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Stochastic Dominance in Wheat Variety Development and Release Strategies

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Variety development and release decisions involve tradeoffs between yields and characteristics valued by end-users, as well as uncertainties about agronomic, quality, and economic variables. In this study, methods are developed to determine the value of varieties to growers and end-users including the effects of variability in economic, agronomic, and quality variables. The application is to hard red spring (HRS) wheat, a class of wheat for which these tradeoffs and risks are particularly apparent. Results indicate two experimental varieties provide improvements in grower and end-user value, relative to incumbents. Stochastic dominance techniques and statistical tests are applied to determine efficient sets and robustness of the results. A risk-adjusted portfolio model, which simultaneously incorporates correlations between grower and end-use characteristics, is also developed to compare the portfolio value of varieties.

Key words: end-user value, grower value, portfolio value, stochastic dominance, tradeoffs, variety development, wheat

Introduction

Fundamental tradeoffs exist in wheat variety development and release decisions, typically involving yields, disease resistance, and quality. Variability and correlations among attributes increase the complexity of variety development and release decisions, with gains in one attribute often associated with losses in another. Breeders confront these tradeoffs in addition to numerous sources of uncertainty as they strive to identify technologies that improve productivity and profitability. These uncertainties include randomness in economic, agronomic, and quality variables—all of which are compounded by the time lag in variety development decisions.

Economic variables include premiums and discounts for wheat characteristics and uncertainty in implicit values of attributes not explicitly measured in the marketing system. Agronomic variables include yield, disease resistance, and adaptability to climatic conditions. Quality attributes include measurable characteristics such as protein and

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test weight, and functional characteristics (absorption, stability, gluten strength, and various other measures that are important to processors) which typically are not measured in grain transactions due to a lack of timely and repeatable tests.

Variety release decisions in the United States are made by public organizations and private breeding firms determined largely by competitive pressures reflected in the market, although public breeding institutions receive guidance from state agricultural experiment stations and national policy (U.S. Congress/Office of Technology Assessment, 1989). These decisions are typically made by committees involving multiple stakeholders, explicitly considering variety performance relative to incumbents and implicitly considering weights ascribed to agronomic and end-use performance. A more explicit representation of the decision process would incorporate uncertainty and risk-averse behavior of growers and end-users.

The objective of this study is to develop methods that can be used to determine *ex ante* value of new varieties relative to incumbents. Methods are applied to the case of hard red spring (HRS) wheat, a class with specialized but uncertain attributes. Extensive agronomic and quality data are used to derive distributions and correlations among characteristics and varieties. These are combined with distributions of economic variables to simulate distributions of the utility of varieties for end-users and growers.

First, traditional stochastic dominance techniques, previously applied by Anderson (1974) on wheat varietal selection, were used to make pairwise comparisons and to create rankings among varieties. Results were used to derive "efficient sets" of varieties, which were defined as those sets which exclude all dominated alternatives. Second, methods for determining statistical significance of stochastic dominance were applied to determine the robustness of the rankings for dominance criteria with less restrictive assumptions. Finally, a risk-adjusted portfolio model was developed to simultaneously incorporate correlation between grower and end-use characteristics to evaluate variety rankings.

The problem addressed in these models could be applied similarly to numerous other grains, locations, and institutional arrangements. The methods extend the current literature on the economics of variety development. Stochastic dominance techniques initially used by Anderson (1974) were extended to incorporate end-user values. The statistical test of dominance allows for robust comparison of varieties with less restrictive assumptions. The portfolio model comprising end-user and grower values provides a framework to incorporate variability and correlations simultaneously for evaluation of variety development and release strategies.

Background

Intense inter-country competition along with advances in breeding technology have resulted in several economic studies focused on variety development and release strategies. Anderson's 1974 investigation was one of the first to compare yields on varieties and used stochastic dominance methods. Other earlier studies on quality were limited to hedonic types of analysis whose purpose was to estimate implicit values of measurable characteristics. Several studies in wheat variety development were conducted by Brennan (1988, 1990, 1997). Yield, quality, and disease resistance are three broad attributes breeders must consider while developing new varieties for cultivation (Brennan, 1988). To estimate the implicit value of a variety, Brennan (1990, 1997) developed a

“quality index” based on a combination of hedonic studies, implicit market valuations, and payments made for quality in different countries. Another wheat attribute is the value of disease resistance. Utilizing prior estimates of the incidence of diseases, Brennan and Murray (1988) incorporated yield and quality (price) impacts to estimate the cost of a disease (value of resistance).

Bañe e Costa, Ensslin, and Costa (1998) examined the value of rice varieties in Brazil. Unnevehr (1986) analyzed consumer preferences for end-use characteristics and compared them to measures used to screen rice varieties in Southeast Asia. Robinson (1995), Fraser (1997), and Petersen (2000) examined wheat varieties in Australia. Fraser investigated the effect of protein premiums on income streams for wheat growers while capturing effects of yield and price variability, as well as yield-protein tradeoffs. Using a weighted goal-programming model, Dahl, Wilson, and Johnson (2004) evaluated values to end-users in wheat. Some of these studies are nonstochastic (with the exceptions of Robinson; Fraser; and Peterson) and/or rely heavily upon subjective expert opinions to assess characteristic values and on biological relationships between crop growth and the environment.

Methods

Two models are developed to estimate the value of a new variety to end-users and to growers. Distributions of values are then compared using traditional stochastic dominance techniques, statistical tests of stochastic dominance, and a portfolio approach to evaluate tradeoffs and risks among varieties.

A weighted goal-programming model is adopted in the end-user value model (Zeleny, 1982). This is a multiple-criterion optimization method utilized to evaluate multiple conflicting objectives. The model evaluates differences between selected end-use characteristics for a new variety and those obtainable from the best blend of incumbent varieties. Deviations of end-use characteristics between a new variety and an optimal blend of existing varieties are assigned weighted values based on the characteristic, size of the deviation, and whether the deviation is positive or negative (i.e., the test variety has higher/lower value for the characteristic than the best blend).

