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The Structure of Models: Understanding Theory Reduction and Testing with a Production Example

George C. Davis

The language of economics is the language of models. Understanding the structure of this language offers many benefits. Unfortunately, the structure is ubiquitous in implementation but absent in documentation. This paper documents the structure of models in the context of the theory reduction and testing process. The structure is used to explain why there are several legitimate ways to deal with nonspherical errors in econometric models and why the recent work on stochastic preferences and technologies is a progressive step forward for the discipline. A production modeling exercise is presented to help illuminate the concepts.

Key words: methodology, models, theory reduction, theory testing, Venn diagrams

Introduction

Economists think and communicate in the language of models. Documenting this fact hardly seems necessary as economic journals and books are filled with many types of models: theoretical models, econometric models, programming models, calibration models, statistical models, empirical models, simulation models, etc. Given the growth in the discipline, this modeling proliferation and specialization is understandable. Though important, this specialization can lead to decreasing returns and model myopia—model inferences are overstated and the growth of the discipline unclear. One way to guard against this problem is to have a better understanding of the general structure of models.

There are many benefits associated with knowing the general structure of models, but four seem noteworthy. First, it improves learning ability. Even a cursory reading of the learning and cognition literature indicates learning is enhanced by organizing related but different concepts under a unifying structure (e.g., Estes, 1994). Second, it improves oral and written communication. Often economists disagree simply because they are using the term "model" in different contexts. Third, it improves the ability to easily recognize important contributions. Many of the contributions in agricultural economics come from importing and applying ideas from general economics. A better understanding of the general structure of models allows for an easier identification of the most important ideas for importation (exportation) from (to) the economics literature. Fourth, it becomes easier to diagnose model problem areas and allocate research time to those areas having the highest epistemological return.

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Not surprisingly, with such benefits, there is a substantial literature on the general structure of models. Unfortunately, this literature has been inaccessible to nonspecialists for years and was spread over the loosely connected fields of formal model theory (e.g., Addison, Henkin, and Tarski, 1965), formal logic (e.g., Suppes, 1957), and methodology and philosophy of science (e.g., Hempel, 1977; Stegmüller, 1976). However, within the last two decades, accounts accessible to nonspecialists have emerged (e.g., Giere, 1979; Suppe, 1989), and some of these accounts have begun to seep into economic discussions (e.g., Davis, 1997, 2000; Hausman, 1992; Rappaport, 1998; Stewart, 1993).

This paper integrates and extends the work of several authors (Cook and Hendry, 1994; Davis, 1997, 2000; Haavelmo, 1944; Hausman, 1992; Stewart, 1993; and Suppe, 1989) into a taxonomy which is easily represented with Venn diagrams. In this regard, the paper is a culmination of an extended inductive struggle to identify the general structure of models that is ubiquitous in implementation but absent in documentation. Those more gifted may not recognize or share this struggle, and may even protest the simplicity of the argument's antecedents. Indeed, perhaps the simplicity of the antecedents has led to a belief that the general argument structure is not worthy of documentation and the consequent is inconsequential. For these individuals, patience is requested. The real insights lie not in the antecedents of an argument, but the consequent. Others, especially those inheriting the profession (i.e., graduate students), will hopefully benefit from the documentation of the entire structure presented.[1]

The paper is divided into two major sections. The first defines the basic components of theory reduction and testing. To be broadly applicable, the concepts must be general and somewhat abstract. The general concepts are illuminated with classical producer theory. The second section expands the basic components to develop a taxonomy that is applied to the cotton production model. To demonstrate the usefulness of the taxonomy, the paper extends Hausman's (1992) Venn diagram (chapter 8) to help explain what is often a puzzle in econometric modeling that is not clearly explained in the literature: Why do some model a violation of spherical errors by changing the estimator (e.g., to FGLS or GMM) and some model the violation by respecifying the conditional mean? Which approach is correct? In answering these questions, the paper also demonstrates why the recent work on stochastic utility/production functions (e.g., McElroy, 1987), demand functions (e.g., Lewbel, 2001), and profit functions (e.g., Pope and Just, 2002) is a significant step forward for the discipline.

The Major Concepts in Theory Reduction and Testing

Theory reduction is the process of transforming a theory into a form that is empirically feasible (Cook and Hendry, 1994). To understand this process, it is important to first be clear on the definitions of some major components of modeling in general. Only the definitions needed are presented.[2] Key terms are given in italics.

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[1] A longer initial draft of the paper was entitled, "A Graduate Student Primer on the Language and Structure of Models: An Application of Theory Reduction and Testing Using Venn Diagrams," and was written for graduate students. The reviewers felt the relevant audience was broader than just graduate students, so the paper was rewritten accordingly.

[2] The paper cited in footnote 1 gives a broader discussion with more references and a glossary. It is located at the author's webpage address: http://agecon.tamu.edu/faculty/gdavis/gdavis.htm.

