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The Inverse Lewbel Demand System

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A new model of consumer preferences is introduced. It is appropriate for modeling perishable commodities which are produced with a lag, where it is reasonable to assume the market-level quantities are fixed by previously made production decisions. The inverse Lewbel system, as it is called, is a flexible nonlinear system of share equations, which nests two other inverse demand systems, the direct translog and the inverse AIDS. Thus, the inverse Lewbel may be employed to test whether these more restrictive preference structures are appropriate. In an application to quarterly U.S. meat consumption, the more restrictive structures are rejected.

Key words: AIDS, demand, functional form, Lewbel, translog, U.S. meat consumption.

Introduction

Recently, interest in inverse demand systems, those which take prices as endogenous and quantities as predetermined, has been rekindled. Christensen, Jorgenson, and Lau developed the direct translog demand system (as well as the indirect system). Both they and Jorgenson and Lau used the direct translog demand system to test demand restrictions.[1] Heien, and Chambers and McConnell developed separable inverse demand systems and applied them to food commodities. Barten and Bettendorf developed an inverse Rotterdam system and applied it to the demand for fish. Huang (1988) used the theoretical development of Anderson and the distance function to generate a system of inverse demands, which was applied to composite food and nonfood commodities (see also Young; Huang 1990). Eales and Unnevehr (1991, 1993) and Moschini and Vissa employed a particular distance function to develop an inverse of the almost ideal demand system model.

Interest in such models stems from the existence of commodities for which the assumption of predetermined prices at the market level may not be viable. Early econometric work in agricultural demand took current supplies as fixed and specified ad hoc inverse demand curves for statistical evaluation. This alternative aggregation story is still employed, especially by those building market models, such as Freebairn and Rausser, and Arzac and Wilkinson. So, for example, if modeling demand for a perishable commodity, the production of which is subject to biological lags, the researcher might employ inverse demands. Production lags are assumed to prevent market-level supply response, while perishability requires the commodity be consumed. Thus, price must adjust.

One difficulty faced in any parametric analysis of demand is that of maintained hypotheses. That is, if the functional form employed is inappropriate, results will be biased by that choice. Such biases might show up in the measurement of elasticities (or flexibilities) or in apparent shifts in consumer preferences (Alston and Chalfant). Thus, it is of interest to have available functional forms that are as flexible as possible. The goal of the current study is to provide a new system of inverse demands, which nests two of the

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previously developed inverse demand systems. This will allow demand analysts to test whether consumer preferences may be modeled with either of the two more restrictive systems and minimize the dangers of biasing results.

In the next section, a new demand system is developed. It is called the inverse Lewbel demand system (ILDS). This is because it is similar to a "normal" demand system developed by Lewbel, which nests the indirect translog and almost ideal demand systems (Christensen, Jorgenson, and Lau; Deaton and Muellbauer 1980a, b). The ILDS nests the direct translog demand system (DTDS) of Christensen, Jorgenson, and Lau and the inverse almost ideal demand system (IAIDS) of Eales and Unnevehr (1991, 1993) and Moschini and Vissa. Share equations for the three demand systems are compared, as are the flexibilities derived from each. Since interpretation of these flexibilities is not as well understood as that of elasticities, it is also discussed. The third section presents and contrasts application of the three inverse demand systems to U.S. meat consumption. The final section summarizes results and offers some concluding observations.

The Inverse Lewbel Demand System

The inverse Lewbel demand system may be derived from the following direct, logarithmic utility function:

$$\ln(U) = \sum_i \beta_i \ln(q_i) + \ln[\ln(Q)], \tag{1}$$

where:

$$\ln(Q) = \alpha_0 + \sum_j \alpha_j \ln(q_j) + .5 \sum_j \sum_i \gamma_{ij} \ln(q_i)\ln(q_j). \tag{2}$$