The objective is to choose shares for a blend of existing varieties that minimize the weighted value of deviations between the new variety and blend of existing varieties across the multiple end-use characteristics. For each iteration of (random) grain quality and price parameters, indexed by k , the best blend is chosen as the solution to the following problem:

$$(1) \quad \text{Min } Z_k = \sum_j (M_{j,k} * P_{j,k}) + (L_{j,k} * N_{j,k})$$

subject to:

$$(2) \quad \sum_{i=1}^n G_{i,j,k} * X_{i,k} - P_{j,k} + N_{j,k} = Y_{j,k}, \quad \sum_{i=1}^n X_{i,k} = 1$$

and nonnegativity constraints $X_{i,k} \geq 0$; $P_{j,k} \geq 0$; and $N_{j,k} \geq 0$. Variables and notation are defined in table 1, and j is assumed to be protein, test weight, absorption, and extraction.

Table 1. Notation for the End-User and Grower Models

Notation	Description
Objective Function Value, End-User Model:	
Z_k	Implicit value (\$/bushel) of a new wheat variety, given draw k
Choice Variables:	
$X_{i,k}$	Share of existing variety (indexed by i) in a blend
$P_{j,k}$	Positive deviation of quality attribute j in blend, relative to new variety
$N_{j,k}$	Negative deviation of quality attribute j in blend, relative to new variety
Parameters, End-User Model:	
$M_{j,k}$	Marginal value/unit of positive quality deviation
$L_{j,k}$	Marginal value/unit of negative quality deviation
$G_{i,j,k}$	Level of quality attribute j in existing variety i
$Y_{j,k}$	Level of quality attribute j in new variety
Parameters, Grower Model:	
I	Income in dollars per acre
P^w	Base price Minneapolis for 14% protein from which transportation and handling are deducted to localize prices
P^P	Premium for protein > 14%
D^P	Discount for protein < 14%
C	Protein content (correlated with yield)
D^T	Test weight discount
TW	Max(58 – Test Weight, 0), i.e., amount test weight is below 58 lbs./bushel
D^{FN}	Discount applied when falling number is less than 300 seconds
FN	Binary variable indicating falling number is lower than limit (300 seconds)
D^{Vom}	Vomitoxin discount
VS	Binary variable indicating vomitoxin exceeds critical limit (2 ppm)
YD	Yield (includes variability due to disease, etc.)

The optimization problem is embedded in a stochastic simulation framework and solved for each iteration (k) where price and quality parameters are drawn from known distributions (table 2). Results from the complete set of k solutions for weighted deviations are used to compute the distribution for the *expected* end-user value of a new variety:

$$(3) \quad V^u = E(Z_k),$$

where V^u represents the end-user's expected value of a new variety relative to the optimal blend of existing varieties.

Grower values were determined by estimating the income for each variety relative to incumbent varieties. This is defined as:

$$(4) \quad V^g = E(I - Target),$$

where V^g is grower value for the new variety, I is income for the new variety, and $Target$ is expected income for all incumbent varieties. Grower income was derived for each variety as:

Table 2. Distributions for Prices, Premiums, and Discounts for End-User and Grower Value Models ($\text{\$/bushel}$)

Description	Base Value	Mean Premium/Discount ($\text{\$/bu.}$)	Std. Dev. ($\text{\$/bu.}$)	Correlation	Distribution
MGE Futures Price		436	77		Normal
Protein 15%	14%	40	34	0.85 w/protein 13%	Normal Truncated at 0
Protein 13%	14%	-14	19	0.85 w/protein 15%	Normal Truncated at 0
Test Weight (lbs./bu.)	58	-4	5		Normal Truncated at 0
Falling Number (seconds)	300	-26	37		Normal Truncated at 0
Vomitoxin (ppm)	Nil	-20	44		Normal Truncated at 0

Sources: Distributions: prices and protein premiums/discounts estimated from Minneapolis Grain Exchange data; premiums and discounts for test weight, falling number, and vomitoxin are from a survey of elevator managers (Wilson and Dahl, 2001).

$$(5) \quad I = \left[P^w + P^P * [\max(0, C - 14)] - D^P * [\min(0, C - 14)] - D^T * TW - D^{FN} * FN - D^{Vom} * VS \right] * YD.$$

This formulation derives the income per acre as the product of prices and yields, where prices are adjusted by random market premiums and discounts (as defined in table 1) for deviations from the base quality.

The grower and end-user models were simulated utilizing the same random draws for quality and agronomic characteristics, and premiums and discounts. This procedure assures the effect of correlations among quality and agronomic characteristics and premiums and discounts between end-user and grower valuations are captured in the simulation. The models were iterated 5,000 times. Results from each were used to generate distributions for grower and end-user values. Paired grower and end-user values were retained for use in the portfolio analysis.

Data

Variety yields, protein content, and other wheat, flour, and end-use characteristics are from results of North Dakota variety trials (North Dakota State University, Department of Cereal Science and Food Technology). Means, standard deviations, and correlations of characteristics were estimated by variety for the years 1989–1997. Although yields by variety were largely positively correlated with test weights and extraction rates, they were negatively correlated with falling number, protein, and absorption. In the end-user model, protein by variety is largely negatively correlated with test weights and extraction, and positively correlated with absorption.

For the end-user model, values for wheat and end-use characteristics were estimated for two groups of varieties. A group of eight incumbent varieties (V_1 – V_8) with observa-

tions throughout the period 1989–1997 was utilized as the base for comparison with newer (with limited observations) and experimental varieties. Means, standard deviations, and correlations were estimated by variety and characteristic. Then values for a second group were estimated, consisting of five newer varieties with limited observations (V_9 – V_{13}) and three experimental varieties that have since been released (V_{14} – V_{16}).

For the grower model, values were estimated for a set of popular varieties in the late 1990s; these included the five newer varieties (V_9 – V_{13}) and six of the eight base incumbent varieties (V_1 – V_6). Data included distributions and correlations for yields, protein, falling number, test weights, and the resistance rating for fusarium head blight (vomitoxin).