Unless a parallel universe is created, a theory will never completely describe the phenomena or actual data-generating process. A *theory* is by definition an abstraction of reality which explains some phenomena with a subset of selected variables. As a simple example, consider classical producer theory. A rational producer is assumed to maximize profit $\pi = \mathbf{p}_o\mathbf{y}_o - \mathbf{p}_i\mathbf{y}_i = \mathbf{py}$, subject to a production technology $T(\mathbf{y})$, where $\mathbf{p}_o$, $\mathbf{y}_o$, $\mathbf{p}_i$, and $\mathbf{y}_i$ represent vectors of output/input prices and quantities, respectively, or more generally netputs, denoted $\mathbf{p}$ and $\mathbf{y}$. This theory ignores many factors that may or may not be important—weather, government policies, marital status of the producer, or the producer's shoe size. To obtain this abstraction, two critical devices are invoked: axioms and assumptions.

An *axiom* is a statement that is considered self-evident or universally true without proof. An *assumption* is a statement presumed to be true without proof. In classical producer theory, rationality is usually considered an axiom, whereas profit maximization is usually considered an assumption (i.e., there may be alternative objective functions). The line between an assumption and an axiom is often blurred. One person's axiom may be another person's assumption. Assumptions can be classified generally as one of three types: theoretical assumptions, ceteris paribus assumptions, and auxiliary assumptions.

Theoretical assumptions are believed to apply within the intended scope of the theory and are necessary for the theory to generate implications. For example, in producer theory, continuity of the production function is a useful theoretical assumption.

The *ceteris paribus assumption* states "all else is the same or constant," but Cartwright (1988) makes the convincing argument that it actual means "all else is right." There are generally two types of ceteris paribus assumptions: internal and external. The *internal ceteris paribus assumption* serves to isolate the effect of a particular variable, holding constant all other variables explicitly specified within the theory. The *external ceteris paribus assumption* separates the theoretical system under study from other possible external factors. The external ceteris paribus assumption is usually implicit and is the complement set to the factors which have been included in the theory. For example, in producer theory, the upward-sloping supply curve is generated by holding constant all other prices included in the theory except the price of the output. This is an internal ceteris paribus assumption. Alternatively, outside of the theory, there is an external ceteris paribus assumption that the legal environment is constant.

Auxiliary assumptions are made in order to apply the theory to the specific phenomena under study. They are often application specific and not based on any universally accepted scientific procedure. Auxiliary assumptions may or may not place restrictions on the theory and may or may not be theory compatible. For example, if the parameter values of a Cobb-Douglas indirect profit function are consistent with the general properties of the indirect profit function, then, though restrictive, this auxiliary functional form assumption is compatible with the theory. Alternatively, if the parameter values are *not* consistent with the properties of the indirect profit function, then the auxiliary functional form assumption is incompatible with the theory. Auxiliary assumptions are crucial and are discussed further in the next section.

From the axioms and assumptions, the laws of the theory are derived. A *theoretical law* is a statement of a specific type of relationship between theoretical variables. In the producer example, the theoretical law of supply states as output price increases, quantity supplied will increase, ceteris paribus (internal and external). Because a theory ignores many possible factors, a theory is a *counterfactual* argument of the form: "If

certain conditions p are satisfied, then q will occur." The implications of this obvious point are easily forgotten but rather profound. A theory *does not* claim to explain an observed phenomenon, but instead claims to explain the counterfactual phenomenon that would occur if the conditions of the theory were satisfied (Suppe, 1989). Therefore, a theory explains counterfactual or *theoretical variables*, which are the variables—and their values—that would occur if the theory were true. *Observational variables* are the variables—and their values—that are observed for a phenomenon and are determined by possibly many more factors than considered by the theory (Haavelmo, 1944, p. 5).

A *hypothesis* is a statement about the type of relationship that exists between observational variables. It is an observational representation of a theoretical law. If a hypothesis has been tested and confirmed in a variety of settings, then it may be called an *empirical law*. The theoretical law of supply suggests the hypothesis that as the (observed) price of a product increases, the (observed) quantity supplied of the product increases, ceteris paribus. If this relationship has been empirically verified in numerous settings, then there is an empirical law of supply for the product. Unfortunately, the terms "theory," "hypothesis," and "law" are often used interchangeably.

Major Levels in Theory Reduction and Testing

An essential component of the previous section is the difference between theoretical variables and observational variables. This difference is called the *theory-data gap*. The theory-data gap must be bridged in order to implement and test a theory. The theory reduction process provides this bridge by implementing a specific class of auxiliary assumptions called *bridging assumptions* (Hempel, 1977). Bridging assumptions are indispensable in model building.

For example, suppose a researcher wants to answer the following question: What is the impact of increasing labor costs on the supply of cotton in the United States? The researcher may proceed as follows. Start by claiming that classical profit maximization implies the supply of cotton is a function of the price of cotton and the price of inputs. Then, let the supply function be represented by the linear function $y = \beta_0 + \beta_1(\text{price of cotton}) + \beta_2(\text{price of inputs})$. Next, gather some data on these variables and estimate this relationship with some statistical procedure. From the statistical output, the researcher will draw inferences and make statements about the supply of cotton in the United States.

