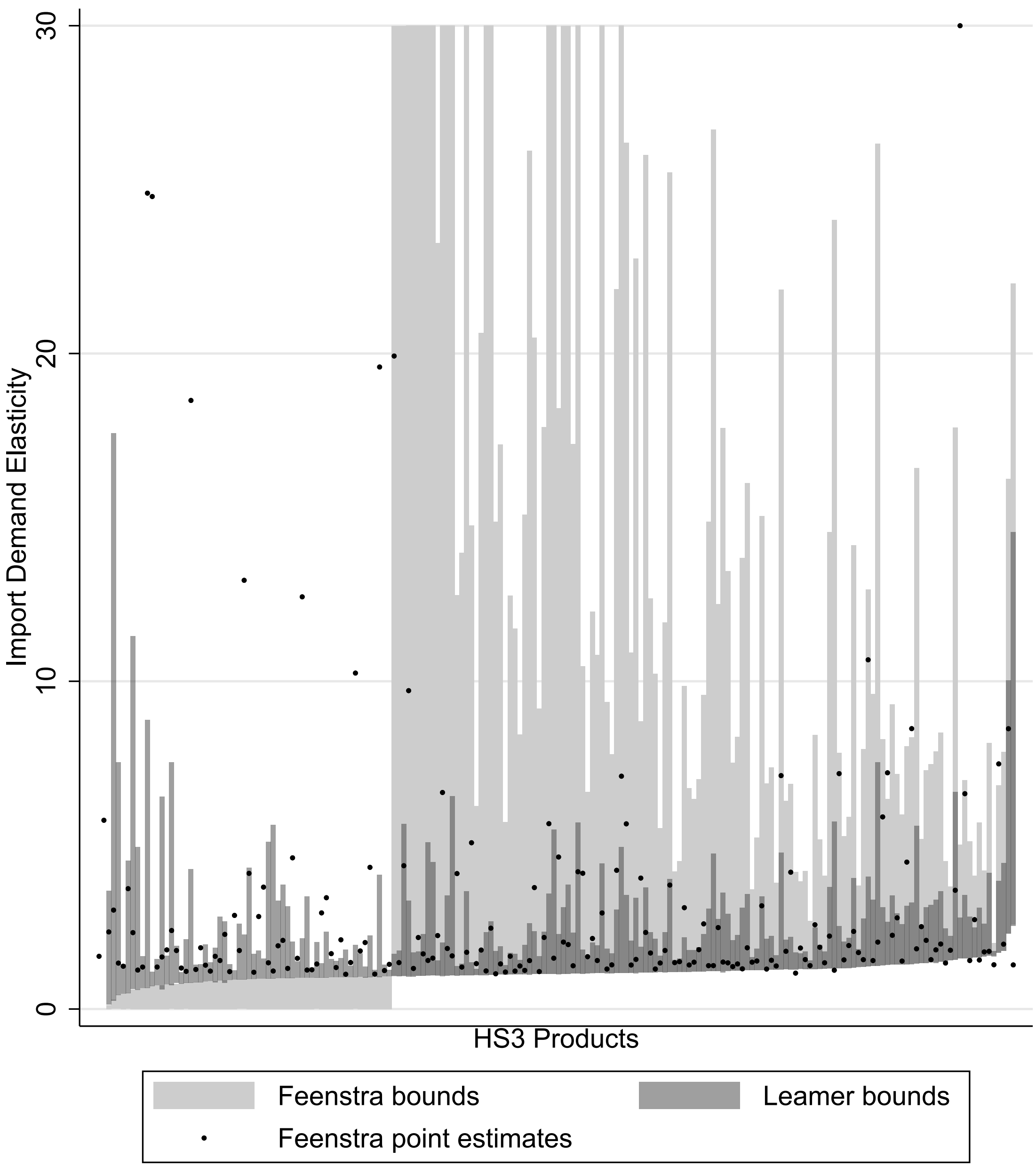

Figure 3: Leamer bounds, Feenstra bounds and point estimates, by 3-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

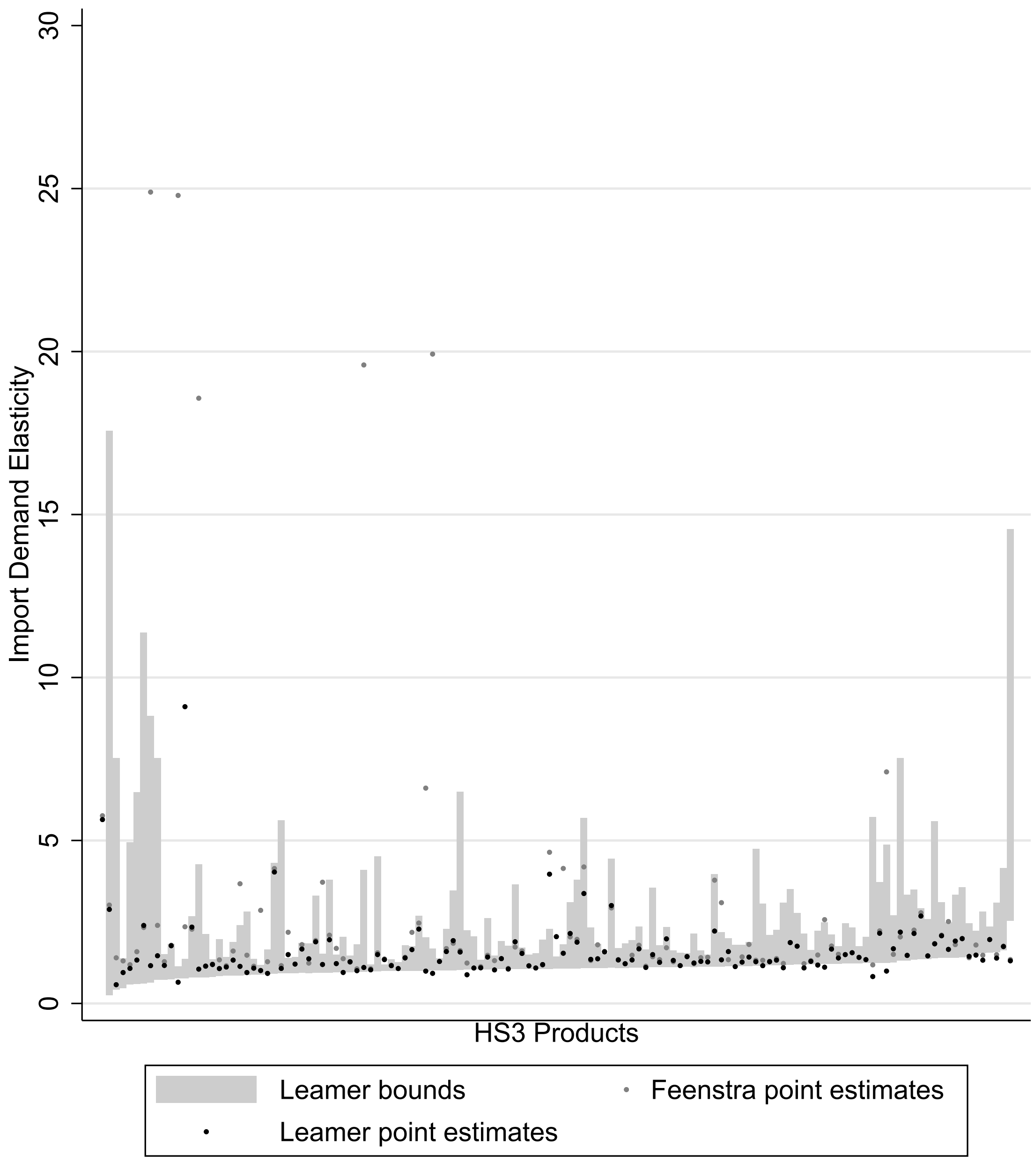

Figure 4: Feenstra point estimates, Leamer point estimates and Leamer bounds by 3-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