Farm prices and protein premiums are average marketing year values, with distributions estimated from daily observations over the 1989–1997 period, localized by deducting shipping and handling costs. Premiums and discounts were random and drawn from the distributions reported in table 2. Protein premiums and discounts are the premiums/discounts for protein relative to 14%, basis Minneapolis from 1989–1997. Discounts for test weight, falling numbers, and vomitoxin (table 2) were taken from a survey of elevator managers on premiums/discounts for hard red spring wheat (Wilson and Dahl, 2001).

To capture the effect of disease resistance for fusarium head blight/vomitoxin, a binary variable (VS), representing presence/absence of vomitoxin in levels exceeding tolerance, was estimated using a two-stage procedure. First, a distribution was estimated for head score (HS) values (Nganje et al., 2001). Head scores are a visual scale used for approximating yield loss due to fusarium head blight in field plots, and represent the percentage of yield loss. In the second step, a functional relationship was estimated between vomitoxin levels and yields, test weights, head scores, and variety susceptibility rankings for fusarium head blight (Stack, 2001). This functional relationship was specified in the simulation to predict vomitoxin levels. If predicted vomitoxin levels exceeded 2 ppm, then VS for vomitoxin was set to 1 and the discount applied. If predicted levels did not exceed 2 ppm, no vomitoxin discount was applied.

For the end-user model, marginal values (M and L) for protein and test weight were assumed to be protein and test weight premiums and discounts from table 2. The marginal value of flour extraction was estimated using Drynan's (1996) valuation model. This model estimates the value of wheat to millers (milling margin) after adjusting for differences in quality characteristics (moisture, foreign material, dockage, extraction rates). The effect of extraction was a 5¢/bushel increase in value for a 1% increase in flour extraction. The marginal value of absorption was estimated assuming additional absorption reduces the amount of flour required to produce a given volume of dough. Using a traditional bread formulation, increasing absorption by 1% (from 62% to 63% absorption) reduces both flour and wheat needs by 0.5%. The marginal value of additional absorption with a wheat cost of 400¢/bushel is approximately 5¢/bushel.

Comparison of Variety Ranking Methods

Several criteria and tests were used to make comparisons among varieties, ultimately with the goal of defining efficient sets of varieties—i.e., those sets of varieties that exclude all dominated alternatives. Traditional stochastic dominance methods are applied first to identify varieties which are not only improved but are also less risky and would be preferred by risk-averse individuals.

Table 3. Risk-Efficient Sets of Varieties Using Stochastic Dominance Criteria

Description	Grower Value	End-User Value
Traditional Stochastic Dominance Analysis:		
FDD	V_{10}, V_{16}	V_5, V_{10}, V_{11}
SDD	V_{10}	V_5, V_{10}
GSD ^a	V_{10}, V_{16}	V_{10}
Tests for Statistical Significance of Stochastic Dominance:		
FDD test	V_{10}	V_5, V_{10}
SDD test	V_{10}	V_5, V_{10}

^aThe generalized stochastic dominance (GSD) solutions were obtained utilizing Meyers' (1980) GSD program updated by Richardson. Results represent the first most preferred efficient set.

Three traditional stochastic dominance criteria [first degree stochastic dominance (FDD), second degree stochastic dominance (SDD), and generalized stochastic dominance (GSD)] were evaluated for each of the grower and end-user values, and are presented in the proceeding section. FDD is a stronger dominance criterion, with fewer restrictive assumptions, followed by SDD and higher order stochastic dominance (Anderson, 1974). GSD incorporates FDD and SDD and higher orders of stochastic dominance (Cochrane, Robinson, and Lodwick, 1985) and should provide consistent rankings. However, since stochastic dominance tests are weak, significance tests were conducted to determine the robustness of the results.¹ The statistical tests provide robust findings with fewer restrictive assumptions compared to lower order stochastic dominance techniques. These methods were applied to grower and end-user values separately, giving rise to potentially different conclusions.

Finally, a portfolio method comprised of end-user and grower values was utilized to compare variety rankings. This portfolio approach provides a framework for incorporating variability and correlations simultaneously to evaluate variety development and release strategies.

Traditional Stochastic Dominance of Grower and End-User Values of Varieties

Distributions of grower and end-user values were evaluated to determine dominance of varieties. FDD, SDD, and GSD were tested through pairwise comparisons of varieties for end-user and grower values. FDD and SDD were analyzed with traditional stepwise stochastic dominance methods (see appendix A). GSD was analyzed using Meyers' (1980) GSD software package updated by Richardson.²

Comparing FDD for grower values indicates many varieties are not dominated by others. Variety V_{10} dominated all other varieties except V_{16} . The risk-efficient set excluded varieties dominated (table 3); for the grower value, the FDD efficient set included V_{10} and V_{16} . Results for all variety comparisons of SDD grower value show more varieties dominate others (table 4). The risk-efficient set using SDD grower value (table 3) only

¹ A major challenge with stochastic dominance analysis is to decrease type II error (larger efficient sets) without increasing type I error (inaccurate rankings) (Cochrane, Robinson, and Lodwick, 1985).

² For a formal presentation of GSD, see Ingersoll (1987, pp. 138–139).