While the share equation for commodity i, w_i, may be derived directly, using:

$$w_i = \frac{\partial \ln(U)/\partial \ln(q_i)}{\sum_j \partial \ln(U)/\partial \ln(q_j)}, \tag{3}$$

it is perhaps more revealing to employ the distance function corresponding to the utility function given in equation (1) (Deaton 1979). That is, the distance function, $d(U, q)$, is a function of the utility level, U, and the vector of quantities, q. It gives the amount by which all quantities must be divided to achieve utility level, U. It is implicitly defined as $u(q/d(U, q)) = U$. The logarithmic distance function, $\ln(d)$, corresponding to the logarithmic utility function, above, is derived by solving the implicit equation:

$$\begin{aligned}(4)\quad \ln(U) &= \sum_i \beta_i \ln\left(\frac{q_i}{d}\right) \\ &\quad + \ln\left\{\alpha_0 + \sum_i \alpha_i \ln\left(\frac{q_i}{d}\right) + .5 \sum_i \sum_j \gamma_{ij} \ln\left(\frac{q_i}{d}\right)\ln\left(\frac{q_j}{d}\right)\right\} \\ &= \sum_i \beta_i(\ln(q_i) - \ln(d)) \\ &\quad + \ln\left\{\alpha_0 + \sum_i \alpha_i(\ln(q_i) - \ln(d)) + .5 \sum_i \sum_j \gamma_{ij}[\ln(q_i) - \ln(d)][\ln(q_j) - \ln(d)]\right\}\end{aligned}$$

for $\ln(d)$. Exponentiating and collecting terms gives:

$$U = \prod_i q_i^{\beta_i} d^{(-\Sigma_i \beta_i)} \left\{\ln(Q) - \ln(d)\left(\sum_j \alpha_j + .5 \sum_i \sum_j \gamma_{ij}[\ln(q_i) + \ln(q_j) - \ln(d)]\right)\right\}, \tag{5}$$

or

$$\text{(6)} \qquad \ln(d) = \frac{\ln(Q) - U \prod_i q_i^{-\beta_i} d^{(\Sigma_i \beta_i)}}{\sum_j \alpha_j + .5 \sum_j \sum_k \gamma_{jk}[\ln(q_j) + \ln(q_k)] - .5\ln(d) \sum_j \sum_k \gamma_{jk}}.$$

Without restrictions, a closed-form solution for the log distance function in equation (6) is not clear. One set of restrictions which makes the solution obvious is:

$$\text{(7)} \qquad \sum_j \alpha_j = 1, \quad \sum_j \beta_j = 0, \quad \sum_i \sum_j \gamma_{ij} = 0, \quad \text{and} \quad \gamma_{ij} = \gamma_{ji} \ \forall \ i \neq j.$$

Imposing these restrictions on equation (6) yields the following form for the log distance function:

$$\text{(8)} \qquad \ln[d(U, q)] = \frac{\ln(Q) - U \prod_i q_i^{-\beta_i}}{1 + \sum_j \sum_k \gamma_{jk}\ln(q_k)}.$$

Compensated inverse demands are derived by differentiation:

$$\text{(9)} \qquad w_i = \frac{\partial \ln(d)}{\partial \ln(q_i)}$$

$$= \frac{\alpha_i + \sum_j \gamma_{ij}\ln(q_j) + \beta_i U \prod_k q_k^{-\beta_k}}{1 + \sum_i \sum_j \gamma_{ij}\ln(q_j)} - \frac{\sum_i \gamma_{ij}(\ln(Q) - U \prod_k q_k^{-\beta_k})}{(1 + \sum_i \sum_j \gamma_{ij}\ln(q_j))^2}.$$

Since, at the optimum, $\ln[d(U, q)] = 0$, the last term vanishes. Substituting for the unobservable utility from equation (1) in the first term results in:

$$\text{(10)} \qquad w_i = \frac{\alpha_i + \sum_j \gamma_{ij}\ln(q_j) + \beta_i\ln(Q)}{1 + \sum_i \sum_j \gamma_{ij}\ln(q_j)}.$$

As indicated above, both the DTDS and IAIDS models are nested within the ILDS. The restrictions on the ILDS to obtain the two more restrictive models depend only on unknown coefficients, and so apply globally. The DTDS is obtained from the ILDS if $\beta_i = 0 \ \forall \ i$. On the other hand, the IAIDS results if $\Sigma_j \gamma_{ij} = 0 \ \forall \ i$. Such restrictions may be tested directly using either a Wald or likelihood ratio test. A comparison of the share equation for the three demand systems is given in table 1.

