MARKETING RESEARCH REPORT NO. 867

A Location-Logistics System for Feed Firm Management

ECONOMIC RESEARCH SERVICE
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PREFACE

This study is part of the U.S. Department of Agriculture's broad program of economic research directed toward expanding market outlets and increasing efficiency in marketing farm products. The farmer has a double interest in the feed industry's efficiency since he produces the feed ingredients and also purchases the finished product.

This is the second phase of a research program for developing techniques useful to management in making decisions. A location-logistics system has been developed for determining the most efficient plant locations while considering the interrelationships among procurement, manufacturing, and distribution.

The first report, MRR-729, "Managerial Aspects of Least-Cost Feed Formulation with Linear Programming," was an appraisal of linear programming in feed formulation and the impact of frequent formula changes on ingredient usage rates.

This research is a cooperative project of the Purdue Agricultural Experiment Station and the Economic Research Service of the U.S. Department of Agriculture.

CONTENTS

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SUMMARY

This study shows that a location-logistics system for feed firm management is feasible. The potential improvements in profits as a result of using the location-logistics system in the analysis more than justified the time required for developing and using the system. The analysis revealed that with properly located plant facilities, the most desirable procurement, processing, and distribution combination could produce 8 to 10 percent higher returns than the poorest combination.

Such a system can be used by feed manufacturers not only in making long-range location decisions but also as an operational control system. It is an application of the systems approach designed to aid management in identifying the relevant decision variables, evaluating the decision alternatives, and determining an optimum course of action. The system is used to analyze the major variables affecting the procurement, processing, and distribution functions of the firm.

A major feed manufacturing firm in the Midwest cooperated in the development and evaluation of the system. To construct the various components of the system, actual data from the case firm's records for 1967 were used along with secondary data.

While the comprehensive nature of the system and size of the problem does require simplification in some areas, the simplifications made do not significantly impair the operational effectiveness of the system.

The techniques of density analysis, linear programming, regression analysis, quadratic programming, and discounted cash flow analysis were integrated in the location-logistics system in such a manner as to be very useful in analyzing the decision variables on which location-distribution decisions are based.

General applicability of the system to other feed manufacturers is both feasible and practical. Other firms considering its use would only need to incorporate input data relevant to their operations. The mathematical design of the system is consistent with current structure of firms within the feed industry.

The major components of this system are: (1) A management review subsystem which uses management analysis and a grid coordinate market density analysis to select potential plant sites, (2) A location-distribution analyzer which uses linear programming to evaluate alternative locations, (3) A plant selection subsystem which uses quadratic programming to evaluate alternative sizes and types of plants to be built at selected sites, and (4) An investment feasibility analyzer which uses present value and discounted cash flow analysis to determine the rate of return on investment for alternative configurations.

Output from the management review subsystem becomes input to the location distribution analyzer. Output from the location distribution analyzer becomes input to the plant selection subsystem. Output from the above three subsystems becomes input to the investment feasibility analyzer, thus making it possible to evaluate the return on investment for each proposed configuration.

A LOCATION-LOGISTICS SYSTEM FOR FEED FIRM MANAGEMENT

By Robert E. Lee and James C. Snyder 1/

INTRODUCTION

The major operating functions of feed manufacturing firms include procurement, manufacturing, and distribution. Too often, feed manufacturers have emphasized efficiency of production at the expense of procurement and distribution. As a result, firms have not effectively served the market and have not optimized profits. Historically, the study of distribution costs has been directed at the level and trend of these costs. Questions concerning the dispersion of warehouse facilities assumed that the location of the producing facility was fixed and that warehouses should be located to minimize costs of serving the customer from the existing plant(s). The question of whether the producing facility is optimally located with respect to procurement and distribution was rarely posed. This question, however, is pertinent to location analysis because location can markedly alter the expected efficiency of a feed firm's total operation. A location-logistics system (LLS) was developed to determine the most efficient plant locations while considering the interrelationships among procurement, manufacturing, and distribution.

In an effort to design a practical system useful to feed industry management, a Midwest-based firm cooperated in the development and testing of the system. It was recognized that one must be cautions in drawing general inferences from case firm studies. The system rather than the case firm results is emphasized because the system can be adopted by most firms in the industry. 2/ At the time this research was conducted, the cooperating firm was manufacturing feeds in three production facilities which were operating at capacity. For this reason, management was faced with a decision of whether to expand production capacity within existing facilities or to build new, and close old, facilities. It was assumed that this expansion should come from building new plants rather than from adding to old, inefficient facilities. When constructing new facilities, it is important that they be properly located; therefore, the LLS was developed to aid management of the feed industry in making optimum location-logistics decisions.

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James C. Snyder is professor of Agricultural Economics, Purdue University, Lafayette, Ind.

2/ For more discussion of case firm results and actual examples see Lee, Robert E., "A Location Distribution System for Feed Firm Management," unpublished Ph.D. thesis, Purdue University, Lafayette, Ind.

The analysis made of the cooperating firm's problem indicated that actual implementation of the LLS was feasible and profitable. The analysis indicated that if plant facilities are located properly, the most desirable procurement, processing, and distribution combinations studied could produce returns that are 8-10 percent higher than the poorest combination analyzed. While the comprehensive nature of the system and size of the problem does require simplification in some areas, when a simplification was made the operational effectiveness of the system was not significantly impaired. The system can be feasibly and practically adapted to fit the structure of most feed manufacturing firms.

THE PROBLEM

In the last half century, the feed manufacturing industry has changed greatly. It has grown from a small group of grain and byproduct mixers to what is now one of the largest manufacturing industries in the United States. As a result, feed firm management has had to reappraise existing plant locations. They must seek new sites and facilities that promise more profitable operations.

Six major changes can be cited as having had significant impact on the location-logistics decision of feed manufacturers:

(1) The industry has become more decentralized. There has been a gradual movement away from large terminal mills toward decentralized or satellite production units.

(2) Feed firms have diversified their operations, becoming more involved in business operations outside of feed manufacturing; for example, integration into the poultry industry.

(3) New technologies available to the industry have changed economies of scale, manpower requirements, and other input and investment requirements.

(4) There has been a trend, more noticeable in some areas than others, toward larger individual order sizes and the production of fewer formulas which decrease start-up, clean-up, and shut-down time.

(5) Customer service has been an increasingly important consideration for all feed firms. Proper location-logistics decisions are, of course, vital to improving customer service. Customers have become more cost conscious, and more capable of evaluating the merits of the service and products they receive.

(6) The changing farm unit, development of rural areas resulting in improved communications, and the emergence of rural service centers have produced many recent changes in the feed industry. For example, the trend toward fewer, larger, and more specialized farms has caused farmers to behave more like businessmen and demand that their inputs be as cheap as possible. Often feed manufacturers have not met

farmers needs, and farmers have started manufacturing their own feed. This kind of reaction by the farmer has caused feed manufacturers to move production closer to the farm and has forced direct delivery of feed.

These major changes have resulted in three basic approaches to distribution. One approach is to ship completely mixed or supplement feeds from large terminal mills through retail dealers to farm consumers. Other firms bypass retail dealers by shipping from decentralized mills or bulk stations to farms. Still other firms are retaining the large terminal mills for mixing supplements and low-volume feeds that are shipped to decentralized locations where they are mixed further before distribution to the farmer.

Today, a thorough analysis of procurement, manufacturing, and distribution needs should be undertaken by the feed firm before the most efficient location can be determined. The LLS discussed below is designed to provide guides to this analysis and aid in effectively evaluating complex problems (see figure 1).

STRUCTURE OF THE LOCATION-LOGISTICS SYSTEM

The LLS is designed to be used specifically for (1) selecting plant sites, (2) analyzing location-distribution patterns, (3) selecting alternative plant sizes and types, and (4) evaluating the return from the configuration 3/ examined so that the most efficient location-logistics system is obtained.