Table 4. Results of Estimated Second Degree Stochastic Dominance for Paired Comparisons of Varieties from Traditional Step Function Methods for Grower and End-User Value

VARIETY Y	VARIETY Z															
	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₉	V ₁₀	V ₁₁	V ₁₂	V ₁₃	V ₁₄	V ₁₅	V ₁₆		
Grower Value:																
V ₁		1	1	3	1	1	1	2	3	3	3	3	3	3	3	
V ₂	2		3	2	3	3	3	2	2	2	2	3	3	3	3	
V ₃	2	3		2	3	3	3	2	3	2	2	3	3	3	3	
V ₄	3	1	1		3	1	1	2	3	3	3	3	3	3	3	
V ₅	2	3	3	3		3	3	2	2	2	3	3	3	3	3	
V ₆	2	3	3	2	3		1	2	3	3	2	3	3	3	3	
V ₉	2	3	3	2	3	2		2	2	2	2	3	3	3	3	
V ₁₀	1	1	1	1	1	1	1		1	1	1	1	1	1	1	
V ₁₁	3	1	3	3	1	3	1	2		3	3	1	3	3	3	
V ₁₂	3	1	1	3	1	3	1	2	3		3	1	3	3	3	
V ₁₃	3	1	1	3	3	1	1	2	3	3		3	3	3	3	
V ₁₄	3	3	3	3	3	3	3	2	2	2	3		2	3	3	
V ₁₅	3	3	3	3	3	3	3	2	3	3	3	1		3	3	
V ₁₆	3	3	3	3	3	3	3	2	3	3	3	3	3		3	
End-User Value:																
V ₁		3	3	2	2	2	1	2	1	1	1	1	2	3	3	
V ₂	3		3	2	2	2	1	2	1	3	3	3	2	2	2	
V ₃	3	3		3	2	3	1	2	1	3	3	3	2	3	3	
V ₄	1	1	3		2	3	1	3	1	1	1	1	3	1	1	
V ₅	1	1	1	1		1	1	3	1	1	1	1	3	1	1	
V ₆	1	1	3	3	2		1	3	1	1	1	1	3	3	3	
V ₉	2	2	2	2	2	2		2	3	2	2	2	2	2	2	
V ₁₀	1	1	1	3	3	3	1		1	1	1	1	1	1	1	
V ₁₁	2	2	2	2	2	2	3	2		2	3	3	2	2	2	
V ₁₂	2	3	3	2	2	2	1	2	1		1	1	2	2	2	
V ₁₃	2	3	3	2	2	2	1	2	3	2		2	2	2	2	
V ₁₄	2	3	3	2	2	2	1	2	3	2	1		2	2	2	
V ₁₅	1	1	1	3	3	3	1	2	1	1	1	1		1	1	
V ₁₆	3	1	3	2	2	3	1	2	1	1	1	1	2		2	

Notes: 1 = variety Y dominates Z, 2 = variety Z dominates Y, and 3 = no dominance identified.

contains variety V₁₀, which dominated all other varieties. The GSD results revealed the most preferred varieties were V₁₀ and V₁₆ (table 3). This is the same grouping as that obtained with FDD, while preferred varieties for SDD only contained V₁₀.

For end-user value, several varieties dominate others using FDD. For those varieties dominating others for grower value, many of these switched from identifying dominance to indicating no dominance. Other comparisons between varieties switched from no dominance for grower value to indicating dominance for end-user value. These changes in dominance suggest tradeoffs between grower and end-user valuations. The risk-efficient FDD set for end-user value includes V₅, V₁₀, and V₁₁. V₅ largely dominates incumbent

varieties, but does not dominate the newer varieties. V_{10} dominates most of the varieties, and V_{11} was not dominated by, nor did it dominate, any variety. The number of variety comparisons having dominance for SDD (as shown in table 4) increased dramatically over those identified using FDD. The risk-efficient set for SDD contained V_5 and V_{10} . Variety V_{11} , which was included in the FDD efficient set, was dominated by most varieties using the SDD criterion. The risk-efficient set for GSD included only V_{10} .³ This compares to traditional FDD with varieties V_5 , V_{10} , and V_{11} , and to SDD with varieties V_5 and V_{10} .

Significance Tests of Stochastic Dominance

Differences in stochastic dominance refinement for grower and end-user values indicate the tradeoffs between groups for selected varieties. It is appealing to use tests of statistical significance because stochastic dominance is a weak test (Pope and Ziemer, 1984) and does not evaluate robustness of the efficient sets. Since GSD incorporates FDD and SDD, the Davidson-Duclos (2000) test for statistical significance was used to evaluate the robustness of efficient sets for FDD and SDD.⁴ Tse and Zhang (2000) examined a number of methods that assess the statistical significance of dominance, compared them, and identified the Davidson-Duclos test as most appropriate based on the power of the test (appendix B).

Davidson-Duclos test statistics were generated, and results of the hypothesis tests are presented in tables 5 and 6 for FDD and SDD, respectively. For FDD, results of tests of hypotheses for grower value show traditional dominance rankings of variety comparisons were not robust. There are several instances where the hypothesis tests indicated varieties dominated others where no dominance had been identified using the traditional FDD analysis. The risk-efficient set (lower portion of table 3) for the statistical test of FDD for grower value only included variety V_{10} . Variety V_{16} , which had been in the FDD grower value efficient set for the traditional FDD, was dominated by V_{10} and was excluded in this efficient set. Significance of variety dominance using SDD for grower value suggested varieties were different, but none dominated for several variety comparisons (e.g., V_1 different from V_6 , V_3 different from V_{12} , among others), where the traditional SDD had indicated there was SDD. Also, several variety comparisons found there was significant SDD (V_1 dominated V_{11} and V_{14}), where the traditional SSD techniques had revealed none.

Results for FDD of end-user values indicate more varieties dominate others than from the traditional analysis. Comparisons show that distributions were different, but dominance was not identified. The risk-efficient set for FDD end-user value included varieties V_5 and V_{10} . For the statistical test of SDD of end-user value, a few variety comparisons identified significant dominance where no such instances were present in the traditional SDD (V_{16} dominated V_1 and V_2). Also, a number of varieties where dominance was identified in the traditional analysis were found to be not significant, or the distributions were different but no dominance was identified. The risk-efficient set for end-user SDD significance (lower portion of table 3) included varieties V_5 and V_{10} .

³ The results of the GSD did not provide consistent refinement when cumulative density functions crossed as in the cases of V_{10} and V_{16} for grower values and V_5 and V_{10} for end-user values. V_{16} was eliminated from the efficient set with SDD and was present in the efficient set with GSD. It is unclear whether this occurrence is due to the fact that stochastic dominance is a weak test. A statistical test (see appendix B) provides more insight into the robustness of FDD and SDD efficient sets, since the GSD results are further refinements of FDD and SDD.