Table 1: U.S. Import Elasticity Descriptive Statistics

	Leamer Point Estimate	Leamer Lower Bound	Leamer Upper Bound	Feenstra Point Estimate	Feenstra Lower Bound	Feenstra Upper Bound
Panel A: 3-digit HS						
count	118	118	118	118	118	77
mean	1.57	1.07	3.07	2.71	1.07	26.32
median	1.35	1.07	2.24	1.53	1.08	9.85
Panel B: 4-digit HS						
count	674	674	674	674	674	473
mean	1.94	1.09	4.51	5.11	1.10	34.61
median	1.33	1.10	2.42	1.63	1.10	10.06
Panel C: 6-digit HS						
count	2754	2745	2745	2754	2751	1830
mean	2.25	1.12	6.86	6.71	1.13	36.42
median	1.40	1.11	2.89	1.74	1.11	9.88
Panel D: 8-digit HS						
count	5044	5026	5026	5044	5037	3256
mean	2.59	1.12	17.72	6.75	1.13	56.72
median	1.44	1.10	3.20	1.79	1.10	10.30

Notes: the sample is restricted to those products for which Leamer and Feenstra point estimates exist. Source: UC Davis Center for International Data, authors' calculations.

Table 2: U.S. Gains from Trade by 4-digit BEA commodity-level data

	Percentile				
	10th	25th	50th	75th	90th
Leamer point estimates	3.3	19.4	43.2	378.9	14935.3
Feenstra point estimates	2.8	12.5	30.8	69.4	581.4
Leamer upper bounds	1.4	5.7	15.7	35.9	200.1
Feenstra upper bounds	0.03	0.6	2.6	5.6	7.6

Source: UC Davis Center for International Data, Bureau of Economic Analysis, authors' calculations.

A Appendix

A.1 Partial Identification using the Quantity–Value Approach

As suggested by Scobie and Johnson (1975), another way to estimate import demand elasticities is regress $\ln x_{ct}$ on $\ln v_{ct}$, thus avoiding the need to construct price data. We again use (6) to transform (1) and (2) into a regression of trade quantities on trade values. The regression equation is

$$\ln x_{ct} = \frac{-\beta}{1-\beta} \ln v_{ct} + \frac{1}{1-\beta} \varepsilon_{ct}, \tag{26}$$

where we denote the OLS coefficient $\hat{\delta}^X$. We define $\hat{\delta}^V$ as the coefficient from the reverse regression of $\ln v_{ct}$ on $\ln x_{ct}$:

$$\ln x_{ct} = \frac{\gamma}{1+\gamma} \ln v_{ct} + \frac{1}{1+\gamma} \eta_{ct}. \tag{27}$$

The corresponding estimates of β are

$$\hat{\beta}^X = \frac{\hat{\delta}^X}{\hat{\delta}^X - 1}, \tag{28}$$

$$\hat{\beta}^V = \frac{1}{1 - \hat{\delta}^V}. \tag{29}$$

The reduced form is given by

$$\ln v_{ct} = \frac{1+\gamma}{\gamma+\beta}\varepsilon_{ct} - \frac{1-\beta}{\gamma+\beta}\eta_{ct},$$
$$\ln x_{ct} = \frac{\gamma}{\gamma+\beta}\varepsilon_{ct} + \frac{\beta}{\gamma+\beta}\eta_{ct}.$$

The probability limit of the OLS estimates of δ^X abd δ^V are thus

$$\texttt{plim}\ \hat{\delta}^X = \frac{\beta(\beta-1)\sigma_\eta^2 + \gamma(1+\gamma)\sigma_\varepsilon^2}{(1+\gamma)^2\sigma_\varepsilon^2 + (1-\beta)^2\sigma_\eta^2}, \tag{30}$$
$$\texttt{plim}\ \hat{\delta}^V = \frac{\beta(\beta-1)\sigma_\eta^2 + \gamma(1+\gamma)\sigma_\varepsilon^2}{\gamma^2\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2}. \tag{31}$$

These direct OLS estimates, when expressed in terms of β^X and β^V, are:

$$\texttt{plim}\ \hat{\beta}^X = \beta + (\beta+\gamma)\frac{(1+\gamma)\sigma_\varepsilon^2}{(\beta-1)\sigma_\eta^2 - (1+\gamma)\sigma_\varepsilon^2}, \tag{32}$$
$$\texttt{plim}\ \hat{\beta}^V = \beta + (\gamma+\beta)\frac{\gamma\sigma_\varepsilon^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2} \gtreqless \beta. \tag{33}$$

The probability limit of the lower bound in this case is equivalent to the Leamer upper bound, while the probability limit of the upper bound is equivalent to the Feenstra upper bound. It follows that the quantity-value lower bound will not hold if the Leamer upper bound holds. It also follows that the union of the Leamer and quantity-value bounds is equal to the Feenstra bounds.

Regressing trade quantities on trade values tends to overestimate the lower bound. In the vast majority of cases where the Leamer upper bound parameter restrictions are met, this implies that the parameter assumptions required for the quantity–value lower bound to hold are unlikely to be met.

In figure A.4 we combine all three approaches, which includes the upper and lower bounds using the regressions of trade value on trade quantity and vice versa, as given by equations (26) and (27), plus the Feenstra point estimates. As predicted from the asymptotic bias of the estimators, the results suggest that the lower bound on the quantity–value lower bound is equal to the Leamer upper bound, and in many cases is larger than the Feenstra point estimates. The Feenstra and quantity–value upper bounds also very similar. It can also be seen that union of the Leamer bounds and the quantity–value bounds are equivalent to the Feenstra bounds, precisely as the theory predicts.

A.2 Estimating Import Demand Elasticities with Measurement Error

We now check whether our theoretical predictions are robust to the presence of measurement error. The observed data are

$$\ln v_{ct} = \ln \tilde{v}_{ct} + u_{ct}$$
$$\ln x_{ct} = \ln \tilde{x}_{ct} + w_{ct}$$

where $\tilde{v}_{ct}$ and $\tilde{x}_{ct}$ denote the true (unobserved) data. The measurement error variances and covariances are σ_u^2, σ_w^2, and σ_{uw}. We assume classical measurement error, i.e., the measurement errors are uncorrelated with the true values.