This apparently seamless transition from a theoretical model to an empirical model hides many methodological norms which involve passing through several levels of generality (Stewart, 1993). Each level contains different types of bridging assumptions. As a road map, the major levels discussed in this section are as follows:

- LEVEL I. Theoretical Conceptual Assumptions and the Full Theoretical Model;
- LEVEL II. Theoretical Bridging Assumptions and Errors, and Partial Theoretical Models;
- LEVEL III. Observational Bridging Assumptions and Errors, and Empirical Models; and
- LEVEL IV. Inferential Bridging Assumptions and Errors, and Estimation Models.

The theme in this section is that at Level I, the theory, assumptions, and concepts being discussed apply more generally than those in Level II, which apply more generally than those in Level III, and so on. Bridging assumptions connect the different levels. The bridging assumptions introduce possible bridging errors which are tentatively assumed to be zero until the model is in a form that may be estimated. If the validity of the estimated model is questionable, then the bridging assumptions, and implicitly bridging errors, may be reconsidered. While there are many assumptions within each level, only the most important will be discussed here. Finer gradations of assumptions could easily be achieved, but the objective is the recognition of the general structure, not the nuances within each level.

LEVEL I. Theoretical Conceptual Assumptions and Full Theoretical Models

Theoretical conceptual assumptions formalize the conceptual theory and generate the implications of the theory. From these assumptions, most economic theories generate a functional relationship, such as

$$\mathbf{Y}^* = \mathscr{F}(\mathbf{X}^*). \tag{1}$$

In equation (1), $\mathbf{Y}^*$ is an m-dimensional set of theoretical variables to be explained, and $\mathbf{X}^*$ is an n-dimensional set of theoretical explanatory variables. The vector valued function $\mathscr{F}$ captures the theorized relationship between $\mathbf{X}^*$ and $\mathbf{Y}^*$. $\mathscr{F}$ is only specified as being a member of a class of functions, $\mathscr{F} \in \mathfrak{F}$, possessing certain general properties implied by the theory (e.g., restrictions on the Jacobian and Hessian matrices). Equation (1) is a *full theoretical model*—a mathematical representation of the complete or full conceptual theory. Let the class of models that satisfy (1) be denoted by M_I.

Returning to the cotton example, to determine the impact of increasing labor costs on cotton production, the researcher may claim that classical profit maximization applies to agricultural outputs and inputs (netputs). Consequently, the indirect profit function $\pi(\mathbf{p}^*)$ results, where $\mathbf{p}^*$ is a vector of the theoretical output and input (netput) prices. Hotelling's lemma then implies a system of supply and input demand functions $\mathbf{Y}^* = \mathscr{F}(\mathbf{X}^*)$, where $\mathbf{Y}^*$ is the vector of theoretical netputs, and $\mathbf{X}^* = \mathbf{p}^*$. The properties of $\mathscr{F}$ generate the implications of the theory, such as the laws of supply and input demand, symmetry, homogeneity of degree zero in prices, etc.

LEVEL II. Theoretical Bridging Assumptions and Errors, and Partial Theoretical Models

Most economic theories do not give the precise dimensions of $\mathbf{Y}^*$ and $\mathbf{X}^*$ or the precise functional form for $\mathscr{F}$. Consequently, these elements must be chosen or assumed.

Perhaps the first theoretical bridging assumption is a *dimension reduction assumption* that reduces the full set of possible theoretical variables, $\mathbf{Y}^*$ and $\mathbf{X}^*$, to some analytically feasible subset, denoted by the union of $y^* \subset \mathbf{Y}^*$ and $\mathbf{x}^* \subset \mathbf{X}^*$, respectively. For simplicity, y^* is a scalar and $\mathbf{x}^*$ a vector. This also implies that the functional relationship F between $\mathbf{x}^*$ and y^* must also be a subvector of $\mathscr{F}$, or $F \subset \mathscr{F}$. Choosing the subsets $\mathbf{x}^*$ and y^* has

an asymmetric effect on the theory reduction process. If the full theory claims to explain $\mathbf{Y}^*$ by the vector explanatory variables $\mathbf{X}^*$, then omitting or aggregating some of these explanatory variables in the theory reduction process introduces the first possible theoretical bridging error, $\varepsilon_d = F(\mathbf{X}^*) - F(\mathbf{x}^*)$, or

$$y^* = F(\mathbf{x}^*) + \varepsilon_d. \tag{2}$$

The *dimension reduction error* ε_d is the possible error induced by invoking the dimension reduction assumption, and is considered to be the only difference between y^* and $F(\mathbf{x}^*)$.

Continuing the cotton supply example, at this level (II) the researcher may decide to focus only on the supply of cotton y^*, ignoring the other equations (outputs and inputs) in the netput system. However, though the other equations can be ignored, the other variables cannot. The theoretical supply of cotton is a function of *all* the prices of outputs and inputs in the netput system. The dimension reduction decision here involves making an explicit or implicit assumption about product jointness and/or netput aggregation, such as discussed in Williams and Shumway (1998). Suppose the researcher assumes cotton is a homogeneous nonjoint product and aggregates all inputs into four categories: capital, labor, energy, and material—implying $\mathbf{x}^*$ = (price of cotton, price of capital, price of labor, price of energy, and price of material).

