Minimum variance hedging: Levels versus first differences

Sören Prehn

Abstract

Nowadays it is widely accepted to estimate minimum variance hedge ratio regressions in first differences. There are both statistical and economic reasons for a first difference approach. However, no study has ever analyzed whether the first difference approach is also consistent with the theory of minimum variance hedging. In this paper we show, on the basis of a simulation study, that the first difference model with intercept does not provide hedge ratio estimates that are in line with the theory of minimum variance hedging. Only a linear regression model in levels provides theoretically consistent results.

Keywords: *Minimum variance hedging, level regression, first differences, hedge ratio.*

JEL classifications: *Q11, Q13.*

1. Introduction

Nowadays, minimum variance hedging (Johnson, 1960; Stein, 1961) is probably one of the best-known concepts in the literature on agricultural futures markets. Less well known, however, is probably that it took more than twelve years for the approach to become established in the literature. The approach only became a standard after Heifner (1972) showed that the minimum variance hedge ratio (i.e., the hedge ratio that minimizes the cash price risk for a hedger) can also be obtained from a linear regression model. One would only have to regress the cash price of a commodity on its futures price, and the slope parameter would then correspond to the minimum variance hedge ratio originally derived by Johnson (1960).

The exact formula of the minimum variance hedge ratio is:

$$\rho_{cf} * \frac{\sigma_c}{\sigma_f}$$

where ρ_{cf} defines the correlation coefficient of the cash and futures price; and, σ_c and σ_f the corresponding standard deviations of the cash and futures price.

Although, the minimum variance hedge ratio regression was initially also estimated in levels (e.g., Ederington, 1979; Berck, 1981; Dale, 1981), this changed quite quickly. Nowadays it is common practice to estimate the minimum variance hedge ratio regression in first differences (e.g., Park and Antonovitz, 1990; Brorsen et al., 1998; Brinker et al., 2009). There are both statistical and economic reasons for a first difference approach. For instance, first differences help to exclude the possibility of a spurious regression (Brown, 1985), and leading agricultural economists (Working, 1953; Peck, 1975) argued that hedgers (i.e., grain merchants or farmers) are more interested in the correlation of price changes than price levels.

However, no study has ever analyzed whether the first difference approach is also compatible with the theory of minimum variance hedging (Johnson, 1960; Stein, 1961). In this paper, however, we want to show on the basis of a simulation study that the last point is quite important, since the first difference model with intercept does not provide hedge ratio estimates that are consistent[1] with the theory of minimum variance hedging. As our results will show, only a linear regression model in levels provides hedge ratio estimates that are in line with the theory of minimum variance hedging.

In the following we will further analyze the last point. We will use a simple simulation study to show that only a linear regression model in levels provides hedge ratio estimates that are consistent with the theory of minimum variance hedging. Finally, we will conclude.

2. Levels versus first differences

For the sake of simplicity, we have limited our simulation study to a single futures contract. We decided to focus on an inverse market[2], as Johnson (1960) did in his original article. However, our results also apply to a carry market.

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[1] When we talk about consistent in the following, we always mean consistent from a theoretical point of view and not from a statistical point of view.

[2] The term inverse market refers to a market situation in which the cash price is higher than the futures price. Inverse markets are known for discounting storage. In fact, short hedgers (i.e. those with a short position on the futures market) would suffer a hedging loss on an inverse market. The opposite of an inverse market is a carry market. Carry markets renumerate storage (Hieronymus, 1977).

For the simulation study, we first created a single cash price series of length 21. We assumed that the cash price would follow a random walk with a mean of 94.75 and a variance of 0.05. We also assumed a drift of 0.25. We repeated the last procedure 100,000 times. In the end, we received 100,000 randomized cash price series. We used a similar approach to construct the basis[3] series. We assumed that the basis would follow a random walk with drift, where the mean is -5.25 and the variance is 0.0025. The drift is again 0.25. Again, we repeated the procedure 100,000 times to obtain 100,000 randomized basis series. The final step was to create the futures price series. Here we always added a cash price series with a basis series to get 100,000 futures price series. See Appendix A for the program code.