Interpretation of results for inverse demand models is not as widely agreed upon as that of "normal" demand models. Anderson clarified the issue to a great extent by showing that the appropriate counterpart of the expenditure elasticity is what will be called the scale flexibility.[2] It can be characterized as the percentage change in the marginal value of good i as the scale of consumption is expanded by one percent. Reference for under-

Table 1. Comparison of Share Equations

ILDS:

$$w_i = \frac{\beta_i \ln(Q) + \alpha_i + \sum_k \gamma_{ik} \ln(q_k)}{1 + \sum_j \sum_k \gamma_{jk} \ln(q_k)},$$

where $\ln(Q) = \alpha_0 + \sum_j \alpha_j \ln(q_j) + .5 \sum_j \sum_k \gamma_{ik} \ln(q_j) \ln(q_k)$.

If $\sum_j \gamma_{ij} = 0 \ \forall \ i$, then ILDS reduces to IAIDS:

$$w_i = \beta_i \ln(Q) + \alpha_i + \sum_k \gamma_{ik} \ln(q_k).$$

If $\beta_i = 0 \ \forall \ i$, then ILDS reduces to DTDS:

$$w_i = \frac{\alpha_i + \sum_k \gamma_{ik} \ln(q_k)}{1 + \sum_j \sum_k \gamma_{jk} \ln(q_k)}.$$

Notes: The w_is are expenditure shares and α's, β's, and γ's are parameters.

standing scale flexibilities is established by realizing that if preferences are homothetic, all scale flexibilities are -1 (Eales and Unnevehr 1991). Necessities have scale flexibilities which are less than -1 and luxuries have scale flexibilities which are greater than -1. A comparison of the share equations and the formulae for price and scale flexibilities from the three models are given in table 2.

Quarterly U.S. Meat Demand

The three demand models are applied to retail demand for meat in the United States. Assuming that beef, pork, and chicken are separable from other consumption goods, a conditional demand system for these three meats is specified.[3] Data employed in this exercise are similar to those employed by Moschini and Meilke, except that their fish category is omitted, and to those employed by Eales and Unnevehr (1991). It consists of 108 quarterly observations on per capita retail consumption of beef, pork, and chicken and retail prices from 1966-Q1 through 1992-Q4. Data are from U.S. Department of Agriculture sources and are available on request from the author.

Estimation was done using Version 7 of the SHAZAM program (White), which employs a Davidon–Fletcher–Powell algorithm for estimation of the nonlinear sets of share equations for each demand model given in table 1. Initial estimation efforts concentrated on the ILDS model. An equation is omitted during estimation, due to singularity of the covariance matrix for a system of shares, and the estimates are invariant to the equation omitted (Barten). Coefficients of the omitted equation can be recovered using the demand restrictions or, as is done below, estimated directly by omitting an alternative equation.[4]

Initial estimates indicated three difficulties with ILDS as specified in equation (10). First, the parameter, α_0, proved impossible to estimate. The iterative algorithm would not converge. In their original article on the AIDS model, Deaton and Muellbauer (1980a) suggest that estimation of this parameter may be "problematic." Deaton (1986, p. 1784) is even stronger.[5] Second, there is strong seasonality in meat consumption. Beef and chicken demands tend to be strongest in the second and third quarters, demand for pork in the first and fourth quarters. To incorporate this into equation (10), the α_is were augmented with three seasonal dummy variables for the second, third, and fourth quarters of the year. Note that, if this is done in $\ln(Q)$ [equation (2)], the model derived will be similar to equation (10), but with the seasonal dummies appearing as intercept shifters in w_i [equation (10)] and as slope shifters in $\ln(Q)$ [equation (2)].[6] The third difficulty