⁴ A statistical test for GSD is unavailable.

Table 5. Results of Hypothesis Tests for First Degree Stochastic Dominance for Grower and End-User Values, by Variety

VARIETY Y	VARIETY Z															
	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₉	V ₁₀	V ₁₁	V ₁₂	V ₁₃	V ₁₄	V ₁₅	V ₁₆		
Grower Value:																
V ₁		1	1	4	1	4	1	2	1	1	4	1	4	4		
V ₂	2		4	2	4	2	4	2	2	2	2	4	4	4		
V ₃	2	4		2	1	2	4	2	4	4	2	4	4	4		
V ₄	4	1	1		1	1	1	2	4	4	4	1	4	4		
V ₅	2	4	2	2		2	2	2	2	2	2	4	4	4		
V ₆	4	1	1	2	1		1	2	4	4	2	1	4	4		
V ₉	2	4	4	2	1	2		2	2	4	2	4	4	4		
V ₁₀	1	1	1	1	1	1	1		1	1	1	1	1	1		
V ₁₁	2	1	4	4	1	4	1	2		4	2	1	4	4		
V ₁₂	2	1	4	4	1	4	4	2	4		4	4	4	4		
V ₁₃	4	1	1	4	1	1	1	2	1	4		1	4	4		
V ₁₄	2	4	4	2	4	2	4	2	2	4	2		2	4		
V ₁₅	4	4	4	4	4	4	4	2	4	4	4	1		4		
V ₁₆	4	4	4	4	4	4	4	2	4	4	4	4	4			
End-User Value:																
V ₁		1	4	4	2	2	1	2	1	4	1	4	2	4		
V ₂	2		4	2	2	2	1	2	1	4	4	4	2	2		
V ₃	4	4		4	2	4	1	2	1	4	4	4	2	4		
V ₄	4	1	4		2	4	1	4	1	4	1	1	4	4		
V ₅	1	1	1	1		1	1	4	1	1	1	1	4	1		
V ₆	1	1	4	4	2		1	4	1	1	1	1	4	1		
V ₉	2	2	2	2	2	2		2	4	2	2	2	2	2		
V ₁₀	1	1	1	4	4	4	1		1	1	1	1	1	1		
V ₁₁	2	2	2	2	2	2	4	2		2	2	2	2	2		
V ₁₂	4	4	4	4	2	2	1	2	1		1	1	2	2		
V ₁₃	2	4	4	2	2	2	1	2	1	2		2	2	2		
V ₁₄	4	4	4	2	2	2	1	2	1	2	1		2	2		
V ₁₅	1	1	1	4	4	4	1	2	1	1	1	1		1		
V ₁₆	4	1	4	4	2	2	1	2	1	1	1	1	2			

Notes: 1 = variety Y statistically dominates Z at $p = 0.05$; 2 = variety Z statistically dominates Y at $p = 0.05$; 3 = no significant statistical difference between varieties Y and Z at $p = 0.05$; and 4 = variety Y is statistically different from Z at $p = 0.05$, however, Z does not dominate Y, and Y does not dominate Z at $p = 0.05$.

Comparisons of statistically significant dominance for end-users and growers reveal differences. The statistical test indicates that V₅ is preferred to V₁, V₃, V₄, V₆, V₉, V₁₁, V₁₂, and V₁₃ for end-users, but is dominated by these varieties for growers. In contrast, other varieties had consistent rankings across grower and end-user values. The statistical tests provide more robust results with fewer restrictive assumptions.

Comparisons of Varieties Using Portfolio Values

Because the risk-efficient sets differ for grower and end-user values, a joint valuation is appropriate. To do so, the joint value of varieties to both end-users and growers was

Table 6. Results of Hypothesis Tests for Second Degree Stochastic Dominance for Grower and End-User Values, by Variety

VARIETY Y	VARIETY Z															
	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₉	V ₁₀	V ₁₁	V ₁₂	V ₁₃	V ₁₄	V ₁₅	V ₁₆		
Grower Value:																
V ₁		1	1	4	1	4	1	2	1	1	4	1	4	4		
V ₂	2		4	2	4	2	4	2	2	2	2	4	4	4		
V ₃	2	4		2	1	2	4	2	4	4	2	4	4	4		
V ₄	4	1	1		1	1	1	2	4	4	4	1	4	4		
V ₅	2	4	2	2		2	2	2	2	2	2	4	4	4		
V ₆	4	1	4	2	1		1	2	4	4	2	1	4	4		
V ₉	2	4	4	2	1	2		2	2	4	2	4	4	4		
V ₁₀	1	1	1	1	1	1	1		1	1	1	1	1	1		
V ₁₁	2	1	4	4	1	4	1	2		4	2	4	4	4		
V ₁₂	4	1	4	4	1	4	4	2	4		4	4	4	4		
V ₁₃	4	1	1	4	1	1	1	2	1	4		1	4	4		
V ₁₄	2	4	4	2	4	2	4	2	4	4	2		2	4		
V ₁₅	4	4	4	4	4	4	4	2	4	4	4	1		4		
V ₁₆	4	4	4	4	4	4	4	2	4	4	4	4	4			
End-User Value:																
V ₁		4	4	4	2	2	1	2	1	4	1	4	2	4		
V ₂	2		4	2	2	2	1	2	1	4	4	4	2	2		
V ₃	4	4		4	2	4	1	2	1	4	4	4	2	4		
V ₄	4	1	4		2	4	1	4	1	4	1	1	4	4		
V ₅	1	1	1	1		1	1	4	1	1	1	1	4	1		
V ₆	1	1	4	4	2		1	4	1	1	1	1	4	1		
V ₉	2	2	2	2	2	2		2	4	2	2	2	2	2		
V ₁₀	1	1	1	4	4	4	1		1	1	1	1	1	1		
V ₁₁	2	2	2	2	2	2	4	2		2	4	2	2	2		
V ₁₂	4	4	4	4	2	2	1	2	1		1	1	2	2		
V ₁₃	2	4	4	2	2	2	1	2	4	2		2	2	2		
V ₁₄	4	4	4	2	2	2	1	2	1	2	1		2	2		
V ₁₅	1	1	1	4	4	4	1	2	1	1	1	1		1		
V ₁₆	4	1	4	4	2	2	1	2	1	4	1	1	2			