To have a reference point, we first calculated the minimum variance hedge ratio using the above formula of Johnson (1960). The average minimum variance hedge ratio for all 100,000 simulations is 0.4982. As expected, this result is exactly the same as the result of a minimum variance hedge ratio regression in levels, see Table 1.[4]

Table 1

Comparison level versus first difference regression (algebraic solution = 0.4982)

	Levels	First differences with intercept
MVHR	0.4982	0.9521
Gross profit	-0.0308	-4.5206

Notes: Calculated averages for 100,000 simulations. MVHR is the abbreviation for minimum variance hedge ratio.

[3] Technically, the basis corresponds exactly to the difference between the cash price and the futures price (i.e., basis = cash price – futures price). If the cash price is relatively low compared to the futures price, the basis is called weak. In the opposite case, the basis is described as strong.

[4] Simulation results for a random walk without drift are shown in Appendix B.

More interesting, however, are the results for the first difference model. As Table 1 shows, the average minimum variance hedge ratio for the first difference model with intercept (column 3) is 0.9521. This value obviously not only differs significantly from the algebraic solution (the level regression) with 0.4982, but also comes (very) close to a full hedge.[5] In fact, if we had not assumed a random walk with drift for the basis but a deterministic trend, we would have obtained a value of one, i.e. a full hedge.

However, a full hedge only minimizes the cash price risk and not the basis risk (Hieronymus, 1977). The basis risk, however, was the main reason why Johnson (1960) originally developed the minimum variance hedge approach. In the 1950s, the New York coffee market, which was mainly investigated by Johnson (1957a,b), was chronically inverse[6], and the majority of the New York coffee importers complained about the seasonally weakening basis, which gave them regular hedging losses as short hedgers; because of the weakening basis, the coffee importers could not hedge without having to buy coffee in Brazil for a stronger basis and resell it in New York for a weaker basis (Gray, 1960a). On the other hand, the coffee importers could not do without hedging either, since the U.S. banks always required a hedge as security for the interim financing of a forward transaction (Gray, 1960b). Johnson's main concern, therefore, was to develop a new hedging approach to minimize not only the cash price risk but also the basis risk causing the hedging losses.

As Table 1 shows, the first difference model with intercept is in clear contradiction to Johnson’s efforts to also minimize the basis risk and thus the hedging losses. If anything, the model only minimizes parts of the basis risk, but not the entire basis risk. In fact, in the concrete example, the New York coffee importers would still make an average hedging loss of -4.5206 (column 3) due to the only partially hedged basis risk.[7] Consequently, we can conclude (at least on the basis of the simulation results) that the first difference model with intercept is not consistent with the theory of minimum variance hedging.

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[5] In fact, hedge ratios in the literature are usually close to one when first differences are used (Brown, 1985). The general rule is that the higher the correlation between the cash price and the futures price, the closer the hedge ratio is to one.

[6] Only once between 1950 and 1957 was the New York coffee market not inverse, compare Figure 2 in Gray (1960b).

[7] For comparison, the full hedge would have caused a hedging loss of -5.

The last point, however, can also be further substantiated econometrically. When two prices (such as the cash price and the futures price) converge[8], the drift between the two prices is known to be captured by the intercept in a first difference model (Enders, 2014). In the concrete example, however, the drift corresponds to the basis, more precisely to the average change in the basis (a proof of this can be found in Appendix C). However, if the non-random, deterministic part of the basis risk (i.e., the average basis change) is already captured by the intercept, the slope parameter of the first difference model only provides a hedge ratio that minimizes the cash price risk and the random basis risk, but not the deterministic basis risk. Johnson's main concern, however, was never to minimize only the random basis risk but also the deterministic basis risk (i.e., the chance in the basis), especially since the deterministic basis risk was primarily responsible for the hedging losses the New York coffee importers experienced in the 1950s.

As it turns out, the only approach that is consistent with Johnson's theory of minimum variance hedging, is the minimum variance hedge ratio regression in levels. Only the level regression minimizes the total basis risk and thus the hedging losses (see Table 1, column 2), which were the main problem of the New York coffee importers in the 1950s (Gray, 1960a,b).[9]

However, we have so far left out one important point, namely the measurement of hedging effectiveness. In the literature it is common practice to use the adjusted R-squared to measure hedging effectiveness (Ederington, 1979). According to Ederington, the R-squared measures the percentage of the cash price risk that is offset by a minimum variance hedge. For instance, in the concrete example, the level regression would offset 98.93 percent of the cash price risk, whereas the first difference model with intercept would only offset 94.70 percent, see Table 2.