Having reduced the dimensions of the dependent and explanatory variables to a feasible set, there potentially is still an infinity of functional forms which may fall within the class $\mathfrak{F}$. Consequently, to implement a parametric model, a specific functional form for F must be chosen. The *functional form assumption* states that the full theoretical model has a specific functional form f, and depends on a vector of parameters β [i.e., $y^* = f(\mathbf{x}^*; \beta)$], where the values of β allow for the possibility that $f \in \mathfrak{F}$. The functional form assumption is another theoretical bridging assumption and may limit the theory in some manner, so a second possible theoretical bridging error is the *functional form error*, $\varepsilon_f = F(\mathbf{x}^*) - f(\mathbf{x}^*; \beta)$.

The dimension reduction and the functional form assumptions are examples of *theoretical bridging assumptions* which help reduce the theoretical model to an empirically feasible form. Consequently, equation (2) can then be written as

$$y^* = f(\mathbf{x}^*; \beta) + \varepsilon_I. \tag{3}$$

The term ε_I is the *theoretical bridging error*, which is cumulative and composed of the potential dimension reduction error and the functional form error:

$$\begin{aligned}\varepsilon_I = \varepsilon_d + \varepsilon_f &= [F(\mathbf{X}^*) - F(\mathbf{x}^*)] + [F(\mathbf{x}^*) - f(\mathbf{x}^*; \beta)] \\ &= F(\mathbf{X}^*) - f(\mathbf{x}^*; \beta).\end{aligned}$$

By reducing the dimensions and choosing a specific functional form, the full theoretical model is now only partially represented. A *partial theoretical model* is a special case of the full theoretical model. Let the set of partial theoretical models represented by the systematic component $f(\mathbf{x}^*; \beta)$ be denoted as $M_{II} \subset M_I$.

Continuing the cotton model example, suppose the researcher chooses a normalized quadratic indirect profit function. Hotelling's lemma implies the partial theoretical model (3) for cotton supply would have the form

$$y^* = \beta_0 + \sum_{i=1}^{4} \beta_i x_i^* + \varepsilon_I,$$

with y^* representing the theoretical supply of cotton, and x_i^* the theoretical normalized netput prices given earlier.

LEVEL III. Observational Bridging Assumptions and Errors, and Empirical Models

To this point, all of the models presented express a relationship between theoretical or counterfactual variables. The father of modern-day econometrics, Trygve Haavelmo, emphasized that a distinction must be made between theoretical variables and observational variables. Assume momentarily that $\varepsilon_I = 0$, or $y^* = f(\mathbf{x}^*; \beta) = F(\mathbf{X}^*)$.[3] As Haavelmo (1944) points out, the value of y^* is determined by evaluating f at some specific $\mathbf{x}^*$, and *does not* exist independent of $f(\mathbf{x}^*; \beta)$. In the context of the cotton production example, this implies that if f and the values of prices are known, substituting the values of the prices into f will generate exactly the output of cotton. But this quantity is the theoretical quantity produced under the assumptions of the theory, not the observed quantity produced.[4] Because interest lies in observed behavior, $f(\mathbf{x}^*; \beta)$ must be modified to tie it to observed variables.

Haavelmo (1944, pp. 55–57) notes that the theoretical variables can always be connected to the corresponding observational variables by including in f a discrepancy variable, ε, which summarizes all of the excluded factors from the theory, or $f(\mathbf{x}, \varepsilon; \beta)$, and $\mathbf{x}$ represents the observational counterpart of the theoretical variables $\mathbf{x}^*$. The observational counterpart to the theoretical variable y^* is then written as $y = f(\mathbf{x}, \varepsilon, \beta)$. Often for convenience this "disturbance" variable is assumed to be additive, such that $f(\mathbf{x}, \varepsilon; \beta) \equiv f(\mathbf{x}; \beta) + \varepsilon$.

Implementing Haavelmo's connection principle requires *observational bridging assumptions*, which identify the observational variables used in the analysis. There are two types of observational bridging assumptions. First, the researcher must make an *observational unit assumption* (e.g., days, quarters, years) because most, if not all, economic theories do not specify the observational units to which they apply. Let t denote the chosen units, which may be years in a time-series context or countries in a cross-sectional context. The observational unit assumption introduces a possible *observational unit error* in the theory reduction process, or $\varepsilon_{ut} = [y_t^* - y^*] + [f(\mathbf{x}^*; \beta) - f(\mathbf{x}_t^*; \beta)]$. Second, even if the observational unit is correct, most economic theories do not indicate precisely how the theoretical variables are to be measured (e.g., an average price versus a more sophisticated price index); therefore, the researcher must also make a *variable measurement assumption*. Let the measurement variables for y_t^* and $\mathbf{x}_t^*$ be denoted by y_t and $\mathbf{x}_t$. The variable measurement assumption introduces a possible *variable measurement error*, $\varepsilon_{mt} = [y_t - y_t^*] + [f(\mathbf{x}_t^*; \beta)] - f(\mathbf{x}_t; \beta)]$.

