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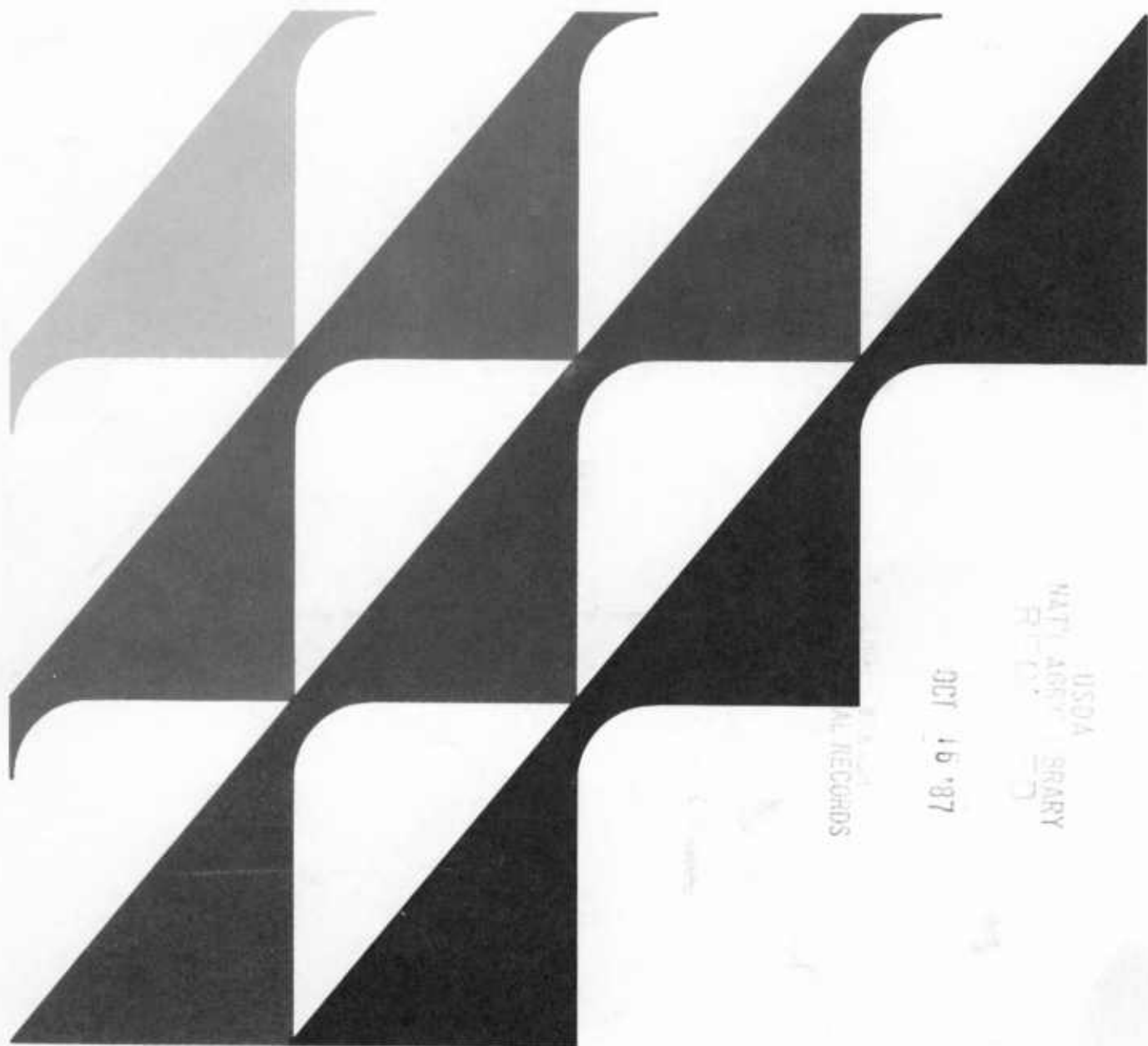
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# A Forecast Evaluation of Capital Investment in Agriculture

Roger Conway  
James Hrubovcak  
Michael LeBlanc



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#### ABSTRACT

A stochastic coefficients model is used to forecast agricultural investment. Variance decomposition attributes the highest portion of the variance to the lagged capital stock variable. In out-of-sample forecasts, the stochastic coefficients model outperforms the nonlinear flexible accelerator and ordinary least squares empirical estimates of agricultural investment for a wide array of risk functions. Our investment model forecasts continued declines in net investment for farm machinery, with greater declines toward the end of the forecast period (1988-90).

Keywords: Forecast, evaluation, agricultural investment, stochastic coefficients

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## SUMMARY

In this report the authors develop a stochastic coefficients model for forecasting investment in agricultural machinery. Over the last two decades, the agricultural sector has experienced large swings in output prices and income, unprecedented increases in manufactured input prices, high inflation rates, and has been subject to extensive government commodity programs. A model that allows for parameter variation, thereby capturing structural change in the farm sector, is likely to forecast agricultural investment more accurately than would a typical fixed coefficient approach.

One cannot test whether the stochastic coefficients model is the correct specification because there is no statistical procedure to identify the "true" model. However, the stochastic coefficients model clearly dominates over six alternative models in a 5-year, out-of-sample forecast. Although not conclusive, the comparison suggests that the stochastic coefficient variant of a logically consistent theoretical framework is a useful forecasting tool.

We forecast an acceleration of current trends in agricultural machinery investment. That is, net investment is forecasted to decline through 1990 from continued weak profits in agriculture. Net investment will decline further before it increases again, reflecting an agricultural sector structurally adjusting to a smaller capital stock. The reliability of the forecast, particularly for later years, is questionable as the estimated parameters are sensitive to new information contained in additional observations. Therefore, as more recent data are used to re-estimate the model, our forecasts for 1989 and 1990 are expected to change. Nevertheless, any renewed near-term growth in machinery stocks is unlikely.

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# A Forecast Evaluation of Capital Investment in Agriculture

Roger Conway  
James Hrubovcak  
Michael LeBlanc\*

## INTRODUCTION

There is a growing need for improved statistical forecasting. In the agricultural sector, economic decisions are made in a volatile environment. During the last decade, there have been large swings in output prices and farm income, unprecedented increases in manufactured input prices, high inflation rates, extensive grain reserve and acreage diversion programs, and significant changes in monetary and fiscal policies. Although some of the agricultural economic literature examines the relationships between the macroeconomy and agriculture, less attention has focused on agricultural investment. What determines agricultural investment remains largely unexplored (17, 24).<sup>1/</sup>

This report develops a logically consistent model for forecasting investment in agricultural machinery. Although many approaches are possible (standard neoclassical, cash-flow, securities value), this analysis is based on Lucas' accelerator model (20, 35). The power of Lucas' work lies in its rendering of the adjustment coefficient whereby, unlike other partial adjustment models, the speed of adjustment depends on economic variables and, therefore, varies through time.

Consistent with Lucas's (19) later work, we propose a stochastic coefficients model which allows economic phenomena to vary all the parameters in the model rather than restrict variability to the adjustment coefficient. Lucas (19) argued "the standard, stable parameter view of econometric theory and quantitative policy evaluation appears not to match several important characteristics of econometric practice, while an alternative general structure, embodying stochastic parametric drifts, matches these characteristics very closely." The stochastic coefficients model presented here was first developed by Swamy and Tinsley (29).

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<sup>1/</sup> Underscored numbers in parentheses refer to literature cited in the References section.

Several explanations in support of parameter variation can be advanced. For example, fixed coefficient econometric models may be inconsistent with the dynamic economic theory of optimizing behavior. Changes in economic or policy variables will result in a new environment that may lead to new optimal decisions and new micro- and macroeconomic structures (19). Both of these explanations are associated with the intuitive notion that the economic structure is dynamic, not static.

In addition to these intuitive explanations, there are econometric or empirical reasons for assuming parameter variability. A nonstationary, or time-varying, random process may generate the "true" coefficients, or omitted variables with nonstationary behavior that are not orthogonal to the included variables may also induce variability in the parameters (7). Furthermore, it is conventional econometric practice to replace unobservable explanatory variables with proxy variables. In most cases, proxy variables imperfectly capture changes in the economic behavior of the true variable. Aggregating over microeconomic units can also induce variation. It is too restrictive to assume the aggregation weights of microeconomic units do not change over time (46, 47). Imposing an incorrect functional form may also induce the coefficients to vary (25). The stochastic coefficients approach advanced by Swamy and Tinsley addresses these problems, has less stringent assumptions, and has excellent forecasting capability.

#### INVESTMENT IN AGRICULTURAL MACHINERY

Capital investment is a way society exchanges the present for the future. Increased net investment expands an economy's productive capacity through technological change. Expanded output, technological change, and substituting capital for labor has significantly increased the use of capital equipment. The capital stock of farm equipment (measured in 1972 dollars) grew from \$5.2 billion in 1922 to \$22 billion in 1985 (fig. 1). However, this growth has been uneven, even in years when the stock of equipment fell. There was negative net investment, implying an eroding capital base, in the early 1930's and 1950's and during 1980-85 (fig. 2). Net investment grew the most during World War II, the Korean War, and during the 1970's. However, there were also sharp downturns during these growth periods.

There has been a large shift from labor to machinery and chemicals in agriculture in the past 25 years. After declining in 1962, the value of agricultural equipment stocks grew at a fairly constant rate until 1979, increasing from \$18.6 billion in 1962 to \$31.4 billion in 1979. The value of the capital stock for tractors and other farm machinery increased from \$4.3 and \$14.2 billion in 1962 to \$7.9 and \$23.5 billion in 1979.

The shift to a capital-intensive agriculture significantly affected the use of variable inputs. While the quantity of labor has declined by about 3.4 percent per year since 1955, use of manufactured inputs such as fertilizers and pesticides increased 6.6 percent per year during 1955-79.

