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Representations of Multi-Attribute Grain Quality

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Grain quality is typically measured via several attributes. As these attributes vary across shipments and time, grain quality can be described using multivariate probability or frequency distributions. These distributions are important in modeling blending opportunities inherent in various grain shipments. For computational reasons, it is usually necessary to represent these distributions with a small set of discrete points and probabilities. In this analysis, we suggest a representation method based on Gaussian quadrature. This approach maintains the blending opportunities available by preserving moments of the distribution. The Gaussian quadrature method is compared to a more commonly used representation in a barley blending model.

Key words: cubature, Gaussian quadrature, grain blending, grain quality

Introduction

Blending models provide useful insight into the economics of grain quality. In a 1976 study, Ladd and Martin used a blending model to derive the implicit values of corn attributes and to analyze alternative grading systems. Wilson and Preszler, in a more recent work, applied a blending analysis to wheat import decisions, focusing on the impact of quality uncertainty. Other studies (such as Johnson and Wilson) have employed regional crop quality data to characterize the blending opportunities of wheat merchandisers. Clearly, blending is an important commercial function, driven by the heterogeneity of grain supplies, quality specifications of end-users, and premiums and discounts for specific quality attributes.

The potential value of blending depends largely on the variability of quality attributes. For example, wheat used for milling has several quality attributes of importance. One of these is protein. Low-protein wheat is normally priced at a discount relative to high-protein wheat, and elevators enhance their margins through careful blending operations. It stands to reason that higher variability in quality attributes would result in greater blending opportunities and, depending on premiums and discounts, higher net revenues for grain handlers. Similarly, end-users of grain can employ blending to reduce acquisition costs. By buying lower quality grain at a discount and blending it with higher quality grain, they may be able to meet quality specifications at a lower cost than by purchasing grain that meets specifications.

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One difficulty in modeling and quantifying the economics of grain blending is representing the distributions of quality attributes. For any given commodity, there may be several attributes that are important to potential purchasers and end-users. Because these attributes vary across shipments and growing seasons, they are best described by a joint probability or frequency distribution. While a researcher may have a large data set with observations on attribute quality, it may be necessary to represent the joint distribution of quality attributes with a small number of points due to computational difficulties (such as the curse of dimensionality). For example, Johnson and Wilson summarize the joint distribution of seven jointly distributed wheat attributes using 10 representative points ("bins") and their associated frequencies ("bushels").

The method of choosing representative points can affect the blending opportunities available. Johnson and Wilson lump wheat shipments of similar quality into a single representative point by averaging the quality across wheat shipments. Our purpose in this investigation is to present an alternative method of representing the joint distributions of grain quality attributes. This alternative method is based on a numerical integration technique called Gaussian quadrature (GQ). The method we propose preserves the blending opportunities inherent in a given set of grain shipments. We compare the GQ approach with the approach used by Johnson and Wilson and other authors. The GQ method is shown to result in lower costs to an end-user who has minimum quality specifications. The GQ method is applied here to the malting barley industry.

Representations of Joint Distributions

Gaussian quadrature is a numerical integration technique that has been advocated by Miller and Rice, and by Preckel and DeVuyst to represent univariate probability distributions. DeVuyst, Preckel, and Liu (DPL) extend the method to apply to any multivariate discrete probability distribution and a large number of multivariate continuous probability distributions. The GQ[1] method involves choosing a discrete set of points and probabilities having lower-order moments that match the moments of the distribution to be approximated or represented. For discrete probability distributions, the representative points can be chosen as a subset of the points of the original distribution.

The DPL method of generating multivariate GQ approximations is as follows. First, compute the lower-order moments about zero, including cross-moments, of the distribution to be approximated. Second, set up a system of linear equations, as in (1), where the left-hand sides are the moments (including cross-moments) about the origin (of order D or less) of the representative distribution and the right-hand sides are the values found in the first step. The X_i's in (1) are the random variables, and the x_{ij}'s are observations of the random variables. In our case, the x_{ij}'s are the points of the original distribution. The p_i's are probabilities and will be solved for in a later step.

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[1] In the numerical integration literature, "quadratures" are a class of numerical integration techniques used with only one independent variable. "Cubatures" are used for multiple independent variables. The method we employ would more properly be called "Gaussian cubature." However, we retain the "quadrature" term as many agricultural economists are familiar with the univariate "Gaussian quadrature" method. The method developed by DPL and employed here is a multivariate extension of the univariate Gaussian quadrature.

$$\sum_i p_i * \prod_j x_{ij}^{d_j} = E\left[\prod_j X_j^{d_j}\right], \quad \forall \sum_j d_j \le D. \tag{1}$$

The moments to be preserved are equal to or lower than some pre-specified order D. Since D determines the order of the approximation,[2] an approximation with $D = 2$ matches all moments through order 2, i.e., means, variances, and covariances. The number of equations in the system (1) is a function of the number of random variables and the moment order to be preserved. So if there are S random variables and D moment orders to be preserved, the number of equations to be solved in (1) is given as C_D^{S+D} (Haber).

The d_j's are exponents on the individual random variables and are set so that their sum is less than or equal to D. The d_j's take only integer values. Because (1) is a system of equations, all combinations of the d_j's that sum to a value less than or equal to D are considered. This includes $d_j = 0 \; \forall j$, which requires the sum of probabilities to equal one. For example, if there are two random variables, X_1 and X_2, and we want to approximate their joint distribution with a second-order approximation (i.e., $D = 2$), the (d_1, d_2) pairs take the values $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, $(0,2)$, and $(2,0)$. These values ensure the sum of probabilities equals one, preserve the mean of X_2, preserve the mean of X_1, preserve the cross-product of the two random variables, preserve the second moment about zero of X_2, and preserve the second moment about zero of X_1, respectively.