Table 2. Comparison of Price and Scale Flexibilities

ILDS:

$$f_{ij} = -\delta_{ij} + \frac{\gamma_{ij} + \beta_i\left(\alpha_j + \sum_k \gamma_{jk}\ln(q_k)\right) - w_i \sum_j \gamma_{ij}}{w_i\left(1 + \sum_j \sum_k \gamma_{jk}\ln(q_k)\right)}$$

$$f_i = -1 + \frac{\beta_i}{w_i} + \frac{\sum_j \gamma_{ij}}{w_i\left(1 + \sum_j \sum_k \gamma_{jk}\ln(q_k)\right)}.$$

If $\sum_j \gamma_{ij} = 0 \ \forall \ i$, then ILDS reduces to IAIDS:

$$f_{ij} = -\delta_{ij} + \frac{\gamma_{ij} + \beta_i\left(\alpha_j + \sum_k \gamma_{jk}\ln(q_k)\right)}{w_i}$$

$$f_i = -1 + \frac{\beta_i}{w_i}.$$

If $\beta_i = 0 \ \forall \ i$, then ILDS reduces to DTDS:

$$f_{ij} = -\delta_{ij} + \frac{\gamma_{ij} - w_i \sum_j \gamma_{ij}}{w_i\left(1 + \sum_j \sum_k \gamma_{jk}\ln(q_k)\right)}$$

$$f_i = -1 + \frac{\sum_j \gamma_{ij}}{w_i\left(1 + \sum_j \sum_k \gamma_{jk}\ln(q_k)\right)}.$$

Notes: The f_{ij}s are own- and cross-price flexibilities; f_is are scale flexibilities; w_is are expenditure shares; δ_{ij} is the Kronecker delta; and α's, β's, and γ's are parameters.

encountered was autocorrelation of the estimated residuals. This suggested the estimation ought to include an autocorrelation correction. The correction employed here uses the same autocorrelation coefficient for all equations, to ensure the system still adds up (Berndt and Savin). SHAZAM employs an approach to such problems due to Pagan.[7]

A final estimation issue is that of structural change. A number of previous studies of U.S. meat demand have found significant shifts in consumer preferences for meats, e.g., Chavas; Dahlgran; Thurman; and Eales and Unnevehr (1988). The most recent study was done by Moschini and Meilke. They employed a linearized version of the almost ideal demand system in a gradual switching regression framework, searching over all possible beginning and ending points for the shifts. They found that the shifts might have happened in a variety of ways, but the pattern which maximized the likelihood started in 1975-Q4 and ended in 1976-Q3. In further testing of this pattern, they found the shifts affected the intercepts and quarterly shift dummies, but not the price or expenditure coefficients. Due to the nonlinear nature of the ILDS, the extensive search carried out by Moschini and Meilke was not attempted here. Instead, the most likely pattern they found, i.e., an abrupt change from 1975-Q4 to 1976-Q3 affecting only the intercepts and quarterly shift dummies, was estimated. A test of this structural change pattern was not significant. The failure to find significant structural change may have been due to different data samples or functional forms, assumptions of endogenous prices or quantities, and/or exclusion of the fish category, among others. Whatever the cause, structural change is ignored in what follows.