Changes in relative input and output prices produced much of the shift from labor to capital and chemicals. During the 1950's and 1960's, farmers could reduce costs by expanding farm size and using lower cost per-unit output farm machinery (rather than higher cost labor). In addition, nonfarm demand for farm labor increased farm wage rates relative to other input prices. Nominal farm labor costs increased 4 percent a year from 1955 to 1970, while machinery

Figure 1

## Machinery capital stock, 1923-85

Billion 1972 dollars

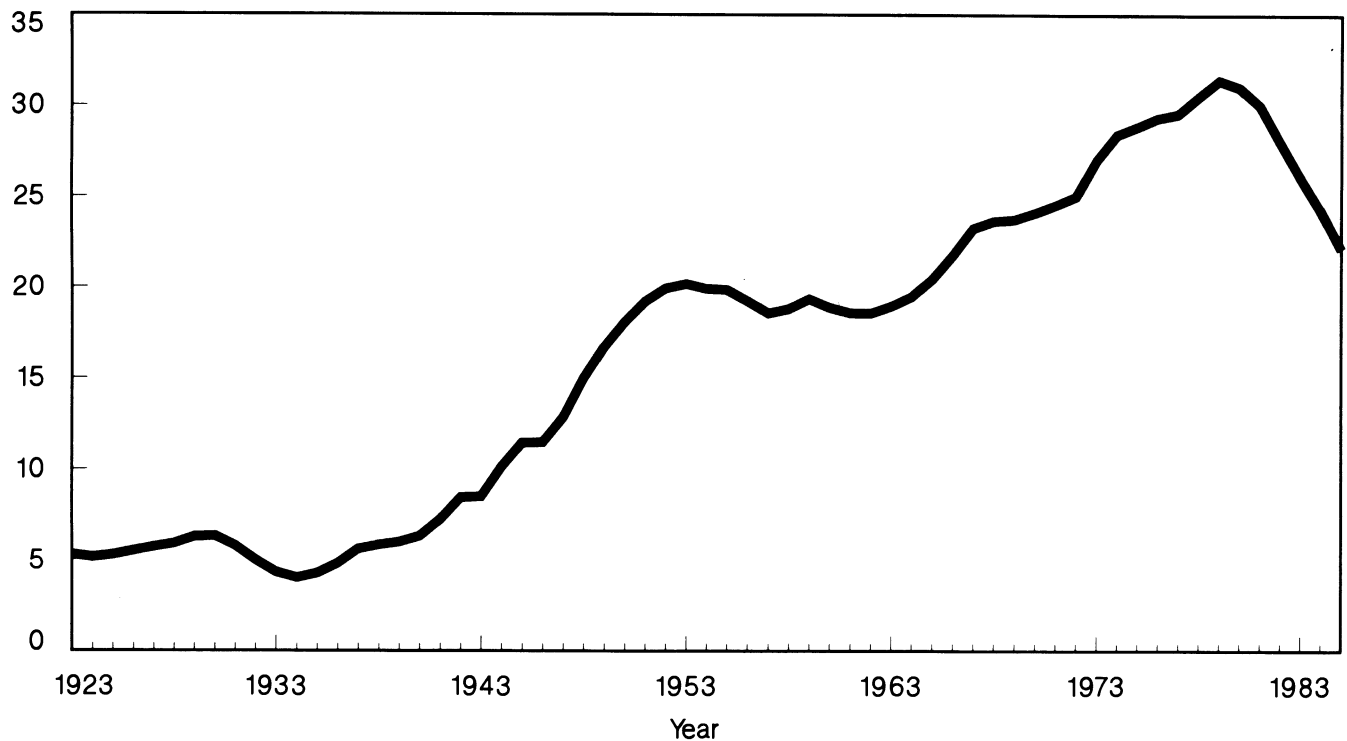
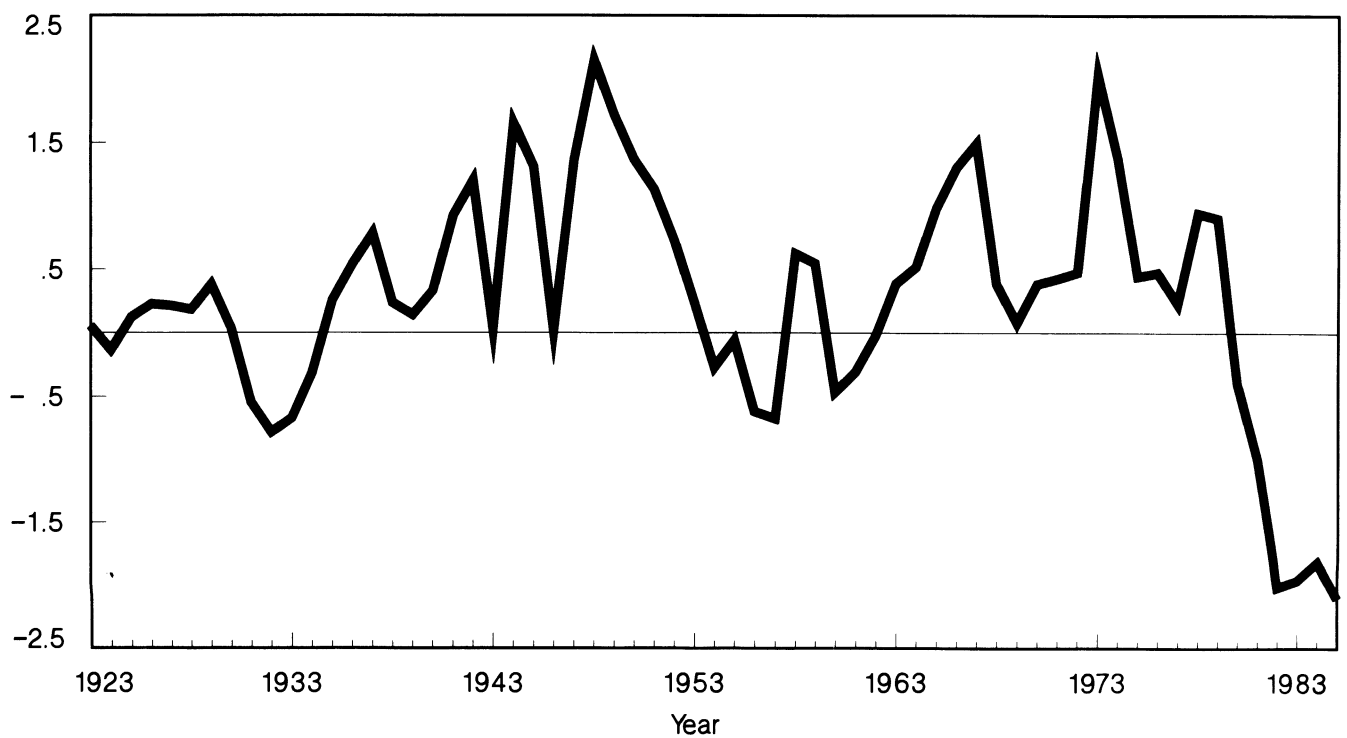


Figure 2

## Net machinery investment, 1923-85

Billion 1972 dollars





prices increased only 2.9 percent. The nominal price of agricultural chemicals declined from 1955 to 1972.

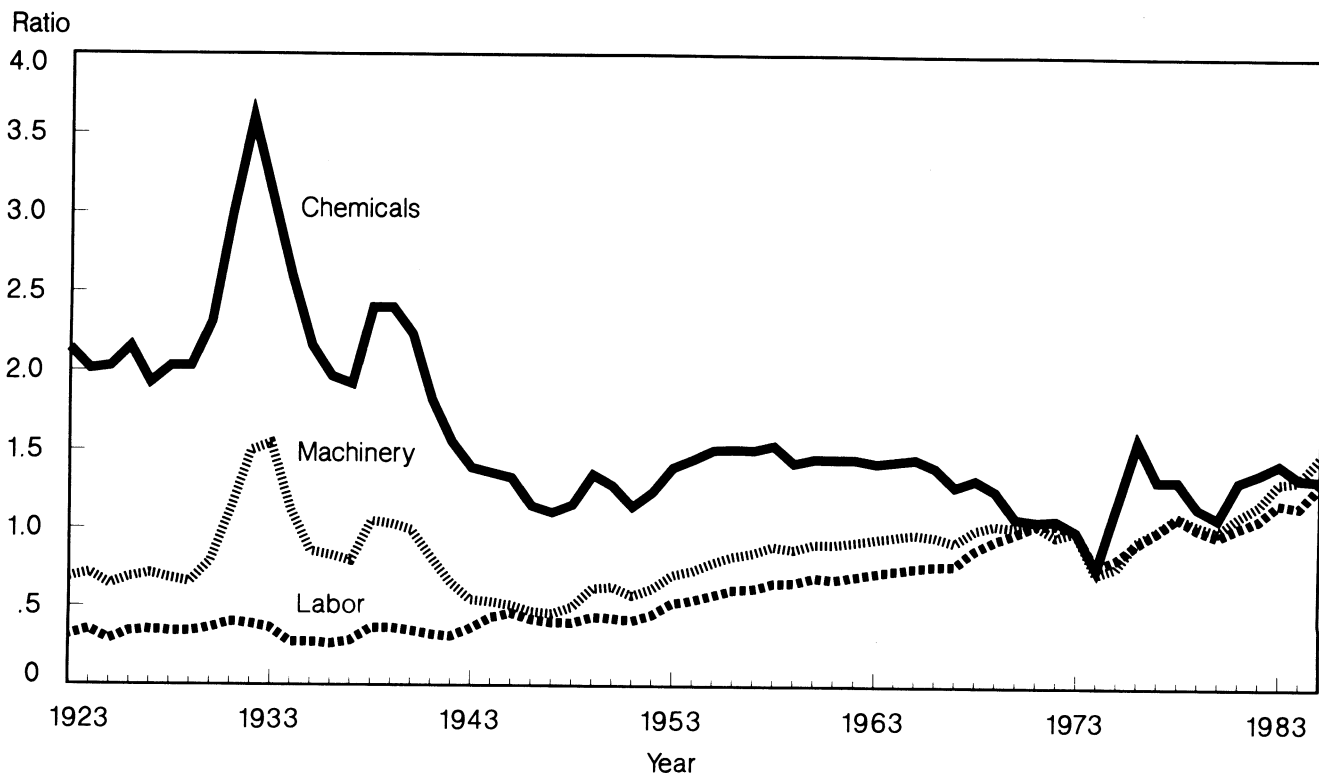
The ratio of chemical input prices to farm output prices fell dramatically between 1955 and 1973, while the ratios of labor prices to output price and machinery prices to output price rose slightly (fig. 3). The decrease in the ratio of chemical input prices to farm output prices increased the demand for agricultural chemicals and the demand for complementary inputs such as farm machinery. Also, the higher demand for chemicals lowered the demand for other inputs (such as labor) that are substitutes for chemicals.

Federal commodity programs created a stable environment by establishing minimum prices for many commodities, thus encouraging long-term investments by reducing uncertainty. Increased demand for output and the resulting higher output prices (from increased export demand during the 1970's) also stimulated the demand for capital inputs. Lower output prices and high real interest rates since 1981 produced successive years of negative net investment through 1986.