A.2.1 Quantity–Price Approach with Measurement Error

We first derive the probability limits on the bounds in the quantity–price specification. Incorporating measurement error, the probability limit of β using (1) is

$$\begin{aligned}\texttt{plim }\hat{\beta} &= \frac{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 + (\beta+\gamma)^2(\sigma_w^2 - \sigma_{uw})}{\sigma_\varepsilon^2 + \sigma_\eta^2 + (\beta+\gamma)^2(\sigma_u^2 + \sigma_w^2 - 2\sigma_{uw})} \\ &= \beta - (\beta+\gamma)\frac{\sigma_\varepsilon^2 + \beta(\beta+\gamma)(\sigma_u^2 - \sigma_{uw}) + (\beta-1)(\beta+\gamma)(\sigma_w^2 - \sigma_{uw})}{\sigma_\varepsilon^2 + \sigma_\eta^2 + (\beta+\gamma)^2(\sigma_u^2 + \sigma_w^2 - 2\sigma_{uw})} \end{aligned} \quad (34)$$

For the reverse regression, we have

$$\begin{aligned}\frac{1}{\texttt{plim }\hat{\beta}^R} &= \frac{\gamma^2\sigma_\varepsilon^2 + \beta^2\sigma_\eta^2 + (\beta+\gamma)^2\sigma_w^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 - (\beta+\gamma)^2(\sigma_{uw} - \sigma_w^2)} \\ &= \beta + (\beta+\gamma)\frac{\gamma\sigma_\varepsilon^2 + \beta(\beta+\gamma)(\sigma_{uw} - \sigma_w^2) + (\beta+\gamma)\sigma_w^2}{\beta\sigma_\eta^2 - \gamma\sigma_\varepsilon^2 - (\beta+\gamma)^2(\sigma_{uw} - \sigma_w^2)} \end{aligned} \quad (35)$$

A.2.2 Value–Price Approach with Measurement Error

We now derive the bounds when using the value-price approach. The probability limit of the lower bound β^P using (7) is identical to (34) in the presence of measurement error. The upper bound $1/\beta^{RP}$, however, is not identical to (35), and takes the following form:

$$\frac{1}{\texttt{plim}\ \hat{\beta}^{R}} = \frac{\gamma\left(1+\gamma\right)\sigma_{\varepsilon}^{2}+\beta\left(\beta-1\right)\sigma_{\eta}^{2}+\left(\beta+\gamma\right)^{2}\sigma_{u}^{2}-\left(\beta+\gamma\right)^{2}\left(\sigma_{u}^{2}-\sigma_{uw}\right)}{\left(\beta-1\right)\sigma_{\eta}^{2}-\left(1+\gamma\right)\sigma_{\varepsilon}^{2}-\left(\beta+\gamma\right)^{2}\left(\sigma_{u}^{2}-\sigma_{uw}\right)}$$
$$= \beta+\left(\beta+\gamma\right)\frac{\left(1+\gamma\right)\sigma_{\varepsilon}^{2}+\left(\beta+\gamma\right)\sigma_{u}^{2}+\left(\beta-1\right)\left(\beta+\gamma\right)\left(\sigma_{u}^{2}-\sigma_{uw}\right)}{\left(\beta-1\right)\sigma_{\eta}^{2}-\left(1+\gamma\right)\sigma_{\varepsilon}^{2}-\left(\beta+\gamma\right)^{2}\left(\sigma_{u}^{2}-\sigma_{uw}\right)}. \quad (36)$$

A.2.3 Quantity–Value Approach with Measurement Error

When regressing traded quantities on trade values, the probability limit of the OLS estimates of δ^X abd δ^V are

$$\texttt{plim}\ \hat{\delta}^{X} = \frac{\beta(\beta-1)\sigma_{\eta}^{2}+\gamma(1+\gamma)\sigma_{\varepsilon}^{2}+(\gamma+\beta)^{2}\sigma_{uw}}{(1+\gamma)^{2}\sigma_{\varepsilon}^{2}+(1-\beta)^{2}\sigma_{\eta}^{2}+(\gamma+\beta)^{2}\sigma_{u}^{2}},$$
$$\texttt{plim}\ \hat{\delta}^{V} = \frac{\beta(\beta-1)\sigma_{\eta}^{2}+\gamma(1+\gamma)\sigma_{\varepsilon}^{2}+(\gamma+\beta)^{2}\sigma_{uw}}{\gamma^{2}\sigma_{\varepsilon}^{2}+\beta^{2}\sigma_{\eta}^{2}+(\gamma+\beta)^{2}\sigma_{w}^{2}}.$$

These direct OLS estimates, when expressed in terms of β^X abd β^V, yield probability limits equal to equations (36) and (35) respectively

$$\texttt{plim}\ \hat{\beta}^{X} = \beta+\left(\beta+\gamma\right)\frac{\left(1+\gamma\right)\sigma_{\varepsilon}^{2}+\left(\beta+\gamma\right)\sigma_{u}^{2}+\left(\beta-1\right)\left(\beta+\gamma\right)\left(\sigma_{u}^{2}-\sigma_{uw}\right)}{\left(\beta-1\right)\sigma_{\eta}^{2}-\left(1+\gamma\right)\sigma_{\varepsilon}^{2}-\left(\beta+\gamma\right)^{2}\left(\sigma_{u}^{2}-\sigma_{uw}\right)}., \quad (37)$$
$$\texttt{plim}\ \hat{\beta}^{V} = \beta+\left(\beta+\gamma\right)\frac{\gamma\sigma_{\varepsilon}^{2}+\beta\left(\beta+\gamma\right)\left(\sigma_{uw}-\sigma_{w}^{2}\right)+\left(\beta+\gamma\right)\sigma_{w}^{2}}{\beta\sigma_{\eta}^{2}-\gamma\sigma_{\varepsilon}^{2}-\left(\beta+\gamma\right)^{2}\left(\sigma_{uw}-\sigma_{w}^{2}\right)}. \quad (38)$$

A.2.4 Discussion

These results suggest that none of the three approaches unambiguously bracket the lower bound of the import demand elasticity in the presence of measurement error. The effect of measurement error on the asymptotic bias depends on the relative magnitudes of the error variance and the true demand and supply elasticities. If the error variance terms are equal in magnitude ($\sigma_u^2 = \sigma_w^2 = \sigma_{uw} \equiv \sigma^2$), then the error terms in the lower bound equation completely cancel out, and (34) is equivalent to the case without measurement error. Under this same special case, the error terms partially cancel for the upper bounds (35) and (36), and measurement error results in an upward biased upper bound.