These observational bridging assumptions applied to the partial theoretical model generate the *empirical model*:

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[3] Haavelmo (1944), and Cook and Hendry (1994) emphasize statistical reduction theory and do not distinguish between partial theoretical models and full theoretical models as is done here.

[4] Pearl (2000, chapter 5) gives a more recent discussion of this issue as well.

$$y_t = f(\mathbf{x}_t; \beta) + \varepsilon_{IIt}, \quad t = 1, 2, ..., T, \tag{4}$$

where T is the sample size and ε_{IIt} is the observational bridging error. The empirical model (4) is only one of many possible representations of the partial theoretical model (3). Let the set of models represented by the systematic component $f(\mathbf{x}_t; \beta)$ be denoted as M_{III}, and $M_{III} \subset M_{II} \subset M_I$.

As before, the *observational bridging error* ε_{IIt} in (4) is cumulative and is now composed of four parts: the possible dimension reduction error ε_d, the possible functional form error ε_f, the possible unit measurement error ε_{ut}, and the possible variable measurement error ε_{mt}, or

$$\begin{aligned}\varepsilon_{IIt} &= \varepsilon_I + \varepsilon_{ut} + \varepsilon_{mt} = \varepsilon_d + \varepsilon_f + \varepsilon_{ut} + \varepsilon_{mt} \\ &= [F(\mathbf{X}^*) - F(\mathbf{x}^*)] + [F(\mathbf{x}^*) - f(\mathbf{x}^*; \beta)] \\ &\quad + [(y_t^* - y^*) + (f(\mathbf{x}^*; \beta) - f(\mathbf{x}_t^*; \beta))] \\ &\quad + [(y_t - y_t^*) + (f(\mathbf{x}_t^*; \beta) - f(\mathbf{x}_t; \beta))].\end{aligned}$$

Thus, even if the full theory is actually true, possible errors are introduced in the theory reduction process. This implies a researcher can always reconcile contradictory evidence with a theory by invoking Duhem's thesis. *Duhem's thesis* states that if, in testing a theory, auxiliary assumptions must be invoked, all that can be rejected is the conjunction of the theory *and* the auxiliary assumptions. The theory cannot be rejected in isolation. Thus, a theory can always be immunized from refutation by attributing empirical anomalies to false auxiliary assumptions rather than a false theory.[5]

Returning to the cotton model, at this level (III) the researcher may let the observational units be years since 1970, and the measurement variables be those found in a data set such as Ball et al. (1997). The empirical model for cotton supply then becomes

$$y_t = \beta_0 + \sum_{i=1}^{4} \beta_i x_{it} + \varepsilon_{IIt}, \quad t = 1970, 1971, ..., 2003, \tag{5}$$

where y_t would be the number of bales produced, x_{1t} the price of cotton relative to the price of material, x_{2t} the price of capital relative to the price of material, x_{3t} the price of labor relative to the price of material, and x_{4t} the price of energy relative to the price of material, all in year t.

LEVEL IV. Inferential Bridging Assumptions and Errors, and Estimation Models

At this point, all that is left unobserved is ε_{IIt} and β. Because the observational bridging error term ε_{IIt} summarizes all the theory reduction assumptions, it is important to interpret it correctly in an empirical setting. Implicitly recognizing Duhem's thesis, Haavelmo (1944, p. 57) discusses this issue in detail and concludes the only viable interpretation is to assume the full theory *and* theory reduction process is correct on average, which may be called the *disturbance bridging assumption*.[6] Under this assumption, the

[5] See Davis (1997) for a proof and more discussion of Duhem's thesis in a specific setting.

[6] Technically this transforms the modeling space from R^N space to a probability space. The details of this move are lengthy and technical, but are inessential for the main point of this paper.

difference between the theoretical dependent variable and the observed dependent variable is ε_{IIt} and $E(\varepsilon_{IIt}) = 0$. An objective function that is some function of this difference is then defined such that the optimal choice of β minimizes the objective function. The most common objective function is the sum of squared errors, which leads to the least squares estimator.

If the functional form is linear in parameters, as in the cotton example, then the empirical model (5) can be written in standard matrix form as $\mathbf{y} = \mathbf{X}\beta + \varepsilon$, with $\mathbf{y}$ and ε each a $\{T \times 1\}$ vector, $\mathbf{X}$ a $\{T \times k\}$ matrix, and β a $\{k \times 1\}$ vector. Assuming there is interest in statistical inference, the classical linear regression model minimizes $\varepsilon'\varepsilon$ subject to three constraints: (*a*) $\varepsilon = \mathbf{y} - \mathbf{X}\beta$, (*b*) $\mathbf{X}$ is of full rank $k \leq T$, and (*c*) $\varepsilon \mid \mathbf{X} \sim N(0, \sigma^2\mathbf{I})$.

