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TRANSPORTATION RESEARCH FORUM

Identifying the Optimum Features of an Urban Bus System

*by Chester G. Wilmot**

1. INTRODUCTION

TRADITIONALLY, the demand for travel on public transport services has been estimated by predicting the demand for a series of alternative public transport systems. The interaction, thus, between supply and demand has been by way of a series of proposals rather than by a dynamic interplay between supply and demand. Aspects of the supply side such as frequencies, route determination and bus scheduling have usually been determined independently (1,2).

In this study, a pseudo-equilibrium model is developed using a disaggregate travel demand model to describe how demand responds to supply and an adapted airline scheduling model is used to identify the optimum supply level given the relationship between supply and demand.

The disaggregate travel demand model is a logit model responsive to supply variables such as travel time, travel cost, service frequency and journey length. The adapted airline scheduling model uses linear programming in identifying the service frequency which would maximize the system-wide difference between revenue and cost given characteristics of the transit system such as fleet size, fare, route lengths, etc.

The central feature of the adapted airline scheduling model is its ability to use a given relationship between service frequency and demand in its input (together with other fixed input data) to identify the optimum service frequency to be chosen. This provides a feedback from demand to supply and an opportunity to identify the optimum value of a supply parameter. Additional supply parameters such as fleet size, fare and route density can be incorporated in the analysis by reapplying the model with different values of these parameters and observing the trend suggested by the discrete result values.

The ability to identify the optimum values of some of the supply parameters of a transit system is a facility that would be useful to authorities investigating the introduction of a new system or those reviewing existing systems.

The models and their use in the above role is described by way of an application in a hypothetical situation using contrived data. The assumptions made in compiling the situation are thought to be reasonably realistic so that the indication of optimum values for the supply parameters identified at the end of the analysis, are not too far from those that would be obtained with real data.

2. THE SITUATION CONSIDERED

A hypothetical situation is created where a small urban area is considering introducing a bus service. The objective is to determine the optimum fleet size, route density, fare level and scheduling pattern for the area using a combined travel demand and scheduling model.

The town is assumed essentially mononucleated with a residential area in the north-eastern region of the city as shown in Figure 1.

It is assumed that a public transport specialist uses personal judgement and a knowledge of the city and its streets to identify an optimum set of bus routes, for three levels of route density, as shown in Figures 2(a), 2(b) and 2(c).

LAYOUT OF HYPOTHETICAL CITY

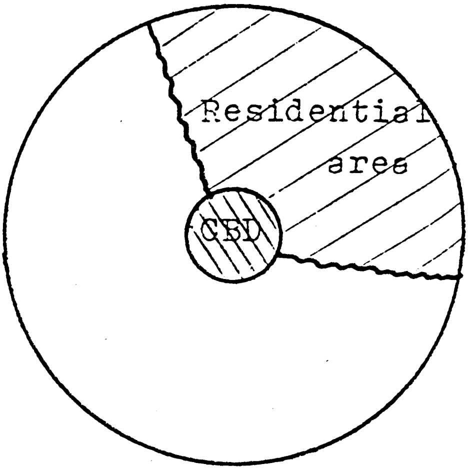

FIGURE 1

**Graduate student, Dept. of Civil Engineering, Northwestern University, Evanston, Illinois.*

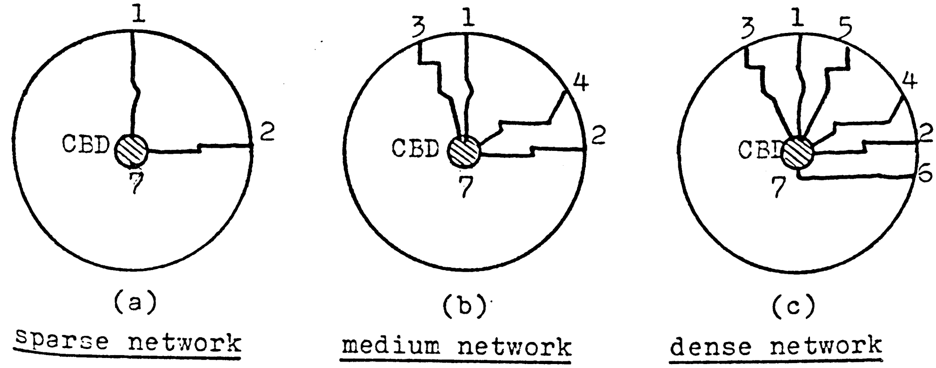

FIGURE 2

As the peak hour operation of public transport is the most critical for the transit operator, the service is designed here for the peak hour. For convenience, only the morning peak is considered in this investigation although a similar exercise could be conducted for the afternoon peak (and off-peak period, if necessary) to provide a fuller picture.