There are four major subsystems in the LLS. These are: (1) The management review subsystem (MRS), which uses sound management analysis and a grid coordinate market density analysis to select potential plant sites. (2) The location-distribution analyzer (LDA), which uses linear programming to evaluate alternative location-distribution configurations. (3) The plant selection subsystem (PSS), which uses regression analysis and quadratic programming to evaluate alternative sizes and types of plants to be built at each site analyzed in each location-logistic configuration examined. (4) The investment feasibility analyzer (IFA), which uses present value and discounted cash flow analysis to determine the rate of return on investment from alternative configurations.

The four subsystems are interrelated as shown in figure 2. The MRS helps delineate the data which are required for the LDA analysis. Output from the LDA analysis provides demand parameters for the PSS analysis; and output from MRS, LDA, and PSS provides the basis for the IFA analysis. Upon completion of this four-step analysis, the major variables will have been considered by the location-logistics analysis.

3/ As used here, "configuration" refers to a group of selected plants at specific sites whose size, number, type, location, capacity constraints, costs of operation, and projected demand are specified.

ELEMENTS OF THE LOCATION—LOGISTICS PROBLEM

PROCUREMENT FUNCTION
1. Large number of alternative ingredients 2. Multiple sources of each ingredient 3. Alternative shipping modes from each source 4. Fluctuations in prices, available quantities, and nutrient analysis
MANUFACTURING FUNCTION
1. Large number of alternative manufacturing plant sites 2. Alternative number and size of plants to be used 3. Type of manufacturing facilities—centralized mother mill, decentralized satellite mills 4. Product line selection—number, concentrate vs complete, bulk vs bag, mash vs pellet
DISTRIBUTION FUNCTION
1. Large number of alternative distribution outlet sites 2. Alternative number and size of distribution outlets to be used 3. Type of distribution—manufacturer direct, retail dealer, combination 4. Multiple shipping modes through each type of distribution network

Figure 1

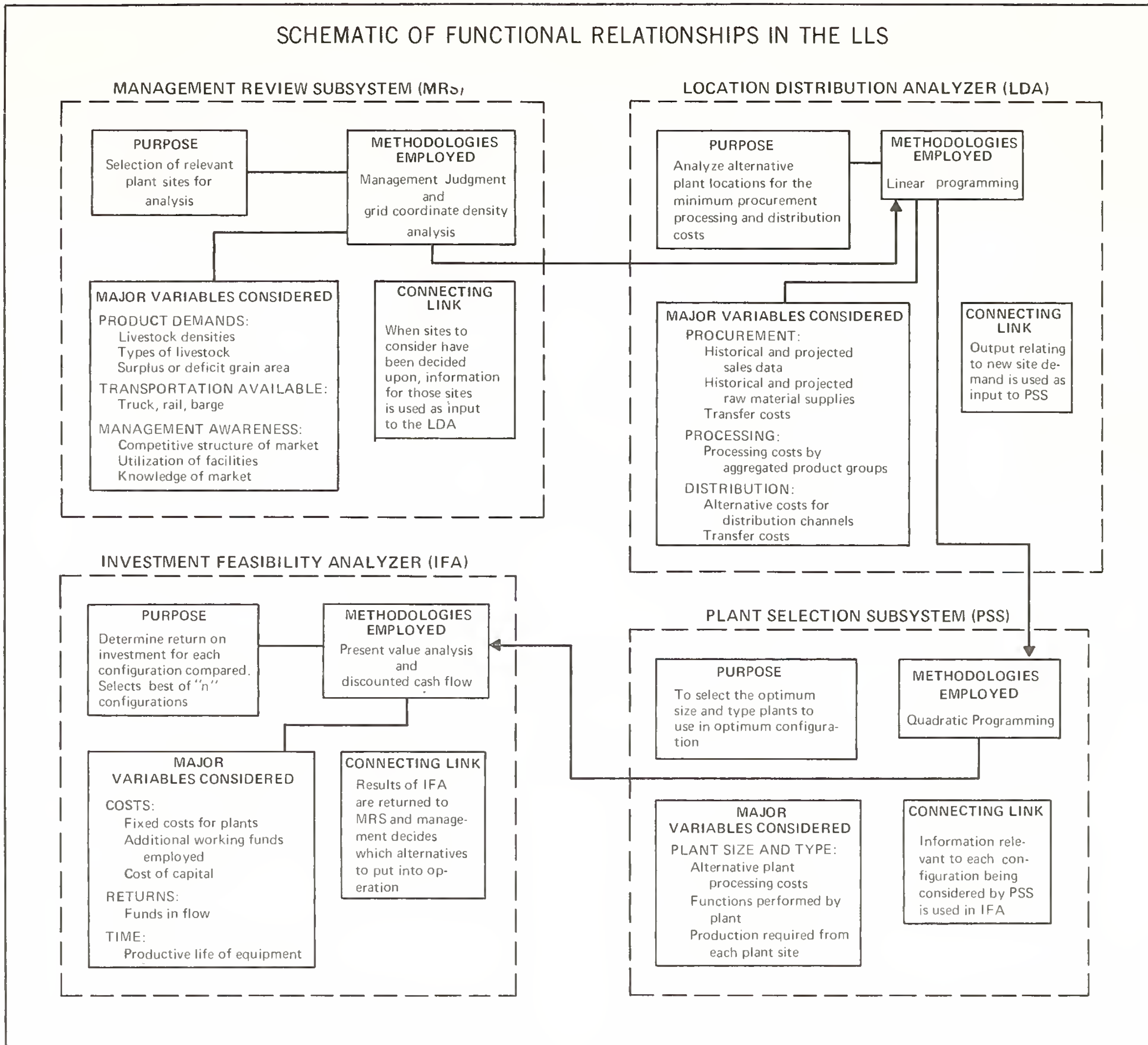
SCHEMATIC OF FUNCTIONAL RELATIONSHIPS IN THE LLS
MANAGEMENT REVIEW SUBSYSTEM (MRS)
PURPOSE
Selection of relevant plant sites for analysis
METHODOLOGIES EMPLOYED
Management Judgment and grid coordinate density analysis
MAJOR VARIABLES CONSIDERED
PRODUCT DEMANDS:
Livestock densities
Types of livestock
Surplus or deficit grain area
TRANSPORTATION AVAILABLE:
Truck, rail, barge
MANAGEMENT AWARENESS:
Competitive structure of market
Utilization of facilities
Knowledge of market
CONNECTING LINK
When sites to consider have been decided upon, information for those sites is used as input to the LDA
LOCATION DISTRIBUTION ANALYZER (LDA)
PURPOSE
Analyze alternative plant locations for the minimum procurement processing and distribution costs
METHODOLOGIES EMPLOYED
Linear programming
MAJOR VARIABLES CONSIDERED
PROCUREMENT:
Historical and projected sales data
Historical and projected raw material supplies
Transfer costs
PROCESSING:
Processing costs by aggregated product groups
DISTRIBUTION:
Alternative costs for distribution channels
Transfer costs
CONNECTING LINK
Output relating to new site demand is used as input to PSS
INVESTMENT FEASIBILITY ANALYZER (IFA)
PURPOSE
Determine return on investment for each configuration compared. Selects best of "n" configurations
METHODOLOGIES EMPLOYED
Present value analysis and discounted cash flow
MAJOR VARIABLES CONSIDERED
COSTS:
Fixed costs for plants
Additional working funds employed
Cost of capital
RETURNS:
Funds in flow
TIME:
Productive life of equipment
CONNECTING LINK
Results of IFA are returned to MRS and management decides which alternatives to put into operation
PLANT SELECTION SUBSYSTEM (PSS)
PURPOSE
To select the optimum size and type plants to use in optimum configuration
METHODOLOGIES EMPLOYED
Quadratic Programming
MAJOR VARIABLES CONSIDERED
PLANT SIZE AND TYPE:
Alternative plant processing costs
Functions performed by plant
Production required from each plant site
CONNECTING LINK
Information relevant to each configuration being considered by PSS is used in IFA

Figure 2

The generalized schematic of the location-logistics system (figure 2), is expressed mathematically as follows:

Minimum Cost LDA:

Minimize

$$CLLC = f\left(\sum_{i=1}^{n} \sum_{j=1}^{m} ZC_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} PC_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{p} DC_{ijk} \right) \quad (1)$$

where

CLLC = Total Cost Location Logistics Configuration.