Notes: 1 = variety Y statistically dominates Z at $p = 0.05$; 2 = variety Z statistically dominates Y at $p = 0.05$; 3 = no significant statistical difference between varieties Y and Z at $p = 0.05$; and 4 = variety Y is statistically different from Z at $p = 0.05$, however, Z does not dominate Y, and Y does not dominate Z at $p = 0.05$.

evaluated using a portfolio approach (McCarl et al., 1987). This approach considers the joint value to growers and end-users, accounts for variability between values of decision makers, and explicitly incorporates the correlation between end-user and grower values.

A portfolio value of a variety (VV_i) was derived as the weighted sum of simulated pairs of end-user and grower values. These were standardized to variables ranging from 0 to 1 prior to use in the portfolio to offset effects of differences in values. Simulated standardized pairs were utilized to retain correlations between valuations of growers and end-users. An initial weight (δ) of 0.5 was assumed for the portfolio value of a variety:

Table 7. Portfolio Means, Variances, and Risk-Adjusted Values, by Variety (base case, $\delta = 0.5$, $\theta = 1.5$)

Variety	Mean Portfolio Value	Portfolio Variance	Risk-Adjusted Value of Portfolio
	<----- (¢/bushel) ----->		
V ₁₀	0.521	0.0028	0.519
V ₁₅	0.500	0.0027	0.498
V ₁₆	0.497	0.0031	0.495
V ₅	0.485	0.0023	0.483
V ₄	0.477	0.0019	0.476
V ₁₁	0.475	0.0030	0.473
V ₁₂	0.473	0.0020	0.472
V ₃	0.472	0.0019	0.470
V ₁	0.471	0.0020	0.470
V ₁₄	0.468	0.0026	0.466
V ₂	0.465	0.0022	0.463
V ₆	0.463	0.0019	0.462
V ₁₃	0.460	0.0020	0.459
V ₉	0.444	0.0026	0.442

$$(6) \quad VV_i = \delta * V_i^g + (1 - \delta) * V_i^u.$$

Values were then compared to determine preferences for varieties. For this procedure, a variety (A_1) was considered to be preferred to an alternative variety (A_2) if:

$$(7) \quad u_{A_1} - \frac{\theta}{2} \sigma_{A_1}^2 \geq u_{A_2} - \frac{\theta}{2} \sigma_{A_2}^2,$$

where u = the mean portfolio value of weighted income for a variety; θ was set using the Pratt risk-aversion parameter and risk premium (McCarl et al., 1987), with $\theta = 2\Lambda/\sigma$; σ^2 = the variance of weighted income of a variety; and A_1 and A_2 represent the prospective varieties compared. Following McCarl et al. (1987), a value of 1.5 was used for Λ . Sensitivities were conducted to examine the effect of alternative weights for end-user and grower value and risk attitude parameters on preferences for varieties.

Portfolio means and variances were calculated for each of the varieties and used to estimate the risk-adjusted portfolio value for each variety (table 7). These values were used to compare and rank varieties. Risk-adjusted values ranged from a high of 0.519 for V_{10} to a low of 0.442 for V_9 . V_{10} was preferred to all other varieties. Sensitivities were conducted for alternative values of δ and summarized for a reduced set of varieties in figure 1. When end-user weights are greater than 0.4, V_{15} is the second ranked variety. As δ approaches 0, this variety drops to third. For lower values of δ , V_{16} is preferred to V_3 , V_4 , and V_5 . As δ increases, V_5 , V_4 , and V_3 become preferred to V_{16} . The portfolio method considers end-user and grower values simultaneously. Across most of the weight values, the portfolio identifies V_{10} as the dominant variety. When examining end-user value alone, the GSD criteria show that V_{10} is dominant for end-users, and V_{10} and V_{16} are dominant for growers.

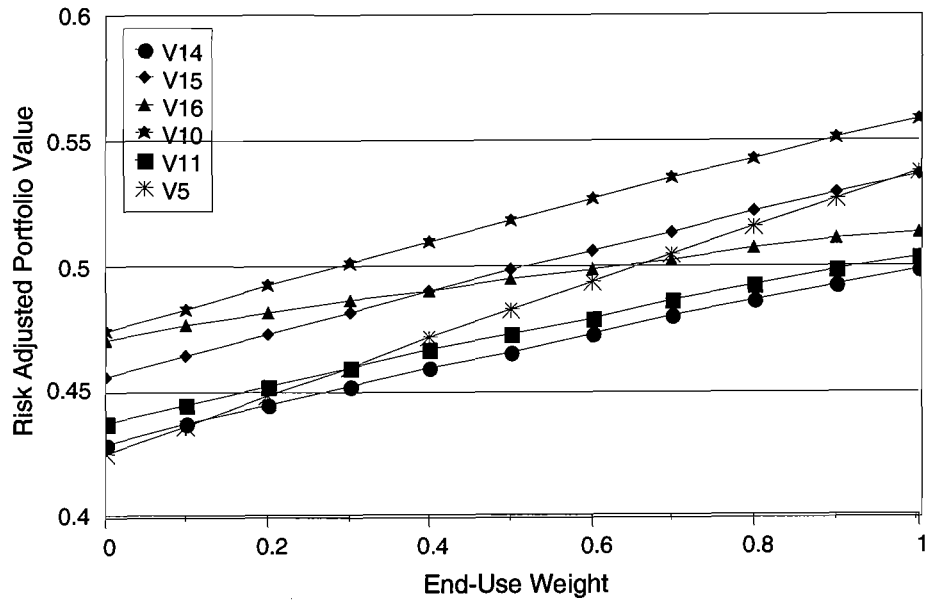


Figure 1. Sensitivity of risk-adjusted portfolio value to end-user weights, for selected varieties