These *inferential bridging assumptions* connect the empirical model to a quantitative procedure that can be used to estimate the unknown quantities of interests and draw inferences. As such, these assumptions lead to the *estimation model*, which in the present setting is $\mathbf{y} \mid \mathbf{X} \sim N(\mathbf{X}\beta, \sigma^2\mathbf{I})$ and comes from coupling the empirical model $\mathbf{y} = \mathbf{X}\beta + \varepsilon$ with the inferential bridging assumption $\varepsilon \mid \mathbf{X} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$, using the transformation method from mathematical statistics. Contrary to the earlier bridging assumptions, these bridging assumptions do not add components to the error, but merely specify its distributional form. Let the set of models consistent with this estimation model be denoted by M_{IV}, so $M_{IV} \subset M_{III} \subset M_{II} \subset M_I$. A single estimate of the estimation model is an *estimated model*, $m_i \in M_{IV}$. *Inferential bridging errors* occur if any of the inferential bridging assumptions are violated in the estimated model. With respect to the cotton production example, at this level (IV) the researcher may apply ordinary least squares to the equation $\mathbf{y} = \mathbf{X}\beta + \varepsilon$.

Though the theory has been connected to the observational data, exactly what economic theory proper says about the statistical aspects of the model can be illuminated with a Venn diagram.[7] Figure 1 shows the theory reduction process in Venn diagram form. The universe of possible models of the phenomenon under study (e.g., cotton production in the United States) is denoted by $\mathbf{M}$. There are four relevant sets: $\mathbf{T}$, the set of all models consistent with the full theoretical model; $\mathbf{B}_T$, the set of all models satisfying the theoretical bridging assumptions; $\mathbf{B}_O$, the set of all models satisfying the observational bridging assumptions; and $\mathbf{B}_I$, the set of all models satisfying the inferential bridging assumptions. If the bridging assumptions are satisfied in these sets, then implicitly the corresponding bridging errors are zero.[8] Insights can be obtained by considering the interrelationships of these sets.

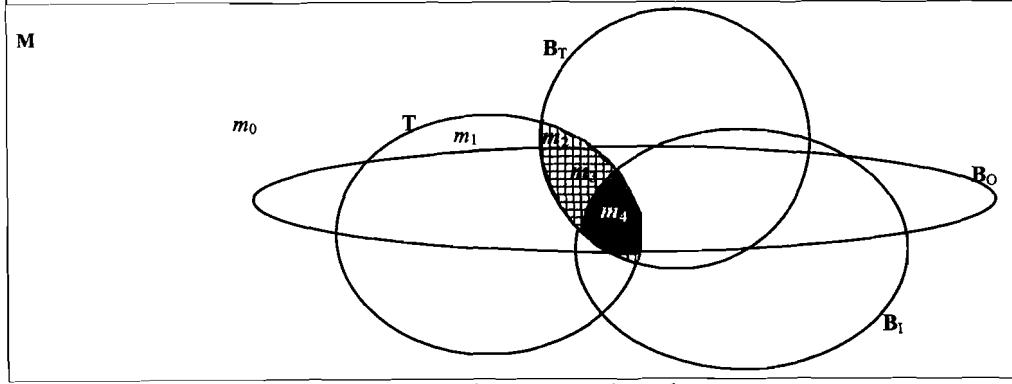
By definition, the full theoretical model considers some factors to be irrelevant (e.g., tractor color), so at Level I the economic theory reduces the set of possible models to $M_I \equiv \mathbf{T}$. If the theory is wrong, then the true model is not an element of the set of full theoretical models (e.g., $m_0 \notin M_I$); but if the theory is true, then the true model lies within M_I (e.g., $m_1 \in M_I$). In moving to Level II, the theoretical bridging assumptions reduce the relevant model space to the set of partial theoretical models $M_{II} \equiv (\mathbf{T} \cap \mathbf{B}_T)$, and the true model is assumed to lie within this set (e.g, $m_2 \in M_{II}$). By definition, the (set) difference between the sets M_I and M_{II} is the theoretical bridging error ε_I. The true

[7] There may be some concern that Venn diagrams are not applicable because in the present setting the sets are not "crisp" (i.e., rules for demarcation are not strict). This concern could be handled using "rough" sets from set theory. However, this added precision adds no new intuition and is viewed as not worth the cost of distracting from the central point of the paper.

[8] Some may prefer to define these sets as where the expected value of the corresponding bridging error is zero. This modification does not substantially affect the discussion that follows.