Last, as (1) is linear in probabilities (p_i), we set up a linear program, without an objective, and solve (1) for the probabilities using an extreme-point solution method, such as the Simplex method. A feasible solution is guaranteed to exist (Tchakaloff). This solution need not be unique. Often, this approach will result in a significant reduction in the number of points with nonzero probability (DeVuyst, Preckel, and Liu).

For the grain-blending issue we are addressing, a second-order (i.e., $D = 2$) approximation is used. Because the variability of grain quality attributes is very important in determining blending opportunities, an approximation of at least order 2 is required. Higher-order approximations are not used to avoid computational difficulties associated with the curse of dimensionality. As discussed below, our model considers four jointly distributed random variables in each time period. The DPL method to generating a second-order GQ approximation for four variables employs 15 points. The number of points employed increases geometrically with the number of jointly distributed variables and the number of moment orders preserved.

The GQ approach is used later in comparison with the conditional mean (CM) approach—the technique used by Johnson and Wilson. The CM approach proceeds as follows. The data are ranked and divided into intervals, with the average of the attributes of the points in the interval used as representative points. Successive rankings are necessary for multiple-attribute commodities. The weight of each representative point is equal to the proportion of observations assigned to the interval. Equally likely intervals are often used.

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[2] The researcher chooses the order of the approximation (i.e., D). While the choice is arbitrary, it is partially governed by how many points can reasonably be evaluated in the economic model and how many moment orders are likely to be of critical importance. In the present context, we clearly need to preserve through at least the second order to capture the blending opportunities.

The CM method has the advantage that it is very easy to generate the representative points and their weights. The CM approach also preserves the means of the original distribution. However, Miller and Rice demonstrate that the CM method biases downward all even-numbered moments, including variances. For the issue at hand, variance is critical in determining blending opportunities. By understating variances, the CM procedure understates blending opportunities and therefore understates the economic advantages of blending. In contrast, GQ approximations by construction exactly preserve the variances of the quality attributes. Higher-order, even-numbered moments may or may not be preserved depending on the order of the GQ approximation.

Malting Barley Quality Attributes

Quality factors are extremely important in the malting barley industry. Since 1993, concerns about crop quality have been heightened by an extended outbreak of fusarium head blight (FHB) in major growing regions of the upper Midwest. FHB is a fungal disease that reduces yields and lowers crop quality. Of particular concern is a chemical by-product of FHB, deoxynivalenol (DON), also known as vomitoxin. DON is water-soluble and heat-stable, so it survives throughout the malting and brewing process. Malt contaminated with DON can create "gushing" (excessive foaming when beer is opened or poured) problems in beer. For that reason, barley with detectable levels of DON—practically, more than 0.5 parts per million (ppm)—is heavily discounted.

DON is an example of a quality attribute that can vary substantially within a single crop year and over time. Other important attributes for the malting and brewing industry are the percentage of plump kernels, test weight (lbs./bu.), and protein levels (%). For each of these, commercial discounts apply at local elevators when the barley delivered by farmers does not meet industry specifications. (Refer to tables 1 and 2 for discounts on DON, protein, and plump.)

Production of six-rowed malting barley, the type preferred by U.S. brewers, is highly variable, and declining. Acres planted to six-rowed malting barley in the three Midwestern states of North Dakota, Minnesota, and South Dakota fell by more than 50% between 1993 and 1999 due to the FHB epidemic and low prices [U.S. Department of Agriculture/National Agricultural Statistics Service (USDA/NASS)]. In the event production of barley of sufficient quality is not produced in the U.S. or available from storage, maltsters must import barley from Canada. Canadian barley is sold through the Canadian Wheat Board, a single-desk seller. Although not explicitly modeled in this analysis, reliance on Canada as a barley supplier exposes the malting industry to risks associated with noncompetitive pricing.

Notation

The notation used in the model below is as follows. Subscripted numbers denote the time period. For example, $Begstk_0$ denotes the beginning or initial period stock of barley. Indices appearing in parentheses are used to denote states of quality or price. For example, $Domestic_2(j,k,p)$ denotes domestic purchases of barley of quality type j given that quality distribution k and price state p are observed. All other notation is defined below:

• $Begstk_0(l)$	initial stocks of barley of quality l;
• $Demand$	annual quantity of malting barley demanded by industry;
• $Discount(\cdot)$	discounts applied to barley with quality $(\cdot)$;
• $Domestic_1(j)$	purchases of domestic barley in period 1 of quality j;
• $Domestic_2(j,k,p)$	purchases of domestic barley in period 2 of quality j given price outcome p and quality distribution k are observed;
• $Domstrd_1(j)$	period 1 domestic barley purchased for storage of quality j;
• $Domstrd_2(j,k,p)$	period 2 domestic barley purchased for storage of quality j by state;
• $Domprod_1(j)$	period 1 domestic barley production of quality j;
• $Domprod_2(j,k,p)$	period 2 domestic barley production of quality j by state;
• $DON(j)$	ppm of DON in barley of j quality;
• $E[\cdot]$	mathematical expectations operator;
• $Endstk_1$	period 1 total ending stocks of domestic barley;
• $Endstk_2(j,k,p)$	period 2 total ending stocks of domestic barley by quality and state;
• $Imports_1$	period 1 imports of Canadian malt barley;
• $Imports_2(k,p)$	period 2 imports of Canadian malt barley by state;
• $Importstrd_1$	period 1 imports stored;
• $Importstrd_2(k,p)$	period 2 imports stored by state;
• $Imputil_1$	period 1 imports utilized;
• $Imputil_2(k,p)$	period 2 imports utilized by state;
• $Plump(j)$	percent plump of barley of j quality;
• $Price_0$	price of initial stocks of barley;
• $Price_1$	period 1 price of malt barley before discounts;
• $Price_2(p)$	period 2 price of malt barley by price p;
• $Prob(k,p)$	joint probability of realizing quality distribution k and price state p;
• $Prot(j)$	percent protein in barley of j quality;
• r	discount rate;
• $Strchg$	storage charge for barley stored across time periods;
• $Trans$	cost of transporting barley from Canada to Minneapolis;
• $tw(j)$	test weight of barley of j quality;
• $Util_{0,1}(l)$	initial stocks of barley of quality l utilized in period 1;
• $Util_{0,2}(l,k,p)$	initial stocks of barley of quality l utilized in period 2; and
• $Util_{1,2}(j,k,p)$	barley of quality j stored in period 1 and utilized in period 2.