$\sum_{i=1}^{n} \sum_{j=1}^{m} ZC_{ij}$ = Summation of Procurement Costs for Products "i" at location sites "j."

$\sum_{i=1}^{n} \sum_{j=1}^{m} PC_{ij}$ = Summation of Processing Costs for Products "i" at location sites "j."

$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{p} DC_{ijk}$ = Summation of Distribution Costs for Products "i" from location sites "j" to destinations "k."

The procurement, processing, and distribution cost components of the above equation are a constant value. This means that procurement, processing, and distribution costs as represented in the equation do not vary with changes in quantity. Therefore, each ton of a specific feed is procured at a constant cost from the origin. Likewise, for a given size plant at a determined capacity, a manufacturing cost is established and is the same for all feeds of a specific type. Similarly, distribution costs are also predetermined and are fixed in the final comparison.

The individual components of procurement, processing, and distribution can be represented mathematically as follows:

Procurement Costs:

$$ZC = f\left[\sum_{i=1}^{n} \sum_{j=1}^{m} (X_{ijk} \cdot C_{ijk}) \right] \quad (2)$$

where

ZC = Procurement Costs

X_{ij} = Quantity of Ingredient "i" received at location "j" from origin "k"

C_{ij} = Cost of ingredient and transportation for ingredient "i" received at location "j" from origin "k."

Processing Costs:

$$PC = f\left[\sum_{i=1}^{n}\sum_{j=1}^{m}(V_{ij}\cdot C_{ij})\right] \qquad (3)$$

where

PC = Processing Costs

V_{ij} = Volume of products "i" produced at locations "j"

C_{ij} = Cost of producing products "i" at locations "j"

Distribution Costs:

$$DC = f\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{p}(Q_{ijk}\, C_{ijk})\right] \qquad (4)$$

where

DC = Distribution Costs

Q_{ijk} = Quantity of Product "i" sold at destination "k" from location "j"

C_{ijk} = Cost of distributing products "i" from location "j" to destination "k"

The specific purpose for which the PSS is designed to be used, is to select from many alternative sizes and types of plants the plant configuration(s) that result in least cost, given fixed demand.

To accomplish this, regression analysis and quadratic programming procedures are used. Knowledge of the cost function permits use of regression analysis to obtain the average total cost equations for each size and type plant analyzed. These equations take the following form:

$$ATC = a_i + b_i g_i + c_i g_i^2 \qquad (5)$$

This quadratic expression can be provided for in the quadratic programming model by adding the coefficient of the first derivative of the above expression to the relevant process vector in the dual equation which relates to the associated manufacturing plant. 4/ 5/ This analysis is structured to analyze

4/ For a more detailed discussion, see Lee, Robert E., "A Location Distribution System for Feed Firm Management," unpublished Ph.D. thesis, Purdue University, Lafayette, Ind., pp. 36-39. 5/ For more detailed discussion on the use of the quadratic programming procedure, see Lee, Robert E., An Introduction to Quadratic Programming, Pennsylvania State University Agricultural Experiment Station, University Park, Pa., A.E. & R.S., No. 41, June 1963.

economies of scale for feed manufacturing plants singularly or in combination. When combinations of plants are analyzed, production in each is allocated in a way that minimizes the combined production costs given fixed demand parameters.

The PSS is solved independently of the LDA and does leave open the possibility for suboptimum solutions; however, by taking the results of the PSS analysis and inputing them into the LDA and running the analysis again, suboptimization can be reduced. If the user so desires, he can continue this interplay between subsystems to reduce suboptimization. The small error due to suboptimum solutions should not limit the usefulness of the results in decisionmaking.

The fourth and final subsystem of the LLS is the IFA. The IFA uses output from the MRS, LDA, and PSS as input. The specific purpose for which the IFA is used is to determine the rate of return on the additional capital employed in constructing and operating the configuration analyzed.

Discounted cash flow analysis is the standard financial tool used for evaluating returns to capital investments and is the principal approach incorporated in the IFA. In gross terms, the IFA is mathematically expressed as follows:

$$\text{Maximize} \quad TNR = (TGR - TC) \qquad (6)$$

where TNR = Total Net Revenue

and

$$TGR = \sum_{i=j}^{n} \sum_{j=1}^{m} (Q_{ij} P_{ij}) \qquad (7)$$

where TGR = Total Gross Revenue

Q_{ij} = Quantity of Product "i" sold at location "j"

P_{ij} = Price for Product "i" at location "j"

TC = Total Cost of Configuration

and the present value procedure for a stream of revenues is mathematically expressed as follows:

$$PV = R_1 \ (1 + r)^{-1} \ \text{---} \ R_t \ (1 + r)^{-t} \qquad (8)$$

A cash flow for the existing situation and the proposed projects are determined. This information accompanied by the time-adjusted analysis provides management with valuable information on which to base investment decisions.

The above mathematical representation of the total LLS summarizes the major functional relationships analyzed by the system. The component parts of the LLS are not solved simultaneously, and as a consequence, suboptimum results for the system are obtained. Through sensitivity analysis, the less than optimum results can be improved upon and solutions obtained that can assist in providing valuable guides to practical decisionmaking, even though the system does not solve for globally optimum solutions.

This publication treats each of the LLS subsystems in the following order: (1) the management review subsystem, (2) the location distribution analyzer, (3) the plant selection subsystem, and (4) the investment feasibility analyzer.

MANAGEMENT REVIEW SUBSYSTEM

The MRS is the first major component of the LLS. Its principal function is to assist management in choosing potential sites at which to locate feed manufacturing facilities. By selecting several feasible sites, the MRS provides the basis for subsequent evaluation of many configurations of those sites to determine the optimum set. This analysis is of value in eliminating many alternative sites as being impractical prior to computational analysis by the other subsystems. Computational analysis could be used to determine the optimum configurations of sites, but this is impractical because of the large number of possible combinations. Moreover, because management is aware of (1) its market, (2) the existing level of facilities utilization, (3) the nature of the livestock concentration in the market area, and (4) the availability of local grains, the number of sites can be reduced to manageable levels without computational analysis by LLS.

The selection of feasible and practical sites was accomplished by systematic management review and demand density analysis. The combined use of the two approaches facilitated the selection process and was valuable as a cross reference. The systematic management review depends on sound managerial judgment, while the demand density analysis is objective and mathematically expressed as follows:

$$\sum_{i=1}^{n} Z_i Y_i = Z_1 Y_1 + Z_2 Y_2 --- Z_n Y_n$$

where

$\sum_{i=1}^{n} ZY$ = Summation of the ZX's

Z = Volume demanded at each destination

Y = Y coordinate value

and

$$\sum_{i=1}^{n} Z_i X_i = Z_1 X_1 + Z_2 X_2 --- Z_n X_n$$

where

$\sum_{i=1}^{n} ZX$ = Summation of the ZX's

Z = Volume demanded at each destination

X = X coordinate value

and

$$\frac{\Sigma\ ZX}{Z} = X \quad \text{center of gravity}$$

$$\frac{\Sigma\ ZY}{z} = Y \quad \text{center of gravity}$$

By plotting the X and Y center of gravity, the point of greatest density is determined; and when used in conjunction with systematic management review, a reasonably objective selection of feasible and practical sites can be made.

Table 1 lists the three existing and seven new plant sites considered in the analysis. Each site was given a code letter. (I, S, and M were the three existing sites.) In addition, table 1 indicates the sector of the firm's market area in which each site is located.

Table 1.--Location of current and potential plant sites in market

Plant site code name	Location in market
I	1/ Central
S	1/ Southeast
M	1/ East
F	West
E	Northeast Central
B	Northeast
R	West
U	North Central
G	Southeast
W	Northeast

1/ Existing plants at inception of study. These plants were operating at maximum capacity when the study was undertaken.