Increased demand for farm capital since 1955 stimulated the demand for farm credit. Total nominal farm debt increased from \$17.7 billion in 1955 to \$195-202 billion in 1985 (37). Interest rates charged to agricultural borrowers are closely related to interest rates in the general economy because the funds come from the same sources. The Farm Credit System (FCS), including

Figure 3

### Input/output price ratio, 1923-85



Federal land banks, production credit associations, and Federal intermediate credit banks, held \$82.9 billion of the 1985 farm debt (37). FCS obtains loanable funds through securities sales in U.S. financial markets. Like other banking organizations, the FCS typically increases interest rates under tight monetary policies or from increases in nonfarm demand for funds. However, interest rates on new FCS loans tend to lag behind those of other lenders when interest rates rise because FCS banks use average-cost pricing (rates based on the average interest rate on all their outstanding bonds) rather than typical marginal-cost pricing.

## THEORETICAL FRAMEWORK

Economists have sought a theoretical framework for the partial adjustment or accelerator model since Nerlove's early applied work (23). Many economists recognized the gap in econometric theory where an elaborate theoretical structure for determining the level of an input was combined with an ad hoc theory of adjustment. Eisner and Strotz developed a more rigorous theory of adjustment by casting the firm (or farm) in a dynamic optimization framework (10). The present value, or net worth, maximized by the firm depends on the optimal level of inputs selected by the firm and the path of the current capital stock to the optimal level.

Lucas, Gould, and Treadway extended Eisner and Strotz's work (13, 20, 35). Their models differed in complexity but had the same underlying structure. Each specified an objective function incorporating factor adjustment costs and a production function. Lucas, Gould, and Treadway also assumed the firm maximizes net worth over a given time, and interpreted adjustment costs as either foregone profits because of shortrun rising prices in the capital-supplying industry or as increasing costs associated with integrating new equipment into production (reorganizing production and training workers). These costs varied with the speed of capital adjustments. They also assumed that the values of the expected input and output prices did not change. This myopic static, or stationary-expectations assumption, is required to define the dynamic maximization problem (22).<sup>2/</sup> Because expectations were static, the firm adjusted to a fixed target considered to be the longrun equilibrium of neoclassical theory. Given these assumptions, a firm that maximizes its present value changes capital stock in the same way the accelerator model suggests.

Incorporating a shortrun restricted profit function into a longrun dynamic optimization framework yields the optimal adjustment paths for the quasi-fixed inputs (2, 3). The assumptions of competitive input and output markets are maintained, as we assume that these competitive real prices are known with certainty and remain stationary over time.

A quasi-fixed input can be varied at a cost  $C(\dot{K})$ , where  $\dot{K}$  equals  $dk/dt$ , and:

$$\dot{K} = I - \delta K \quad (1)$$

---

<sup>2/</sup> This assumption probably could be relaxed if a more general approach to forming expectations were allowed. See (27) for a description of a coherent subjective Bayesian reinterpretation of rational expectations.

where:  $I$  is the gross addition to capital stock and  $\delta$  is the rate of exponential depreciation. The cost of adjustment is:

$$C(\dot{K}) = qI + qD(\dot{K}) \quad (2)$$

where:  $q$  is the purchase price of the quasi-fixed asset,  $D(\dot{K})$  is a twice differentiable function, and  $D''(\dot{K}) > 0$ . Adjustment costs at the initial time  $t=0$  are:

$$C(0) = q\delta K \quad (3)$$

This formulation assures constant marginal costs of replacement with increasing marginal costs of net change.

Net receipts can be written:

$$R(t) = PG(W, K) - C(\dot{K}) \quad (4)$$

where:  $G(W, K)$  is the unit-output-price (UOP) restricted profit function,  $P$  is the unit price of output,  $K$  is a quasi-fixed capital input, and  $W$  is a vector of input prices normalized on output prices.<sup>3/</sup>

If the firm requires a rate of return,  $r$  (a weighted average of the rate of return to equity and the cost of external financing), the present value of net receipts at time  $t=0$  is:

$$V(0) = e^{-rt} \int_0^{\infty} R(t) dt \quad (5)$$

The firm's longrun dynamic problem is to choose time paths for variable inputs,  $X(t)$ , and the quasi-fixed input,  $K(t)$ , to maximize  $V(0)$  given  $K(0)$ ,  $X(t)$ , and  $K(t)$  are greater than 0. That is, because  $G$  assumes shortrun optimizing behavior conditional on  $P$ ,  $W$ , and  $K$ , the optimization problem facing the firm is to find the time paths of  $X(t)$  and  $K(t)$  among all the possible  $G(W, P)$  combinations, thereby maximizing the present value of net receipts.

The results are linked to the partial adjustment and flexible accelerator literature. The shortrun demand for the quasi-fixed factor can be generated as an approximate solution in the neighborhood of  $K^*(t)$ , which is the steady-state, or longrun, profit-maximizing demand for the vector of quasi-fixed factors in time  $t$  (20). The approximate solution is the linear differential equation:

$$\dot{K} = B[K^*(t) - K(t)] \quad (6)$$

where:

$$B = -0.5(r - [r^2 - 4H''(K^*)/C''(0)]^{0.5}) \quad (7)$$

---

<sup>3/</sup> The restricted profit function is the locus of shortrun maximized profit of a firm as a function of output prices, input prices, and quantities of fixed factors (18). The profit function is nonincreasing and convex in  $W$  (normalized input prices) and nondecreasing in  $P$  and  $K$ .

The adjustment coefficient, B, now depends on economic forces: the discount rate, the adjustment cost, the production relationship embodied in the profit function, and the profit-maximizing behavior of the firm.

### EMPIRICAL MODEL

Additional assumptions are required before the theoretical framework can be applied. The adjustment relationship must be recast as a difference equation, and functional forms for the adjustment cost and profit functions must be selected.<sup>4/</sup> Furthermore, we linearized the nonlinear flexible accelerator to allow for a structural form that can be estimated in a stochastic coefficients framework. The stochastic coefficients model is more general because it allows variation in all parameters of the model, while the flexible accelerator model allows parameter variation only in the adjustment coefficient.

We used a first-order variant of the generalized ARIMA stochastic coefficients process model developed by Swamy and Tinsley (29) and Havenner and Swamy (15) for the investment model. This approach is a generalization of other stochastic coefficients models, such as the Kalman filter and Cooley-Prescott procedures (29).

Investment is assumed to be generated by a linear version of the flexible accelerator:

$$K_t - K_{t-1} = b_{kt} + b_{wkt}W_t + b_{ut}U_t + B_t K_{t-1} \quad (8)$$

$$= \underline{X}_t' \underline{\beta}_t \quad (t=1,2,\dots,T)$$

where: W is the ratio of input to output prices and U is rental rate. Thus:

$$E\underline{\beta}_t' = \underline{\beta}' = (\bar{b}_k, \bar{b}_{wk}, \bar{b}_u, \bar{B}) \quad (9)$$

$$(\underline{\beta}_t - \underline{\beta}') = \Phi(\underline{\beta}_{t-1} - \underline{\beta}') + \underline{a}_t \quad (10)$$

where:  $\Phi$  may or may not be diagonal, all the characteristic roots of  $\Phi$  are less than 1 in absolute value, and  $\underline{a}_t$  is a vector of errors such that:

$$E\underline{a}_t = 0 \quad (11)$$

$$E\underline{a}_t \underline{a}_t' = \begin{cases} \Delta_a & \text{if } t=t' \\ 0 & \text{if } t \neq t' \end{cases} \quad (12)$$

$\Delta_a$  is positive definite; it may or may not be diagonal. The observation vectors and matrices are:

$$\underline{Y} = (Y_1, Y_2, \dots, Y_T) \quad (13)$$

$$\underline{X} = (\underline{X}_1, \underline{X}_2, \dots, \underline{X}_T) \quad (14)$$

$$D_X = \text{diag}(\underline{X}_1, \underline{X}_2, \dots, \underline{X}_T) \quad (15)$$

<sup>4/</sup> We used a quadratic approximation, normalized on output price, for the profit function so the model can be estimated without placing a priori restrictions on the elasticities of substitution (12). In addition, the optimal path is globally rather than locally valid (34).

The unobservables are:

$$\underline{\beta}_t = \underline{\bar{\beta}} + \underline{\varepsilon}_t \quad (16)$$

$$\underline{\varepsilon} = (\underline{\varepsilon}_1, \underline{\varepsilon}_2, \dots, \underline{\varepsilon}_T)' \quad (17)$$

The variance-covariance matrices are:

$$E \underline{\varepsilon}_t \underline{\varepsilon}_t' = \Gamma_0 = \Phi \Gamma_0 \Phi' + \Delta_a \quad (18)$$

$$E \underline{a}_t \underline{a}_t' = \Delta_a$$

$$E \underline{\varepsilon} \underline{\varepsilon}' = \Sigma_\beta = \begin{bmatrix} \Gamma_0 & \Gamma_0 \Phi' & \Gamma_0 \Phi'^2 & \Gamma_0 \Phi'^{T-1} \\ \Phi \Gamma_0 & \Gamma_0 & \Gamma_0 \Phi'^3 & \dots & \Gamma_0 \Phi'^{T-2} \\ \vdots & \vdots & \vdots & & \vdots \\ \Phi^{T-1} \Gamma_0 & \Phi^{T-2} \Gamma_0 & \Phi^{T-3} \Gamma_0 & \dots & \Gamma_0 \end{bmatrix} \quad (19)$$

$$E(\underline{Y} - \underline{X}\underline{\beta})(\underline{Y} - \underline{X}\underline{\beta})' = \Sigma_y = D_x \Sigma_\beta D_x' \quad (20)$$

Following (1), one can show that  $\Sigma_\beta$  is positive definite if the eigenvalues of  $\Phi$  are less than 1 in absolute value.  $\Gamma_0$  is positive definite if  $\Delta_a$  is positive definite.

$$\underline{\beta} \sim ws(\underline{\bar{\beta}}, \Sigma_\beta) \quad (21)$$

$$\underline{Y} \sim ws(D_x \underline{\bar{\beta}}, \Sigma_y) \quad (22)$$

The conditional expected value and variance of the dependent variable vary with the conditioning variables. One may decompose the variance in the dependent variable into its contributing factors. Allowing the independent variable to influence the variance of the dependent variable is important because an independent variable may significantly influence the variance of the dependent variable even though it has only a relatively slight influence on the mean. This decomposition is analogous to allocations of the multiple  $R^2$  among the explanatory variables in a conventional regression equation (31).