The Model

We develop a model of the U.S. malt industry to demonstrate the importance of preserving variability in attribute quality by comparing the GQ method to the CM method. Purchases of domestic barley, imports, and expected costs are compared when the joint distribution of quality attributes for malting barley are approximated or represented using GQ and CM.

For simplicity, we represent the U.S. malt industry as a single buyer of U.S. six-rowed malting barley. The industry's objective (2) is to minimize the expected discounted cost of meeting the U.S. demand for malt:

$$
\begin{aligned}
(2)\quad \mathrm{Min}\ E[Cost] = {} & \\
& \sum_{l} Begstk_0(l) * \big(Price_0 - Discount(l)\big) + \sum_{j} Domestic_1(j) \\
& * \big(Price_1 - Discount(j)\big) + Imports_1 * \big(Price_1 + Trans\big) + \frac{1}{1+r} \\
& * \Bigg(\sum_{k} \sum_{p} Prob(k,p) * \bigg(\Big(\sum_{j} \big(Domestic_2(j,k,p) - Endstk_2(j,k,p)\big) \\
& \qquad\qquad * \big(Price_2(j,k,p) - Discount(k)\big) + Imports_2(k,p) \\
& \qquad\qquad * \big(Price_2(p) + Trans\big) - Importstrd_2(k,p) * Price_2(p)\Big)\bigg) \\
& \quad + \big(Endstk_1 + Importstrd_1\big) * Strchg \Bigg).
\end{aligned}
$$

A two-year time horizon is assumed. Initially, the industry's beginning stocks of malting barley are assumed to equal twice the annual quantity demanded for malting barley. The joint distribution of attributes of the beginning stocks is assumed to be known and later varied. In the first (or current) year, the industry observes the quality and quantity of U.S. barley available and then must decide how much grain is to be used from stored stocks and how much new crop barley will be purchased for both storage and current-year use. Both stocks and new barley vary in quality, but quality and quantity are assumed to be known. In the event that insufficient quantities of malting barley are grown in the U.S., malting barley can be imported from Canada. Canadian barley is assumed to meet exactly the industry's minimum quality specifications, as data regarding the distribution of the quality of Canadian malting barley are not readily available. The Canadian barley can be used immediately or stored for future use.

Prices, production, and the quality distribution in the second year are unknown. To represent the distribution of possible prices and production, we develop two regression models, specified in (3) and (4). First, changes in production of malting barley are regressed on lagged prices of feed barley and the malting price premium, using annual data from 1985 through 1999 (t-ratios are in parentheses):

$$
\begin{aligned}
(3)\qquad Y(t) = {} & -1.17260 + 0.00499FP(t-1) + 0.00705PM(t-1) \\
& \ (-2.896) \qquad\ (1.909) \qquad\qquad\quad (1.920) \\
& R^2 = 0.538, \quad \mathrm{DW} = 2.216,
\end{aligned}
$$

where $Y(t)$ is the ratio of year t production to year $t-1$ production of six-rowed malting barley in North Dakota, Minnesota, and South Dakota; $FP(t)$ is the price of feed barley (¢/bu., marketing year average) received by producers in North Dakota; and $PM(t)$ is the average malting premium in North Dakota (¢/bu., marketing year average), defined as malting barley price minus feed barley price.

The North Dakota malting premium is taken to be representative of premiums received by barley producers in the three-state region. Data on malting premiums in Minnesota and South Dakota are not available. The premium is regressed on regional production of malting barley and a time trend, using data from 1984 through 1999:

$$
\begin{aligned}
(4)\qquad PM(t) = {} & 153.9473 - 0.5569R(t) - 2.7097T(t) \\
& \ \ (8.700) \qquad (-7.067) \qquad (-3.290) \\
& R^2 = 0.802, \quad \mathrm{DW} = 2.347,
\end{aligned}
$$

Table 1. Discounts for DON (Vomitoxin) Contamination in Malt Barley

DON (ppm)	Discount (¢/bushel)	DON (ppm)	Discount (¢/bushel)
≤ 0.5	0.00	≤ 3.0	62.25
≤ 1.0	4.45	≤ 4.0	70.50
≤ 2.0	57.25	> 4.0	NA[a]

Source: Derived from Johnson and Nganje.

[a] Discounts for DON above 4 ppm are larger than premiums for malting barley, so producers receive a higher price by selling their barley as feed.

