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Mathematical Modeling of World Grain Trade Restrictions

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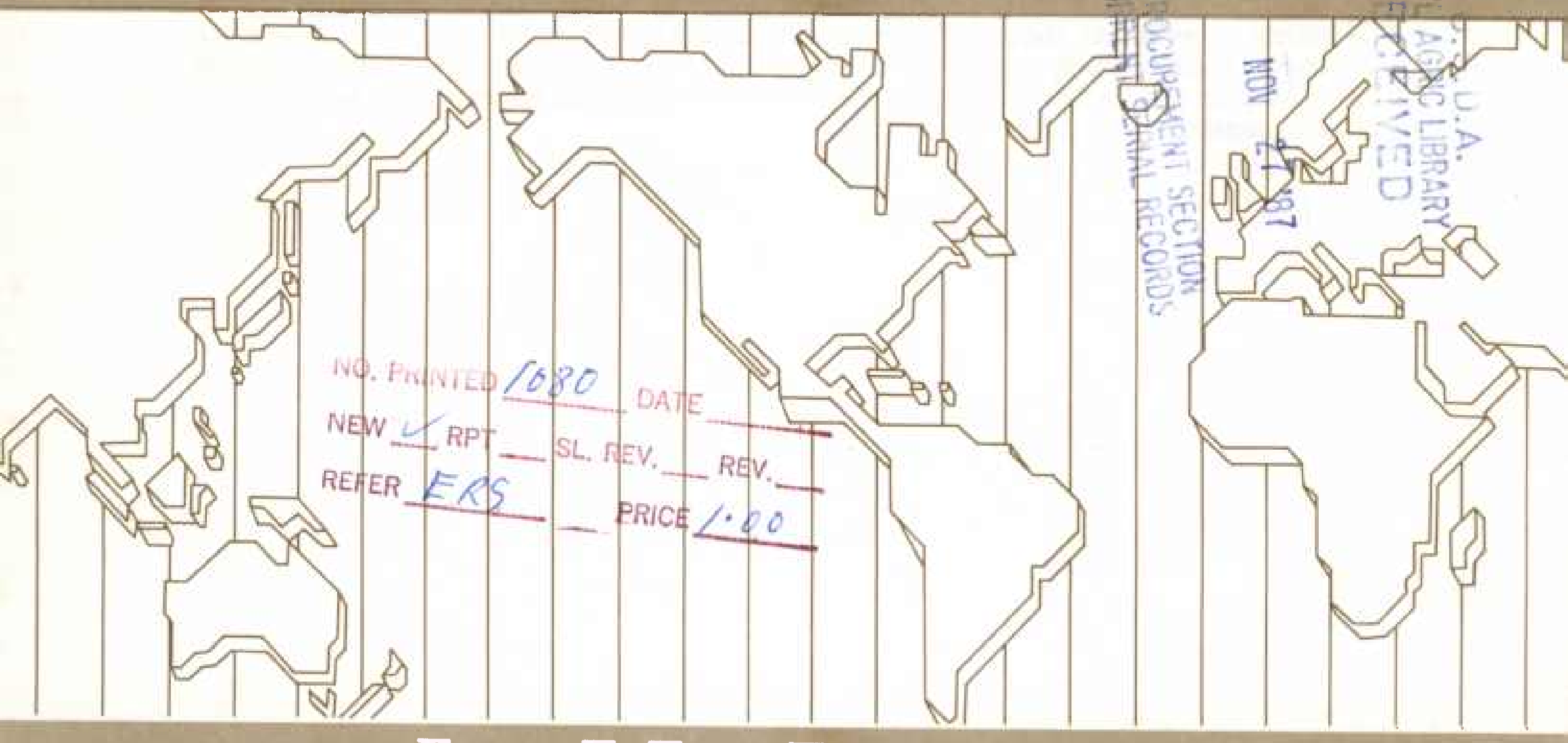
$$(4) \quad X_i = c_i(P_{wi})^{d_i}$$