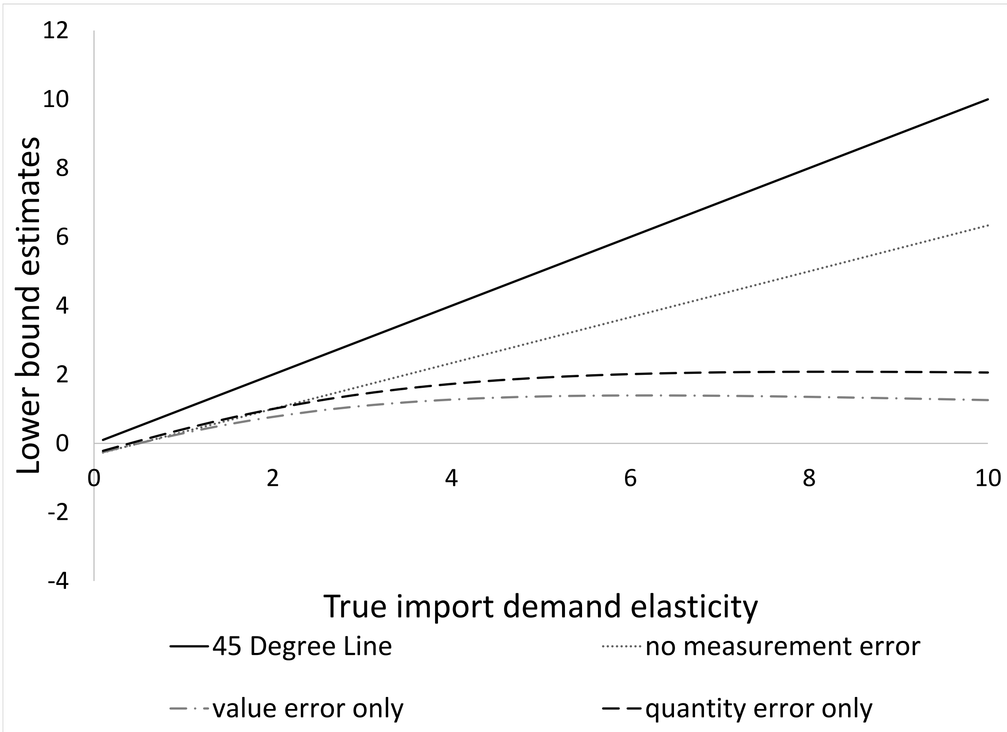

Figure A.1: Lower bounds with measurement error

Notes: $\gamma = 1, \sigma_\varepsilon^2 = 0.5, \sigma_\eta^2 = 1$ in all cases. $\sigma_u^2 = 0, \sigma_w^2 = 0.05$ in quantity measurement error case. $\sigma_u^2 = 0.05, \sigma_w^2 = 0$ in value measurement error case. Source: authors' calculations

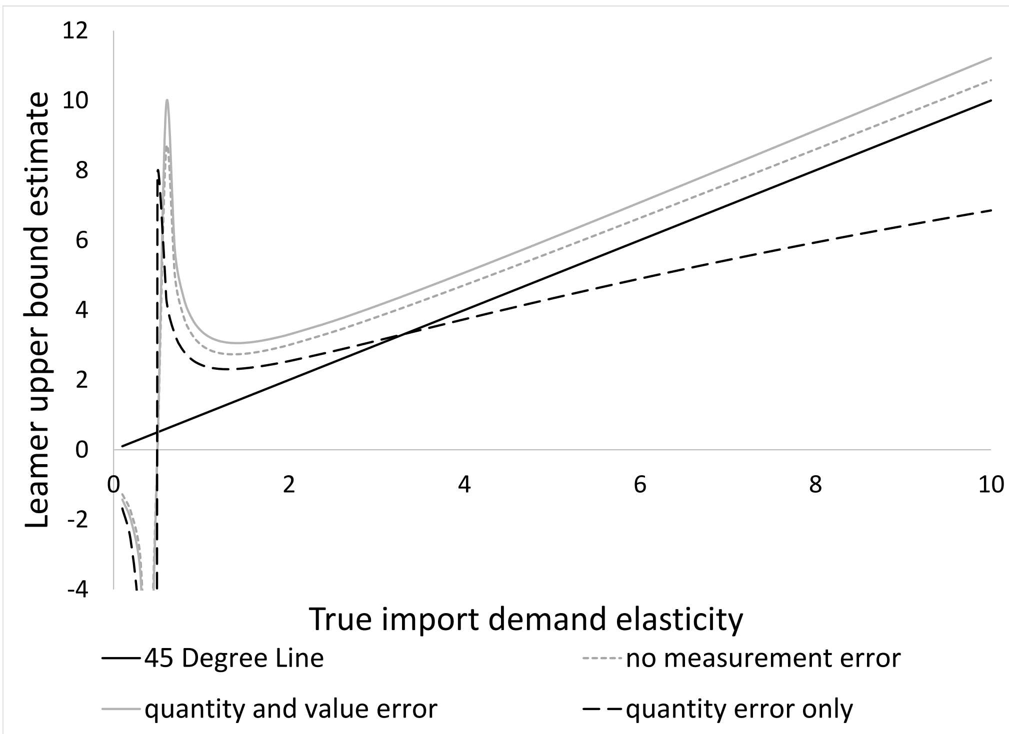

Figure A.2: Leamer upper bounds with measurement error

Notes: $\gamma = 1, \sigma_\varepsilon^2 = 0.5, \sigma_\eta^2 = 1$ in all cases. $\sigma_u^2 = \sigma_w^2 = \sigma_{uw} = 0.05$ in identical measurement error case. $\sigma_w^2 = 0.05, \sigma_u^2 = \sigma_{uw} = 0$ in quantity measurement error case.
Source: authors' calculations

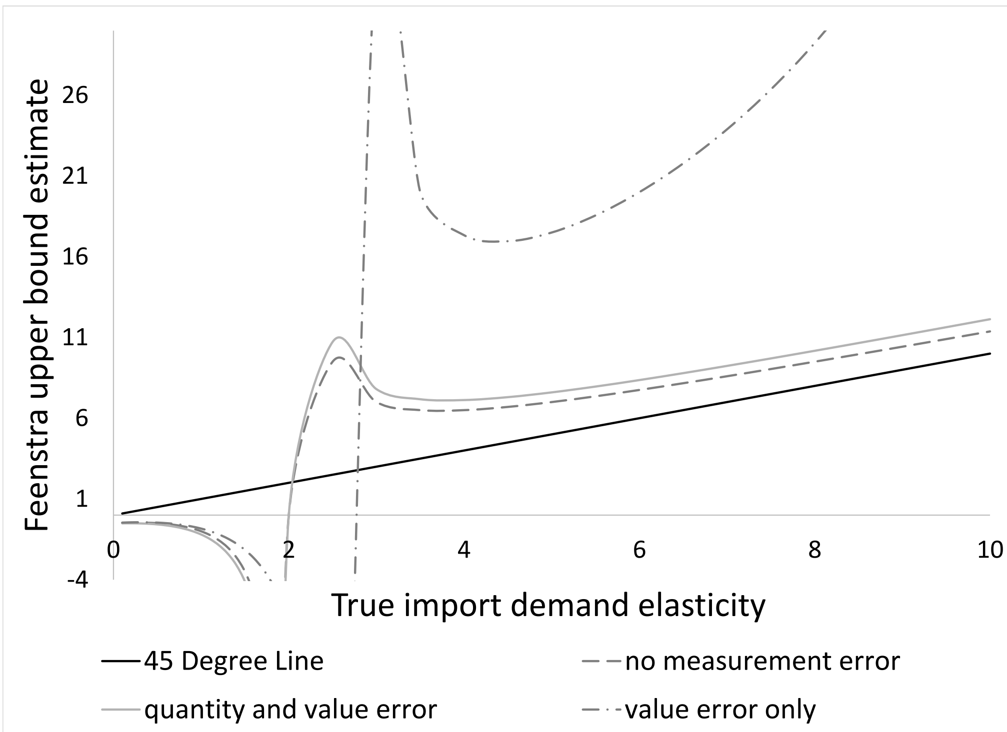

Figure A.3: Feenstra upper bounds with measurement error

Notes: $\gamma = 1, \sigma_{\varepsilon}^2 = 0.5, \sigma_{\eta}^2 = 1$ in all cases. $\sigma_u^2 = \sigma_w^2 = \sigma_{uw} = 0.05$ in identical measurement error case. $\sigma_u^2 = 0.05, \sigma_w^2 = \sigma_{uw} = 0$ in value measurement error case. Source: authors' calculations