The performance of the above three networks is established for varying fleet size and fare level in each case. This is done by establishing the patronage resulting from optimum scheduling on each route for each of the conditions (i.e., route density, fleet size and fare level) and measuring performance of each situation in terms of the difference between costs (fixed and operating) and revenue. While performance of a public transport system should not be measured only in economic terms, the decisionmakers are free to trade off economic performance against the achievement of social, environmental and energy goals.

It is considered that ½ mile on either side of a route is the reasonable upper limit of the catchment area for that route (3). The number of riders who could make use of the bus (i.e., the number of potential patrons who reside within the catchment area, work within the CBD and are not captive to car) are assumed as being those shown in Table 1 overleaf. Also included is the average distance from the catchment area to the CBD and the estimated turnaround time for buses during an entire cycle on each route. While being hypothetical values, the entries in the table are realistic with the sparse network routes serving larger populations and the populations decreasing on these same routes when other routes provide competition on the outer limits of their catchment areas. This is also reflected in the reduction in the average distance from residence to the nearest bus stop.

3. PROCEDURE FOLLOWED IN CONDUCTING THE ANALYSIS

Two models are borrowed from related transport fields to produce a scheduling of buses which maximizes the profit of the system and estimates bus ridership on each route. The net profit is used as a measure of performance of the system considered.

The first model is a disaggregate choice model which provides estimates of bus patronage in response to specified values of:

(i) population served,
(ii) distance to the bus route,
(iii) frequency of bus service,
(iv) bus fare,
and (v) average distance to the CBD.

Items (i), (ii) and (v) above are specified in Table 1 and thus by varying (iii) and (iv) (fare and frequency) over feasible ranges, estimates of patronage by route coinciding with these may be obtained. The model is described in more detail in the next section.

The second model is an aircraft scheduling model which determines the optimum frequency on a given network by route when fleet size and fare levels are specified. Thus, considering a given fare and fleet size, the second model establishes a frequency and demand by route which maximizes the profit for the whole system. This second model which utilizes the linear programming procedure is described more fully in section 5.

The alternatives which are processed by these models are all the combinations

TABLE 1

ASSUMED VALUES IN THE HYPOTHETICAL CITY

ROUTE DENSITY	ROUTE	POP. SERVED	AVE.DIST. TO BUS RTE (feet)	AVE.DIST. TO CBD (feet)	TURNAROUND TIME FOR BUS (hrs.)
sparse	17	300	1300	10000	0.8
	27	250	1300	10000	0.7
medium	17	270	1000	10000	0.8
	27	210	1000	10000	0.7
	37	200	1000	10000	1.05
	47	160	1000	10000	0.85
dense	17	240	700	10000	0.8
	27	170	700	10000	0.7
	37	200	700	10000	1.05
	47	160	700	10000	0.85
	57	120	700	10000	1.15
	67	110	700	10000	1.0

of 3 route densities, 3 fleet sizes and 2 fare levels, or 18 individual cases. It was hoped that by obtaining solutions at a few points along the three dimensions, a distinguishable trend may be identified that would provide the means of determining the optimum choice of route density, fleet size and fare level. The outcome of the investigation is documented in section 6.

4. ESTABLISHING THE DEMAND FUNCTION

Individual choice models provide a probabilistic estimate of an individual's choice in terms of the utility of the alternative chosen and the utility of all the other alternatives in the choice set. For a binary choice, the logit individual choice model may be written as:

$$P_i^1 = \frac{\exp(u_i^1 - u_i^2)}{1 + \exp(u_i^1 - u_i^2)}$$

where,

u_i^1 = the utility of mode 1 for individual i

u_i^2 = the utility of mode 2 for individual i

exp = the exponential function

If individual values of utility are not available but only average zonal values, then the total demand from that area may be obtained by using the naive aggregation method (4) as follows:

Demand for mode 1 from the area = population of the area x P^1

where P^1 = the probability that mode 1 will be chosen in that area.