LOCATION-DISTRIBUTION ANALYZER

In the selection of optimum plant locations from the set of potential sites, the LDA makes use of an advanced linear programming model to evaluate comparative procurement, processing, and distribution costs. The following factors are considered in the LDA analysis: (1) Ingredient purchase and transport cost from alternative sources via different shipping modes, (2) Alternative formulation, manufacturing, and related costs for feeds (bag or bulk, pelleted or mash) from different sizes and types of manufacturing plants which have different production costs, (3) Product distribution costs for different forms of feed from points of production to points of distribution, (4) Expected final product demand, and (5) Production plant capacities.

The LDA was designed in a modular fashion to facilitate the addition and deletion of activities and constraints. The modular design also improves the ease with which coefficient revisions can be made. This ability to revise coefficients facilitates updating the system to reflect changes in plant numbers, location, production technology, and other market and production parameters.

Structure of the LDA

Readers not familiar with the mathematical notation can safely ignore this discussion for the general discussion in the text explains the key components of the system.

Figure 3 represents the generalized structure of the LDA for one plant at one site in the system of plants. Multiples of six sites with a plant at each site can be used in the present system before program capabilities become constraining. Each site and plant combination incorporated into the system would take on the same basic structure presented in figure 3. The discussion describing the model in figure 3 would be the same for every additional site and plant combination considered by use of the LDA.

Activities

Ingredient Cost Summation Activities.--The activities associated with the first column of submatrices

A_{i1} (i = 1,---,n)

are set up to provide a means for summarizing the total cost of each product in each plant during the time period planned. The model has one such activity for each of the major products used by the firm.

Processing Cost Summation Activities.--The activities associated with the submatrices

A_{i2} (i = 1,---,n)

are set up to summarize the costs of the three processing functions: manufacturing, pelleting, and bagging of feed during the period being considered. The model has one activity for each of the three functions for each of the plants considered by the firm. The number of plants considered can vary for successive simulated planning periods.

Total Product Usage Activities.--The activities associated with the submatrices

A_{i3} (i = 1,---,n)

represent the quantities of feed products to be transferred from production unit to final consumption point. Each plant has a set of activities to perform this function. Within each plant, the model has one total usage activity for each major feed product used by the firm.

THE GENERAL FORM OF THE LINEAR PROGRAMMING LDA

CONSTRAINTS \ ACTIVITIES	Total Product Cost Summation	Manufacturing, Pelleting, and Bagging Cost Summation	Total Product Usage	Manufacturing, Pelleting, and Bagging Usage	Transportation Cost Summation	Product Form Control Bulk Meal Feed	Product Form Control Bagged Meal Feed	Product Form Control Bulk Pelleted Feed	Product Form Control Bagged Pelleted Feed	Demand Requirements by Product by Demand Area		
Transfer Cost Equation For Ingredients by Product	$-I_{1,1}$		$D^{+a,I}_{1,3}$								=	
Transfer Cost Equations for Manufacturing, Pelleting, and Bagging		$-I_{2,2}$		$D^{+a,I}_{2,4}$							=	
Total Product Usage Transfer Equations			$-I_{3,3}$			$I_{3,6}$	$I_{3,7}$	$I_{3,8}$	$I_{3,9}$		=	
Manufacturing, Pelleting, and Bagging Usage Transfer				$-I_{4,4}$		$A^{+1}_{4,6}$	$A^{+1}_{4,7}$	$A^{+1}_{4,8}$	$A^{+1}_{4,9}$		=	
Transportation Cost Equations					$-I_{5,5}$					$A^{+a}_{5,10}$	=	
Truck Availability Constraints										$A^{+a}_{6,10}$	$\leq$	B_6
Mash Feed (Bulk) Transfer Equations by Product to Demand Areas						$-I_{7,6}$				$A^{+a}_{7,10}$	=	
Mash Feed (Bagged) Transfer Equations by Product to Demand Areas							$-I_{8,7}$			$A^{+a}_{8,10}$	=	
Pelleted Feed (Bulk) Transfer Equations by Product to Demand Areas								$-I_{9,8}$		$A^{+a}_{9,10}$	=	
Pelleted Feed (Bagged) Transfer Equations by Product to Demand Area									$-I_{10,9}$	$A^{+a}_{10,10}$	=	
Capacity Constraints on Manufacturing, Pelleting, and Bagging				$A^{+1}_{11,4}$							$\lesseqgtr$	B_{11}
Demand Constraints by Demand Area										$I_{12,10}$	=	B_{12}
Objective Function	C_1	C_2			C_3						=	Minimum

A is a general submatrix with some nonzero elements. Superscripts when shown indicate the only types of elements in the submatrix.

I is a identity submatrix of dimension k = the number of feed products used by the firm.

-I is a negative identity submatrix of dimension k.

I' is an identity submatrix of dimension r = the number of components in the processing function, production, bagging, and pelleting.

-I' is a negative identity submatrix of dimension r.

D is a diagonal submatrix of dimension k. The diagonal elements are of the type indicated by the superscript.

D' is a diagonal submatrix of dimension r. The diagonal elements are of the type indicated by the superscript.

B is a columnar submatrix or vector.

C is a row submatrix or vector.

Blank squares represent submatrices. All of those elements are zero.

Figure 3

Total Manufacturing Usage Activities.--The activities associated with the submatrices

$$A_{i4} \ (i = 1,---,n)$$

represent the quantities of feed having each of the three functions (manufacturing, pelleting, and bagging) performed on them. Within each plant, the model has one total usage activity for each of the three functions.

Transportation Cost Summation Activity.--The activity associated with the submatrices

$$A_{i5} \ (i = 1,---,n)$$

is set up to summarize the cost of transporting the finished product from the plant to the respective final demands. Each plant considered in the model by the firm in successive simulated planning periods has one transportation cost; summarization activity into which all the costs of transportation from that plant to specified demand points are summarized.

Product Form Control Bulk Meal.--The activities associated with the submatrices

$$A_{i6} \ (i = 1,---,n)$$

act as transfers for all products in the bulk meal form from product production to a control vector which allocates the products of that particular form to a particular demand. Each plant considered by the firm in the model has similar submatrices. The model has one activity for each of the major products in bulk meal form used by the firm.

Product Form Control Bagged Meal.--The activities associated with the submatrices

$$A_{i7} \ (i = 1,---,n)$$

act as transfers for all products in the bagged meal form from product production to a control vector which allocates the products of that particular form to a particular demand point. Each plant considered by the firm in the model has similar submatrices. The model has one activity for each of the major products in bagged meal form used by the firm.

Product Form Control Bulk Pelleted.--The activities associated with the submatrices

$$A_{i8} \ (i = 1,---,n)$$

act as transfers for all products in the bulk pelleted form from product production to a control vector which allocates the products of that particular form to a particular demand point. Each plant considered by the firm in the model has similar submatrices. The model has one activity for each of the major products in bulk pelleted form used by the firm.

Product Form Control Bagged Pelleted.--The activities associated with the submatrices

A_{i9} (i = 1,---,n)

act as transfers for all products in bagged pelleted form from product production to a control vector which allocates the products of that particular form to a particular demand point. Each plant considered by the firm in the model has similar submatrices. The model has one activity for each of the major products in bagged pelleted form used by the firm.

Product Demands.--The activities associated with the submatrices

A_{i10} (i = 1,---,n)

are the quantities of product, by product, type, and form, demanded by a particular demand point in the case firm system. These activities provide the final control mechanism for integrating the product type and form requirements. Each plant may serve a subset or the entire group of demand points, depending on how the cost relationships compare. For each demand point, the model has one activity for the type and form of product being demanded.

Constraints

Product Cost Transfers.--The submatrices

A_{1j} (j = 1,---,n)

are used to transfer the cost for each ton of feed demanded and produced to the total product cost for ingredients summarization activities. Each constraint row sets total ingredient cost for a given product equal to the value obtained when total ingredient cost for the products is multiplied by the number of produced tons of product in each product group. The model has one constraint row for each major product produced by the firm. The model is constructed in such a way as to equate demand of product with supply of product produced in the plants operated by the firm.