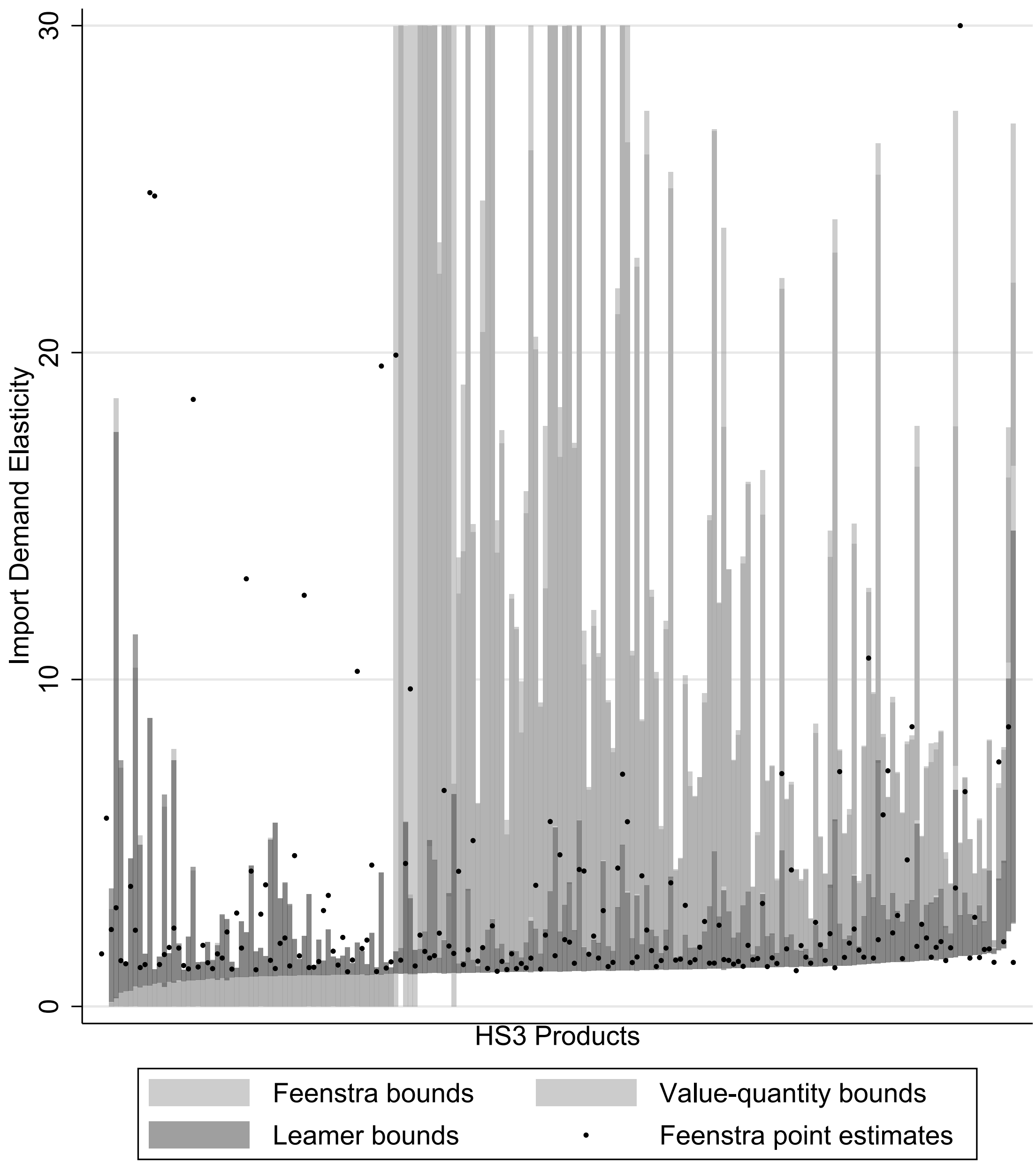

Figure A.4: Bounds using all three approaches and Feenstra point estimates, by 3-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