Table 2. Discounts for Protein and Plump

Protein (%)	Discount Formula (¢/bushel)	Plump (%)	Discount Formula (¢/bushel)
≤ 13.5	0.00	≥ 70	0.00
≤ 14	(% Protein − 13.5) ∗ 20	≥ 60	(70 − % Plump) ∗ 2
> 14	(% Protein − 14) ∗ 30 + 10	< 60	(60 − % Plump) ∗ 4 + 20

Source: Derived from Johnson and Nganje.

where t-ratios are in parentheses, $R(t)$ is the regional production of six-rowed malting barley in year t (million bu.), and $T(t)$ is a time trend (1984 = 1, 1985 = 2, etc.).

To represent uncertainty about production and prices, the residuals from the two regressions are combined with contemporaneous data on feed barley prices. We do not use a regression model for the latter. The resulting data set has three variables and 15 observations for years 1985 through 1999. Using Gaussian quadrature, we identify 10 representative observations which, with suitable probability weights, match the first and second moments of the entire sample. These are incorporated into the programming model as alternative "states of nature" in year 2.

In the second year, the industry chooses a purchase plan to meet quality and quantity constraints at the lowest possible expected cost. Barley stored previously can be blended with new crop purchases. Shortfalls in U.S. production and quality can be offset by imports from Canada. At the end of the two-year planning horizon, ending stocks are constrained to be equal to beginning stocks. Otherwise, the model would reduce cost by driving stocks to zero. Further, the average level of DON in stored grain cannot exceed 0.5 ppm. This constraint prevents the program from utilizing high-quality barley from storage and replacing it with low-quality, low-price barley.

Barley purchases and barley removed from storage can be blended to meet quality specifications for DON, protein, test weight, and plump. This blending introduces the possibility of cost savings. Low-quality barley is purchased at discounted prices. Low-quality barley can then be blended with higher-quality barley to satisfy quality constraints. The discounts for DON, protein, and plump are derived from Johnson and Nganje and are reported in tables 1 and 2.

In (5), model constraints for quality are given. These quality constraints are derived from Johnson and Nganje. The constraints require that protein in utilized grain not exceed 13.5% in both years and in all states of nature. The level of DON in utilized grain, in both years and all states of nature, is constrained to be equal to or less than 0.5 ppm. Plump is constrained to be at least 70% in utilized grain, and test weight is required to be at least 43 pounds.

$$
\begin{aligned}
(5)\quad & \sum_l Util_{0,1}(l) * Prot(l) + \sum_j Util_{1,1}(j) * Prot(j) + Imports_1 * 0.135 \\
& \quad \leq 0.135 * Demand; \\[6pt]
& \sum_l Util_{0,2}(l,k,p) * Prot(l) + \sum_j Util_{1,2}(j,k,p) * Prot(j) + \sum_k Util_{2,2}(j,k,p) \\
& \quad * Prot(k) + Imports_2(k,p) * 0.135 \leq 0.135 * Demand \ \forall (k,p); \\[6pt]
& \sum_l Util_{0,1}(l) * DON(l) + \sum_j Util_{1,1}(j) * DON(j) + Imports_1 * 0.5 \\
& \quad \leq 0.5 * Demand; \\[6pt]
& \sum_l Util_{0,2}(l,k,p) * DON(l) + \sum_j Util_{1,2}(j,k,p) * DON(j) + \sum_k Util_{2,2}(j,k,p) \\
& \quad * DON(k) + Imports_2(k,p) * 0.5 \leq 0.5 * Demand \ \forall (k,p); \\[6pt]
& \sum_l Util_{0,1}(l) * Plump(l) + \sum_j Util_{1,1}(j) * Plump(j) + Imports_1 * 0.70 \\
& \quad \geq 0.70 * Demand; \\[6pt]
& \sum_l Util_{0,2}(l,k,p) * Plump(l) + \sum_j Util_{1,2}(j,k,p) * Plump(j) \\
& \quad + \sum_k Util_{2,2}(j,k,p) * Plump(k) + Imports_2(k,p) * 0.70 \\
& \quad \geq 0.70 * Demand \ \forall (k,p); \\[6pt]
& \sum_l Util_{0,1}(l) * tw(l) + \sum_j Util_{1,1}(j) * tw(j) + Imports_1 * 43 \geq 43 * Demand; \\[6pt]
& \sum_l Util_{0,2}(l,k,p) * tw(l) + \sum_j Util_{1,2}(j,k,p) * tw(j) + \sum_k Util_{2,2}(j,k,p) \\
& \quad * tw(k) + Imports_2(k,p) * 43 \geq 43 * Demand \ \forall (k,p).
\end{aligned}
$$

Additional constraints (6) are added to require that stored barley averages no more than 0.5 ppm DON. In addition to preventing the program from driving down the quality of stored grain, this constraint reflects the industry's uncertainty about effects of FSB infestations. Even in 1999 (which is considered a high-quality crop year), 80 out of 163 malt barley samples (49%) had DON higher than 0.5 ppm.

$$
\begin{aligned}
(6)\quad & \sum_l Util_{0,2} * DON(l) + \sum_j Domstrd_1(j) * DON(j) + Importstrd_1 * 0.5 \\
& \quad \leq 0.5 * Endstk_1; \\[6pt]
& \sum_j \left(Domstrd_1(j) - Util_{1,2}(j)\right) * DON(j) + \sum_j Domstrd_2(j,k,p) * DON(j) \\
& \quad + Importstrd_2(k,p) * 0.5 \leq 0.5 * Endstk_2(k,p) \ \forall (k,p).
\end{aligned}
$$