Consider one of the modes as bus and one of the variables in the bus utility function as frequency of service. Changing frequency of service will alter the bus utility and therefore the demand. This would provide the relationship between demand and frequency of bus service in the scheduling program described in the next section.

In order to provide realistic values in the relationship, utility functions from a logit model calibrated for a bus feeder service to a rail station in one of the suburbs of Chicago (5) are adopted for use here. The modes considered in that study were "bus" and "all other modes" with the utility of these latter modes forming the base mode and all the variables in the specification being bus specific variables. Thus, the relative utility of "all other modes" was assumed to be zero.

The utility function for bus developed in the above study was:

$$U_{bus} = 2.5238 - 0.00072B - 0.0419F - 0.042H + 0.0032S$$

where

B = average distance to nearest feeder bus top (feet)

F = bus fare (cents)

H = bus headway (minutes)

S = distance to train station (in hundreds of feet)

Since S was the length of the trip in the borrowed model, a value of 10,000 feet (the average distance to the CBD) was assumed for S in all the cases analyzed in this study. Values for B (the distance to the bus top) were proposed in Table 1 (column 4) and thus by varying fare and service frequency (the reciprocal of bus headway H) through reasonable ranges, relationships between service frequency and demand could be established for each route using the transferred model. The relationships established for route 2 and 25¢ fare are shown below; similar relationships for other routes and other fare levels were also obtained.

5. THE SCHEDULING MODEL

The model used here is one developed at the MIT Flight Transportation Laboratory (6). Using linear programming, the model maximizes the difference between revenue and operating cost (in the adaptation this is modified to include some fixed costs) subject to a number of operational constraints. Explicitly, the formulation is as follows:

Objective function:

$$\text{Max} \left(\sum_{pq} y_{pq}P_{pq} - \sum_{pq} C.n_{pq}\right)$$

subject to:

1. Load factor not to be exceeded.
   $$LF.S.n_{pq} - P_{pq} \geqslant 0 \text{ for all pq}$$
2. Total passenger flow is sum of segment flows.
   $$\sum_{k} a_{pq}{}^{k}.n_{pq}{}^{k} - P_{pq} \geqslant 0 \text{ for all pq}$$
3. Total frequencies is sum of segment frequencies.
   $$n_{pq} - \sum_{k} n_{pq}{}^{k} = 0 \text{ for all pq}$$
4. Segment frequencies are bounded at breakpoints.
   $$0 \leqslant n_{pq}{}^{k} \leqslant n_k{}^{pq} - n_{k-1}{}^{pq} \text{ for all pq}$$
5. Fleet availability must not be exceeded.
   $$\sum_{pq} T_{pq}.n_{pq} \leqslant U.A.$$
6. Minimum daily frequency on each route.
   $$n_{pq} \geqslant N_{pq} \text{ for all pq}$$

where:

p , q = symbols for station origin and destination.

y_{pq} = bus fare on route pq (\$/passenger).

P_{pq} = number of passengers on route pq (passengers/hour).

C = marginal direct operating cost (\$/bus).

n_{pq} = service frequency on route pq (buses/hour).

LF = load factor (i.e., proportion of passenger bearing capacity that may be used).

S = seat capacity of bus (seats/bus).

$a_{pq}{}^{k}$ = slope of demand curve for segment k on route pq (pass/bus).

$n_{pq}{}^{k}$ = frequency difference on segment k on route pq (buses/hr).

$n_k{}^{pq}$ = frequency at upper end of segment k on route pq (buses/hr).

T_{pq} = block time to travel from p to q (hours).

U = utilization rate (i.e., proportion of full time that a bus can be considered in service).

A = fleet size.

N_{pq} = minimum daily frequency on route pq (buses/hour).

k = linear segment on demand curve.

In order that the scheduling model may use the demand function established with the disaggregate demand model, the curved functions such as those in Figure 3 must be approximated by a series of straight lines or segments, as they are referred to in the terminology of the supply model above. The linearization of the demand curves for route 2 and a 25 cent fare is shown in Figure 4 overleaf. This provides the interaction between the demand and scheduling model.