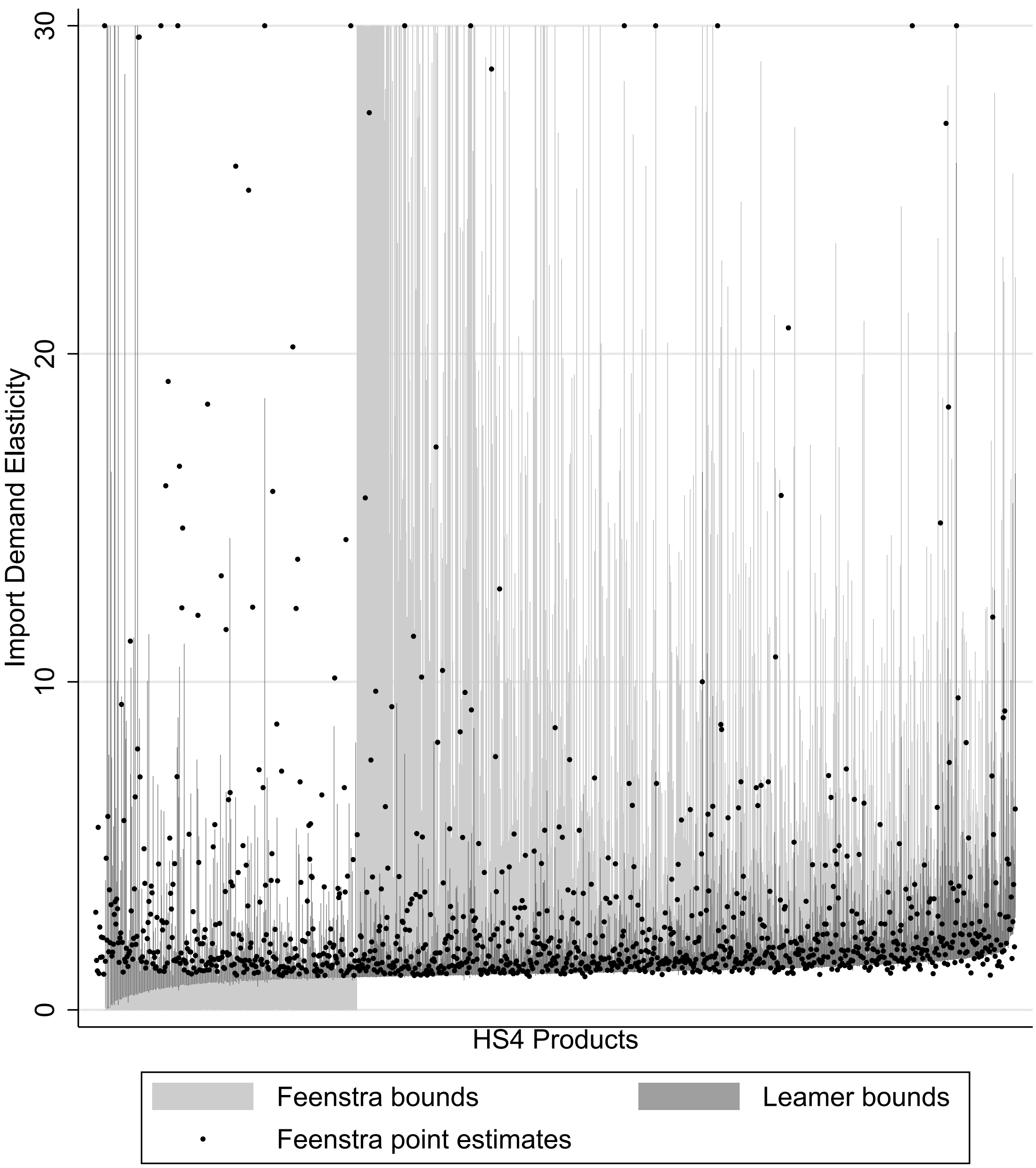

Figure A.5: Leamer bounds, Feenstra bounds and point estimates, by 4-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

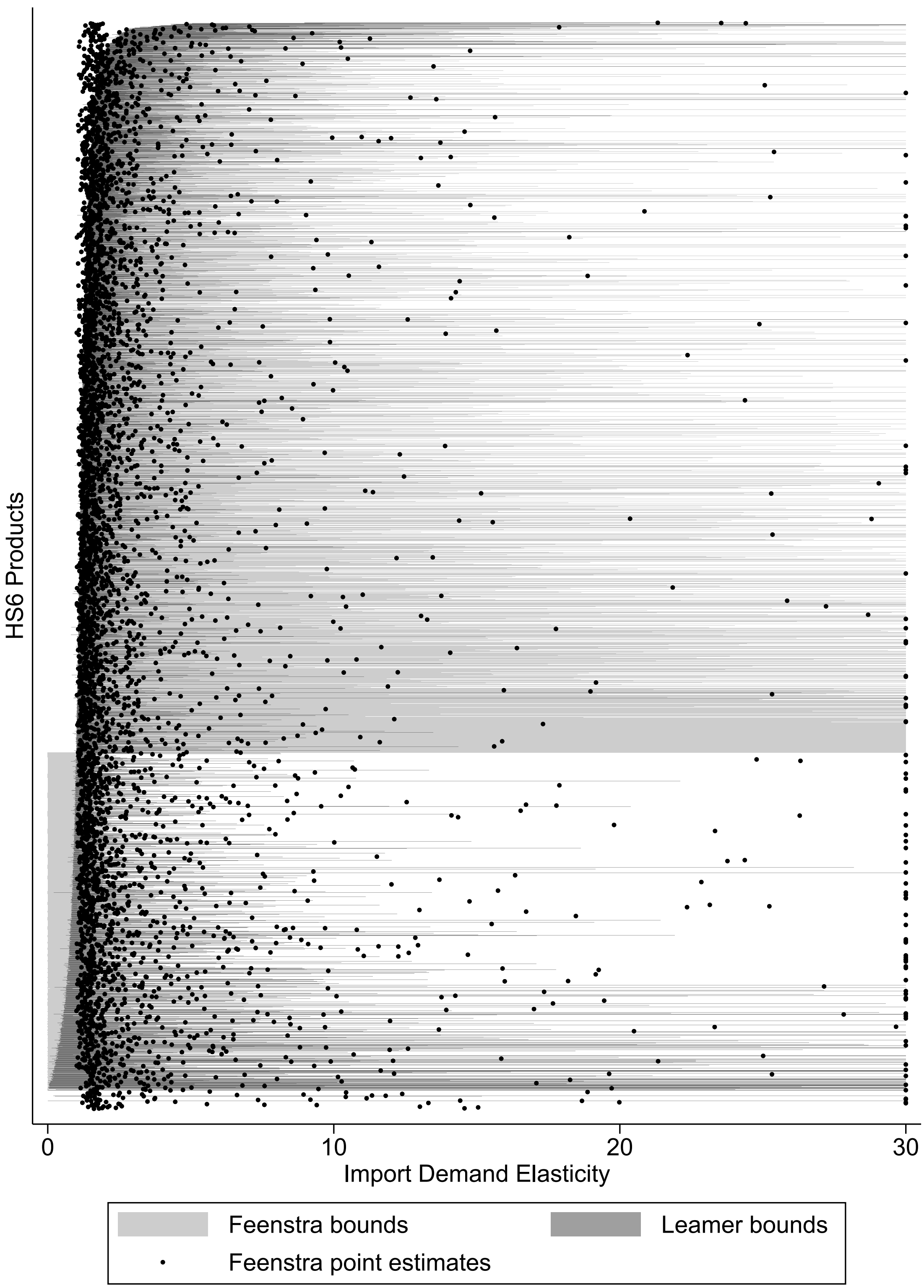

Figure A.6: Leamer bounds, Feenstra bounds and point estimates, by 6-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