Manufacturing, Pelleting, and Bagging Cost Transfers.--The submatrices

A_{2j} (j = 1,---,n)

are used to transfer the cost for manufacturing, pelleting, and bagging each ton of feed demanded and produced to the manufacturing, pelleting, and bagging cost summarization activities. Each constraint row sets total costs for the product mix being used by the firm equal to total cost of manufacturing, pelleting, and bagging, respectively. The model is set up with one constraint row for each of the three functions. The model is structured in such a way as to equate manufacturing, pelleting, and bagging required with the product manufactured, pelleted, and bagged.

Product Usage Transfers.--The submatrices

A_{3j} (j = 1,---,n)

are used to transfer the products produced by the firm into the four respective form groups of bulk meal, bagged meal, bulk pelleted, and products used by the firm.

Manufacturing, Pelleting, and Bagging Usage Transfers.--The submatrices

A_{4j} (j = 1,---,n)

are used to transfer the products produced into one, two, or three of the manufacturing functions of manufacturing, pelleting, and bagging. The model is structured to make this transfer by product type and product form. One constraint row is provided for each of the three functions.

Transportation Cost Transfer.--The submatrices

A_{5j} (j = 1,---,n)

are used to transfer the cost of moving a final product of a specific form from a production unit to a series of demand points; then into an activity which combines the total cost of transportation into one aggregate figure. The model has one row constraint provided to handle the transportation summation.

Truck Availability Constraints.--The submatrices

A_{6j} (j = 1,---,n)

are set up to confine the analysis to the number of truck hours available for distributing the final product.

Product Form Transfers Bulk Meal.--The submatrices

A_{7j} (j = 1,---,n)

are used to summarize and transfer the demand requirements for the bulk meal products used by the firm. The summarized product demands of the bulk meal type are then transferred to the appropriate ingredient and manufacturing cost centers. The model has one row constraint for each product used by the firm.

Product Form Transfers Bagged Meal.--The submatrices

A_{8j} (j = 1,---,n)

are used to summarize and transfer the demand requirements for bagged meal products used by the firm. The summarized product demands of the bagged meal type are then transferred to the appropriate ingredient, manufacturing, and bagging cost centers. The model has one row constraint for each product used by the firm.

Product Form Transfers Bulk Pelleted.--The submatrices

A_{9j} (j = 1,---,n)

are used to summarize and transfer the demand requirements for the bulk pelleted products used by the firm. The summarized product demands of the bagged pelleted type are then transferred to the appropriate ingredient, manufacturing, and pelleting cost centers. The model has one row constraint for each product used by the firm.

Product Form Transfers Bagged Pelleted.--The submatrices

A_{10j} (j = 1,---,n)

are used to summarize and transfer the demand requirements for the bagged pelleted products used by the firm. The summarized product demands of the bagged pelleted type are then transferred to the appropriate ingredient, manufacturing, and pelleting cost centers. The model has one row constraint for each product used by the firm.

Manufacturing, Pelleting, and Bagging Capacity Constraints.--The submatrices

A_{11j} (j = 1,---,n)

are used to specify maximum and minimum availabilities of manufacturing, pelleting, and bagging capacity in the plant being considered. The model has one constraint row for each restriction placed on the three functions.

Demand Requirements.--The submatrices

A_{12j} (j = 1,---,n)

are used to specify the quantities of various products demanded at individual demand points. More than one plant can serve a demand point with a specific product. The model has been structured in such a way that one constraint row can be set up for each plant. This plant will satisfy a given demand point with a specific form of product.

Costs used in the LDA

The LDA as used in the case firm analysis includes three major types of variable costs: procurement, processing, and distribution. These costs were divided into the following categories:

- Procurement:
  - Transfer costs
  - Ingredient costs
- Processing:
  - Manufacturing costs
  - Pelleting costs
  - Bagging costs

Distribution:
Transportation costs from plant to destination

Procurement Costs

Transfer and ingredient costs.--Projected product sales volume and computed least cost product formulas were used to establish raw material needs by ingredient type. The procurement cost used in the analysis was based on the market price for ingredients and transfer costs from ingredient source to processing plant.

Processing Costs

Manufacturing costs.--Manufacturing costs were based on case firm data and reflect current operations. Manufacturing costs were accounted for by an activity separate from pelleting and bagging costs. This permitted analysis by the three major cost centers of processing. Analysis of case firm and related data indicated that for manufacturing costs the linearity assumption was valid for the volume ranges considered.

Pelleting cost.--This cost was applied only to pelleted product, and was based on case firm and engineering economics data.

Bagging cost.--The cost of bagging feed was applied to bagged product. It consists largely of the cost for labor and bags.

Distribution Costs

Transportation costs.--The cost of transportation for distribution is the cost involved in moving the final product from a production plant to a final demand point. This kind of transfer cost is applied to product shipped from each processing plant to demand point.

Cost Summary

All procurement, processing, and distribution costs used in the LDA are included in the categories discussed above. The modular nature of the system permits more detailed categorization of costs if needed.

Results of the LDA

The main objective of this discussion is to demonstrate the efficacy of using the LDA to determine the least-cost combination of plants and sites to serve the firm's market; the LDA was used to analyze 13 alternative configurations (see table 2). The table includes two major categories: (1) existing and proposed plants, and (2) simulated configurations. Demand forecasts for each configuration were based on projections supplied by the firm. In most cases, as shown in the table, plants were assumed to be restricted to their normal capacity. The LDA analysis considered no more than six sites per configuration, because this was the maximum number that the firm contemplated

Table 2.--Plant configurations and capacity constraints

Configuration	Capacity constraints
A. Existing and proposed	
$(I,S,M)_1$	None
$(I,S,M,R,W,G)_1$	Normal
(I,S,M,R,W,U)	Normal
(I,S,M,R,U,G)	Normal
(I,S,M,U,W,G)	Normal
(I,S,M,R,W)	Normal
(I,W,M,R,U)	Normal
(I,S,M,W,U)	Normal
(I,S,G,E,B)	Normal
(I,S,G,F,B)	Normal
(I,S,F,E,B)	Normal
B. Simulated	
$(I,S,M)_2$	Simulated
$(I,S,M,R,W,G)_2$	Simulated

operating. In addition to varying numbers of plants, the analysis considered seven feed products and four product forms. The results are summarized in table 3.

Plant Configuration Analysis

Base Configuration

Analysis was made of the existing plant and site configuration to provide a benchmark of potential savings from investment in new plants at new sites. A base configuration $(I,S,M)_1$ consists of using expanded current facilities to meet projected demand. Procurement, processing, and distribution costs to meet projected demand using this base configuration were $4,377,711 or $11.83 per ton (table 3). In calculating these costs, it was assumed that the existing production facilities were operating at capacity, and that because of this, unit operating costs probably were understated. (As production increases to meet the new product demand, the existing facilities would operate less efficiently and costs per unit would be higher.) For this reason, the potential savings obtained from new plant investment were felt to be greater than that indicated by the differentials given in table 3.

In addition to the substantial savings, there are other important benefits from investment in new facilities in addition to cost reduction. These include: (1) expansion of sales to meet projected growth goals of the firm, and (2) increases in the number of plants to improve market penetration. Both of the benefits were considered in the MRS analysis before new facilities investment was made.

Table 3.--Cost summary for selected configurations

Configuration	Procurement, processing, and distribution costs		Difference from base	
	Dollars	Dollars/ton	Dollars/ton	Dollars
A. Existing and proposed				
$(I,S,M)_1$	4,377,711	11.83	1/.00	--
$(I,S,M,R,W,G)_1$	4,225,094	11.44	+.39	144,070
(I,S,M,R,W,U)	4,186,544	11.33	+.50	184,734
(I,S,M,R,U,G)	4,203,773	11.36	+.47	173,900
(I,S,M,U,W,G)	4,269,044	11.55	+.28	103,451
(I,S,M,R,W)	4,254,926	11.52	+.31	114,517
(I,S,M,R,U)	4,233,439	11.41	+.42	155,400
(I,S,G,W,U)	4,296,548	11.63	+.20	73,894
(I,S,G,E,B)	4,330,920	11.72	+.11	40,635
(I,S,G,F,B)	4,200,778	11.35	+.48	177,600
(I,S,F,E,B)	4,178,247	11.29	+.54	199,800
B. Simulated				
$(I,S,M)_2$	3,986,334	10.77	+1.06	392,200
$(I,S,M,R,W,G)_2$	4,045,619	10.95	+.88	325,081

1/ Base case includes existing plant and site configuration and provides a benchmark against which to measure proposed new configuration.