ESTABLISHED RELATIONSHIP BETWEEN FREQUENCY AND DEMAND WHEN THE FARE IS SET AT 25 CENTS — ROUTE 2 —

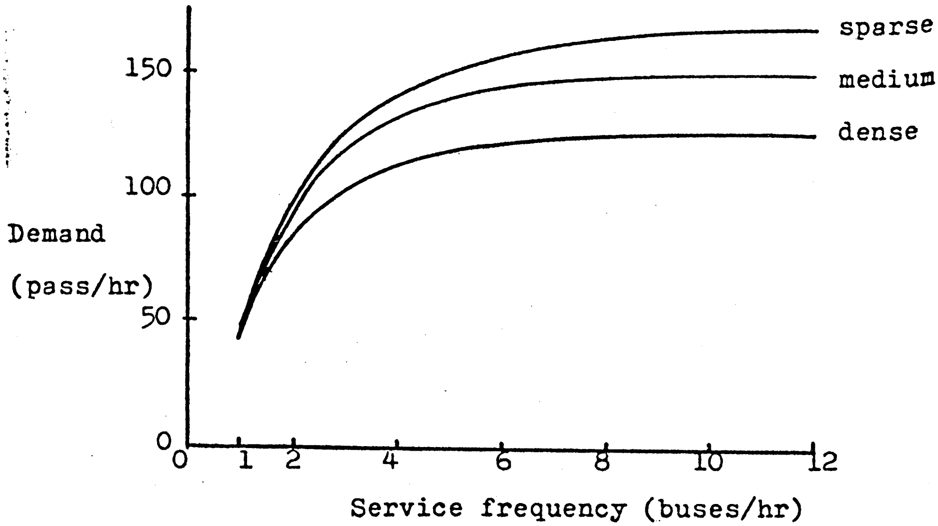

FIGURE 3

APPROXIMATION OF DEMAND CURVE WITH A SERIES OF STRAIGHT LINES — FARE 25 CENTS AND ROUTE 2

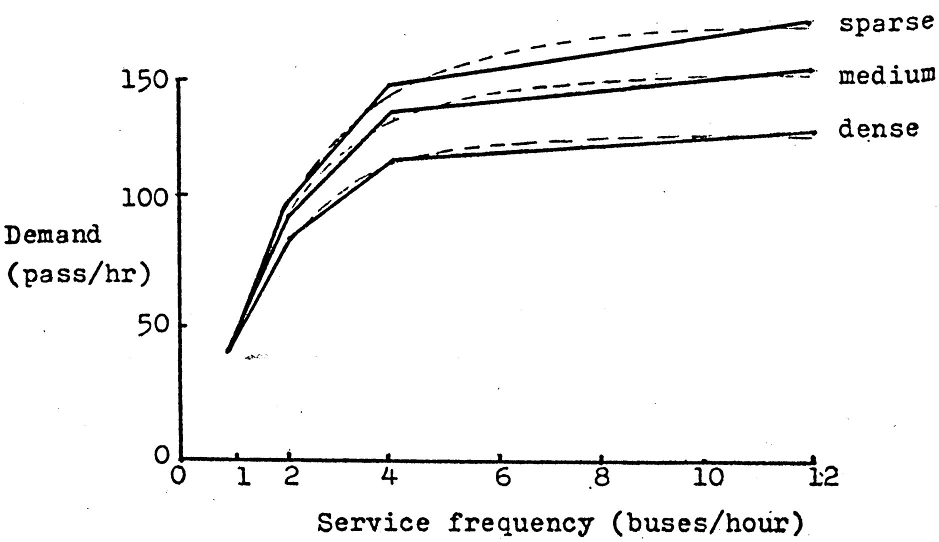

FIGURE 4

Certain assumptions had to be made regarding the operation of the system and these are itemized below:

1. The period of analysis is 1 hour and during that time demand is considered to remain uniform.
2. Frequency relates to trips in one direction only and 1 trip is considered as a journey from the residential area to the CBD and back to the residential area.
3. The demand on the return journey was chosen as being one-half of that on the inbound journey.
4. The operating cost of a bus is assumed to be $15/hour.
5. The minimum allowable frequency on any route is 1 bus/hr.
6. The utilization rate (i.e., the proportion of time a bus is in service) is set at 0.8 throughout.
7. The load factor is set at a maximum of 1.
8. Seating capacity of each bus is set at 50 and no standees were considered.
9. All buses were assumed to be identical.