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[8] If the cash and futures price did not converge at contract maturity, arbitrageurs could make a risk-free profit. They could buy on the market with the lower price and sell on the market with the higher price (Hieronymus, 1977).

[9] In order to avoid a spurious regression, we recommend always testing for cointegration in advance. If the latter is given, there should be no statistical problems with a level regression. The slope parameter should measure the long-run minimum variance hedge ratio.

Table 2

Comparison hedging effectiveness cash price risk versus basis risk

Hedging effectiveness cash price risk		Hedging effectiveness basis risk	
Levels	First differences with intercept	Levels	First differences with intercept
98.93 %	94.70 %	99.38 %	9.59 %

Notes: Calculated averages for 100,000 simulations.

Instead of the R-squared, we would propose an alternative measure for measuring the hedging effectiveness. We would propose to focus on the basis risk rather than the cash price risk. The basis risk had been the main problem of the New York coffee importers, and unfortunately the R-squared does not provide specific information on how much of the basis risk is offset by the respective minimum variance hedge regression. Alternatively, we would propose following measure of hedging effectiveness:

$$\frac{gross\ profit\ full\ hedge - gross\ profit\ minimum\ variance\ hedge}{gross\ profit\ full\ hedge}$$

Our approach takes advantage of the fact that there is a one-to-one relationship between the basis risk and the gross profit of a hedge. For instance, it is known that a full hedge only minimizes the cash price risk, but not the basis risk (Hieronymus, 1977). Therefore, the gross profit of the full hedge can be directly attributed to the unhedged basis risk. Similarly, the gross profit of the first difference model can be attributed to the unhedged non-random, deterministic basis risk (see above). In the way the measure is constructed, the numerator always corresponds to the hedged basis risk, and the denominator to the total basis risk (i.e., the random and deterministic basis risk).

In contrast to the R-squared, our measure measures how much of the basis risk is offset by the respective minimum variance hedge regression. For example, the level regression offsets 99.38 percent of the basis risk, whereas the first difference model with intercept only offsets 9.59 percent of the basis risk, see Table 2. Unlike the R-squared, our new measure clearly shows what the main problem of the first difference model is: The first difference model only partially minimizes the basis risk, which is in clear contradiction to Johnson's theory of minimum variance hedging.

The previous results are quite interesting, as they challenge the common practice of estimating minimum variance hedge ratio regressions in first differences (e.g., Brorsen et al., 1998; Brinker et al., 2009). As we have shown above, first differences are clearly in contradiction with the theory of minimum variance hedging. If minimum variance hedge ratios are really of interest, then there is no other way than to estimate the minimum variance hedge ratio regression in levels.[10] First differences would not provide hedge ratio estimates that are consistent with the theory of minimum variance hedging.

Our results, however, should not be misunderstood. We do not deny that there may be situations in which hedgers want to minimize only the random basis risk, but not the deterministic basis risk. In these situations, the first difference approach is clearly the right choice. Our point is, when the first difference approach is used for hedging, it should not be called a minimum variance hedge, which it clearly is not, but a random basis risk hedge. In particular, potential users of the hedge should be made aware that they are still exposed to a significant basis risk that may or may not work out in their favor.

3. Conclusions

In this paper we have investigated whether it is theory conform to estimate the minimum variance hedge ratio regression in first differences, as it is common in the literature. Interestingly, we were able to show that the first difference model with intercept does not provide hedge ratio estimates that are compatible with the theory of minimum variance hedging (Johnson, 1960). The first difference model minimizes only the random basis risk, but not the total basis risk. The total basis risk, however, was actually the original reason for Johnson to develop the minimum variance hedge approach. The only approach that provides hedge ratio estimates consistent with the theory of minimum variance hedging is a linear regression model in levels.

[10] Note that to estimate the minimum variance hedge ratio consistently, it is necessary to use spread-adjusted data (i.e., spread-adjusted continuous futures and cash prices). Non-adjusted data would deliver inconsistent estimates (Prehn, 2020).