Balance constraints, specified in (7), are also imposed:

$$
\begin{aligned}
(7)\quad & Imports_1 = Imputil_1 + Importstrd_1;\\
& Imports_2(k,p) = Imputil_2(k,p) + Importstrd_2(k,p)\ \forall\,(j,k\,p);\\
& Begstk_0(l) = Util_{0,1}(l) + Util_{0,2}(l,k,p)\ \forall\,(l,k,p);\\
& Domestic_1(j) = Util_{1,1}(j) + Domstrd_1(j)\ \forall j;\\
& Domestic_2(j,k,p) = Util_{2,2}(j,k,p) + Domstrd_2(j,k,p)\ \forall\,(j,k,p);\\
& Domestic_1(j) \le Domprod_1(j)\ \forall j;\\
& Domestic_2(j,k,p) \le Domprod_2(j,k,p)\ \forall\,(j,k,p);\\
& Domstrd_1(j) \ge Util_{1,2}(j,k,p)\ \forall\,(k,p);\\
& \sum_l Util_{0,1}(l) + \sum_j Util_{1,1}(j) + Imputil_1 = Demand;\\
& \sum_l Util_{0,2}(l,k,p) + \sum_j Util_{1,2}(j,k,p) + Util_{2,2}(j,k,p) + Imputil_2(k,p)\\
& \qquad = Demand\ \forall\,(k,p);\\
& Endstk_1 = \sum_l \big(Begstk_0(l) - Util_{0,1}(l)\big) + \sum_j Domstrd_1(j) + Importstrd_1;\\
& Endstk_2(k,p) = \sum_j \big(Domstrd_1(j) - Util_{1,2}(j,k,p) + Domstrd_2(j,k,p)\big)\\
& \qquad + Importstrd_2(k,p)\ \forall\,(k,p).
\end{aligned}
$$

Data and Representative Points

Seven years (1993–99) of quality data from the Midwestern growing region (North Dakota, Minnesota, and South Dakota) are used to represent the possible quality distribution of the second year's crop. These data are collected as part of an annual survey of regional crop quality conducted by the Department of Cereal Science, North Dakota State University. Each of the seven distributions are approximated using both the CM and GQ methods. The number of data points varies across years with a low of 147 observations for 1995 and a high of 194 observations for 1999.

First, the data are approximated using the CM approach. Fifteen equally likely representative intervals are chosen. Similar quality grain is chosen by ranking the barley first on DON, then plump, protein, and test weight. In cases where the number of observations is not divisible by 15, individual observations are proportioned between two adjacent intervals. Assume, for example, there are 172 observations for a given year. Each interval would then have 11.47 observations. The 12th observation is proportioned between the first (0.47) and the second (0.53) intervals. The second interval then contains 53% of the 12th observation, observations 13 through 22, and 93% of the 23rd observation. The 1999 data are given in appendix table A1, and the CM approximation of 1999 data is presented in table 3.

Next, each year's quality distribution is represented using a second-order GQ approximation. The GQ approximations each contain 15 unequally weighted points. Table 4 presents the GQ approximation of the 1999 quality distribution. As can be seen in table 4, these points are a subset of the actual data points from appendix table A1.

Table 3. Conditional Mean (CM) Representation of 1999 Malting Barley Quality Distribution

Plump (%)	Test Weight (lbs.)	Protein (%)	DON (ppm)	Probability[a] (Weight)
78.57583	46.28589	12.28160	0.00000	0.06667
69.05890	44.99264	12.56074	0.00000	0.06667
58.92454	45.53252	12.71288	0.03865	0.06667
69.06871	46.52822	13.31104	0.20000	0.06667
77.10245	45.70982	12.28712	0.29509	0.06667
64.62209	43.93988	12.80184	0.32025	0.06667
71.62270	45.50613	12.40184	0.45583	0.06667
63.90245	44.43742	12.94110	0.53620	0.06667
68.10675	44.58160	12.75337	0.65337	0.06667
60.34417	44.07178	12.98221	0.77669	0.06667
68.46748	44.39202	12.48405	0.96012	0.06667
68.08037	45.67117	12.84540	1.22638	0.06667
71.79509	45.28282	12.39571	1.61902	0.06667
66.06012	44.67853	12.96933	2.34294	0.06667
65.03865	44.83436	12.63374	6.46810	0.06667

[a] Probabilities do not sum to one due to rounding.

Table 4. Second-Order Gaussian Quadrature (GQ) Approximation of 1999 Malting Barley Quality Distribution

Observ. No.[a]	Plump (%)	Test Weight (lbs.)	Protein (%)	DON (ppm)	Probability[b] (Weight)
27	52.90000	50.00000	14.50000	0.00000	0.02293
12	72.70000	45.10000	13.20000	0.00000	0.08720
31	73.80000	47.80000	11.70000	0.10000	0.17796
63	53.00000	43.80000	12.70000	0.30000	0.08047
65	79.10000	45.00000	11.50000	0.40000	0.07961
86	71.20000	41.90000	12.70000	0.60000	0.07770
91	57.10000	38.90000	12.80000	0.60000	0.09715
85	77.60000	47.20000	13.70000	0.60000	0.16761
92	44.40000	45.60000	13.20000	0.60000	0.05174
106	36.00000	41.30000	10.40000	0.80000	0.00278
107	43.10000	45.70000	15.60000	0.80000	0.00008
154	52.90000	44.10000	13.40000	4.00000	0.03326
155	71.20000	45.40000	12.20000	4.70000	0.11036
161	62.90000	46.50000	14.00000	8.90000	0.00697
162	66.50000	42.10000	13.60000	9.40000	0.00346

[a] Observation numbers correspond to data in appendix table A1.