Averaging over the sample period makes  $\text{var}(u_t)$  unit-free and one obtains:

$$1 = \frac{1}{T} \sum_{t=1}^T \left[ \sum_{i=1} \sum_{j=1} X_{it} X_{jt} \Gamma_{ij} \right] \quad (i, j=1, \dots, k) \quad (23)$$

$$\underline{x}'_t \Gamma_x \underline{x}_t$$

$\Delta_a$  and  $\Phi$  will collapse to scalar characteristics of the intercept (constant term) coefficients when the coefficients do not vary. One may obtain t-tests of the individual components to test the significance of the uncertainty allocations to slope coefficients by using an asymptotic approximation of the covariance matrix of the estimated column stack,  $\text{vec}(\Delta_a)$ .

The first regressor,  $x_{1t}$ , is usually a unit vector intercept with a stochastic component of its coefficient, serving as the analogue of the additive disturbance familiar to fixed coefficient specifications. The stochastic coefficients model will have a total residual,  $a_t$ . This residual is a weighted sum of the stochastic elements of the coefficients of the intercept regressor and the time-varying regressors, where  $a_t \equiv x_t' \epsilon_t$ .

The residual,  $(a_t)$ , does not necessarily increase when performing stochastic coefficients estimation. Estimates of  $a_t$  (where  $t=1,2,\dots,T$ ) from the two estimators will converge as the sample size increases if ordinary least squares is a consistent estimator of the means of the coefficient vector,  $\bar{\beta}$ .

## DATA

The analysis uses aggregate time-series data for 1923 through 1985. The ratio of prices paid for farm inputs to prices received for farm outputs, the implicit rental rate of capital inputs, and the lagged capital stock explain changes in the stock of farm machinery.

The U.S. Department of Agriculture (USDA) produced the ratio of prices paid to prices received (38, 39). The prices paid index includes allowances for interest, taxes, wage rates, and production items, such as feed, seed, and fertilizer. The prices received index is an aggregate index of prices received for all farm products.

Implicit rental rates for tractors and long-lived farm equipment are estimated and then aggregated into a single rental rate for farm machinery. Rental rates for each machinery category are functions of the asset prices, service lives, depreciation rates, the tax treatments of assets in each category, and discount rates.

A single price index series for farm machinery categories is from a capital stock survey by the Bureau of Economic Analysis (BEA), U.S. Department of Commerce (40). The service lives for each equipment category amounted to 85 percent of tax form Bulletin F depreciation lives (42). The service lives for tractors and long-lived equipment are 9 and 13 years, respectively. We determined the rate of economic depreciation for assets in each category from the double declining-balance depreciation method, where the capacity of assets in the  $i$ th category in year  $t$  is:

$$n_i(t) = [1 - (2/L_i)]^{t-1} \quad (i=1,2,\dots,m) \quad (24)$$

where:  $1 \leq T \leq L_i$ ,  $n_i(t) = 0$ , and  $t \geq L_i$

The tax treatment of assets in each category is based on the tax savings over the service life of the asset. Tax depreciation allowances were limited to the straight-line rate, and tax lives were set equal to average Bulletin F lives before 1955. From 1955 to 1980, assets in each category were

depreciated under the sum-of-year's digits method. In 1962, the minimum allowable tax lives were shortened. The tax life of long-lived equipment fell from 15 to 10 years. In 1975, the asset depreciation range (ADR) system was introduced and the allowable tax lives were again reduced: tax lives of tractors and long-lived equipment fell from 10 to 8 years. In 1981, the Economic Recovery Tax Act (ERTA) introduced the accelerated cost recovery system (ACRS), which depreciated tractors and long-lived equipment over 5 years.

The marginal ex-ante Federal income tax rates developed for this analysis were the expected tax rates an investor or firm would pay on an additional dollar of income before undertaking any new investment. These ex-ante rates were estimated for sole proprietorships during 1962-79 (43). Before the Revenue Act of 1964, the lowest marginal tax rate applied to all taxable income below \$2,000. We assumed that the appropriate marginal tax rate corresponds to the lowest tax bracket. Post-1979 estimates of marginal income tax rates contain the statutory tax brackets, but employ USDA data for farm and off-farm income to develop proxies for taxable income.

We assumed that all capital purchases were completely debt-financed. Nominal interest rates equaled rates charged by Federal land banks on new farm loans (37). Nominal interest rates were adjusted for inflation and the tax-deductible amount of interest charges to compute the real required after-tax rate of return (the real discount rate).

An aggregate index of the stock of tractors and long-lived equipment was developed from USDA estimates of farm capital purchases (38) and converted into constant dollars by deflating with price indices from the BEA capital stock study. The constant dollar investment series was then depreciated with the appropriate service lives to estimate a constant dollar machinery stock from the perpetual inventory method.

## RESULTS AND INTERPRETATION

The stochastic coefficients model is estimated using a first-order variant of a generalized ARIMA stochastic coefficients process model (29). Table 1 shows means of the estimated coefficients and their associated asymptotic standard

Table 1--Parameter estimates and associated statistics for the stochastic coefficients model 1/

Parameter	:	Mean	Asymptotic	Asymptotic	Coefficient
	:	value	standard error	t-statistic	of variation
	:				
Constant	:	2,459.9232	250.6532	9.8140	6.6E-04
Input/output	:				
price ratio	:	-2,600.5522	300.7220	-8.6477	7.93E-04
Rental rate	:	-3,643.2544	1,158.4516	-3.1449	8.50E-05
Lagged stock	:	.49877E-01	.11380E-01	4.3828	.5301
	:				

1/ Parameter estimates are mean values conditioned on the estimates (second iteration) of  $\Delta_a$  and  $\phi$ .

errors and t-statistics. Although the value of each t-statistic exceeds 2, the coefficients may or may not be statistically significant because the t-statistics depend on large sample properties. The small sample properties of these asymptotic statistics are unknown.<sup>5/</sup> The estimated parameters suggest a reasonable model structure. All estimated parameters have the expected sign, based on the theoretical model discussed above.

The estimated parameters for the stochastic coefficients model attribute important explanatory roles for all the model's variables. Changes in the input/output price ratio have the largest effect on net investment.

The rental rate also significantly affects machinery investment. A 1-percent increase in the rental rate produces a 2.8-percent decrease in net investment. However, changes in the relative profitability of production, manifested in the input/output price ratio, are the most important economic determinants of net investment. The parameter associated with the lagged capital stock also significantly affects investment in agricultural machinery. Furthermore, empirical results suggest that adjustment in agriculture is slow when we interpret the lagged capital stock parameter as the rate of capital stock adjustment to optimal levels. Agricultural machinery stocks adjusted at an average 5-percent annual rate during 1923-85.

Figures 4-7 present estimated parameter values for 1923-85. The constant and the parameter associated with the input/output price vary symmetrically like a first-order autoregressive process [AR (1)] around mean values. Both time series dampen to mean values, particularly in the post-1960 era. The parameter associated with the rental rate also shows little variability and no apparent historical pattern (table 1).

The adjustment rate, the parameter associated with lagged capital stock, shows the greatest variability. The coefficient of variation (0.5301) is much greater than that for the other parameters. Large increases in the adjustment rate since 1950 parallel major changes in farm policy and the general farm economy. For example, significant increases in the late 1950's coincide with the major farm legislation, including the Agricultural Trade and Assistance Act of 1954 (P.L.-480) and the Agricultural Act of 1956. Adjustment rate increases in the 1960's parallel the Food and Agricultural Act of 1965 and new tax legislation which allows for faster amortization, accelerated depreciation, and the investment tax credit. Increases in 1973 coincide with world crop shortages and global inflation.

There are periods when the time path for the stochastic coefficients adjustment variable is negative. The agricultural sector may overadjust or fluctuate the adjustment of actual to desired capital stock over time because of factors such as risk and uncertainty, imperfect information, weather shocks, or dramatic changes in Government programs. Therefore, the adjustment coefficient may be greater than 1, and sometimes be negative. This view of the time-varying adjustment coefficient is consistent with Griliches insight that restricting a time-dependent adjustment coefficient between 0 and 1 generally cannot be derived from the properties of the solution of the optimal

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<sup>5/</sup> Classical theories of inference are also unclear unless the parameters represent a real physical entity (28). This difficulty is not unique to the model considered here.



Figure 4

## Constant, 1923-85

Parameter value (thousands)

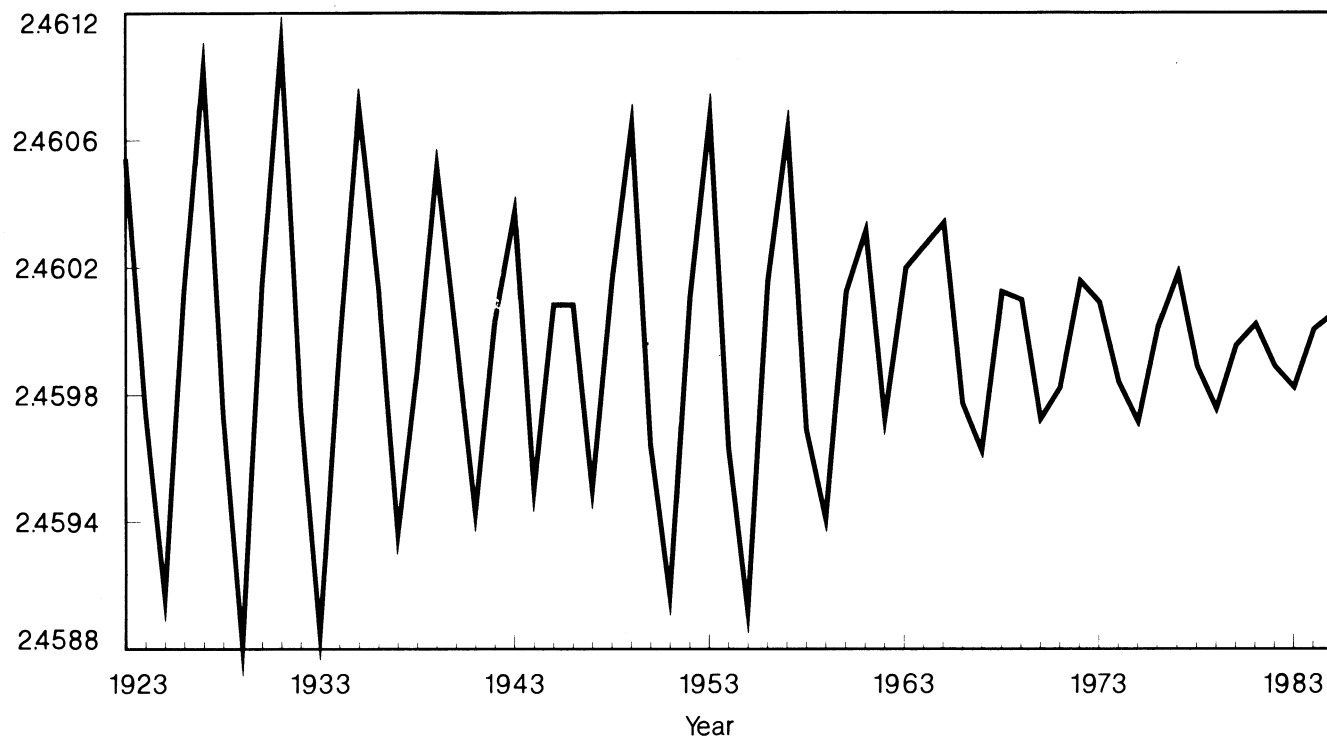


Figure 5

## Coefficients for input/output price ratio, 1923-85

Parameter value (thousands)