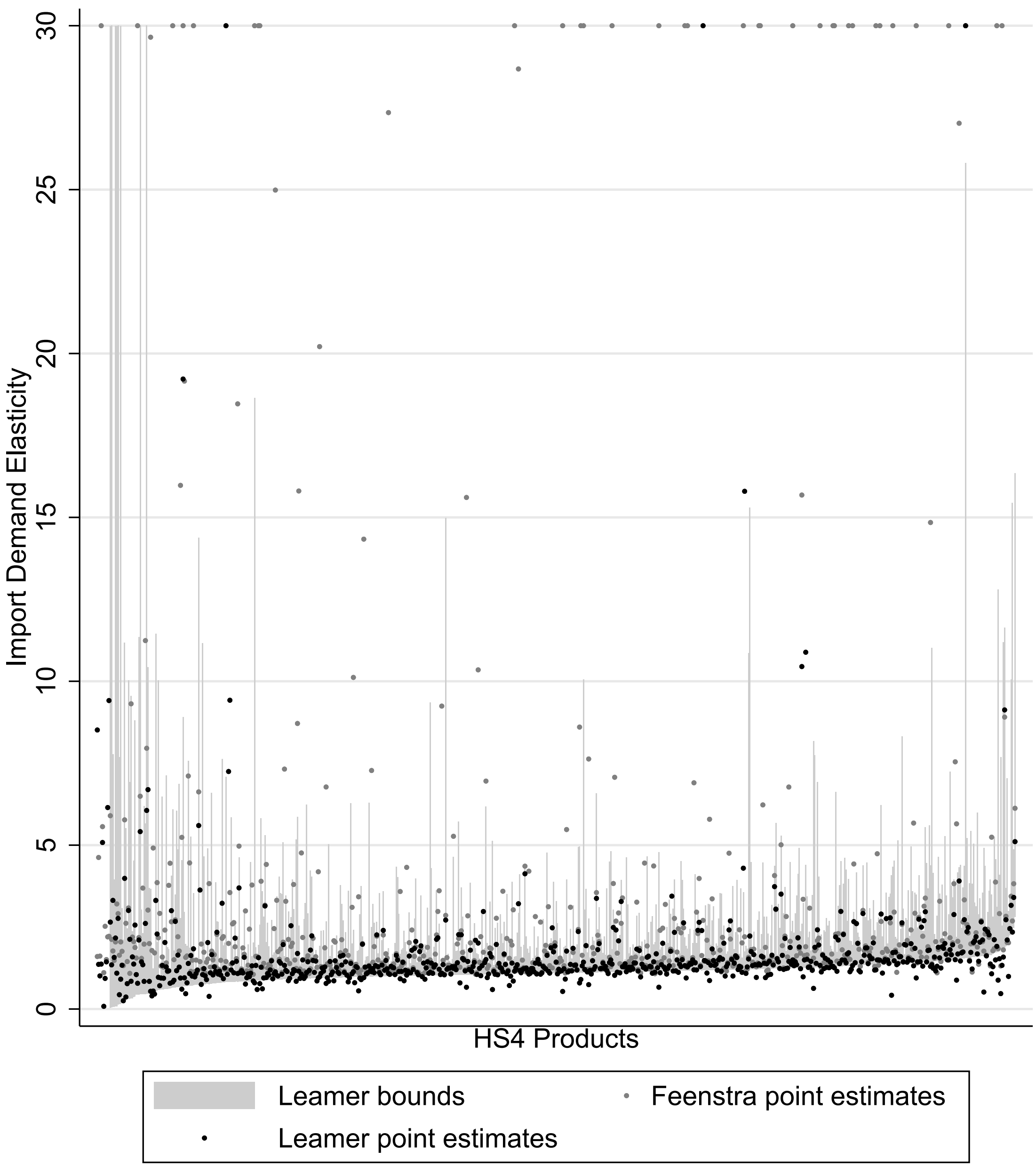

Figure A.7: Feenstra point estimates, Leamer point estimates and Leamer bounds by 4-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

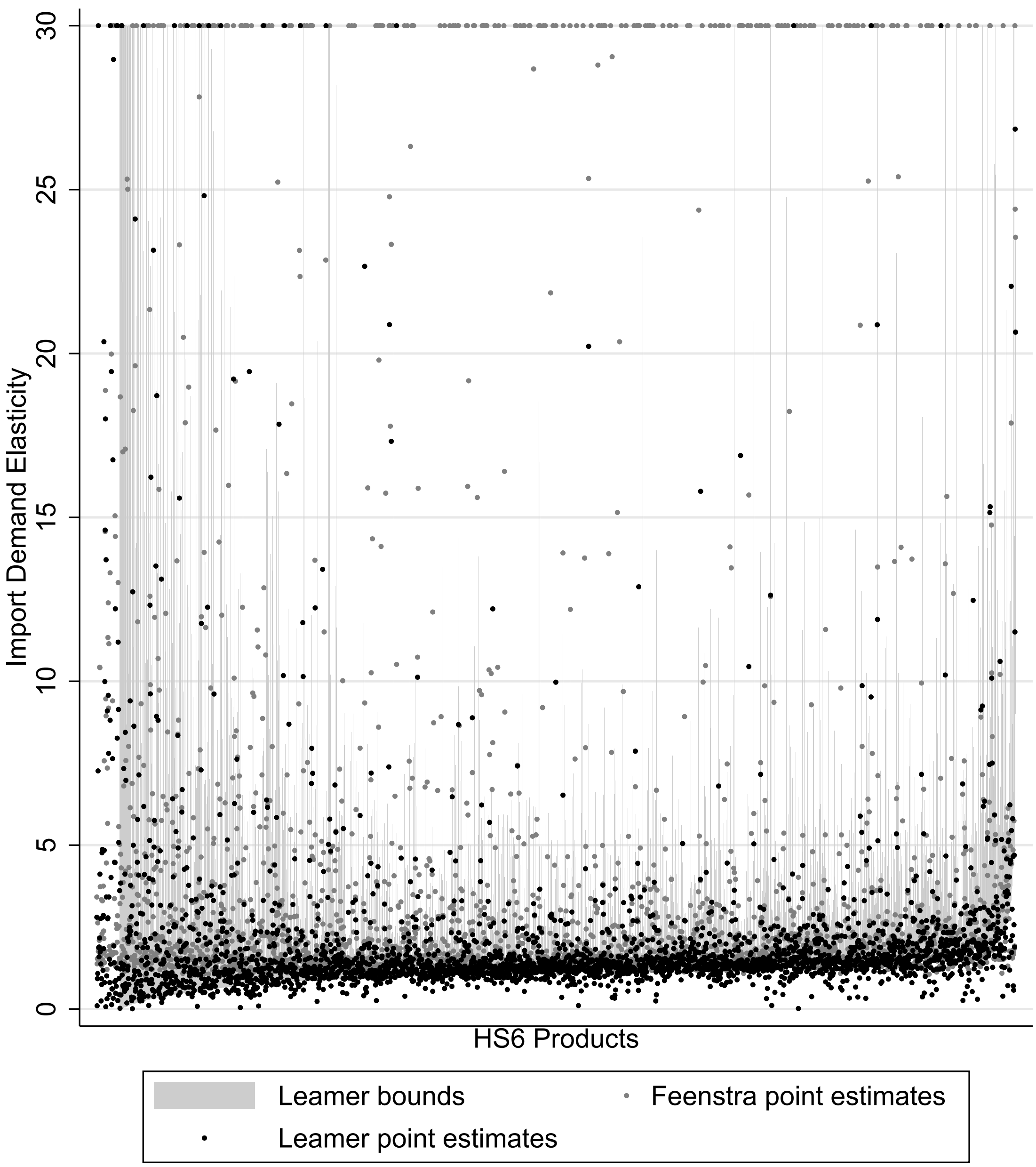

Figure A.8: Feenstra point estimates, Leamer point estimates and Leamer bounds by 6-digit HS, U.S., 1993-2006. Source: UC Davis Center for International Data, authors' calculations