The results summarized in table 3, part A, indicate that all proposed plant and site configurations offered a saving over the base case. The base case costs were $0.11 per ton higher than the next most inferior configuration and $0.54 per ton higher than the "optimum" configuration.

Existing and Proposed Configurations

Proposed plant and site configurations were selected by management through the use of MRS criteria and were then analyzed to determine potential savings from alternative configurations. The most superior configuration was ascertained by comparing the proposed configurations to the base configuration and to each other. The plants used in the proposed-configuration analysis included: (1) the existing facilities, and (2) general-line plants capable of producing 52,000 tons annually. 6/

A comparison of alternative new configurations indicated that improper site selection could cost $159,165 annually. This conclusion is based on the analysis of configurations with five or six plants. Conceivably, a firm with

6/ A "general-line plant" refers to a plant capable of producing all possible feed lines. In this study, a general-line plant is capable of producing 50 percent pelleted and bagged feed and distributes the product in both bagged and bulk form.

more than six plants would incur a greater cost in choosing an inferior location. The four configurations from table 3-A that provided the greatest savings were (I,S,M,R,W,U), (I,S,M,R,U,G), (I,S,G,F,B), and (I,S,F,E,B).

Study of the four superior configurations indicated that plants should be built in the western, north central, southeast, or northeast sections of the market. Further study of configurations (I,S,M,R,W,U) and (I,S,F,E,B), the two superior configurations, suggested that new plants should be built in the western, north central, or northwest segments. The "optimal" configuration suggested that the firm's oldest and least efficient plant be closed. This plant was salvaged in figuring the analysis.

The LDA analysis also indicated that the place to construct one, two, or three plants would be in the western, north central, and northeast sectors of the market, respectively. Each of the four best configurations included three new plant sites. However, the crucial sites appeared to be those in the western or north central segments of the market: an analysis of (I,S,M,R,U) and (I,S,M,W,U) indicated that building a plant in either of these sectors rather than in the northeast sector could contribute up to $81,506 savings on an annual basis.

Simulated Configurations

The LDA can be used to simulate desired plant and site configurations. This allows examination of outcomes to certain actions before the actions are taken. The simulated configurations were designed to analyze comparative operational costs of three large, centrally located plants versus six smaller, decentralized plants. This cost comparison was based on most recent manufacturing and distribution technology. The analysis indicated that it was more efficient to operate the three large centralized plants than six small decentralized plants (table 3, part B). The larger centralized plants cost $0.18 per ton less to operate than did the six decentralized plants. The distribution saving from operating six decentralized plants did not appear to offset the lower manufacturing costs of the three centralized plants.

Concluding Comments

The above analysis of one feed manufacturer's operations partially demonstrates potential usefulness of the LDA. More extensive use of sensitivity analysis would enhance the value of the system, particularly in the simulation of alternative market conditions and technological innovations. This type of analysis would provide additional guides for evaluating different marketing strategies and long-range planning objectives.

PLANT SELECTION SUBSYSTEM

Output from the LDA is used as input to the PSS. The demand allocated to individual plants in a proposed plant site configuration is determined by use of the LDA. This information is used to provide the demand parameters for the

PSS. This demand is then allocated among the new plants until the optimum size and type plant at each site are ascertained.

Selecting the optimum size and type of plants is a difficult task in light of the large number of technological alternatives and complex economic interrelationships. Mills can be built to handle a general line of feeds or specialized groups of feeds. They can be built to handle pelleted or mashed feed. With the many combinations of plants which could conceivably be built, feed firm management must exercise intelligence in deciding upon plant size and type. The PSS is designed to be used as an aid. In addition, the PSS can be used as a planning tool, guiding management in proper selection of manufacturing plants to satisfy long-range sales objectives.

Methodology

To select the optimum size and type plant to be erected at each site, it must be possible to evaluate a feed processing plant's curvilinear total cost function. To make this evaluation, the quadratic programming technique is used as an essential feature of the PSS. Because the total cost function is cubic, the average total cost function which is quadratic is used in the analysis. This use of the average total cost function is permissible and results in a solution equivalent to that obtained when using the total cost function, provided product demand is held constant. For mathematical proof, see the appendix.

Technically, the entire problem for which the LDA and PSS is designed to provide answers which can be formulated as a single subsystem and solved, using quadratic programming. Unfortunately, current computer programs are not capable of solving a quadratic programming problem of this size. For this reason, the two systems were developed to solve the problem as two separate solutions. This sequential analysis may cause suboptimization problems. However, the results of the LDA-PSS approach are superior to a conventional linear programming analysis which could be employed to answer similar questions.

Structure of the PSS

The question answered by the PSS is accomplished by solving a single or multiple set of quadratic functions. In the PSS, the primal-dual structure for formulating the model is used to accomplish this physical task. Using the primal-dual leads to a means of stating quadratic programming problems similar to linear programming problems. This allows for structuring the PSS in a modular fashion, and as such, facilitates coefficient changes and provides needed flexibility in doing sensitivity or comparative analysis. The structure of the PSS, in macro form, is illustrated in figure 4. Each submatrix within the PSS is described to indicate how it relates to the subsystem and to the determination of the optimum processing cost for a specific size and type feed plant at varying levels of production.

Submatrix "A'" is a submatrix containing the coefficients of the first derivative of the average total cost function. This, when added to the

STRUCTURE OF PSS, MACRO FORM

Cj* / Ci**				Po
M^{***} Z_1^{****}	A	B	O	O′
M Z_2	A′	B′	I	Y

* Costs on activities
** Costs on constraints
*** Large negative numbers
**** Processes

Submatrix 'A' is the submatrix representing the primal vectors in the formulation; i.e., the primal problem of a linear programming model.

Figure 4

relevant process vector of the primal in the proper dual equation of the respective manufacturing plant, provides for the quadratic expression in the PSS.

Submatrix "B" is a submatrix of artificial variables for transforming equal to or less than constraints to equalities in the primal-dual formulation of the PSS structure.

Submatrix "B'" and submatrix "I" are submatrices of artificial variables for transforming the primal and dual systems to equality systems within the PSS structure.

Submatrix "O" and submatrix "O'" are empty set submatrices; that is, no entries are made into these submatrices.

Submatrix "Y" is the submatrix representing the dual vectors in the formulation, i.e., the dual problem of a linear programming model.

Submatrix "Po" is a submatrix of problem constraints for system equations in the PSS structure.

A detailed discussion of how the quadratic expression is arrived at will not be presented here. 7/ All that needs understanding here is that with the

7/ But can be found in Hutton, Robert E., "An Introduction to Quadratic Programming," Department of Agricultural Economics and Rural Sociology, Pennsylvania State University, University Park, Pa. A.E. & R.S. No. 41, June 1963.

primal-dual structure, the objective function can be expressed in quadratic form.

APPLICATION OF THE PLANT SELECTION SUBSYSTEM

Because so many different plant sizes and types can be built, one must first narrow the choice to the most relevant sizes and types to be analyzed. Three factors are of special importance in this selection: (1) the structure of the market in which the site is located, (2) the current and projected demand levels at selected sites, and (3) the number of shifts operated. These criteria are further discussed below.

Structure. The market of the cooperating feed manufacturer was characterized by (1) many types of livestock, (2) grain surpluses, (3) intense competition, and (4) strategic proximity to ingredient sources. These characteristics implied a need for a general-line processing plant that could produce a large percentage of its feed as concentrates and deliver it both in bulk and bagged form. These needs arise where many types of livestock use many lines of feed; they also arise where grain is in surplus and farmers use concentrates to mix with their own grain. Because completely mixed concentrates are used in smaller quantity, much of the feed is bagged.