$$(17) \quad P^*_{wj} = P_{wj}(1 + t_j).$$

$$(24) \quad M_j = f_j(R_j P_{w*j})\text{-}g_j$$

$$(1a) \quad Q_{Dij} = Q_{Dij}\ (Pd_{ij})$$



$$F_{1j} = -E_{Ij}E_{pw1j}/E_{x1}$$

$$F_{1j} = -E_{Ij}E_{pw1j}/E_{x1}$$

$$F_{2m} = (E_{xm}E_{pw2m}/E_{x2})\ c_m d_m(P_{wm})^{(d_m-1)}$$

$$(7a) \quad E_{pwij} = E_{Iij} \cdot E_{pij}$$

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Abstract

This paper develops a mathematical model of trade as a means for developing explicit functions for measuring the effects of trade restrictions. Assuming a constant elasticity of export demand and supply, we formulate the model by using the implicit function theorem. This allows us to derive directly how export demand and supply of any commodity will change given a specific market intervention. This is a general analysis assuming m exporters and n importers. We use the model to derive the effects on U.S. exports of government interventions in the trade sector such as the imposition of an import quota, trade embargo, import tariff, exchange rate change, or price intervention.

Keywords: trade, government intervention, import quotas, tariffs, exchange rates, trade embargoes.

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Introduction

One of the most pervasive features of the international agricultural trade environment is the very large degree of government intervention. These interventions usually take the form of one or more of the following trade restrictions: Quotas and licensing of trade, trade embargoes, import tariffs and import taxes, and exchange rate intervention. Governments often intervene directly by controlling the marketing of traded goods (4, 6, 8).1/ In this paper, we restrict ourselves to an analysis of only the former type of trade restrictions.

The pervasiveness of the restrictions is indicated by the following. Of 21 less-developed countries studied in a joint ERS-University of Minnesota project (3), 19 had some form of trade controls on wheat and 18 had trade control on rice. In addition, 17 countries had trade controls on one or more coarse grains.

There is evidence that these interventions lead to serious welfare losses. Bale and Lutz (1) found that government intervention in the trade sector resulted in welfare loses of from 10.6 percent of gross national product for Egypt to a low of 1.5 percent for Argentina. Gerrard and Roe (4) found similar effects for Tanzania and Riethmuller and Roe (7) found similar welfare losses in Japan.

A central issue for international trade analysts is to quantify the effects of various trade restrictions on import quantities. A spatial equilibrium model developed by Samuelson (9) and formulated by Takayama and Judge (10) as a quadratic programming (QP) model has been widely used in international trade to quantify the effects of trade restrictions. However, the solutions obtained from this basic model are often far from what we observe, and this model does not reflect the price distortion resulting from government intervention. Furthermore, the demand and supply equations in a QP spatial equilibrium model are assumed to be linear, and data for the transportation costs connecting all sources and destinations are needed to solve the model.

Therefore, a simple mathematical model reflecting not only what we observe but incorporating linear and/or nonlinear demand and supply equations and which does not require extensive data on transportation costs is needed to measure the effects of various trade restrictions by importing countries on exporting countries. In this paper, utilizing

1/ Underscored numbers in parentheses refer to items in the references.

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the implicit function theorem, we develop a system of equations for estimating the effects on U.S. export price and quantity of various trade restrictions imposed by countries around the world. This model can also be used for estimating the effect of trade liberalization.

The Model

Following Tweeten (11, 12, 13), Johnson (5), and Bredahl and others (2), and as shown in the appendix, the export demand elasticity of the kth exporting country can be written as:

$$(1) \quad E_{xk} = \Sigma_j(E_{Ij}E_{Pwkj}I_{kj}) - \Sigma_{i \neq k}(E_{xi}E_{Pwki}I_{ki})$$

for i, k = 1, 2, ...,m and j = 1, 2, ..., n

where:

E_{xk} = the export demand elasticity of the kth country

E_{xi} = the excess supply elasticity of the ith country

E_{Ij} = the excess demand elasticity of the jth country

I_{kj} = $(M_j/X)(X/X_k)$ – relative market weight of the kth country

I_{ki} = X_i/X_k – the relative export market weight of the ith compared with the kth country

E_{Pwkj} = the jth country price transmission elasticity with respect to a change in the kth country export price

X_k = the export demand of the kth country

X = world exports

M_j = the imports of the jth country

M = world imports

m = number of exporting countries

n = number of importing countries

From equation (1), the export demand elasticity is clearly a composite of domestic supply and demand elasticities, price transmission elasticities, and market shares. Thus, it represents the outcome of the interaction of fundamental consumer and producer behavior, government intervention, and policies as well as the effect of marketing and transportation systems. By multiplying both sides of (1) by X_k/E_{xk} and rearranging terms, we get the following:

$$(2) \quad F_k = X_k + \Sigma_{i \neq k}(E_{xi}E_{Pwki}X_i/E_{xk}) - \Sigma_j(E_{Ij}E_{Pwkj}M_j/E_{xk}) = 0 \text{ for all k.}$$