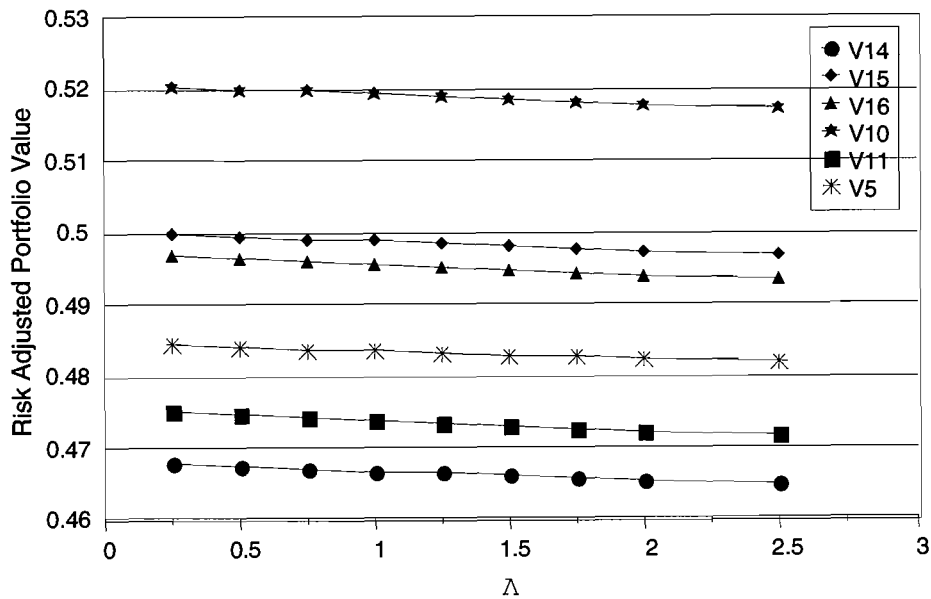


Figure 2. Sensitivity of risk-adjusted portfolio value to risk parameter, by variety

For newer releases, V_{14} is dominated by V_{10} , V_{11} , V_{12} , and V_{15} for grower value and does not dominate other incumbents. For end-user values, V_{14} only dominates V_9 . The portfolio method indicates that V_{14} dominates V_{13} and V_9 . Of the other two new releases (V_{15} and V_{16}), both were higher ranked varieties. Using SDD (table 4), V_{16} dominates V_{15} for grower value, while V_{15} dominates V_{16} for end-user values. Under the portfolio analysis, both V_{15} and V_{16} rank higher than most incumbent varieties across a wide range of end-user weights. However, as end-user weights increase to higher levels, V_5 , V_4 , and V_3 rank higher. Both V_{15} and V_{16} have lesser value than V_{10} . Of the two newer releases, V_{15} consistently ranks higher than V_{16} unless end-user weights are less than 0.4.

Changes in θ , the risk-aversion parameter, affect the estimated value of the risk-adjusted portfolio for individual varieties, but have little impact on rankings of varieties (figure 2). As Λ increases (becomes more risk averse), risk-adjusted portfolio values for varieties decrease.

Summary and Conclusions

Variety development and release decisions involve tradeoffs between growers and end-users as well as significant uncertainties about economic, agronomic, and quality variables. The breeding function has always confronted these issues, but their importance has likely increased in recent years. Methods are developed in this study that can be used to compare and rank ex ante values of varieties to growers and end-users separately, and jointly under risk considerations. Values of individual varieties were determined for growers and end-users using stochastic methods. The models were applied to HRS wheat, a class with numerous varieties, which typically commands premiums for various attributes, has been vulnerable to severe disease problems, and for which there are many sources of uncertainty in variety development and adoption.

Results clearly suggest differences among varieties and rankings which would occur between growers and end-users. Based on the stochastic dominance criteria and statistical tests, dominance varies depending on whether grower and end-user values are utilized, confirming there are tradeoffs among varieties. Differences in relative valuations of growers and end-users suggest the need to jointly model both values in a portfolio analysis. The portfolio analysis identified one variety with higher utility relative to others. This variety, as it turns out, has higher protein and other end-use attributes, a greater resistance to disease, and became one of the more popular varieties planted. These rankings changed slightly with different weights applied to end-users, but were largely unaffected by changes in the risk-aversion coefficient. Portfolio rankings provide a less abstruse framework, especially in evaluating the sensitivity of key parameters.

The analysis focused on a particular wheat class which has some interesting features making the analytic framework appealing. The methods could also be used to evaluate valuations in targeted geographic areas and/or to examine valuations for specific end-use market segments (e.g., pizza dough, hearth breads). The results provide perspective to breeders on the value of varieties across the continuum of grower/end-user weights. They provide insight into the value of a variety relative to others and may also be used to identify prospective future breeding opportunities. Similar problems confront breeders and economists evaluating variety development strategies for other grains and oilseeds. Thus, the framework could be applied elsewhere.

The methodology makes several contributions to the evolving literature on the economics of breeding. First, it analyzes release decisions in a stochastic framework, allowing for numerous uncertainties and correlations among key grower and end-user variables. Second, it uses a statistical test to refine stochastic dominance results with less restrictive assumptions. Third, it uses a portfolio method with an explicit utility function which simultaneously incorporates valuations for both end-users and growers when ranking individual varieties.

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Appendix A: Traditional Stochastic Dominance

Following Hadar and Russell (1969) and Moss (2001), the decision maker has a utility function $U(x)$, defined on the outcomes of a random variable i , and chooses between two actions. For growers, this is the choice of producing either varieties a_1 or a_2 , and i is income per acre. The returns for growing a variety are defined by the probability density function $f(x)$ for variety a_1 , and $g(x)$ for variety a_2 . FDD implies variety a_1 is preferred to a_2 if it is always expected to yield income at least equal to a_2 , with a greater probability of earning income higher than a_2 for at least one income level. Specifically, variety a_1 dominates variety a_2 if:

$$(A1) \quad \bar{\Delta}_1 = G(x) - F(x) \geq 0 \quad \forall x,$$

with at least one strict inequality, where G and F are cumulative distribution functions. FDD is a fairly weak criterion and tends to eliminate few alternatives from the choice set.