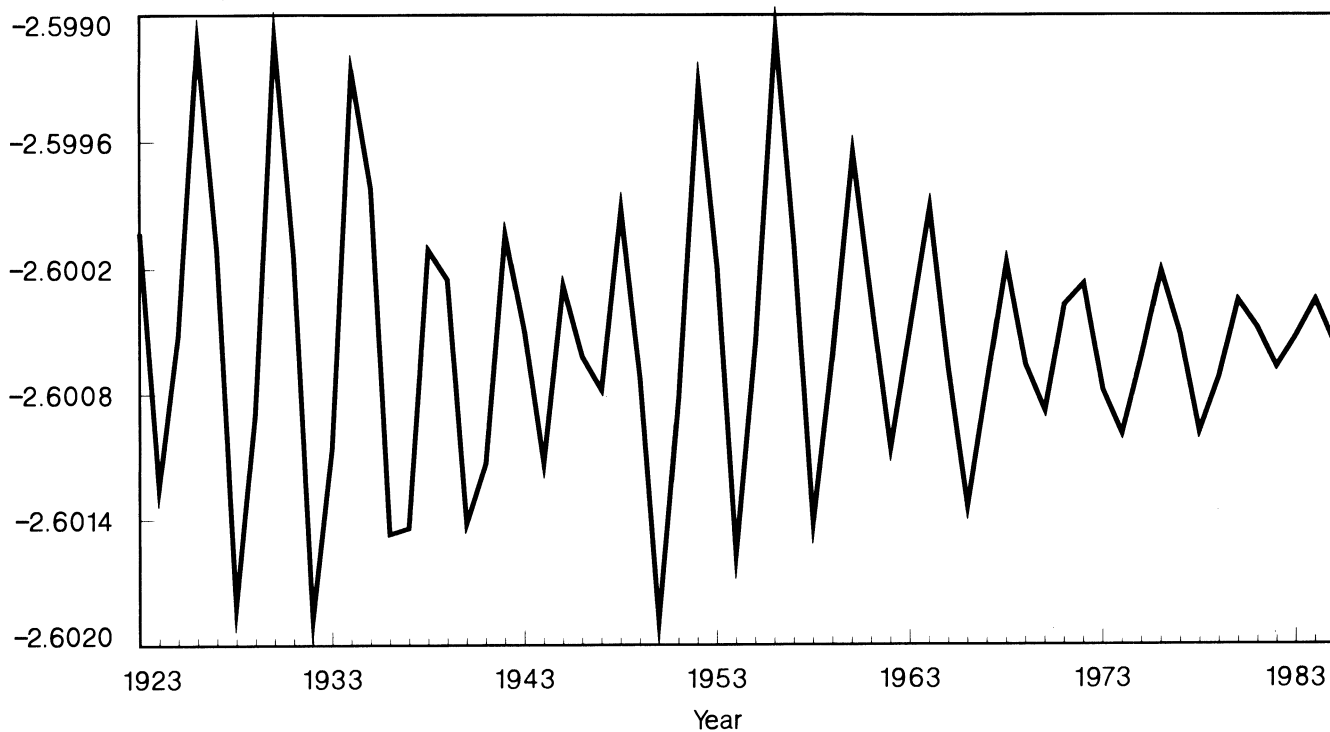


Figure 6

## Rental rate, 1923-85

Parameter value (thousands)

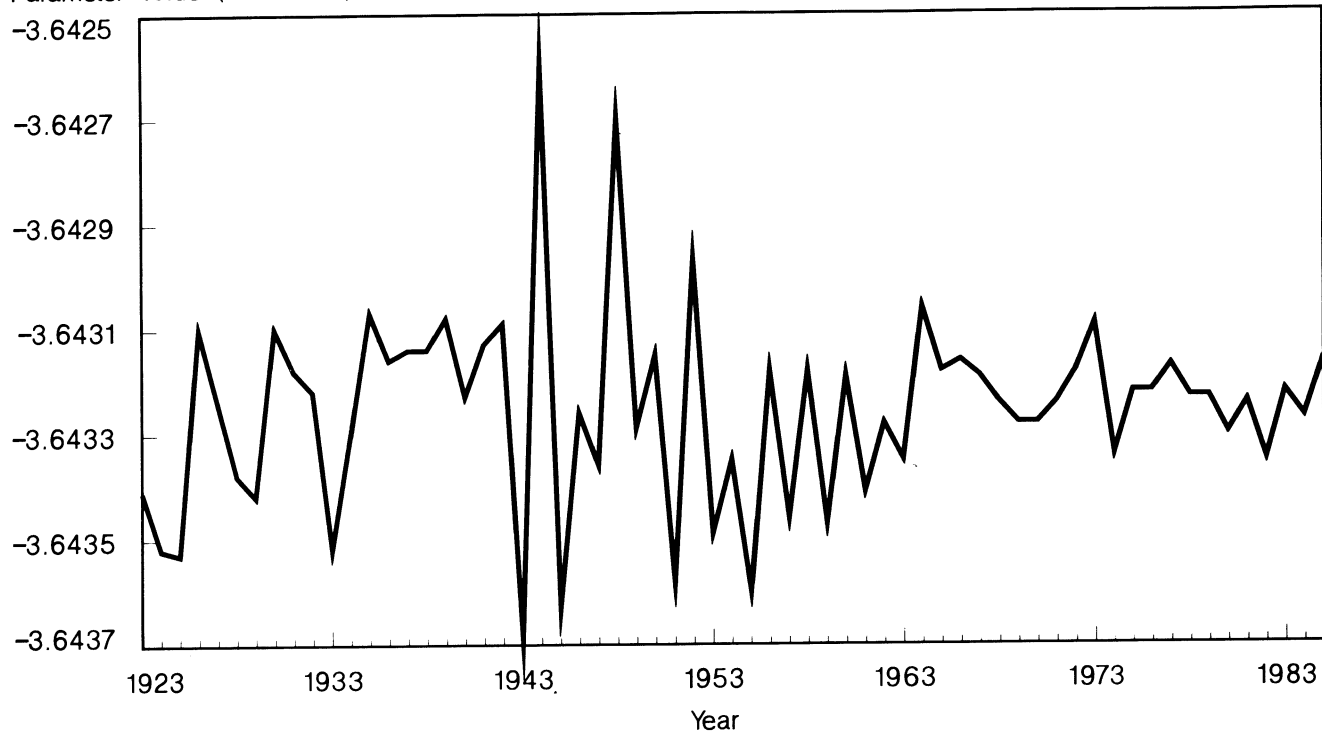
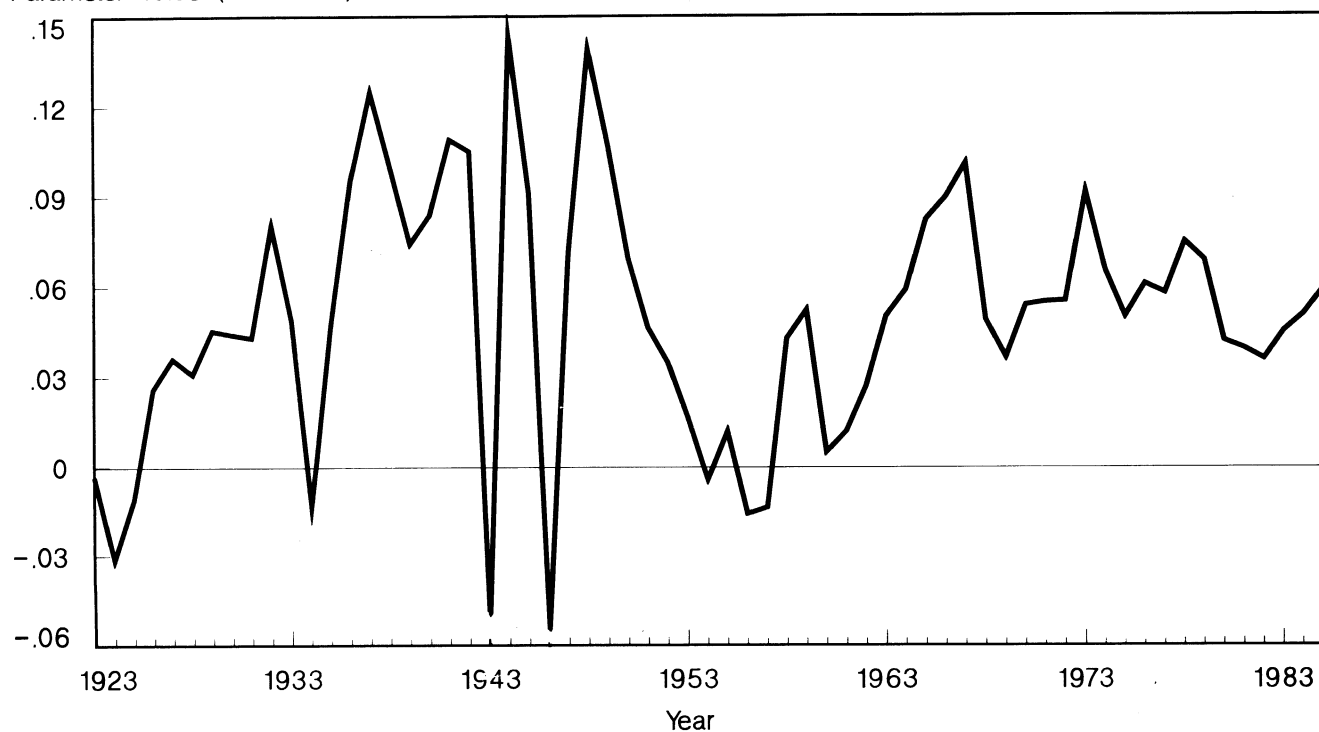


Figure 7

## Adjustment rate, 1923-85

Parameter value (thousands)



adjustment path toward an uncertain, continuously changing equilibrium level (14). The variation in the estimates of the adjustment coefficients supports earlier work by showing that those restrictions on the adjustment coefficients are inappropriate when the equilibrium value is uncertain.