Variables X_k and X_i $(i \neq k)$ in equation (2) are the export demand of the kth country and the export supply of the ith country, respectively. Assuming a constant elasticity, we can represent the export demand of the kth country by:

$$(3) \qquad X_k = a_k(P_{wk})^{-b_k} \qquad \text{for } k = 1, 2, \ldots, m$$

where:

$a_k, b_k > 0$ for all k

$-b_k = E_{xk}$

and

P_{wk} = the export demand price for the kth exporter.

The export supply of the ith country is represented by:

$$(4) \qquad X_i = c_i(P_{wi})^{d_i}$$

where:

$c_i \gtreqless 0$ and $d_i > 0$, for all i = k

$d_i = E_{xi}$

and

P_{wi} = the export supply price of the ith exporter.

The assumption of constant elasticities in (3) and (4) guarantees that the equality in (1) always holds. Substituting equations (3) and (4) in equation (2) gives our basic working model.

$$(5) \quad F_k = a_k(P_{wk})^{-b_k} + \Sigma_{i \neq k}(E_{xi}E_{Pwki}(c_i(P_{wi})^{d_i})/E_{xk}) - \Sigma_j(E_{Ij}E_{Pwkj}M_j)/E_{xk}$$
$$= 0 \qquad \text{for all } k = 1, 2, \ldots, m.$$

Import Quotas

If a country imposes a system of import quotas, then it is using imports (M_j) as a policy instrument. Our concern is what the change in country k exports (X_k) will be when country j sets an import quota (M_j^*). Thus, taking the partial derivation of (3) with respect to M_j^* yields:

$$(6) \qquad \partial X_k/\partial M_j^* = \partial X_k/\partial P_{wk}(\partial P_{wk}/\partial M_j^*) = -a_k b_k(P_{wk})^{-(1+b_k)}(\partial P_{wk}/\partial M_j^*)$$

Therefore, the effects on export demand of a change in the jth country's imports can be evaluated by:

$$(7) \qquad (\partial X_k/\partial M_j^*)dM_j^* = -a_k b_k(P_{wk})^{-(1+b_k)}(\partial P_{wk}/\partial M_j^*)dM_j^* \qquad \text{for all k.}$$

To determine the effect of the quota (M_j^*) on exports, one needs to determine the effect which the change in the jth country imports has on the export prices of k; that is, the right hand side of equation (6). We obtain this by applying the implicit function theorem to a system of m equations (5) and then solving the resulting system of equations.2/ We indicate this result using matrix notation as follows:

2/ See any advanced calculus textbook for the generalized version of the implicit-function theorems for systems of equations.

$$(8)\quad \begin{pmatrix} \partial P_{w1}/\partial M_j \\ \partial P_{w2}/\partial M_j \\ \cdot \\ \cdot \\ \partial P_{wm}/\partial M_j \end{pmatrix} = \begin{pmatrix} \partial F_1/\partial P_{w1} & \partial F_1/\partial P_{w2} & \cdots & \partial F_1/\partial P_{wm} \\ \partial F_2/\partial P_{w1} & \partial F_2/\partial P_{w2} & \cdots & \partial F_2/\partial P_{wm} \\ \cdots & & & \\ \cdots & & & \\ \partial F_m/\partial P_{w1} & \partial F_m/\partial P_{w2} & \cdots & \partial F_m/\partial P_{wm} \end{pmatrix}^{-1} \begin{pmatrix} -\partial F_1/\partial M_j \\ -\partial F_2/\partial M_j \\ \cdot \\ \cdot \\ -\partial F_m/\partial M_j \end{pmatrix}$$

$$= \begin{pmatrix} F_{11} & F_{12} & \cdots & F_{1m} \\ F_{21} & F_{22} & \cdots & F_{2m} \\ \cdots & & & \\ \cdots & & & \\ F_{m1} & F_{m2} & \cdots & F_{mm} \end{pmatrix}^{-1} \begin{pmatrix} -F_{1j} \\ -F_{2j} \\ \cdot \\ \cdot \\ -F_{mj} \end{pmatrix}$$

where:

$F_{11} = -a_1 b_1 (P_{w1})^{-(1+b_1)}$

$F_{12} = (E_{x2} E_{pw12}/E_{x1})\ c_2 d_2 (P_{w2})^{(d_2-1)}$

$F_{1m} = (E_{xm} E_{pw1m}/E_{x1})\ c_m d_m (P_{wm})^{(d_m-1)}$

$F_{21} = (E_{x1} E_{pw21}/E_{x2})\ c_1 d_1 (P_{w1})^{(d_1-1)}$

$F_{22} = -a_2 b_2 (P_{w2})^{-(1+b_2)}$

$F_{2m} = (E_{xm} E_{pw2m}/E_{x2})\ c_m d_m (P_{wm})^{(d_m-1)}$

$F_{m1} = (E_{x1} E_{pwm1}/E_{xm})\ c_1 d_1 (P_{w1})^{(d_1-1)}$

$F_{m2} = (E_{x2} E_{pwm2}/E_{xm})\ c_2 d_2 (P_{w2})^{(d_2-1)}$

$F_{mm} = -a_m b_m (P_{wm})^{-(1+b_m)}$

$F_{1j} = -E_{Ij} E_{pw1j}/E_{x1}$

$F_{2j} = -E_{Ij} E_{pw2j}/E_{x2}$

$F_{mj} = -E_{Ij} E_{pwmj}/E_{xm}$

Equation (8) can be written in compact notation as follows:

$$(9)\qquad \partial P_{wj} = G^{-1} \partial FM_j \qquad \text{for } j = 1, 2, \ldots n.$$

Since the determinant of G in (9) is the particular Jacobian determinant /J/ which is nonzero under the implicit-function theorem and since the system of (8) must be nonhomogeneous, there should be a unique solution to (8) or (9). Substituting (9) into (7) yields:

$$(10) \qquad dX = (\partial Xk/\partial M_j^*)dM_j^* = BG^{-1}\partial FM_j dM_j^*$$

where X is the vector of export demands and B is the (m X m) diagonal matrix

$$(11) \qquad B = \begin{pmatrix} -a_1b_1(P_{w1})^{-(1+b_1)} & \cdots & & & 0 \\ 0 & -a_2b_2(P_{w2})^{-(1+b_2)} & \cdots & & 0 \\ 0 & & \cdots & & 0 \\ 0 & & \cdots & & 0 \\ 0 & \cdots & & & -a_mb_m(P_{wm})^{-(1+b_m)} \end{pmatrix}$$

Note, this implies that we are not considering cross–price effects.

Trade Embargoes

To understand how to derive the effect of a trade embargo, we need only to recognize this as the special case of the import quota where country j reduces its imports from the kth country to zero.

Recognizing that the imports of the jth country is the sum of imports from all other countries:

$$(12) \qquad M_j = \Sigma_i M_{ij}$$

and substituting (12) into (5) yields:

$$(13) \qquad F_k = a_k(P_{wk})^{-b_k} + \Sigma_{i\neq k}(E_{xi}E_{Pwki}/E_{xk})c_i(P_{wi})^{d_i} - \Sigma_j(E_{Ij}E_{Pwkj}/E_{xk})(\Sigma_i M_{ij}) = 0 \quad \text{for all } k = 1, 2, \ldots, m.$$

Proceeding as in the previous case by applying the implicit function theorem to (13), the direct effect of a trade embargo by the kth exporter to the jth importer on the export prices can be measured by:

$$(14) \qquad \begin{pmatrix} \partial P_{w1}/\partial M_{kj} \\ \partial P_{w2}/\partial M_{kj} \\ \cdot \\ \cdot \\ \partial P_{wm}/\partial M_{kj} \end{pmatrix} = \begin{pmatrix} \partial F_1/\partial P_{w1} & \partial F_1/\partial P_{w2} & \cdots & \partial F_1/\partial P_{wm} \\ \partial F_1/\partial P_{w1} & \partial F_2/\partial P_{w2} & \cdots & \partial F_2/\partial P_{wm} \\ \cdots & & & \\ \cdots & & & \\ \partial F_m/\partial P_{w1} & \partial F_m/\partial P_{w2} & \cdots & \partial F_m/\partial P_{wm} \end{pmatrix}^{-1} \begin{pmatrix} -\partial F_1/\partial M_{kj} \\ -\partial F_2/\partial M_{kj} \\ \cdot \\ \cdot \\ -\partial F_m/\partial M_{kj} \end{pmatrix}$$

This can similarly be written in compact matrix notation as:

(15) $\partial P_{wkj} = G^{-1}\partial FM_{kj}$

Thus, the effects of changes in imports of the jth importing country on the kth exporter of a trade embargo can be calculated with (7) and (15) as:

(16) $dX_k = (\partial X_k/\partial M_{kj})(dM_{kj}) = BG^{-1}\partial FM_{kj}(dM_{kj})$

where X_k is a vector of export demands and $dM_{kj} = -M_{kj}$.