[b] Probabilities do not sum to one due to rounding.

Table 5. Baseline Model Parameter Values

Parameter	Baseline Value
Initial and ending stocks	128.2 mil. bu.
Current year barley quality distribution	1999
Quality (year) of initial barley stocks	1998
Current-year domestic production	64.2 mil. bu.
Annual industry demand for malt barley (*Demand*)	85 mil. bu.
Initial price of malt barley ($Price_0$)	\$2.60/bu.
Price in period 1 for malt barley ($Price_1$)	\$2.60/bu.
Interest rate (*r*)	0.09%
Annual storage cost per bushel (*Strchg*)	\$0.36/bu.
Cost per bushel of transport barley from Canada to U.S. (*Trans*)	\$0.27/bu.

Table 6. Sensitivity to Current-Year Domestic Production of Barley

Domestic Production (mil. bu.)	Year 1 Domestic Purchases (mil. bu.)		Year 1 Imports (mil. bu.)		Objective ($ mil., US)	
	CM	GQ	CM	GQ	CM	GQ
30	24.00	25.27	139.10	135.98	753.57	667.98
40	32.00	33.69	130.83	122.01	730.34	626.77
50	40.00	42.12	122.56	108.04	707.12	585.64
60	48.00	50.54	114.29	94.07	683.89	544.57
70	56.00	59.22	106.02	81.35	660.67	513.60
80	64.00	67.68	97.75	71.06	637.45	501.87
90	72.00	76.14	89.47	60.77	614.24	490.16
Avg. % difference	-5.20%		22.80%		22.38%	

Model Results

The model is solved using GAMS (Brooke, Kendrick, and Meeraus). Baseline model parameters are given in table 5. Several of these parameters are varied over a wide range to allow comparison between the two approximation methods. In table 6, we report the results from varying domestic production of malting barley from 30 million bushels to 90 million bushels. Table 7 shows the results of varying the quantity of initial and ending stocks from 21 to 191 million bushels. Finally, tables 8 and 9 report the results of varying the quality distribution of initial stocks and the quality distribution of current-year domestic barley production, respectively.

In all optimizations performed, the CM approximation overstates the value of the objective function (expected cost) relative to the GQ approximation. The average difference in the objective function ranges from 20.86% to 25.60%. Higher costs reflect the understatement of blending opportunities by the CM representations. The highest expected costs occur when either the stored grain or purchased grain has a 1995 quality

Table 7. Sensitivity to Initial and Ending Stocks Requirement

Initial/ Ending Stocks (mil. bu.)	Year 1 Domestic Purchases (mil. bu.)		Year 1 Imports (mil. bu.)		Objective ($ mil., US)	
	CM	GQ	CM	GQ	CM	GQ
21	51.36	54.60	41.86	34.56	441.40	411.54
43	51.36	54.31	51.63	41.03	459.22	415.77
64	51.36	54.31	61.49	47.61	500.80	422.10
85	51.36	54.31	71.36	54.19	526.27	431.40
106	47.08	54.31	83.14	61.68	568.11	446.00
128	51.36	54.31	91.09	70.23	619.18	467.65
149	51.36	54.31	100.95	78.77	674.14	493.60
170	51.36	54.08	110.82	88.21	731.25	527.33
191	51.36	54.08	120.68	99.82	788.36	579.18
Avg. % difference	−5.92%		24.03%		25.60%	

Table 8. Sensitivity to Quality Distribution of Stored Barley

	Year 1 Domestic Purchases (mil. bu.)		Year 1 Imports (mil. bu.)		Objective ($ mil., US)	
Quality (Year)	CM	GQ	CM	GQ	CM	GQ
1993	51.36	54.31	150.52	125.59	816.70	642.20
1994	51.36	54.31	138.66	110.05	771.51	639.00
1995	52.49	54.31	145.85	154.95	799.88	788.67
1996	51.36	55.04	104.75	71.90	655.27	502.94
1997	53.01	63.53	111.33	77.97	676.75	518.05
1998	51.36	54.08	110.82	85.00	674.14	527.33
1999	51.36	56.00	60.14	49.53	555.08	514.43
Avg. % difference	−7.25%		25.75%		20.86%	

Table 9. Sensitivity to Quality Distribution of Current-Year Barley

	Year 1 Domestic Purchases (mil. bu.)		Year 1 Imports (mil. bu.)		Objective ($ mil., US)	
Quality (Year)	CM	GQ	CM	GQ	CM	GQ
1993	19.74	19.98	144.59	129.31	767.64	639.30
1994	21.40	12.16	140.27	135.31	754.26	654.62
1995	21.40	8.19	142.82	151.68	762.83	712.72
1996	34.24	43.00	127.11	89.67	718.95	510.90
1997	29.96	41.21	129.78	90.00	726.13	513.63
1998	34.24	11.66	129.52	148.76	727.03	702.28
1999	51.36	54.08	110.82	88.21	674.14	527.33
Avg. % difference	53.86%		15.47%		22.26%	

distribution. In 1995, DON contamination was very high, averaging 5.9 ppm across samples. In contrast, 1999 DON averaged less than 1.1 ppm. In these cases, the industry must import large amounts and blending opportunities are limited.