The lack of extensive parameter variability somewhat supports fixed coefficient renderings of the flexible accelerator. Although the more general stochastic coefficients model allows for variation in all the parameters, only the adjustment variable shows appreciable variation. Therefore, fixed coefficient approaches, which allow the adjustment rate to vary in response to changes in economic variables, may not seriously distort the analysis with unduly restrictive assumptions. Nevertheless, the stochastic coefficients model still seems to be the superior forecast tool.

Table 2 presents the decomposition of normalized variance. The lagged capital stock variable has the highest proportion of the variance of agricultural investment. The input/output price variable has the second highest, followed by the constant and the rental rate of capital. This result suggests that attributing all the variance in the dependent variable to the constant term, implicit in a constant coefficients model, may be inappropriate. This relative ranking contrasts with the ranking based on asymptotic t-statistics, where the constant had the greatest influence on the dependent variable, followed by the input/output price ratio, rental rate, and lagged capital stock.

We cannot identify the specific causal factors inducing variability in the adjustment coefficient because there are many explanations for parameter variability. However, allowing for alternative sources of variation, including variation in the rental rates and input/output prices, is a major difference between the stochastic coefficients and flexible accelerator model.

Table 2--Average decomposition of normalized variance of  
agricultural investment, 1923-85

Item	:	Constant	Input/output price ratio	Rental rate	Lagged stock
Constant	:	0.2487E-04	0.25946E-04	0.51949E-06	-0.4177E-03
Input/output price ratio	:	.25946E-04	.31807E-04	.67838E-06	-.80709E-03
Rental rate	:	.51949E-06	.67838E-06	.9129E-08	.91211E-04
Lagged stock	:	-.4177E-03	-.80709E-03	.91211E-04	1.001
Net contribution	:	-.36637E-03	-.74866E-03	.91242E-04	.9981

## FORECAST EVALUATION

Because there is no statistical procedure to identify the "true" model, one cannot test whether the stochastic coefficients model is the correct specification.<sup>6/</sup> Instead, we developed a useful predictive tool. To identify the usefulness of our model, we compare its accuracy over five periods (1981-85) with out-of-sample forecasts from six other models. Our instrumentalist approach is consistent with Boland's view that predictive superiority is a sufficient condition for favoring one model over another (5). Although the six alternatives do not exhaust all possible models, thereby solving the problem of induction, they provide a basis for evaluating the predictive capability of the stochastic coefficients investment model.

One of the six models is the fixed coefficient analogue of the stochastic coefficients investment model. Net investment is regressed on a constant, an input/output price ratio, a rental rate, and lagged capital stock. Two other models are variants of the fixed coefficient model. One model includes net farm income (income) as a regressor, the other includes a time trend (time). A fourth model is a fixed coefficient, nonlinear flexible accelerator and takes the form given by equations 6 and 7, where  $K^*$  is a function of the ratio of input to output prices and the rental rate of capital. The final two models are atheoretical. Investment is assumed to be a stochastic process following both a first-order autoregressive (AR1) process and a second-order autoregressive (AR2) process.<sup>7/</sup>

Forecasting net investment with the Swamy-Tinsley stochastic coefficients model is a two-step process (29). Because  $\beta_t$  follows a stationary first-order vector autoregressive process, the time-varying component of  $\beta_t$  must be predicted for 1981-85. This information is then combined with the mean parameter,  $\bar{\beta}$ , to forecast net investment. Table 3 presents out-of-sample forecasts for the Swamy-Tinsley stochastic coefficients model and the six alternative models. Out-of-sample forecasts do not use information past 1980 to estimate or modify parameters for the stochastic or fixed coefficients models.

Table 3 shows that the stochastic coefficients model is the superior predictor. However, an unambiguous indicator of forecast accuracy does not exist. Each indicator has its own risk function. For example, a mean absolute error criterion is based on a linear loss function, while a mean square error criterion is based on a quadratic loss function. Therefore,

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<sup>6/</sup> Although Aristotelian principles may be applied to determine the validity of economic theory under certainty, the truth of its conclusions so constructed cannot be determined simply by inspecting empirical results. The problem of induction is impossible to solve and the uncertainty theories and approximations (commonly adopted in econometrics) violate Aristotle's axiom of the excluded middle. Econometricians cannot establish the truth or falsehood of economic theories unless they are logically inconsistent, in which case they are false. However, econometricians can develop sufficient and logically consistent theories and impose them on their empirical models. Care should be taken that the behavioral assumptions of economic theory and the statistical assumptions made do not contradict each other. See (28) for further discussion of this issue.

<sup>7/</sup> Other ARIMA models were identified, estimated, and used for forecasting. Their results were uniformly poor and thus excluded.

Table 3--Out-of-sample net investment forecasts, 1981-85

Year	Actual	Stochastic coefficient	Fixed coefficient	Income	Time	Flexible accelerator	AR1	AR2
<u>Billion 1972 dollars</u>								
1981	-993	-385	354	623	602	308	610	566
1982	-2,017	-1,169	321	-359	987	120	543	500
1983	-1,962	-1,369	343	-818	1,349	135	502	472
1984	-1,815	-1,359	449	614	1,764	157	478	460
1985	-2,104	-1,845	33	198	1,580	169	463	460

different analysts may prefer different models, depending on their assumed loss function. We suggest analysts consider a wide variety of forecast and other criteria, including goodness of fit and tracking measures.

Table 4 presents each model's forecast for 1981 through 1985. The forecast statistics, based on years with dramatic declines in agricultural investment, provide an excellent test of forecast accuracy. The evaluation statistics for each model are mean absolute error (MAE), mean absolute percentage error (MAPE), root mean square error (RMSE), and Theil's U2 coefficient. Table 4 shows that the stochastic coefficients model is the most accurate out-of-sample forecaster. The mean absolute error statistic (MAE) is representative of the stochastic coefficients dominance over its competitors. The nearest competitor, the flexible accelerator model, is more than three times greater in MAE. The absolute error shows the stochastic coefficients model dominates each year. After missing the actual value by a relatively wide margin in 1982 (\$849 million), the stochastic coefficients' forecast improves through 1985, where the absolute error is \$258 million. The stochastic coefficients model outperforms the other six models for nearly any sensible risk function.

The stochastic coefficients model is sensitive to additional information. The predictive ability of the model is enhanced as more current information is added. The 5-year out-of-sample forecast is compared with a 5-year rolling horizon forecast (fig. 8). The rolling horizon forecast is computed by sequentially estimating the parameters of the model with information from 1923 to year  $t$  and forecasting year  $t+1$ , where  $t$  runs from 1980 through 1984. Therefore, we compare forecasts from five data sets, with each containing more information than the previous year.

The rolling horizon more closely predicts actual net investment than the 5-year out-of-sample forecast. But recall that additional information is important. Consider, for example, the ex-post out-of-sample forecasts for 1982 (fig. 8). The 1982 rolling horizon forecast includes information through

Table 4--Forecast evaluation statistics 1/

Model	:	MAE	MAPE	RMSE	Theil's U2
	:				
Stochastic coefficient	:	533	34	1.89	0.89
Fixed coefficient	:	2,297	133	2.17	1.03
Fixed coefficient	:				
with income	:	2,269	131	2.17	1.02
Fixed coefficient	:				
with time	:	2,078	119	2.15	1.02
Flexible accelerator	:	1,829	109	2.13	1.00
AR1	:	3,034	170	2.24	1.06
AR2	:	1,956	112	2.14	1.01
	:				

1/ Mean value for net investment during 1981-85 is -\$1,778 million.

1981. The fixed horizon forecast contains information only through 1980. The additional information improves 1981 forecast accuracy by 45 percent. Over the 5-year horizon, the additional information leads to a 49-percent mean gain in forecast accuracy.

#### NET INVESTMENT EX-ANTE FORECAST

We use the stochastic coefficients model to forecast net investment in agricultural machinery for 1986-90. The forecast is particularly interesting because of the deflationary economic environment in the agricultural sector since 1980, where agricultural machinery capital stock eroded from a little over \$31 billion (1972 dollars) in 1980 to about \$24 billion in 1985.

Our 5-year forecast suggests net agricultural machinery investment will remain negative, further depleting the capital stock (fig. 9). The declines in net investment are forecast to be greater in the second half of the 1980's than in the first half, and even greater in 1990.

Continuing negative net investment is driven largely by agriculture's attempt to depreciate its capital stock because of a persistently weak farm economy. Our assumption regarding the time paths of the input/output price ratio and the various components of the rental rate are consistent with the notion of a continued weak farm economy (table 5). No available evidence suggests that our exogenous variable forecast assumptions should be reconsidered anytime soon. We are not likely to see any near-term increase in net investment unless output prices increase significantly relative to input prices. It takes time for even significant increases in output prices to alter the course of net investment because the capital structure of the agricultural sector adjusts slowly. There must be a 25-percent increase per year in output prices before positive net investment can be observed by 1990.