In cases where the kth exporter imposes export embargoes on the jth importer as we have considered above, the jth importer switches the source of its imports from the kth exporter to the other exporting country, say the ith exporter ($i \neq k$). The effects on the exports and export prices of switching the source of imports can be measured by replacing M_{kj} (and therefore dM_{kj}) in equations (14), (15), and (16) with M_{ij}. It should be noted that, first, behavioral changes of importing countries resulting from export embargo are not reflected in the model. One example is the effect of the U.S. embargo on grain shipments to the Soviet Union. One hypothesis is that the willingness of the United States to impose such an embargo establishes a worldwide attitude that the United States is not a reliable supplier. This change in attitude can have a significant effect over the long run, encouraging importers to treat the United States as a residual supplier rather than as the principle supplier (Tweeten, 1987). Second, if the United States imposes an export embargo on the jth importer, while the lth importer increases imports by the same quantity from the United States, such switching may not lead to a zero effect on the U.S. exports and price. A zero effect would be obtained when:

$(E_{IJ} E_{pwkj}/E_{xk})dM_{kj} = (E_{Ir}E_{pwkr}/E_{xk})dM_{kr}$ where k = U.S.

Import Tariffs

A tariff in its usual form is computed as a percentage of the border price (t_jP_{wj}). It thus has the effect of changing the border price so that:

(17) $P^*_{wj} = P_{wj}(1+t_j)$.

Assuming a constant elasticity import function of the following form:

(18) $M_j = f_j(P^*_{wj})^{-g_j}$

where f_j, $g_j > 0$ and considering t_j as a policy variable, then:

(19) $M_j = f_j(P_{wj})^{-g_j}.(1+t_j)^{-g_j}$

and $P_{wj}dt_j = dP^*_{wj}$ from (17).

Substituting (19) into equation (5) yields:

$$F_k = a_k(P_{wk})^{-b_k} + \Sigma_{i\neq k}(E_{xi}E_{Pwki}/E_{xk})c_i(P_{wi})^{d_i} - \Sigma_j(E_{Ij}E_{Pwkj}/E_{xk})f_j(P_{wj})^{-g_j}.(1+t_j)^{-g_j} = 0 \quad \text{for all } k = 1, 2, \ldots, m. \qquad (20)$$

Proceeding as in the previous case, we apply the implicit function theorem to (20) to get:

$$(21)\quad \begin{pmatrix} \partial P_{w1}/\partial t_j \\ \partial P_{w2}/\partial t_j \\ . \\ . \\ \partial P_{wm}/\partial t_j \end{pmatrix} = \begin{pmatrix} \partial F_1/\partial P_{w1} \cdots & \partial F_1/\partial P_{wm} \\ \partial F_2/\partial P_{w1} \cdots & \partial F_2/\partial P_{wm} \\ \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots \\ \partial F_m/\partial P_{w1} \cdots & \partial F_m/\partial P_{wm} \end{pmatrix}^{-1} \begin{pmatrix} -\partial F_1/\partial t_j \\ -\partial F_2/\partial t_j \\ . \\ . \\ -\partial F_m/\partial t_j \end{pmatrix}$$

or equivalently:

$$(22)\quad \partial P_{tj} = G^{-1}\partial Ft_j$$

Therefore, the effect of a change in tariffs imposed by the jth country on all exporters can be estimated by:

$$(23)\quad dX = (\partial X/\partial t_j)dt_j = BG^{-1}\partial Ft_j(dt_j)$$

Exchange Rate

We now turn to the effect of a change in the exchange rate in country j on the exports of country k. The dollar price of the commodity (P_{wj}) times the exchange rate R_j equals the local price of the commodity (P_{w*j}) such that $R_jP_{wj} = P_{w*j}$. Differentiating the import demand equation with respect to the exchange rate yields:

$$(24)\quad \partial M_j/\partial R_j = -f_j^* g_j(P_{w*j})^{-g_j-1}\partial P_{w*j}/\partial R_j$$
$$= -M_j g_j/R_j$$