Projected Demands. A demand projection for the early 1970's was provided by the feed manufacturer and was projected forward to 1982 (the last year of the processing equipment's useful life) for the selected sites (table 4). The total projected demand to be satisfied was 272,384 tons.

Table 4.--Projected demand for sites considered, 1968-82

Year	Plant site		
	F	E	B
	Tons per year		
1968	42,337	45,817	30,139
1969	45,465	49,201	32,365
1970	48,593	52,585	34,591
1971	51,721	55,969	36,817
1972	54,849	59,353	39,043
1973	57,883	62,635	41,202
1974	63,570	68,772	45,092
1975	68,937	74,582	48,916
1976	74,304	80,391	52,739
1977	79,671	86,201	56,563
1978	83,249	90,074	59,112
1979	86,777	93,947	61,661
1980	90,355	97,820	64,210
1981	93,933	101,693	66,759
1982	98,378	104,000	70,006

Number of Shifts. Data for production facilities were based on an eight-hour shift. The cost function used in the PSS analysis was also based on a one-shift operation. 8/

Configuration Selection

The optimal configuration of the LDA analysis (I,S,F,E,B) was felt to meet the required criteria and was selected for further analysis by PSS. The procurement, processing, and distribution costs for this particular configuration were \$.54/ton less than those of the firm's existing configuration of plants (table 3). The analysis was initially limited to consideration of plants capable of producing 150,200, 250, or 300 tons of feed per eight-hour shift. A plant of smaller capacity would not meet the required production, even on a two-shift basis. The choice was subsequently narrowed to three plants with capacities of 200 tons per shift (52,000 tons/year), because these could satisfy demand on two-shift operation without running at extremely inefficient levels (table 4).

The output from the LDA permitted calculation of the production level needed at each site, and this information was used as input in the PSS subsystem to determine the cost of production. As an example of the results, a production level of 45,397 tons/year per shift yielded a cost of \$4.71 per ton at each plant.

Any number of combinations of cost functions could be considered and the results compared to find the least-cost configuration; but for illustrative purposes, this analysis shows how PSS is used to compare plant sizes and types and it indicates how the PSS subsystem is related to the overall LLS.

INVESTMENT FEASIBILITY ANALYZER

The IFA is used to compare configurations selected by LLS to determine which one provides the highest return on the capital employed. The IFA consists of four components: (1) A fixed capital request and funds employed schedule is used to provide all the information regarding the cost of an investment, (2) A cash flow analysis is used to compare the proposed investment cash flow against an existing operations cash flow, (3) A present value analysis is used to determine the present value of a stream of revenues for the proposed investment, (4) A determination of rate of return is used to compute the rate of return for the proposed investment by the discounted cash flow method.

8/ To conduct the PSS analysis, it is necessary to use a continuous function (single shift allows for this), but to meet the projected demand, the 52,000 ton/year plant must operate two shifts. Therefore, to make the PSS analysis, the projected demand was halved in order that cost functions for a single shift operation could be used. This is why the results should be generalized to a two-shift operation and why it may be somewhat limiting, because the second shift operation may not have the same cost function.

First, an investment opportunity request is prepared to determine the construction costs for the investment. Then a funds employed schedule is prepared to determine the additional working capital needed for the investment.

Second, a cash flow analysis is prepared. The cash flow analysis is an analysis of the proposed inflow of cash from both the proposed investment and the existing configuration. The inflows from both configurations are computed and the differences obtained.

Third, the present value analysis is calculated at a minimum acceptable interest rate set by the firm. The difference between the present value obtained and the cost of the investment indicates the discounted dollar gain or loss given the minimum rate of return specified.

Fourth, the actual discounted rate of return may be calculated and compared with the minimum acceptable rate.

Using either approach (3) or (4), the IFA can be used to determine the more efficient configurations from those selected by the LLS.

Results of IFA Analysis

The results of the IFA analysis are illustrated by evaluating the return to the capital employed in configuration (I,S,F,E,B) (table 2).

Assumptions of Analysis

The investment analysis was based on conservative estimates of future happenings. The factors on which the analysis is based can be divided into (1) projected sales, (2) construction costs, (3) fixed capital requirements, and (4) operating margins and costs.

Sales Projections. Sales were estimated to 1973 by the cooperating firm. These estimates indicated an annual increase of 20,000 tons. Because it is difficult to exploit the same market at a constant rate of increase in sales, the expansion in sales was decreased to an annual rate of 15,000 tons for the years 1974 to 1977 and 10,000 tons annually to 1982. This sales increase estimate was considered to be conservative.

Construction Costs. The analysis was based on the construction costs of building the three 52,000-ton plants mentioned earlier. The type of plant considered was a general-line plant--that is, one which can produce all lines of feed and distribute in both bulk and bag. The equipment, facility, and installation costs of a 200-ton/day general-line feed plant are summarized in table 5.

When this basic information was adjusted for salvage value of the replaced facility and the investment credit benefit on equipment, the net investment cost for the three new facilities approximated $2,000,000.

Table 5.--Equipment, facility, and installation costs for 200 tons per day general-line feed mill 1/

Item	Mill cost
	Dollars
Equipment	232,580
Facilities	259,690
Installation	179,715
Total	671,985

1/ Vosloh, C. J., "Costs and Economies of Scale in Feed Manufacturing," Marketing Research Report No. 815, ERS, U.S. Dept. Agr., 1968.

The depreciation costs for the new plants were based on data published by the U.S. Department of Agriculture, and a representative rate supplied by the cooperating firm was used for the old plants (table 6).

Table 6.--Depreciation costs for new plants

Item	Cost
	Dollars
Equipment	27,560
Building	10,400
Total	37,960

Fixed Capital Requirements.--The total $2,000,000 investment in fixed facilities was assumed to have been incurred in year zero for purposes of running the financial analysis. The differential between the cash flow in the original situation and the proposed situation were determined (see table 7). The difference represents the amount of returns which can be applied to repayment of the funds borrowed to construct the new facilities. With a knowledge of the returns over the 15-year life of the equipment, it is possible to determine the rate of return on the investment by discounted cash flow procedures.

Operating Margins and Costs.--In addition to the fixed costs involved in constructing the new facilities, additional working capital is required. In an investment feasibility analysis, only the additional requirements for working capital resulting from the new investment are applied to the new investment.

Table 7.--Comparative analysis of net profit for existing facilities, and proposed facilities

Year	Cash flow: Existing facilities	Cash flow: Proposed facilities	Difference
	-Dollars-		
1967	1,713,146.76	1/NA	--
1968	1,802,606.00	1,827,957.19	25,351.19
1969	1,908,206.04	1,980,527.18	72,321.14
1970	1,973,156.12	2,133,534.12	160,378.00
1971	1,973,156.12	2,285,412.09	312,255.97
1972	1,973,156.12	2,443,787.95	470,631.83
1973	1,973,156.12	2,593,004.43	619,848.31
1974	1,973,156.12	2,758,942.74	785,786.62
1975	1,973,156.12	2,823,788.05	850,631.93
1976	1,973,156.12	2,939,799.54	966,643.42
1977	1,973,156.12	3,055,299.31	1,082,143.19
1978	1,973,156.12	3,241,639.14	1,268,483.02
1979	1,973,156.12	3,209,595.91	1,236,439.79
1980	1,973,156.12	3,291,622.04	1,318,465.92
1981	1,973,156.12	3,364,166.11	1,391,009.99
1982	1,973,156.12	3,441,091.73	1,467,935.61
Total difference	--	--	12,028,325.93

1/ Not applicable. Proposed facilities were not contributing to profits in 1967.

Only sales in excess of what the old facilities were capable of producing can be applied to the new investment. The increased revenue figure was obtained by multiplying the increase in sales as a result of the new investment by the average selling price for the weighted mix of feed products produced ($110).