6. RESULTS FROM THE HYPOTHETICAL EXAMPLE

A standard linear programming computer program was used to obtain solutions for the 18 different cases (2 fare, 3 fleet size and 3 route density) analyzed. The results are summarized in Tables 2 and 3.

The general indication from these tables is that the objective function (i.e., net profit per morning peak hour in dollars) is positive and considerably higher with the higher fare while the corresponding load factors seem to be only marginally lower. From this, it would seem that a fare of 35 cents would be more desirable than a fare of 25 cents. There is not a clear indication which route density is more desirable while larger fleet size seems clearly superior. However, the fact that fixed costs are not included in the objective function, distorts the picture and these should be included before conclusions are drawn.

The inclusion of the fixed cost of the bus is arranged by amortizing the cost of the bus and converting these annual costs to hourly costs for the period when the bus is in operation. This has been done by assuming the purchase price of a bus is $100,000, its life is 10 years, interest rate is 10% per annum and the average bus operates 6 hours per day. This results in a capital recovery cost of $7.43 per hour of the buses' operation. Adding this to the information in Tables 2 and 3, the values in Table 4 are obtained. These are also displayed in Figures 5 and 6.

7. CONCLUSIONS

The results as depicted in Figures 5 and 6 indicate a number of things quite dramatically. While these emerge from hypothetical data and some of the values assumed are low in relation in 1981 figures, the data is within reasonable bounds of reality and the drawing of conclusions seems to be in order. The conclusions are:

(i) Considering that many of the concave curves in Figure 6 have clear minimum points, there seems to be an optimum fleet size for each route density and fare level considered.

(ii) Change in fare level seems to make relatively little difference to the optimum fleet size for each route density considered.

(iii) Considering the envelope of the set of curves for each fare level (in Figures 5 or 6), it seems as though a family of curves could be generated at different fare levels which would describe deficit per passenger in terms of fleet size. Note that each point along such a curve would represent a different route density. This family of curves could conceivably be of the shape shown in Figure 7 overleaf.

The envelope to the curves in Figure 7 would provide an optimum value for fare, route density and fleet size.

The findings in this study point towards a possible fruitful area of investigation. Use of actual data, the use of a locally estimated individual choice model and the investigation of a wider range of alternative supply variables would provide results with greater credibility than those produced here. If similar results can be produced with real data then a procedure will have been developed which can assist decision-makers with establishing optimum values for supply parameters which heretofore were established in an ad hoc or even arbitrary fashion.

REFERENCES

1. Lampkin, W. and P. D. Saalmans, "The design of routes, service frequencies and schedules for a municipal bus undertaking: A case study," Operational Research Quarterly, Vol. 18, No. 4.
2. Scheele, S., "A supply model for public transit services." Prepared for presentation at the International symposium on traffic supply models, University of Montreal, Nov., 1977.

3. Bandi, F., "Length of walking distances and distance between stops: Their influence on the attractiveness of public transport," UITP Review, No. 1, 1974. pp 163-188.
4. Koppelman, F. S., "Travel prediction with models of individual choice behavior," Ph.D. dissertation, Dept. of Civil Engineering, M.I.T., 1975.
5. Spear, B. D., "Applications of new travel demand forecasting techniques to transportation planning: A study of individual choice models," Federal Highway Administration, March, 1977. Reference to the feasibility study of feeder bus service to a suburban railroad station in Homewood, Illinois.
6. Simpson, R. W., "Scheduling and routing models for airline systems," NTIS publications PB 196528, M.I.T. Flight Transportation Laboratory, 1969.