Therefore,

$$(25)\quad \partial F_k/\partial R_j = (E_{Ij}E_{Pwkj}/E_{xk})M_j g_j/R_j$$
$$= 0 \qquad \text{for all } k = 1, 2, \ldots, m.$$

Application of the implicit function theorem to (25) results in the following:

$$(26)\quad \begin{pmatrix} \partial P_{w1}/\partial R_j \\ \partial P_{w2}/\partial R_j \\ . \\ . \\ \partial P_{wm}/\partial R_j \end{pmatrix} = \begin{pmatrix} \partial F_1/\partial P_{w1} \cdots & \partial F_1/\partial P_{wm} \\ \partial F_2/\partial P_{w1} \cdots & \partial F_2/\partial P_{wm} \\ \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots \\ \partial F_m/\partial P_{w1} \cdots & \partial F_m/\partial P_{wm} \end{pmatrix}^{-1} \begin{pmatrix} -\partial F_1/\partial R_j \\ -\partial F_2/\partial R_j \\ . \\ . \\ -\partial F_m/\partial R_j \end{pmatrix}$$

or in compact matrix notation:

$$(27)\quad \partial P_w R_j = G^{-1}\partial FR_j.$$

Thus, we can compute the effects of change in the jth importing country exchange rate on the export demand of all exporters by estimating:

$$(28) \qquad dX = (\partial X/\partial R_j)dR_j = BG^{-1}\partial FR_j(dR_j).$$

Price Insulation

Many governments of importing countries influence domestic prices by insulating domestic prices from international market forces. This tends to reduce the elasticity of price transmission toward zero. The effects of the removal of such price distortions on exporters are evaluated in this section.

Consider the import price transmission elasticities, E_{pwkj} for all k and j, in equation (5). Application of the implicit function theorem on equation (5) results in the following:

$$(29) \qquad \begin{pmatrix} \partial P_{w1}/\partial E_{Pwkj} \\ \partial P_{w2}/\partial E_{Pwkj} \\ \cdot \\ \cdot \\ \partial P_{wm}/\partial E_{Pwkj} \end{pmatrix} = \begin{pmatrix} \partial F_1/\partial F_{Pw1} & \cdots\cdots & \partial F_1/\partial P_{wm} \\ \partial F_2/\partial F_{Pw1} & \cdots\cdots & \partial F_2/\partial P_{wm} \\ & \cdots\cdots\cdots\cdots\cdots\cdots & \\ & \cdots\cdots\cdots\cdots\cdots\cdots & \\ \partial F_2/\partial F_{Pw1} & \cdots\cdots & \partial F_2/\partial P_{wm} \end{pmatrix}^{-1} \begin{pmatrix} \partial F_1/\partial E_{Pwkj} \\ \partial F_2/\partial E_{Pwkj} \\ \cdot \\ \cdot \\ \partial F_2/\partial E_{Pwkj} \end{pmatrix}$$

or in compact matrix notation:

$$(30) \qquad \partial E_{Pwkj} = G^{-1}\partial FE_{pwkj}$$

The effects of changes in degree of price distortion by the jth importing country from the kth exporter on the export demand of all exporters can then be estimated by:

$$(31) \qquad dX = \partial X/\partial E_{pwkj}dE_{pwkj} = BG^{-1}\partial FE_{pwkj}dE_{pwkj}$$

Conclusion

Evidence (3) suggests that most importing countries engage in some form of government intervention in their grain trade. Using the basic relationship underlying the derivation of the foreign demand elasticity, we develop a mathematical trade model which can be used to estimate the effects of government intervention on export prices and quantities. The model is partial equilibrium in form, focusing on a single commodity with m exporters and n importers. The model has several limitations which restrict its application. Although the model can be used to estimate the effects of many types of government trade intervention, acquiring appropriate data is a serious problem. Furthermore, it is difficult to incorporate the interrelationship between exporters and importers in a quadratic spatial programming model. This last limitation, however, is common to most trade models.