Table 3. Comparison of Demand Models for U.S. Meats

	BFQ	*PKQ*	*CKQ*	*Q*	CONST	*Q2*	*Q3*	*Q4*	R^2/DW
Inverse Lewbel:									
Beef	.383*	−.163*	.054	−.234*	1.041*	.006*	.001	−.009*	.964
	(.084)	(.067)	(.048)	(.106)	(.151)	(.001)	(.002)	(.001)	1.868
Pork	−.163*	−.369*	.082	.612*	−.337*	−.010*	−.007*	.009*	.940
	(.067)	(.074)	(.067)	(.048)	(.114)	(.001)	(.001)	(.001)	1.772
Chicken	.054	.082	.040	−.249*	.297*	.003*	.006*	.000	.967
	(.048)	(.067)	(.025)	(.094)	(.136)	(.001)	(.001)	(.001)	1.757
Inverse AIDS:									
Beef	.096*	−.057*	−.039*	.080*	.671*	.007*	.001	−.009*	.963
	(.017)	(.010)	(.010)	(.031)	(.032)	(.001)	(.002)	(.001)	1.826
Pork	−.057*	.076*	−.018*	−.006	.273*	−.010*	−.007*	.010*	.935
	(.010)	(.011)	(.009)	(.024)	(.025)	(.001)	(.002)	(.001)	1.759
Chicken	−.039*	−.018*	.057*	−.074*	.056*	.003*	.006*	.000	.967
	(.013)	(.009)	(.014)	(.026)	(.027)	(.001)	(.001)	(.001)	1.767
Direct Translog:									
Beef	.207*	−.039*	−.079*		.800*	.007*	.001	−.009*	.963
	(.033)	(.014)	(.014)		(.069)	(.001)	(.002)	(.001)	1.823
Pork	−.039*	.072*	−.044*		.253*	−.010*	−.007*	.010*	.935
	(.014)	(.016)	(.010)		(.057)	(.001)	(.002)	(.001)	1.723
Chicken	−.079*	−.044*	.045*		−.054	.003*	.006*	.000	.967
	(.014)	(.010)	(.014)		(.060)	(.001)	(.001)	(.001)	1.763

Notes: Asymptotic standard errors are in parentheses. All three systems were corrected for first-order autocorrelation, using one ρ for all equations in each demand system (Berndt and Savin). Estimates of ρ were: ILDS, .905 (.032); IAIDS, .888 (.031); and DTDS, .882 (.032). Standard errors are obtained for all coefficients by re-estimating with an alternative equation omitted. Quantity vectors were normalized to unit length prior to estimation and all models are estimated with homogeneity and symmetry imposed.
* Indicates the coefficient exceeds twice its standard error.

Results for the ILDS model are given in the top third of table 3. Estimates incorporate autocorrelation as suggested by Berndt and Savin and restrictions required by theory. The share equations fit well and show no evidence of continued difficulties with autocorrelation. The intercept and dummy variables are all significant, except for the third-quarter dummy for beef and the fourth-quarter dummy for chicken. The own-quantity effects for beef and pork, cross-quantity effects between beef and pork, and the q-indices are significant. As noted earlier, both the DTDS and IAIDS are nested within the ILDS. Wald tests of the parameter restrictions associated with DTDS and IAIDS were 37.33 and 33.84, respectively. Each is distributed asymptotically as a chi-square random variable with two degrees of freedom. Thus, from a statistical point of view, neither the DTDS nor the IAIDS is appropriate for modeling U.S. meat demands using quarterly data. Even though the more restrictive demand systems were rejected, both the IAIDS and DTDS models were estimated for comparison. Results are given in the bottom two-thirds of table 3. The results for the constants and quarterly dummies are similar for all three specifications. Effects of imposing the IAIDS and DTDS restrictions are seen in the estimates of the quantity effects. The ILDS model is considerably more sensitive to quantity changes in either beef or pork than either of the other more restrictive models. However, the IAIDS and DTDS models show more significance of the quantity effects, because imposition of either set of restrictions dramatically improves the efficiency of the estimators (standard errors of estimated quantity effects in the ILDS model are as much as six times the size of those in the other two models).

Coefficients from share equation models such as those in table 3 are difficult to interpret. Therefore, the flexibility formulae in table 2 are evaluated at the sample means of the data for all three demand specifications. Findings are presented in table 4. The results are qualitatively similar for the three models, even though the two more restrictive models were rejected. All meats are more own-price flexible than would be expected based on