TABLE 2

SUMMARY OF SCHEDULING RESULTS WITH 25¢ FARE

ROUTE DENSITY	FLEET SIZE	OBJECTIVE FUNCTION	ROUTE	ROUTE FREQ.	ROUTE LOADING	LOAD FACTOR
sparse	2	12.56	17 27	1.125 1.0	56.25 45.5	1 0.91
	6	34.07	17 27	3.33 2.2	166.7 100.1	1 0.91
	10	34.07	17 27	3.33 2.2	166.7 100.1	1 0.91
medium	6	22.03	17 27 37 47	1.975 2.0 1.0 1.0	93.75 100.0 50.0 50.0	0.95 1 1 1
	8	29.41	17 27 37 47	3.33 2.0 1.0 1.51	166.7 100.0 50.0 75.5	1 1 1 1
	16	37.15	17 27 37 47	3.33 2.0 2.57 2.0	166.7 100.0 129.57 100.09	1 1 1 1
dense	7	11.14	17 27 37 47 57 67	1.0 1.07 1.0 1.0 1.0 1.0	50.0 53.57 50.0 50.0 33.0 36.0	1 1 1 1 0.67 0.72
	12	29.55	17 27 37 47 57 67	3.33 2.13 1.51 2.0 1.0 1.0	166.7 106.56 75.7 100.09 33.0 36.0	1 1 1 1 0.66 0.72
	24	33.52	17 27 37 47 57 67	3.33 2.13 2.57 2.0 1.0 2.0	166.7 106.57 128.57 100.09 33.0 36.0	1 1 1 1 0.66 0.72

TABLE 3

SUMMARY OF SCHEDULING RESULTS WITH 35¢ FARE

ROUTE DENSITY	FLEET SIZE	OBJECTIVE FUNCTION	ROUTE	ROUTE FREQ.	ROUTE LOADING	LOAD FACTOR
sparse	2	21.90	17 27	1.125 1.0	56.25 46.0	1 0.92
	6	49.30	17 27	2.67 2.0	133.3 92.0	1 0.92
	10	49.30	17 27	2.67 2.0	133.3 92.0	1 0.92
medium	5	42.41	17 27 37 47	1.75 1.0 1.0 1.0	87.5 46.0 46.0 37.0	1 0.92 0.92 0.92
	8	70.29	17 27 37 47	2.67 2.0 1.92 1.0	133.3 92.0 88.3 37.0	1 0.92 1 0.74
	16	75.45	17 27 37 47	2.67 2.0 2.0 2.0	133.3 92.0 92.0 74.0	1 0.92 0.92 0.74
dense	7	34.07	17 27 37 47 57 67	1.06 1.0 1.0 1.0 1.0 1.0	53.12 46.0 46.0 37.0 29.0 27.0	1 0.92 0.92 0.74 0.58 0.54
	12	74.97	17 27 37 47 57 67	2.67 2.0 2.0 2.0 1.10 1.0	133.3 92.0 92.0 74.0 31.9 27.0	1 0.92 0.92 0.74 0.58 0.54
	24	75.07	17 27 37 47 57 67	2.67 2.0 2.0 2.0 2.0 1.0	133.3 92.0 92.0 74.0 58.0 27.0	1 0.92 0.92 0.74 0.58 0.54

TABLE 4

SUMMARY OF WORKED RESULTS

FARE	ROUTE DENSITY	FLEET SIZE	OBJECTIVE FUNCTION $/hr.	CAPITAL RECOVERY COST $/hr.	DAILY DEFICIT $/hr. (- for surplus	TOTAL PASSENGER pass/hr.	DEFICIT PER PASS $/hr. (- for surplus)
25c	sparse	2	12.56	14.86	2.30	101.75	0.0226
		6	34.07	44.58	10.51	266.7	0.0394
		10	34.07	74.30	40.23	266.7	0.1508
	medium	6	22.03	44.58	22.35	293.75	0.0767
		8	29.41	59.44	30.03	392.2	0.0767
		16	37.15	118.88	81.73	496.36	0.1646
	dense	7	11.14	52.01	40.87	272.57	0.1499
		12	29.55	89.16	59.61	518.05	0.1150
		24	33.52	178.32	144.80	570.93	0.2536
35c	sparse	2	21.80	14.86	-6.94	102.25	-0.0678
		6	48.30	44.58	-3.72	225.33	-0.0165
		10	48.30	74.30	26.0	225.33	0.1154
	medium	5	42.41	37.15	-5.26	226.5	-0.0232
		8	70.29	59.44	-10.85	350.68	-0.0309
		16	75.45	118.88	43.43	391.33	0.1109
	dense	7	34.08	52.01	17.93	238.13	0.0753
		12	74.97	89.16	14.19	449.33	0.0315
		24	75.07	178.32	103.25	477.33	0.2163