Figure 1. Venn Diagram of the Theory Reduction Process

Additional Assumption Set	Level, Element, and Model Set	Area	Producer Theory Systematic Component
$\mathbf{T} \equiv$ Theoretical Conceptual Assumptions (e.g., profit maximization)	I. $m_1 \in M_I \equiv \mathbf{T}$: Full Theoretical Model		Complete Netput system: $\mathbf{Y}^* = \mathcal{F}(\mathbf{X}^*)$
$\mathbf{B}_T \equiv$ Theoretical Bridging Assumptions (e.g., dimension reduction, functional form)	II. $m_2 \in M_{II} \equiv (\mathbf{T} \cap \mathbf{B}_T) \subset M_I$: Partial Theoretical Model		Normalized quadratic based supply function with 4 theoretical explanatory variables $y^* = \beta_0 + \sum_{i=1}^{4} \beta_i x_i^*$
$\mathbf{B}_O \equiv$ Observational Bridging Assumptions (e.g., observational units, measurement units)	III. $m_3 \in M_{III} \equiv (\mathbf{T} \cap \mathbf{B}_T \cap \mathbf{B}_O) \subset M_{II}$: Empirical Model		Supply Function with observational variables $y_t = \beta_0 + \sum_{i=1}^{4} \beta_i x_{it}$ $t = 1, 2, \ldots, T$ or $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}$
$\mathbf{B}_I \equiv$ Inferential Bridging Assumptions (e.g., error distribution)	IV. $m_4 \in M_{IV} \equiv (\mathbf{T} \cap \mathbf{B}_T \cap \mathbf{B}_O \cap \mathbf{B}_I) \subset M_{III}$: Estimation Model		Empirical model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}\|\mathbf{X} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$.



M = Universe of all possible systematic models for phenomenon under study.
T = Set of models where the theoretical conceptual assumptions are satisfied.
$\mathbf{B}_T$ = Set of models where the theoretical bridging assumptions are satisfied and the theoretical bridging error is zero.
$\mathbf{B}_O$ = Set of models where the observational bridging assumptions are satisfied and the observational bridging error is zero.
$\mathbf{B}_I$ = Set of models where the inferential bridging assumptions are satisfied and the form of the error is correct (no inferential bridging errors).

model could lie within the set of full theoretical models, but not within the set of partial theoretical models (e.g., m_1). In this case, the theory is true but the selected model false because some important theoretical variables have been erroneously aggregated (e.g., skilled and unskilled labor), omitted (e.g., an input price), or another functional form may be more appropriate (e.g., translog).

In moving from Level II to Level III, the observational bridging assumptions reduce the relevant model space from M_{II} to the set of empirical models $M_{III} \equiv (\mathbf{T} \cap \mathbf{B}_T \cap \mathbf{B}_O)$, and the true model is assumed to lie within this set (e.g, $m_3 \in M_{III}$). Because of the cumulative nature of the bridging errors, the difference between the sets M_{II} and M_{III} is the sum of the observational unit error ε_{ut} and the variable measurement error ε_{mt}. If the partial theory is true but the empirical model false, then the true model could lie within the set of partial theoretical models, but not within the set of empirical models (e.g., m_2).

The reduction from Level III to Level IV involves implementing the inferential bridging assumptions and reduces the model space from M_{III} to the set of estimation models $M_{IV} \equiv (\mathbf{T} \cap \mathbf{B}_T \cap \mathbf{B}_O \cap \mathbf{B}_I)$, and the true model is assumed to lie within this set (e.g., $m_4 \in M_{IV}$). Prior to implementing these assumptions, no specific structure is assumed for the error term, and thus the difference between the sets M_{IV} and M_{III} is that the models within M_{III} are not required to have a spherical error structure as in M_{IV} (e.g., M_{III} allows for heteroskedasticity or serial correlation). Again, the inferential bridging assumptions may be the only part of the theory reduction process that is flawed (e.g., nonspherical error), in which case the true model could be an element such as m_3, where all the other assumptions are satisfied.

Implications for Estimating a Classic Producer Theory Model

Suppose the researcher estimates the cotton model by OLS. It initially looks great in terms of parameter signs: positive for the own price and negative for the input prices. However, a test for first-order serial correlation rejects the spherical error assumption, so the initial significance levels of the parameter estimates are suspect. What should be done? There are two alternatives: (*a*) respecify the (conditional) mean until the disturbance term is spherical, or (*b*) use a more sophisticated estimator which allows for a nonspherical error, such as feasible generalized least squares (FGLS) or generalized method of moments (GMM).[9] Which is correct? It depends on your modeling philosophy, and specifically which underlying assumption you think is questionable and which you think is unquestionable. Respecifying the mean until the error is spherical assumes the spherical error assumption is valid but the partial theoretical model is invalid. Invoking a more sophisticated estimator assumes the partial theoretical and empirical model is valid but the spherical error assumption is invalid. Can economic theory proper or econometric theory proper help in choosing between these two options? Not as they are normally practiced.

[9] Respecifying the conditional mean could involve reassessing any of the bridging assumptions before the inferential bridging assumptions (e.g., functional form, observational units and measurement, etc.). Usually the first attempt is to add or delete variables, including lags. This is implicitly a reevaluation of the dimension reduction assumption and often is not compatible with the stated theoretical model.