In most of the scenarios (tables 6, 7, and 8), the CM approach relative to GQ understates the amount of domestic purchases and overstates the level of imports. In these three tables, the CM representation understates domestic purchases by 5.2% to 7.3% and overstates imports by 22.8% to 25.75%. Higher imports lead to higher expected cost because low-quality domestic barley is priced lower than imports but, by understating variability, the CM approach does not allow blending of more extreme quality grains. In table 9, the CM approach results in purchases which average 53.9% more for domestic barley and 15.5% more for imports than with the GQ approach. This reflects the added cost of maintaining feasibility across both time periods with the CM approximation.

Summary and Conclusions

The values of commodities are determined by their various quality attributes (Ladd and Martin). In the grain handling and processing industries, these values are often realized through blending activities. Grain qualities can vary between shipments and across time; hence, they are best characterized by a joint probability or frequency distribution. Researchers must often summarize or represent the multivariate distribution with a small set of points and probabilities.

In this study, we present a method representing or approximating joint attribute quality distributions using a numerical integration technique called Gaussian quadrature. The GQ method is compared to a more widely used representation, the conditional mean (CM) approach, in a malting barley blending model. Relative to the GQ method, the CM method is shown to understate variability in attribute quality, resulting in reduced blending opportunities. Based on our findings, the GQ method results in lower expected costs when compared to the CM method.

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References

Brooke, A., D. Kendrick, and A. Meeraus. *GAMS: A User's Guide*, Release 2.25. South San Francisco CA: The Scientific Press, 1992.

DeVuyst, E. A., P. V. Preckel, and S. Liu. "Approximating Mathematical Expectations Using Multivariate Gaussian Quadrature." Dept. of Agr. Econ., North Dakota State University, April 2000. [This paper is available upon request by contacting Eric A. DeVuyst at edevuyst@ndsuext.nodak.edu.]

Haber, S. "Numerical Evaluation of Multiple Integrals." *SIAM Review* 12(October 1970):481–526.

Johnson, D. D., and W. Nganje. "Impacts of DON in the Malting Barley Supply Chain: Aggregate Costs and Firm-Level Risks." Paper presented at The Economics of Quality Control in Agriculture Conference, Saskatoon, Saskatchewan, 2–4 December 1999.

Johnson, D. D., and W. W. Wilson. "Wheat Cleaning Decisions at Country Elevators." *J. Agr. and Resour. Econ.* 18(December 1993):198–210.

Ladd, G. W., and M. B. Martin. "Prices and Demands for Input Characteristics." *Amer. J. Agr. Econ.* 58(February 1976):21–30.

Miller, A. C., and T. R. Rice. "Discrete Approximations of Probability Distributions." *Manage. Sci.* 29(1983):352–62.

Preckel, P. V., and E. A. DeVuyst. "Efficient Handling of Probability Information for Decision Analysis Under Risk." *Amer. J. Agr. Econ.* 74(August 1992):655–62.

Tchakaloff, V. "Formulas de Cubatures Mecanique a Coefficients Non Negatifs." *Bulletin des Sciences Mathématiques* 18(1957):123–34.

U.S. Department of Agriculture, National Agricultural Statistics Service. Various malting barley data, 1993–99. USDA/NASS website. Online at http://nass.usda.gov:81/ipedb/. [Accessed February 15, 2000].

Wilson, W. W., and T. Preszler. "End-Use Performance Uncertainty and Competition in International Wheat Markets." *Amer. J. Agr. Econ.* 74(August 1992):556–63.

Appendix

Table A1. 1999 Malting Barley Quality Distribution

Observ. No.	Plump (%)	Test Wgt. (lbs.)	Protein (%)	DON (ppm)
1	83.7	49.1	12.9	0.0
2	82.9	45.0	12.2	0.0
3	80.6	45.8	12.2	0.0
4	80.0	50.8	13.4	0.0
5	79.5	47.8	12.5	0.0
6	78.6	48.0	12.3	0.0
7	77.5	48.8	13.1	0.0
8	76.8	49.6	11.7	0.0
9	76.6	37.2	11.5	0.0
10	73.9	43.0	11.0	0.0
11	73.6	43.7	12.3	0.0
12	72.7	45.1	13.2	0.0
13	71.4	44.3	13.2	0.0
14	71.4	43.8	11.7	0.0
15	70.2	44.7	12.7	0.0
16	70.1	41.7	12.5	0.0
17	68.7	47.1	13.9	0.0
18	68.5	44.7	13.1	0.0
19	68.1	47.2	12.7	0.0
20	67.5	43.8	12.3	0.0
21	64.8	46.3	10.9	0.0
22	64.4	46.9	11.8	0.0
23	64.4	43.5	12.9	0.0
24	60.1	43.2	12.9	0.0
25	55.4	43.3	13.4	0.0
26	53.6	45.4	13.4	0.0
27	52.9	50.0	14.5	0.0
28	48.0	45.6	11.0	0.0
29	47.7	47.9	12.7	0.0
30	42.3	41.4	12.2	0.0
31	73.8	47.8	11.7	0.1
32	78.2	47.9	12.2	0.2
33	77.9	43.8	13.5	0.2
34	75.2	46.4	13.5	0.2
35	73.3	44.4	12.9	0.2
36	72.9	48.6	12.3	0.2
37	70.3	49.1	14.2	0.2
38	69.6	48.6	12.3	0.2
39	68.0	44.8	13.3	0.2
40	67.7	47.0	13.6	0.2
41	67.4	45.0	12.8	0.2
42	66.5	45.9	14.5	0.2
43	65.2	47.1	14.2	0.2
44	49.9	45.4	12.1	0.2
45	89.9	48.6	12.5	0.3
46	88.1	46.3	11.2	0.3
47	83.4	49.0	13.1	0.3
48	82.0	46.8	11.7	0.3
49	79.3	44.8	12.0	0.3
50	75.1	42.9	12.0	0.3
51	73.3	48.0	13.7	0.3
52	72.6	40.9	12.0	0.3
53	72.0	44.5	12.9	0.3
54	71.8	46.5	11.8	0.3
55	71.2	42.6	12.5	0.3
56	64.9	45.7	13.1	0.3
57	64.8	47.5	12.5	0.3
58	62.9	45.7	13.5	0.3
59	59.9	44.7	13.3	0.3
60	58.5	41.2	12.8	0.3

(continued . . .)