As with any forecast, this net investment forecast's accuracy is difficult to assess. There are several sources of forecast errors, including errors in the explanatory variables, regime changes, and specification error. Perhaps the most important source for errors is the regime change, or change in model

Figure 8

## Forecasted net investment for actual, fixed, and rolling horizons, 1981-85

Billion 1972 dollars

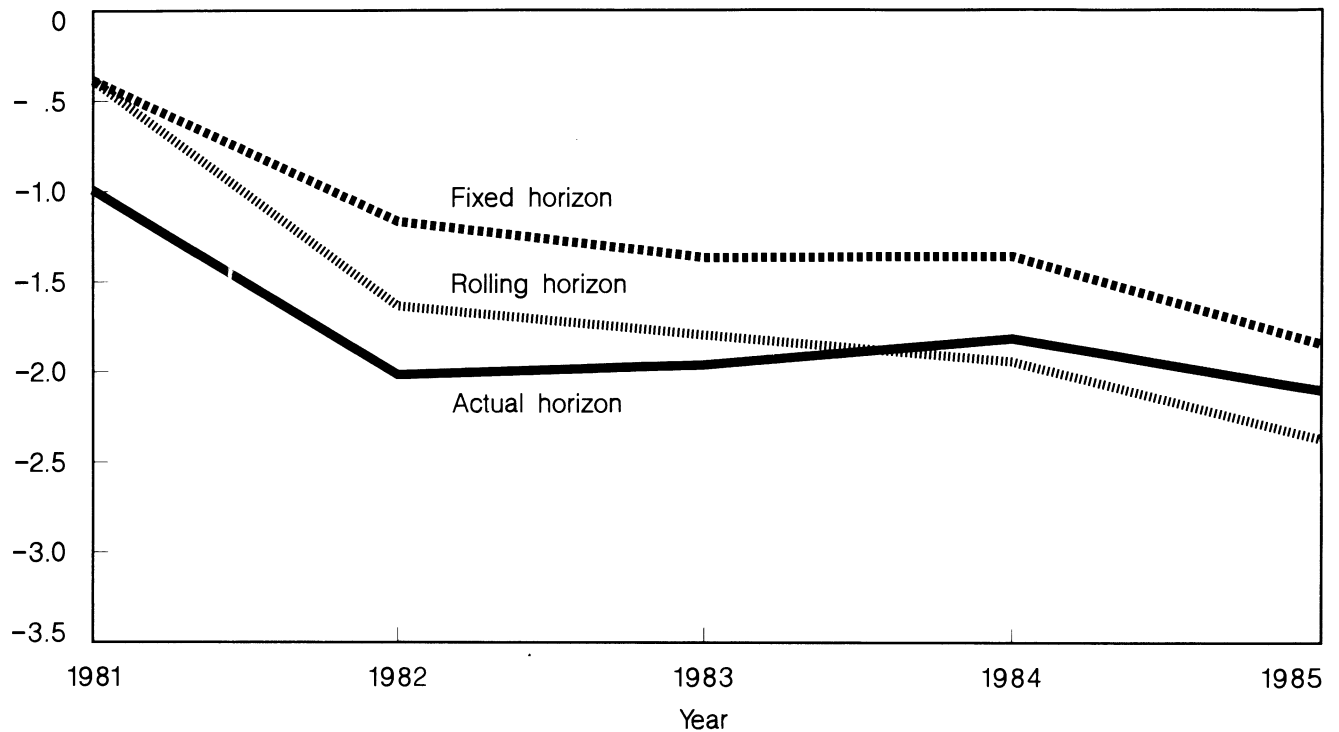


Figure 9

## Forecasted net investment, 1986-90

Billion 1972 dollars

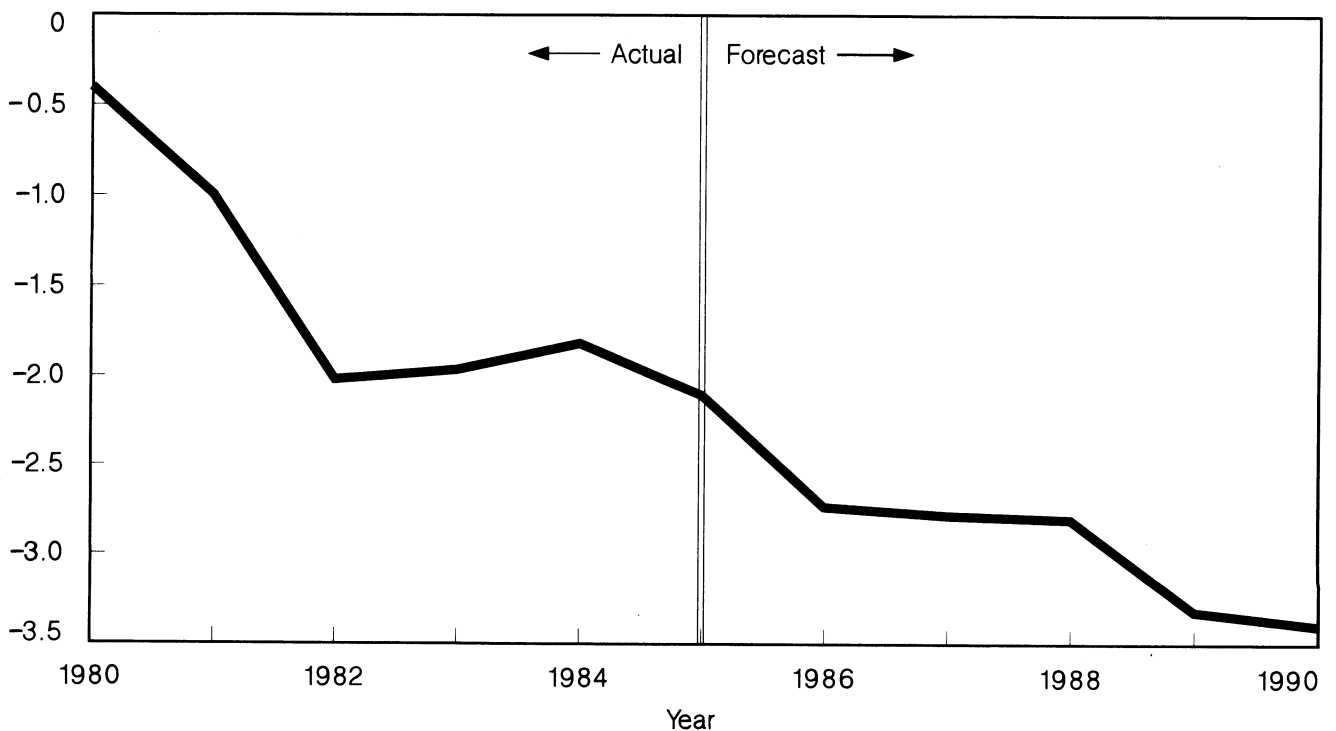


Table 5--Exogenous variable forecast assumptions, 1986-90

Year	Input/output price ratio	Capital rental rate	Interest rate	Inflation rate	Machinery price index
	<u>Ratio</u>	-----	<u>Percent</u>	-----	<u>Index</u>
1986	1.488	0.664	10.3	1.6	2.813
1987	1.471	.651	10.9	3.0	2.852
1988	1.471	.614	10.3	4.7	2.935
1989	1.478	.672	12.4	5.3	3.061
1990	1.482	.692	10.9	3.8	3.125

Sources: The input/output price ratios are from unpublished USDA estimates. The interest and inflation rates and the machinery price index are from a private econometric consulting firm's unpublished forecasts. The capital rental rates are computed using equation 32 and the interest rate and inflation rate forecasts.

structure. As Maddala said, "The accuracy of predictions will depend on the stability of the coefficients between the period used for estimation and the period used for prediction" (21). However, this model somewhat lessens this source of error because the stochastic coefficients approach attempts to capture structural change by allowing for parameter variation.

Caution is warranted, however, in placing too much confidence in the investment forecasts for 3-5 years ahead. Because the stochastic coefficients model is sensitive to additional information, forecasts for 1988-90 may change significantly as information for 1986 and 1987 is used to re-estimate the model. Five years is a long time to measure economic change: forecasts are less reliable as one moves further from what is known.

In addition, the model is estimated in a partial equilibrium framework. As the capital stock erodes, real output will at some point show declines, thus transmitting into higher output prices. This price feedback effect likely will occur before the capital stock reaches \$9 billion, the level predicted for 1990. A more complete modeling system and a greater understanding of the relationship between capital and output is needed before more confidence can be placed in agricultural net investment forecasts.

## CONCLUSIONS

An important question for manufacturers of agricultural machinery and policymakers is: how will the agricultural capital structure look in the future? We have attempted to answer this question by developing and applying a stochastic coefficients forecasting model of agricultural investment. The model's theoretical structure is based on Lucas' work leading to the development of the flexible accelerator. In a comparison with six alternative models, the stochastic coefficients model clearly outperforms its competitors in a 5-year out-of-sample forecast. Although not conclusive, the stochastic coefficients variant of a logically consistent theoretical framework is a useful forecasting tool.



Our forecast of net investment in agricultural machinery investment shows an acceleration of current trends. Net investment is forecasted to decline throughout the forecast period, with greater declines at the period's end, implying continued erosion of the capital stock. Any renewed near-term growth in machinery stocks is unlikely.

However, the forecast's reliability, particularly for the later years, may be questionable due to the model's sensitivity to additional information, uncertainty with respect to the exogenous variables, and the partial equilibrium model structure.

#### REFERENCES

- (1) Anderson, T.W. The Statistical Analysis of Time Series. NY: John Wiley and Sons, 1971.
- (2) Berndt, E., M. Fuss, and L. Waverman. Dynamic Models of the Industrial Demand for Energy. Palo Alto, CA: Electric Power Research Institute, 1978.
- (3) Berndt, E., C. Morrison, and G. Watkins. "Dynamic Models of Energy Demand: An Assessment and Comparison," Modeling and Measuring Natural Resource Substitution. E. Berndt and B. Field, eds. Cambridge, MA: Massachusetts Institute of Technology Press, 1981, pp. 259-89.
- (4) Bernanke, B., H. Bohn, and P. Reiss. "Alternative Nonnested Specification Tests of Time Series Investment Models." Technical working paper no. 49. National Bureau of Economic Research, June 1985.
- (5) Boland, L.A. "A Critique of Friedman's Critics," Journal of Economic Literature, Vol. 17 (1979), pp. 503-22.
- (6) Christensen, L., and D. Jorgenson. "The Measurement of U.S. Real Capital Input, 1929-1967," The Review of Income and Wealth, Series 15 (1969), pp. 293-320.
- (7) Duffy, W. "Parameter Variation in a Quarterly Model of the Post-War U.S. Economy." Unpublished Ph.D. dissertation. University of Pittsburgh, 1969.
- (10) Eisner, R., and R. Strotz. "Determinants of Business Investment," Impacts of Monetary Policy. Englewood Cliffs, NJ: Prentice-Hall, 1963.
- (11) Fletcher, R., and M. Powell. "A Rapidly Convergent Descent Method for Minimization," Computer Journal, Vol. 6 (1963), pp. 163-68.
- (12) Fuss, M., D. McFadden, and Y. Mundlak. "A Survey of Functional Forms in the Economic Analysis of Production," Production Economics: A Dual Approach to Theory and Applications, Vol. 1. M. Fuss and D. McFadden, eds. Amsterdam: North-Holland, 1978.
- (13) Gould, J. "Adjustment Costs in the Theory of Investment of the Firm," Review of Economic Studies, Vol. 35 (1968), pp. 47-55.