Consider first what economic theory proper says about the distribution of $\mathbf{y} \mid \mathbf{X}$. As stated, the distribution of $\mathbf{y} \mid \mathbf{X}$ is $N(\mathbf{X}\beta, \sigma^2\mathbf{I})$ and comes from $\mathbf{y} = \mathbf{X}\beta + \varepsilon$, $\varepsilon \mid \mathbf{X} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$, and the method of transformations. The mean or first moment of $\mathbf{y} \mid \mathbf{X}$ is $\mathbf{X}\beta$ and comes from the empirical and therefore partial theoretical model in conjunction with the inferential bridging assumption that the expected value of the disturbance is zero. However, the variance or second moment of $\mathbf{y} \mid \mathbf{X}$, $\sigma^2\mathbf{I}$, comes *solely* from the inferential bridging assumption that the error term is spherical. In the example presented here and in general, economic theory proper says nothing about the inferential bridging assumptions, and therefore has nothing to say about the variance of $\mathbf{y} \mid \mathbf{X}$. From the economic theory proper perspective, the disturbance may be spherical, nonspherical, or whatever is necessary to make $\mathbf{y} = \mathbf{X}\beta + \varepsilon$, which holds by definition.

Yet knowing the structure of the variance is critical for drawing valid statistical inferences. What does econometric theory proper tell us about this problem? Nothing, except that nonspherical errors are no big deal and can be easily handled by either FGLS or GMM, the latter of which produces a heteroskedastic autocorrelation consistent covariance estimator. In terms of the Venn diagram, taking the empirical model as being correct and implementing a more sophisticated estimator is equivalent to saying the true model is an element of the empirical model set M_{III} but not an element of the estimation model set M_{IV}. Alternatively, taking the empirical model as being incorrect and respecifying the mean is equivalent to saying M_{III}, and perhaps M_I and M_{II}, need to be altered (moved) until the estimated model falls within the area of models with spherical errors, $\mathbf{B}_I$.

In accounting for nonspherical errors, both approaches take a common tack: one moment in the distribution of $\mathbf{y} \mid \mathbf{X}$ is assumed correct and held fixed while the other moment is adjusted until the desired statistical properties are obtained. Because economic theory proper, as it is normally practiced, has no implications beyond the mean of the phenomenon being modeled, rejecting an assumption about the variance, such as no serial correlation, technically has no implication for the economic theory. Consequently, it is actually logically impossible to eliminate theoretical models based on violations of a priori assumptions about the variance (Davis, 2000). The root of this problem stems from the weak theoretical link between the disturbance term and the full or partial theoretical model. As Bentzel and Hansen (1955, p. 167) stated almost 50 years ago, "... the whole discussion about the appropriateness of different estimation procedures seems futile ... as long as we have no 'economic theory of disturbances.'" However, there are encouraging signs.

Several authors have recently explored the implications of including disturbance terms in the agent's optimization problem (e.g., McElroy, 1987; Lewbel, 2001; Pope and Just, 2002). An important theme emerging from this work is that homoskedastic disturbances are inconsistent with optimizing behavior, except under some strong assumptions. The discussion above indicates why this result is important. This approach to economic theorizing provides information about the first (mean) *and* second (variance) moments of the probability distribution of the phenomenon under consideration. Consequently, the number of ways for dealing with second-moment problems is reduced because it is not possible for *any* second moment (e.g., homoskedastic) to be compatible with the economic theory. This is just as Haavelmo advocated over a half century ago: the auxiliary bridging assumptions about probability distributions should be brought into the scientific domain of economic theory proper.

Conclusions

The language of economics is the language of models. This paper presents a general structure of this language which is ubiquitous in implementation but absent in documentation. Specifically, the paper breaks the modeling problem of theory reduction and testing down into four levels that are easily recognized and represented with a Venn diagram. A production modeling exercise is presented to help illuminate the concepts.

Within this framework, two important questions are answered. First, why are there presently several legitimate ways to deal with nonspherical errors in econometric models? Second, why does the idea of stochastic preferences or technology modeling represent a progressive step forward for the discipline? The answer to the first question comes from using the framework to illuminate what theoretical models do and do not say about empirical models. In particular, the domain of empirical discourse in economics will always involve analyzing multiple moments of probability distributions. Yet, most economic theories make only vague statements about the first moment. Consequently, these economic theories are technically immunized from model specification debates stemming from problems associated with violated assumptions about higher moments. This then leads to the answer of the second question.

The theoretical work on stochastic preference and technology represents a progressive step forward for the discipline because these theories generate more empirical content. They make statements about the first *and* second moments of the variables being modeled. This is consistent with recommendations from the methodology literature. As discussed and demonstrated by Davis (1997) in an alternative setting, science usually progresses more rapidly by reformulating or altering a theory than by altering some type of bridging assumption, such as functional form. This is also consistent with Haavelmo's (1944) advice that the auxiliary bridging assumptions about probability distributions should be brought into the scientific domain of economic theory proper.

Other modeling issues could be explored as well using the structure presented. Most likely, different issues will generate different-looking Venn diagrams. Regardless, the general framework should provide useful insights for those struggling with understanding the structure of models.

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