Results

A cash flow analysis of the original and new plant configuration was made. The difference between the two cash flow streams was calculated (table 8), and a present value analysis was computed to determine the present value of the future stream of cash inflow (tables 9, 10). The present value of the future stream was compared to the cost of the investment ($2,000,000) to determine whether or not the investment met the firm's 10 percent standard. The rate of return on the capital invested was also generated to determine whether it was more desirable than other alternatives for the capital funds invested by the firm.

Table 8.--Present value of stream of revenues for proposed investment

Year	Annual cash flow	Present value factor 1/	Present value at 10 percent
	Dollars	Percent	Dollars
1	25,351	0.909	23,044
2	72,321	.826	59,737
3	160,378	.751	120,443
4	312,256	.683	213,270
5	470,631	.621	292,261
6	619,848	.564	349,594
7	785,787	.513	403,108
8	850,632	.467	397,245
9	966,643	.424	409,856
10	1,082,143	.386	417,707
11	1,268,483	.350	443,969
12	1,236,440	.319	394,424
13	1,318,466	.290	382,355
14	1,391,010	.263	365,835
15	1,467,936	.239	350,836
Total present value of stream of revenues	--	--	4,623,690

1/ A 10 percent rate of return is assumed.

Table 9.--Present value of additional receivables and cash used to cover working capital of investment

Item	Funds returned to firm at end of project life	Present value factor 1/	Present value at 10 percent
	Dollars	Percent	Dollars
Cash	362,402	0.239	86,614
Receivables	1,812,000	.239	433,068
Inventories	628,000	.239	150,092
Salvage value			
Nondepreciation tax credit	138,679	.239	33,144
Land	150,000	.239	35,850
Equipment	66,000	.239	15,774
Total	3,157,081	--	754,542

1/ A 10 percent rate of return is assumed.

Table 10.--Determination of rate of return by discounted cash flow method

Year	Annual cash inflow	Present value factor at 22 percent	Present value at 22 percent	Present value factor at 24 percent	Present value at 24 percent
	Dollars	Percent	Dollars	Percent	Dollars
1	25,351	0.820	20,787	0.806	20,433
2	72,321	.672	48,599	.650	47,009
3	160,378	.551	88,368	.524	84,038
4	312,256	.451	140,827	.423	132,084
5	470,631	.370	174,133	.341	160,485
6	619,848	.303	187,813	.275	170,458
7	785,787	.249	195,660	.222	174,445
8	850,632	.204	173,528	.179	152,263
9	966,643	.167	161,429	.144	139,197
10	1,082,143	.137	148,253	.116	125,529
11	1,268,483	.112	142,070	.094	119,237
12	1,236,440	.092	113,752	.076	93,969
13	1,318,466	.075	98,884	.061	80,426
14	1,391,010	.062	86,242	.049	68,159
15	1,467,936	.051	74,864	.040	58,717
Discounted cash flow	--	--	1,855,218	--	1,626,449
	-Discounted cash flow from facilities at end of project life-				
15	3,157,081	.051	161,011	.040	126,283
Total discounted cash flow	--	--	2,016,229	--	1,752,732

To obtain the rate of return on an investment by the discounted cash flow method, alternative discount rates were examined by sequential methods to discount the inflow of funds back to the present (table 10). Through sequential analysis and use of interpolation, the rate of return was calculated to be 22.07 percent.

CONCLUSIONS

This study has shown that the development and use of a mathematical system to analyze location-logistics problems is feasible. Potential improvements in profits more than justify the time and cost involved in developing the system. The case firm analysis suggested that the optimum location-logistics configuration would save 8 to 10 percent of the investment in new facilities annually.

Applying the system to other feed manufacturers is both feasible and practical, and the mathematical design of the system is consistent with the current structure of firms within the feed industry.

The LLS is a systematic analysis providing superior location-logistics guides which can accompany management judgment in making decisions. Through several simulations, more appropriate findings can be obtained. With these evaluations, the system supplements traditional analytical procedures.

Specific Uses of the LLS

A. Selecting

(1) Specific sites at which to locate facilities.
(2) The optimum set of site configurations.
(3) The proper sizes and types of plants to construct.
(4) The sources from which ingredients should be purchased.
(5) The destinations to which feed is delivered from given plants.
(6) The optimum channels of procurement and distribution.
(7) The configuration of plants that will best serve the feed manufacturers' changing procurement, production, and distribution patterns and provide the highest rate of return on the investment involved.

B. Use as

(1) A guide in making long-range planning decisions on procurement, processing, and distribution problems.
(2) An operational control system.
(3) An aid in arriving at the optimum procurement strategy.
(4) An aid in determining the optimum distribution system required to meet market penetration goals of the firm.
(5) An aid in analyzing the interplay between transportation modes to determine the most economical distribution pattern and optimum use of transportation modes.

MATHEMATICAL APPENDIX

The purpose of this appendix is to verify that the procedure used in the PSS is valid. It intends to show that instead of minimizing the total cost function, it is legitimate to use the average total cost function in determining the optimum plant size and type to satisfy a given product demand. The average total cost function is of the following form:

$$ATC_i = a_i + b_iQ_i + C_iQ_i^2 \qquad i=1,---,n \qquad (E.1)$$

subject to $q_i \geq 0$

where ATC_i = average total cost

Q_i = quantity produced in plant "i"

and

$$a_i,\ c_i > 0 > b_i.$$

The objective function to be minimized is:

$$G = \sum_i ATC_i + \lambda(\sum_i Q_i - Q) \qquad (E.2)$$

Q = total quantity produced.

Taking the partial derivatives of G with respect to Q_i and λ the following equations result:

$$\frac{\partial G}{\partial Q_i} = b_i + 2c_iQ_i + \lambda = 0; \quad i=1,---,n \qquad (E.3)$$

$$\frac{\partial G}{\partial \lambda} = \sum_i Q_i - Q = 0. \qquad (E.4)$$

Taking the total cost function, on the other hand,

$$TC_i = a_iQ_i + b_iQ_i^2 + C_iQ_i^3 \quad i=1,---,n \qquad (E.5)$$

subject to $Q_i \geq 0$

where: TC = total cost

Q_i = quantity produced in plant "i"

and $a_i;\ c_i > 0 > b_i.$

The following objective function to be minimized is obtained:

$$H = \sum_i TC_i + \lambda^* (\sum_i Q_i - Q) \qquad (E.6)$$

and taking the partial derivatives of H with respect to Q_i and λ^*.

The following equations result:

$$\frac{\partial H}{\partial Q_i} = a_i + 2b_iQ_i + 3c_iQ_i^2 + \lambda^* = 0; \quad i=1,---,n \qquad (E.7)$$

$$\frac{\partial H}{\partial \lambda^*} = \sum_i Q_i - Q = 0. \qquad (E.8)$$

Clearly conditions (E.4) and (E.8) for the minimization of average and total costs are the same.

We can write λ^* in terms of λ as:

$$\lambda^* = K\lambda. \qquad (E.9)$$

Hence it follows that conditions (E.3) and (E.7) will be the same if

$$a_i + 2b_iQ_i + 3c_iQ_i^2 = Kb_i + 2Kc_iQ_i; \quad i=1,---,n \tag{E.10}$$

or

$$(Kb_i - a_i) + 2(Kc_i - b_i)Q_i + 3c_iQ_j^2 = 0 \quad i=1,---,n. \tag{E.11}$$

Equation (E.11) represents a quadratic relationship for each plant.

If K is negative, this indicates that the plants are still operating under conditions of average falling cost, and at most one plant should be used.

If K is positive, then writing (E.11) in the form:

$$A_{Qi}^2 + B_{Qi} + C = 0 \tag{E.12}$$

where

$$A = 3c_i > 0,$$

$$B = (Kc_i - b_i) > 0, \text{ and}$$

$$C = (Kb_i - a_i) < 0$$

it can be seen that $4AC < 0$ in the square root formula:

$$Q_i = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{E.13}$$

and (E.11) has positive and negative real roots. The positive root corresponds to the output at which both (E.3) and (E.7) hold.

Thus, given that total overall output is fixed, it is legitimate to minimize total cost by minimizing average total cost. This permits the use of quadratic rather than cubic expressions.

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