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Appendix: The Derivation of Foreign Demand Elasticity

Assume in a partial equilibrium context that we are concerned with deriving the effect of a change in export price (P_{wik}) on export quantity (X_{ik}) of a particular commodity (i) for an arbitrary exporting country (k). Assume further, for simplicity, that only a single domestic price (Pd_{ij}) prevails in all countries (j).1/ Furthermore, let us assume that there are not stocks held or accumulated. Under these assumptions and the generalized functions specified below, a generalized export demand elasticity can be derived. Let the domestic demand (Q_{Dij}), supply (Q_{Sij}) and market equilibruim conditions be specified as follows:

(1a) $$Q_{Dij} = Q_{Dij}(Pd_{ij})$$

where $\partial Q_{Dij}/\partial Pd_{ij} < 0$.

(2a) $$Q_{sij} = Q_{sij}(Pd_{ij})$$

where $\partial Q_{sij}/\partial Pd_{ij} > 0$.

Furthermore, let Pd_{ij} be the market-clearing price so that

(3a) $$Q_{Iij} = Q_{Eij} = Q_{Dij}(Pd_{ij}) - Q_{sij}(Pd_{ij}) = Q_{Iij}(Pd_{ij})$$

If we let Q_{Iij} be both positive (imports) and negative (exports), then this reflects the environment for all countries and commodities. Q_{Iij} is clearly domestic excess demand for all countries.

If we assume that free trade prevails so that there are no transport cost, marketing costs, or unbiased government intervention, then the excess demand elasticity (E_{Iij}) is derived by taking the partial derivative of (3a) and normalizing by the price quantity ratio.2/

(4a) $$E_{Iij} = (\partial Q_{Dij}/\partial Pd_{ij})(Pd_{ij}/Q_{Dij})(Q_{Dij}/Q_{Iij}) - (\partial Q_{sij}/\partial Pd_{ij})(Pd_{ij}/Q_{sij})(Q_{sij}/Q_{Iij})$$

Letting:

$$E_{Dij} = (\partial Q_{Dij}/\partial Pd_{ij})(Pd_{ij}/Q_{Dij})$$

$$E_{Sij} = (\partial Q_{sij}/\partial Pd_{ij})(Pd_{ij}/Q_{sij})$$

$$I_{Dij} = Q_{Dij}/Q_{Iij}$$

$$I_{sij} = Q_{sij}/Q_{Iij})$$

the demand and supply elasticities and market shares respectively, we get:

(5a) $$E_{Iij} = E_{Dij}I_{Dij} - E_{sij}I_{sij}$$

1/ This does not preclude government intervention in the domestic market, but only that intervention does not separate producers and consumers.

2/ It is clear that we could introduce other variables into the demand and supply without changing the outcome as long as there are no cross-price effects.

which is the excess demand elasticity. To get the usual form of the import demand elasticity, we allow the domestic (Pd_{ij}) and border price(P_{wij}) to differ.3/ We thus introduce a border price transmission function.

(6a) $$Pd_{ij} = Pd_{ij}(P_{wij})$$

where $\partial Pd_{ij}/\partial P_{wij} \gtreqless 0$.

Substituting (6a) into (3a), taking partial derivatives, and putting the equation into elasticity form yields the import demand elasticity (E_{Ipwij}).

(7a) $$E_{pwij} = E_{Iij} \cdot E_{pij}$$

where E_{pij} is the price transmission elasticity:

(8a) $$E_{pij} = \partial Pd_{ij}/\partial P_{wij})(P_{wij}/Pd_{ij}).$$

We derive the generalized export demand elasticity by recognizing that the sum of exports equals the sum of imports, assuming no commodities in transit, and introducing an international price transmission to allow for international transportation costs. Thus:

(9a) $$P_{wij} = P_{wij}(P_{wik})$$

and

(10a) $$\Sigma_n Q_{Iin} = \Sigma_j Q_{Iij}$$

where n refers to exporters and j refers to importers. Solving (10a) for the kth exporter yields.

(11a) $$Q_{Iik} = \Sigma_j(Q_{Iij}) - \Sigma_{n \neq k}(Q_{Iin})$$

for k = 1, 2, ...m.

Taking the partial derivatives with respect to the kth export price (P_{wik}) and normalizing for elasticities yields the export demand elasticity.

(12a) $$E_{xik} = \Sigma_j(E_{Iij}E_{pwk}I_{kj}) - \Sigma_{n \neq k}(E_{xin}E_{pwk}I_{kn}).$$

3/ This could come about for a variety of reasons including transportation costs and government intervention in the trade sector.

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