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Appendix A

We used the software R for our simulation study.

```
### Simulation study ###

set.seed(123)


### Specify random walk function ###

RW <- function(N, x0, mu, variance) {

  z <- cumsum(rnorm(n = N, mean = 0, sd = sqrt(variance)))
  t <- 1:N
  x <- x0 + t * mu + z
  return(x)
  }


### Construction of vector ###

n <- 100000

vec_full_hedge_profit <- rep(NA, n)

vec_mvhr <- rep(NA, n)

vec_coef_levels <- rep(NA, n)
vec_levels_adj_r2 <- rep(NA, n)
vec_levels_profit <- rep(NA, n)

vec_coef_differences_with <- rep(NA, n)
vec_differences_with_adj_r2 <- rep(NA, n)
vec_differences_with_profit <- rep(NA, n)


### For loop ###

for(i in 1:n){

        ### construction futures price ###

        drift <- 0.25
        cash_price <- RW(21, 94.75, drift, 0.05)

        ### construction basis ###

        basis <- RW(21, -5.25, 0.25, 0.0025)

        ### construction cash price ###
```

```r
        futures_price <- cash_price + basis

        ### Calculating profit full hedge ###

        vec_full_hedge_profit[i] <- -(cash_price[1] - futures_price[1]) + (cash_price[21] - futures_price[21])


        ### Calculating MVHR algebraically ###

        vec_mvhr[i] <- cor(cash_price, futures_price) * (sd(cash_price)/sd(futures_price))

        ### level regression ###

        fit_levels <- lm(cash_price ~ futures_price)
        vec_coef_levels[i] <- coef(fit_levels)[2]
        vec_levels_adj_r2[i] <- summary(fit_levels)$adj.r.squared

        ### profit level regression ###

        fit_aux <- lm(I(diff(cash_price) - (coef(fit_levels)[2] * diff(futures_price))) ~ 1)
        vec_levels_profit[i] <- coef(fit_aux)[1] * 20

        ### first difference regression, with intercept ###

        fit_differences_with <- lm(diff(cash_price) ~ diff(futures_price))
        vec_coef_differences_with[i] <- coef(fit_differences_with)[2]
        vec_differences_with_adj_r2[i] <- summary(fit_differences_with)$adj.r.squared

        ### profit first difference regression, with intercept ###

        vec_differences_with_profit[i] <- coef(fit_differences_with)[1] * 20
}


### Calculating average values ###

### profits ###

mean(vec_levels_profit)
mean(vec_differences_with_profit)
mean(vec_full_hedge_profit)


### MVHRs ###

mean(vec_coef_levels)
mean(vec_coef_differences_with)


### Adjusted R2 ###

mean(vec_levels_adj_r2)
mean(vec_differences_with_adj_r2)
```

```
### Hedging effectiveness basis risk ###

mean( (vec_full_hedge_profit - vec_levels_profit)/vec_full_hedge_profit )
mean( (vec_full_hedge_profit - vec_differences_with_profit)/vec_full_hedge_profit )
```

Appendix B

Table B1

Comparison level versus first difference regression (algebraic solution = -0.0129)

	Levels	First differences with intercept
MVHR	-0.0129	0.9520
Gross profit	-0.1423	-4.7603

Notes: Calculated averages for 100,000 simulations. MVHR is the abbreviation for minimum variance hedge ratio.

Appendix C

In principle it is quite easy to prove that the intercept parameter of the first difference model captures the average change in the basis. For this purpose, first look at the definition of the intercept parameter estimator $\hat{\alpha}$:

$$\hat{\alpha} = \overline{\Delta p}_{c,t} - \hat{\beta}\,\overline{\Delta p}_{f,t},$$

where $\overline{\Delta p}_c$ and $\overline{\Delta p}_f$ correspond to the mean value of the cash price and the futures price in first differences. $\hat{\beta}$ is the estimator of the slope parameter. And, the index t = 1, …, n, where n is the number of observations.

This definition can be further simplified by assuming that $\hat{\beta} = 1$ and the definitions of the differenced cash and futures price are included. The definition of $\hat{\alpha}$ then is

$$\hat{\alpha} = \bar{p}_{c,t} - \bar{p}_{c,t-1} - \left(\bar{p}_{f,t} - \bar{p}_{f,t-1} \right).$$

Furthermore, if one considers the definition of the basis, $b = p_c - p_f$, and rearranges the former definition, then

$$\hat{\alpha} = \bar{p}_{c,t} - \bar{p}_{f,t} - (\bar{p}_{c,t-1} - \bar{p}_{f,t-1}) = \overline{\Delta b}_t.$$

According to this definition, the intercept parameter α captures the average change of the basis over the time period of the hedge.