TOTAL DAILY DEFICIT AS A FUNCTION OF SYSTEM FEATURES

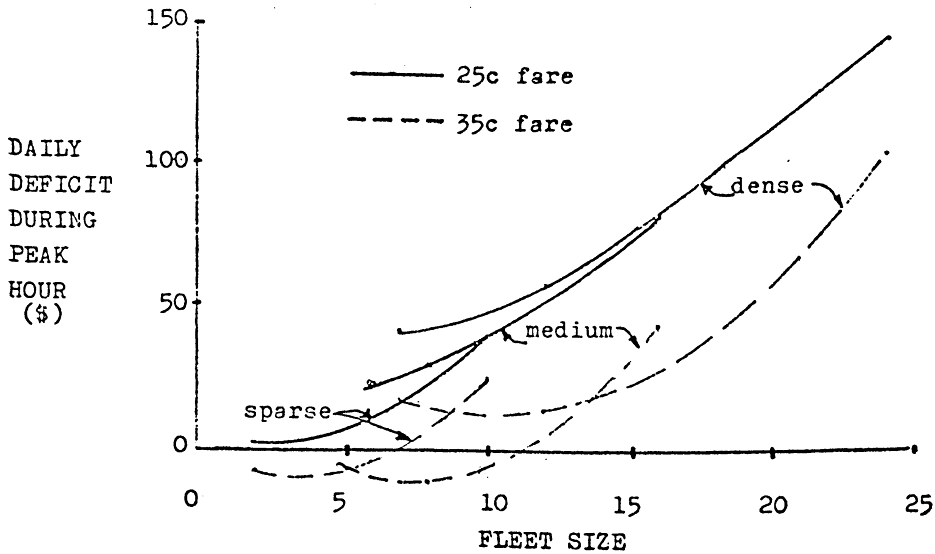

FIGURE 5

DEFICIT PER PASSENGER AS A FUNCTION OF SYSTEM FEATURES

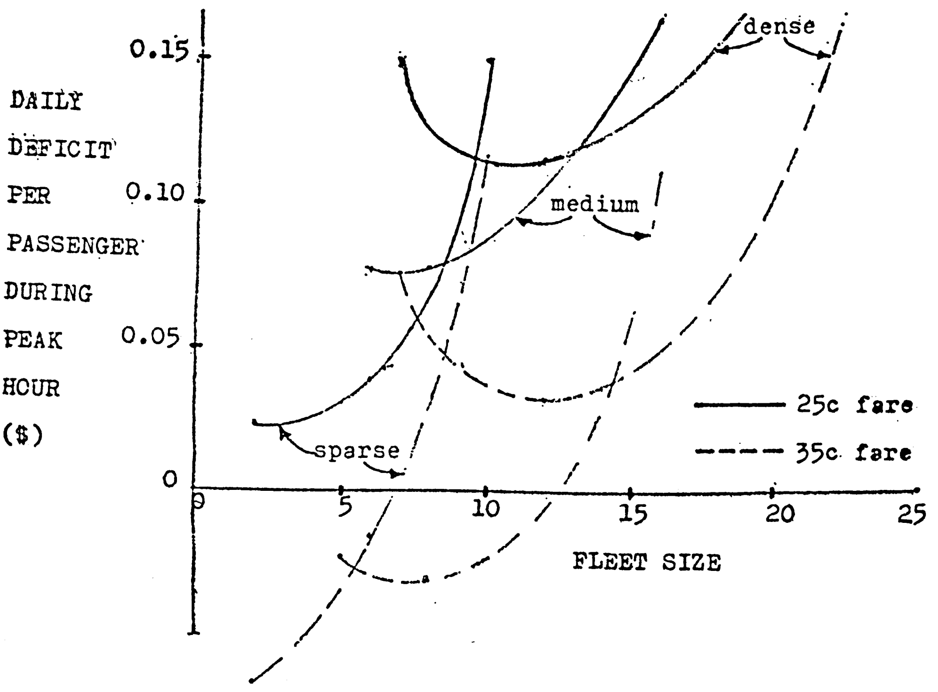

FIGURE 6

POSSIBLE FAMILY OF ENVELOPE CURVES

FIGURE 7

Fare 1 < Fare 2 < Fare 3 < Fare 4