- (14) Griliches, Z. "Distributed Lags: A Survey," Econometrica, Vol. 35 (1967), pp. 16-49.
- (15) Havenner, A., and P.A.V.B. Swamy. "A Random Coefficient Approach to Seasonal Adjustment of Economic Time Series," Journal of Econometrics, Vol. 15 (1981), pp. 177-210.
- (16) Jorgenson, D. "Econometric Studies of Investment Behavior: A Survey," Journal of Economic Literature, Vol. 9 (1971), pp. 1,111-47.
- (17) Lamm, R. "Investment in Agriculture: An Empirical Analysis," Agricultural Finance Review, Vol. 42 (1982), pp. 16-23.
- (18) Lau, L. "Applications of Profit Functions," Production Economics: A Dual Approach to Theory and Applications, Vol. 1. M. Fuss and D. McFadden, eds. Amsterdam: North-Holland, 1978.
- (19) Lucas, R. "Econometric Policy Evaluation: A Critique," The Philips Curve and Labor Markets: Carnegie-Rochester Conference Series, Vol. 1. Karl Brunner and Allen Meltzer, eds. 1976. Supplement to Journal of Monetary Economics.
- (20) \_\_\_\_\_. "Optimal Investment Policy and Flexible Accelerator," International Economic Review, Vol. 8 (1967), pp. 78-85.
- (21) Maddala, G.S. Econometrics. NY: McGraw-Hill. 1977.
- (22) Nerlove, M. "Estimates of the Elasticities of Supply of Selected Agricultural Commodities," Journal of Farm Economics, Vol. 38 (1956), pp. 301-08.
- (23) \_\_\_\_\_. "Lags in Economic Behavior," Econometrica, Vol. 40 (1972), pp. 221-51.
- (24) Penson, J., R. Romain, and D. Hughes. "Net Investment in Farm Tractors: An Econometric Analysis," American Journal of Agricultural Economics, Vol. 63 (1981), pp. 629-35.
- (25) Rausser, G., Y. Mundlak, and S. Johnson. "Structural Change, Updating, and Forecasting," New Direction in Econometric Modeling and Forecasting in U.S. Agriculture. G.C. Rausser, ed. Amsterdam: North-Holland, 1983.
- (26) Resler, D.H., J.R. Barth, P.A.V.B. Swamy, and W.D. Davis. "Detecting and Estimating Changing Economic Relationships: The Case of Discount Window Borrowing," Applied Economics, Vol. 17 (1985) pp. 509-27.
- (27) Swamy, P., J. Barth, and P. Tinsley. "The Rational Expectations Approach to Economic Modeling," Journal of Economic Dynamics and Control, Vol. 4 (1982), pp. 125-47.
- (28) Swamy, P.A.V.B., R.K. Conway, and P. von zur Muehlen. "The Foundation of Econometrics--Are There Any?" Econometric Reviews, Vol. 4 (1985) pp. 1-61.

- (29) Swamy, P., and P.A. Tinsley. "Linear Prediction and Estimation Methods for Regression Models with Stationary Stochastic Coefficients," Journal of Econometrics, Vol. 12 (1980), pp. 103-42.
- (30) Swamy, P., P. Tinsley, and G. Moore. An Autopsy of a Conventional Macroeconomic Relation: The Case of Money Demand, Special Studies Report 167. Fed. Res. Brd. Apr. 1982.
- (31) Theil, H. Principles of Econometrics. NY: John Wiley and Sons, 1971.
- (32) Tideman, T., and D. Tucker. "The Tax Treatment of Business Profits Under Inflationary Conditions," Inflation and the Income Tax. Henry Aaron, ed. Washington, DC: The Brookings Institution, 1976.
- (33) Tinsley, P., P. Swamy, and B. Garrett. "The Anatomy of Uncertainty in a Money Market Model." Unpublished working paper. Fed. Res. Brd. 1981.
- (34) Treadway, A. "The Globally Optimal Flexible Accelerator," Journal of Economic Theory, Vol. 7 (1974), pp. 17-39.
- (35) \_\_\_\_\_. "The Rational Multivariate Flexible Accelerator," Econometrica, Vol. 39 (1971), pp. 845-56.
- (36) U.S. Department of Agriculture, Economic Research Service. Agricultural Finance: Outlook and Situation Report, AF0-26. March 1986.
- (37) \_\_\_\_\_. Economic Indicators of the Farm Sector: National Financial Summary. Annual issues.
- (38) \_\_\_\_\_. Financial Characteristics of U.S. Farms, January 1, 1986, A1B-500. Aug. 1986.
- (39) U.S. Department of Agriculture, Statistical Reporting Service. Agricultural Statistics. Annual issues.
- (40) U.S. Department of Commerce, Bureau of the Census. Bulletin F--Income, Tax, Depreciation and Obsolescence, Estimated Useful Lives, and Depreciation Rates. Rev., Jan. 1942.
- (41) \_\_\_\_\_. 1969 Census of Agriculture, 1970 Farm Finance Survey, Vol. 5. Special Reports, Part 2, 1974.
- (42) \_\_\_\_\_. 1978 Census of Agriculture, 1979 Farm Finance Survey, Vol. 5. Special Reports, Part 6, 1982.
- (43) U.S. Department of Commerce, Bureau of Economic Analysis. Fixed Reproducible Wealth in the United States, 1925-1979. Mar. 1982.
- (44) U.S. Department of Treasury, Internal Revenue Service. Business Income Tax Returns. Annual issues, 1957-80.
- (45) Wisley, T., and S. Johnson. "An Evaluation of Alternative Investment Hypotheses Using Nonnested Tests," Southern Economic Journal, Vol. 52 (1985), pp. 422-30.

- (46) Zellner, A. "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests of Aggregation Bias," Journal of the American Statistical Association, Vol. 62 (1962), pp. 348-68.
- (47) \_\_\_\_\_. "On the Aggregation Problem: A New Approach to a Troublesome Problem," Economic Models, Estimation, and Risk Programming: Essays in Honor of Gerhard Tinter. K. Fox, ed. NY: Springer-Verlag, 1969.

## APPENDIX

We developed implicit rental rates from the equality between the purchase price of the asset and the present value of the future rents generated by the asset (16). Assuming constant new asset price expectations and allowing for alternative depreciation patterns, the basic relationship is:

$$q_i = \int_0^{L_i} e^{-rt} u_i n_i(t) dt \quad (i=1,2,\dots,m) \quad (25)$$

where:  $q_i$  is the purchase price of the  $i$ th asset when new,  $L_i$  is the service life,  $u_i$  is the rental rate expressed in terms of an undepreciated unit of capital,  $n_i(t)$  is the capacity of the asset available in year  $t$  of its service life, and  $r$  is the discount rate.

Equation 25 ignores all tax considerations. When capital income is subject to an income tax, the term to the right of the equal sign in equation 25 is modified to include the effects of the tax. The modified term includes the present value of the rents generated by the asset, and the present value of the tax savings produced by the investment tax credit and the tax depreciation deductions. Assuming the firm's marginal tax rate remains constant as  $T$ , equation 25 is respecified to accommodate the tax system:

$$q_i = (1 - T)u_i N_i + \theta_i q_i + T(1 - h\theta_i) Z_i q_i \quad (i=1,2,\dots,m) \quad (26)$$

where:  $(1 - T)u_i N_i$  is the present value of the future rents,  $\theta_i q_i$  is the present value of the investment tax credit, and  $T(1 - h\theta_i) Z_i q_i$  is the present value of the future tax depreciation deductions.

If the price expectations and the marginal tax rate are constant, the rental rate remains constant over the life of the asset. The productive capacity of the asset, however, declines over the life of the asset so that:

$$N_i = \int_0^{L_i} e^{-rt} n_i(t) dt \quad (i=1,2,\dots,m) \quad (27)$$

where:  $r$  is the discount rate, the real after-tax rate of return required by the firm.

Although the firm pays taxes on the rents generated by each asset, the firm can deduct the decline in the value of the asset as an expense. The tax system does not distort the asset mix if the present value of depreciation deductions claimed for tax purposes equals the true decline in capacity for each asset.

If  $z_i(t)$  is a fraction of the price of the  $i$ th asset deducted from income in year  $t$  of the asset's tax life ( $M_i$ ), then the present value of tax depreciation is  $TZ_i q_i$ , where  $p$  is the rate of inflation, and:

$$Z_i = \int_0^{M_i} e^{-(r+p)t} z_i(t) dt \quad (i=1,2,\dots,m) \quad (28)$$

However, when the tax depreciation base declined by the amount of the investment tax credit, the real value of the tax depreciation deduction is  $T(1 - h\theta_1)Z_1q_1$ , where  $h$  is the percentage of the credit that reduces the depreciation base.

In addition to the depreciation deductions, firms may also be eligible to claim an investment tax credit. If firms claim the credit at the end of the first year of the asset's service life, the present value of the credit is  $\theta_1q_1$ , where:

$$\theta_1 = e^{-(r+p)}\theta_1 \quad (i=1,2,\dots,m) \quad (29)$$

The discount rate is more realistic when it is a weighted average of the longrun real after-tax interest rate (external financing) and the longrun real after-tax return to equity (internal financing). Because nominal interest charges are deductible from taxable income, the real cost of external debt-financing ( $r_d$ ) is:

$$r_d = [r_n(1 - T) - p]/(1 + p) \quad (30)$$

where:  $r_n$  is the nominal interest rate. After combining the real costs of equity and debt-financing, the real cost of the capital or real after-tax discount rate becomes:

$$r = f_{rd} + (1 - f)r_e \quad (31)$$

where:  $f$  is the fraction of debt financed,  $r_d$  is the real after-tax debt financed, and  $r_e$  is the real after-tax return to equity (32).

Given the market price of the asset, equation 25 becomes:

$$u_1 = q_1[1 - \theta_1 - T(1 - h\theta_1)Z_1]/N_1(1 - T) \quad (i=1,2,\dots,m) \quad (32)$$

which is the rental rate the firm must charge to earn the required real after-tax rate of return.







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