Table A1. Continued

Observ. No.	Plump (%)	Test Wgt. (lbs.)	Protein (%)	DON (ppm)
61	58.3	37.5	13.9	0.3
62	56.3	39.6	13.0	0.3
63	53.0	43.8	12.7	0.3
64	81.6	48.4	12.0	0.4
65	79.1	45.0	11.5	0.4
66	77.3	49.9	12.4	0.4
67	75.3	41.6	13.4	0.4
68	73.1	45.7	12.3	0.4
69	61.7	45.1	10.6	0.4
70	42.3	39.9	13.6	0.4
71	87.0	45.2	12.5	0.5
72	79.5	44.4	12.0	0.5
73	77.2	48.4	12.3	0.5
74	75.5	47.8	11.2	0.5
75	71.2	44.8	13.0	0.5
76	69.2	48.5	13.1	0.5
77	66.9	47.7	12.7	0.5
78	66.8	47.4	12.5	0.5
79	64.9	46.3	12.8	0.5
80	58.8	40.3	13.4	0.5
81	57.0	41.8	11.9	0.5
82	44.8	40.6	13.8	0.5
83	42.5	42.9	13.8	0.5
84	83.5	46.1	12.6	0.6
85	77.6	47.2	13.7	0.6
86	71.2	41.9	12.7	0.6
87	69.5	47.0	12.4	0.6
88	68.1	46.8	11.9	0.6
89	68.0	39.8	12.6	0.6
90	58.0	40.5	13.6	0.6
91	57.1	38.9	12.8	0.6
92	44.4	45.6	13.2	0.6
93	81.5	45.9	12.4	0.7
94	78.2	47.6	14.0	0.7
95	76.2	47.4	11.7	0.7
96	74.5	47.8	12.4	0.7
97	73.3	44.3	13.0	0.7
98	70.2	45.9	12.7	0.7
99	67.3	41.2	12.8	0.7
100	66.8	42.1	12.3	0.7
101	64.3	48.7	13.8	0.7
102	79.6	45.1	13.1	0.8
103	73.4	46.1	12.5	0.8
104	60.8	45.5	13.8	0.8
105	51.6	43.7	12.5	0.8
106	46.0	38.2	13.0	0.8
107	43.1	45.7	15.6	0.8
108	36.0	41.3	10.4	0.8
109	79.2	48.2	13.1	0.9
110	71.0	45.2	12.7	0.9
111	70.6	47.5	13.3	0.9
112	56.1	40.8	12.6	0.9
113	53.0	44.6	13.1	0.9
114	82.2	43.6	11.7	1.0
115	74.6	44.5	11.6	1.0
116	73.2	44.6	12.1	1.0
117	70.7	42.2	12.7	1.0
118	67.4	44.9	11.5	1.0
119	66.6	47.2	12.9	1.0
120	60.4	39.8	13.3	1.0
121	60.2	46.8	12.1	1.0
122	49.0	45.3	13.7	1.0
123	75.9	50.9	12.3	1.2
124	73.4	43.9	12.6	1.2
125	73.0	46.6	12.4	1.2
126	74.5	45.2	14.1	1.3
127	73.6	47.1	13.4	1.3
128	62.0	44.2	12.9	1.3
129	76.2	43.9	12.6	1.4
130	70.1	46.3	12.6	1.4
131	59.3	43.8	11.7	1.4
132	63.6	41.3	11.9	1.5
133	57.2	43.0	12.4	1.5
134	81.9	47.2	12.3	1.6
135	76.8	45.4	13.0	1.6
136	74.5	47.6	12.3	1.6
137	83.9	46.7	12.4	1.7
138	75.7	46.2	12.4	1.7
139	73.2	46.1	12.3	1.7
140	72.8	45.6	12.9	1.7
141	69.9	45.2	12.5	1.7
142	56.6	43.1	12.3	1.7
143	70.5	45.9	12.8	1.8
144	58.5	47.1	14.1	2.1
145	74.3	44.5	13.3	2.2
146	71.8	45.9	13.2	2.2

(continued . . .)

Table A1. Continued

Observ. No.	Plump (%)	Test Wgt. (lbs.)	Protein (%)	DON (ppm)
147	62.1	37.7	12.9	2.2
148	67.7	48.9	13.0	2.3
149	57.1	43.9	13.1	2.3
150	68.3	44.6	12.5	2.8
151	61.0	44.8	12.9	2.9
152	79.1	45.0	12.3	3.0
153	44.6	42.0	13.6	3.1
154	52.9	44.1	13.4	4.0
155	71.2	45.4	12.2	4.7
156	78.5	44.1	12.1	4.8
157	69.2	48.7	13.1	5.4
158	68.7	45.0	11.9	5.6
159	68.6	46.9	11.2	5.6
160	68.4	46.1	11.0	8.6
161	62.9	46.5	14.0	8.9
162	66.5	42.1	13.6	9.4
163	61.2	41.9	13.0	10.6