Table 4. Comparison of Price and Scale Flexibilities

	Beef	Pork	Chicken	Scale
Inverse Lewbel:				
Beef	−.752*	−.062	−.046	−.860*
	(.033)	(.144)	(.066)	(.051)
Pork	−.238	−.730	−.073	−1.041*
	(.233)	(.382)	(.126)	(.081)
Chicken	−.622	−.308	−.626	−1.556
	(.437)	(.432)	(.353)	(1.636)
Inverse AIDS:				
Beef	−.746*	−.060*	−.058*	−.864*
	(.033)	(.013)	(.012)	(.053)
Pork	−.220*	−.737*	−.067*	−1.023*
	(.058)	(.023)	(.010)	(.086)
Chicken	−.693*	−.303*	−.584*	−1.579*
	(.152)	(.071)	(.099)	(.196)
Direct Translog:				
Beef	−.739*	−.030*	−.081*	−.850*
	(.032)	(.014)	(.029)	(.049)
Pork	−.227*	−.988*	.087*	−1.129*
	(.029)	(.012)	(.030)	(.049)
Chicken	−.706*	.112*	−.816*	−1.409*
	(.028)	(.014)	(.265)	(.243)

Notes: Flexibilities are calculated for each model using the formulae in table 2 at the sample means. Standard errors are given in parentheses and are calculated as in Mood, Graybill, and Boes (p. 181).
* Indicates the flexibility exceeds twice its standard error.

previous work with "normal" demand systems.[8] Chicken, in particular, is very flexible. All meats are gross q-substitutes (negative cross-price flexibilities) for the ILDS and IAIDS, while pork and chicken show some complementarity in the DTDS estimates (Hicks). There are differences, however. The most obvious difference is that the ILDS flexibilities are considerably more variable than those of the IAIDS or DTDS. Some of the standard errors are larger by a factor of 10 or more. Flexibilities vary by as little as .5% between the ILDS and IAIDS estimates of beef scale flexibility to as much as a 230% difference in the pork-chicken cross-price flexibility between the ILDS and DTDS estimates. Thus, the evidence from both the coefficients and the flexibilities shows that imposition of the IAIDS or DTDS restrictions produces results which give a much less variable picture of consumer preferences for meats than are obtained from the more flexible ILDS.

Conclusions

A new model of consumer preferences at the market level is introduced. It is similar to a demand system proposed by Lewbel, except the share equations are derived from the primal specification. That is, starting from a utility or distance function, the inverse Lewbel demand system (ILDS) is derived. Demands relate expenditure shares to quantities and an inverse AIDS quantity index (Eales and Unnevehr 1991). The ILDS model nests both the direct translog demand system (DTDS) and inverse almost ideal demand system (IAIDS). This makes it possible to test whether either set of restrictions corresponding to the DTDS or IAIDS is consistent with the data at hand. The derived model is appropriate when one can assume that at the market level, quantities are fixed and that prices adjust so that the fixed quantities are consumed. One would expect the ILDS (or DTDS, or IAIDS) to be appropriate when modeling perishable commodities where supply response lags for either biological or other reasons.

In terms of relative advantages among the three demand systems, the DTDS does not require the researcher to cope with α_0 in the IAIDS quantity index. It does, however, require nonlinear estimation. The IAIDS model has been shown to be well approximated, in practice, by substitution of a Stone's quantity index for the IAIDS index (Eales and Unnevehr 1991). This avoids the necessity of dealing with both α_0 and nonlinear estimation. The substitution of the Stone's quantity index could be employed for estimation of the ILDS model as well. However, the remaining model, while simplified, is still nonlinear. The ILDS does allow for increased flexibility over either the DTDS or the IAIDS, in that it nests them both. Restrictions required for homogeneity, symmetry, DTDS, or IAIDS depend only on unknown parameters and may be tested or imposed.

As an example, all three demand systems are applied to the problem of modeling quarterly U.S. meat consumption. All three models fit well and give qualitatively similar characterizations of U.S. preferences for meats. Wald tests of the ILDS results suggested both the DTDS and the IAIDS models are rejected. The coefficients of the ILDS are more variable than those of the more restrictive models. Practical differences between results of the three specifications were examined through the flexibilities implied by each. For both the ILDS and IAIDS, all meats are gross q-substitutes, as one would expect. Pork and chicken are gross q-complements for the DTDS. For all three specifications, meats are more own-price flexible than would be suggested by previous studies, which assumed prices and expenditures were predetermined. The flexibilities of the IAIDS are closer to those of the ILDS than are those of the DTDS.