SDD is more discriminating because it includes higher moments of the distribution of returns and considers risk preferences of decision makers. SDD implies that the area under the cumulative density functions for f are always less than for g . Variety a_1 dominates variety a_2 in the second degree if:

$$(A2) \quad \bar{\Delta}_2 = \int_a^x [G(z) - F(z)] dz \geq 0 \quad \forall x,$$

with at least one strict inequality.

An "efficient set" of varieties is formed by eliminating varieties that are dominated by others. For FDD, this involves a sequence of binary comparisons:

$$(A3) \quad \begin{aligned} \bar{\Delta}_1^i &= \inf_x G(x) - F(x), \\ \bar{\Delta}_1^s &= \sup_x G(x) - F(x), \end{aligned}$$

where, if the signs of inf and sup are positive, then f dominates g ; if they are negative, then g dominates f ; and if the signs are opposite, then there is no FDD. For SDD, the binary comparisons are:

$$(A4) \quad \begin{aligned} \tilde{\Delta}_1^i &= \inf_x \int_a^x [G(z) - F(z)] dz, \\ \tilde{\Delta}_1^s &= \sup_x \int_a^x [G(z) - F(z)] dz, \end{aligned}$$

where the same rules apply.

Following Goh et al. (1989), a stepwise cumulative probability density function is assumed:

$$(A5) \quad F(x) = \frac{N^*[y \leq x]}{N},$$

where $F(x)$ is the cumulative density function, $N^*[y \leq x]$ is the number of observations less than or equal to the index value, and N is the sample size. A similar distribution is estimated for $G(x)$, which allows comparisons of the two alternatives. The inf and sup statistics for pairwise comparisons of varieties were derived and compared using procedures by Goh et al.

Appendix B: Davidson-Duclos Test for Statistical Significance of Stochastic Dominance

Consider the following sample statistics for comparison of the distributions for values of wheat varieties Y and Z :

$$(A6) \quad \begin{aligned} \hat{D}_Y^s(\mathbf{x}) &= \frac{1}{N(s-1)!} \sum_{i=1}^N (\mathbf{x} - y_i)_+^{s-1}, \\ \hat{D}_Z^s(\mathbf{x}) &= \frac{1}{N(s-1)!} \sum_{i=1}^N (\mathbf{x} - z_i)_+^{s-1}, \\ \hat{V}_Y^s(\mathbf{x}) &= \frac{1}{N} \left[\frac{1}{((s-1)!)^2} \frac{1}{N} \sum_{i=1}^N (\mathbf{x} - y_i)_+^{2(s-1)} - \hat{D}_Y^s(\mathbf{x})^2 \right], \\ \hat{V}_Z^s(\mathbf{x}) &= \frac{1}{N} \left[\frac{1}{((s-1)!)^2} \frac{1}{N} \sum_{i=1}^N (\mathbf{x} - z_i)_+^{2(s-1)} - \hat{D}_Z^s(\mathbf{x})^2 \right], \\ \hat{V}_{Y,Z}^s(\mathbf{x}) &= \frac{1}{N} \left[\frac{1}{((s-1)!)^2} \frac{1}{N} \sum_{i=1}^N (\mathbf{x} - y_i)_+^{s-1} (\mathbf{x} - z_i)_+^{s-1} - \hat{D}_Y^s(\mathbf{x}) \hat{D}_Z^s(\mathbf{x}) \right], \end{aligned}$$

where s = degree of dominance test, N is number of samples, \mathbf{x} is a vector representing the cumulative stepwise increment of the distribution of value (grower or end-user) up to the step examined, and the following normalized statistic:

$$(A7) \quad T^s(\mathbf{x}) = \frac{\hat{D}_Y^s(\mathbf{x}) - \hat{D}_Z^s(\mathbf{x})}{\sqrt{\hat{V}^s(\mathbf{x})}},$$

where

$$(A8) \quad \hat{V}^s(\mathbf{x}) = \hat{V}_Y^s(\mathbf{x}) + \hat{V}_Z^s(\mathbf{x}) - 2\hat{V}_{Y,Z}^s(\mathbf{x}).$$

Assuming observations from the two distributions being compared are independent, then

$$\hat{V}^s(\mathbf{x}) = \hat{V}_Y^s(\mathbf{x}) + \hat{V}_Z^s(\mathbf{x}),$$

and the normality results still hold.

Using these estimated statistics, Tse and Zhang (2000) suggest the following hypotheses to test for significance:

1. H_0 : $\hat{D}_Y^s(x_i) = \hat{D}_Z^s(x_i)$ for all x_i ,
2. H_A : $\hat{D}_Y^s(x_i) \neq \hat{D}_Z^s(x_i)$ for some x_i ,
3. H_{A1} : $Y > sZ$,
4. H_{A2} : $Z > sY$.

The following decision rules were used to assess each of the hypotheses:

1. If $|T^s(x_i)| < M_{\infty, \alpha}^K$ for $i = 1, \dots, K$, accept H_0 ;
2. If $-T^s(x_i) > M_{\infty, \alpha}^K$ for some i , and $T^s(x_i) < M_{\infty, \alpha}^K$ for all i , accept H_{A1} ;
3. If $T^s(x_i) > M_{\infty, \alpha}^K$ for some i , and $-T^s(x_i) < M_{\infty, \alpha}^K$ for all i , accept H_{A2} ;
4. If $T^s(x_i) > M_{\infty, \alpha}^K$ for some i , and $-T^s(x_i) > M_{\infty, \alpha}^K$ for all i , accept H_A ,

where $M_{\infty, \alpha}^K$ is the studentized maximum modulus statistic with K and infinite degrees of freedom.