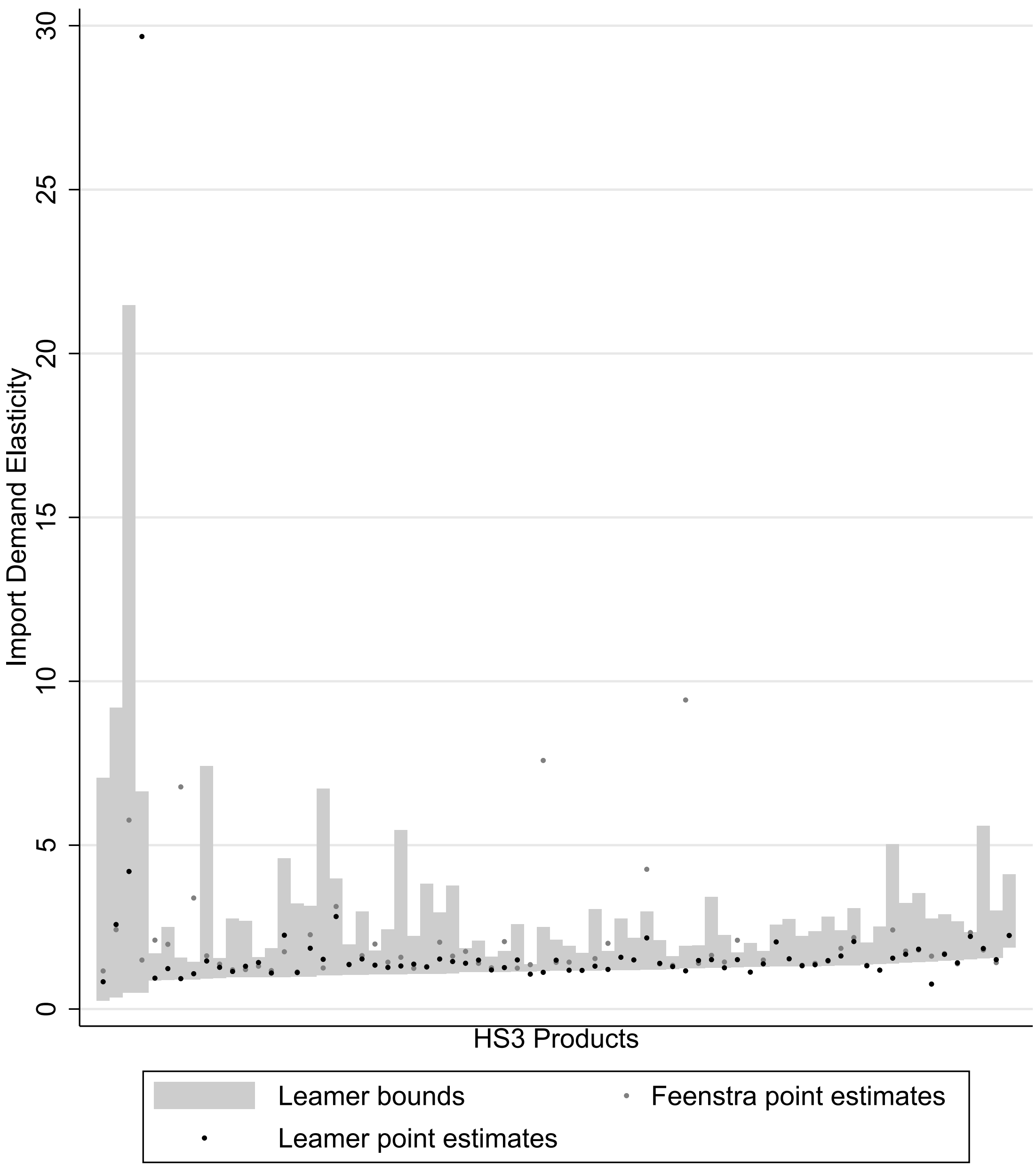

Figure A.9: Feenstra point estimates, Leamer point estimates and Leamer bounds by 3-digit HS, U.S., 1991-2015. Source: Comtrade, authors' calculations

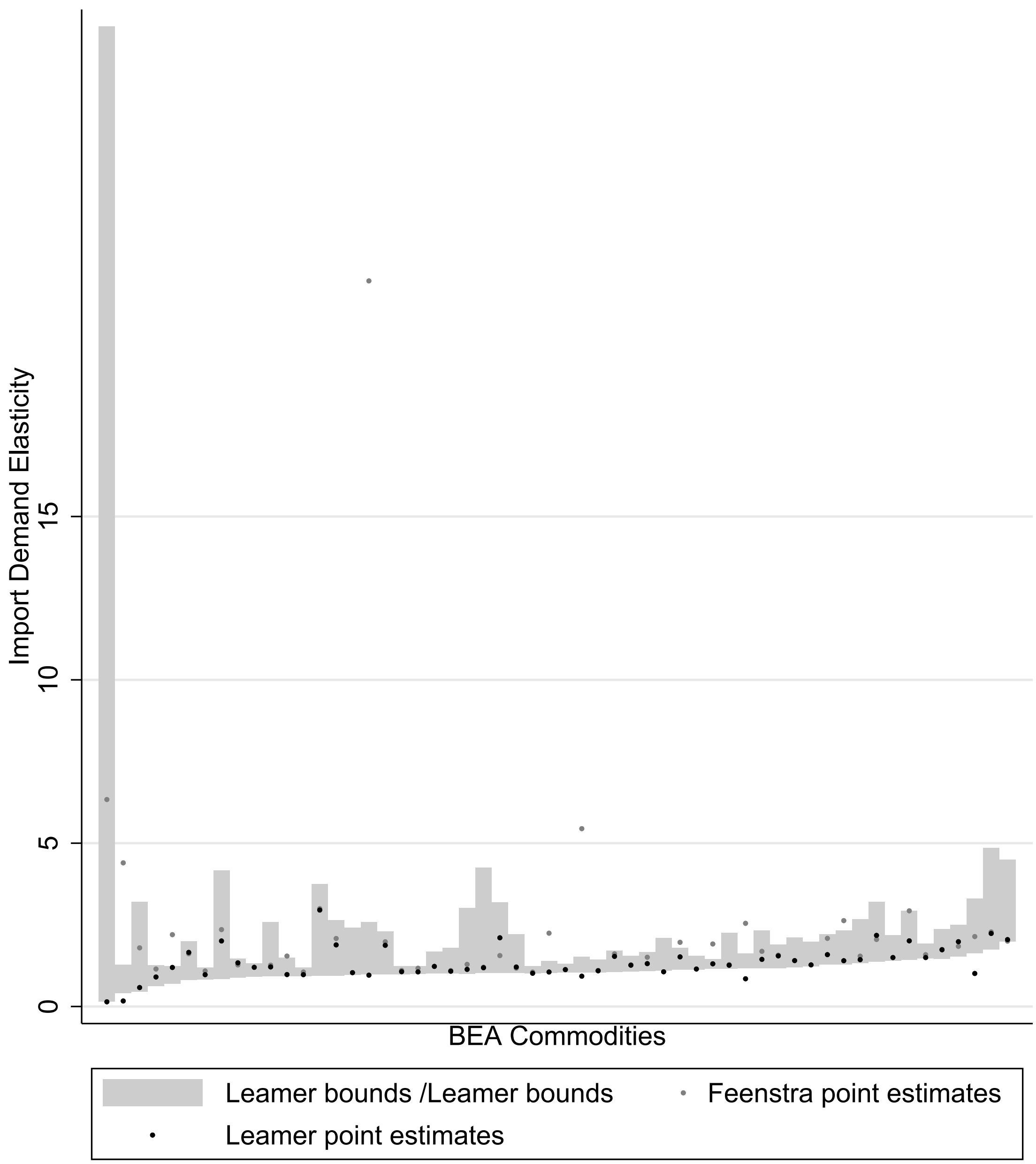

Figure A.10: Feenstra point estimates, Leamer point estimates and Leamer bounds by 4-digit BEA commodity, U.S., 1993-2006. Source: UC Davis Center for International Data and BEA, authors' calculations