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Notes

[1] Apologies are made to the reader for the diversity of terms employed in this article when describing demand systems. However, it is dictated by developments in the demand literature. The "direct" and "indirect" translogs of Christensen, Jorgenson, and Lau are so called depending on whether the derivation began with the direct or indirect utility function. "Normal" is employed in this study to denote demands which take quantities as endogenous. "Inverse" is employed when prices are assumed endogenous and is adopted to reflect usage in the literature.

[2] Anderson actually calls it a scale elasticity. In keeping with agricultural economics literature, it and the "quantity elasticities" will be called scale and price flexibilities, respectively.

[3] Implicitly, it is assumed that within a calendar quarter, the quantities of beef, pork, and chicken are fixed, due to production lags. Because meats are perishable, prices adjust so that the available quantities are consumed. Note that if this is so, then all the right-hand-side variables in equation (10) are predetermined. This allows applied researchers to avoid the problem of "normal" conditional demand systems raised by LaFrance, i.e., even if prices are predetermined in such systems, expenditures cannot be. It begs the empirical question, however, of whether either prices or quantities can be taken as predetermined in analysis of quarterly U.S. meat demand. To answer this question would require the development of appropriately specified supply models for beef, pork, and chicken or a list of believable instruments to be used in nonlinear 3SLS estimation and compared to the nonlinear SUR using a Wu–Hausman type test. Autocorrelation in the demand residuals, noted below, would complicate the comparison. While an interesting issue, the present purpose is merely to introduce the inverse Lewbel demand system. Thus, nonlinear SUR is employed to estimate all demand models.

[4] This approach has the added benefit of providing more evidence that the nonlinear estimator has converged to the global minimum of the sum of squares. That is, what had originally appeared to be a global minimum when the chicken share equation was dropped, turned out to be a local minimum when the pork share was dropped. Another approach employed to verify that the optimum was indeed a global minimum was to use a uniform random number generator to generate starting values between -1 and 1, since it seemed likely that the coefficients would lie in this range, and then to re-estimate the model 100 times. Several inferior minima were found, but none better than those reported.

[5] Deaton and Muellbauer (1980b) suggest determining α_0 (for the AIDS model), *a priori,* by noting it is the "outlay required for a minimal standard of living" in the base year, when all prices are one (p. 316). Examination of equation (1) shows that α_0 scales the conditional utility function for meat (i.e., in a base year, when all quantities are one, utility is equal to α_0). An alternative employed here is to search over α_0 values, in the range from 0 to 2 by .01. The value which produced the highest likelihood was 1.35. Subsequently, α_0 is fixed at 1.35 and estimates of other parameters are conditional on that value. The sensitivity of the other parameter estimates to values of α_0 in this range was small. Its value had some impact on the other α's, but little on the quarterly dummies, γ's, or β's.

[6] To account for seasonality of demand for meats, the α_js in equation (2) are augmented with three seasonal

dummy variables ($D_k = 2, 3, 4$), whose associated coefficients must sum to zero over i for adding up. This results in:

$$w_i = \frac{\beta_i \ln(Q^s) + \alpha_i + \sum_k \theta_{ik} D_k + \sum_k \gamma_{ik} \ln(q_k)}{1 + \sum_j \sum_k \gamma_{jk} \ln(q_k)}, \tag{10'}$$

with $\ln(Q^s)$ given by:

$$\ln(Q^s) = \alpha_0 + \sum_j \left(\alpha_j + \sum_k \theta_{jk} D_k\right) \ln(q_j) + .5 \sum_i \sum_j \gamma_{ij} \ln(q_i) \ln(q_j). \tag{2'}$$

[7] Of course, the cause of the meat demand dynamics is another issue of some interest. An extension of the error-correction model of Anderson and Blundell would be a natural alternative to autocorrelation correction employed below. This topic likely would be a fruitful one for future research efforts.

[8] To be consistent with the notions of elastic and inelastic as applied to "normal" demand systems, an inverse demand is said to be flexible if its own-price flexibility is between 0 and −1, and inflexible if it is less than −1.

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