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Using Satellite Imagery in Predicting Kansas Farmland Values

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Can remotely sensed imagery improve hedonic land price models? A remotely sensed variable was added to a hedonic farmland value model as a proxy for land productivity. Land cover data were used to obtain urban and recreational effects as well. The urban and recreational effects were statistically significant but economically small. The remotely sensed productivity variable was statistically significant and economically large, indicating that knowing the "greenness" of the land increased the explanatory power of the hedonic price model. Thus, depending upon the cost of this information, including remotely sensed imagery in traditional hedonic land price models is economically beneficial.

Key words: hedonic models, land values, remote sensing, satellite imagery

Introduction

Nearly 75% of agricultural assets are in land (U.S. Department of Agriculture/Economic Research Service). Because profitability in production agriculture is capitalized into land values, changes in land values are important indicators of economic well-being of the agricultural sector. Conceptually, land values are determined by the capitalized expected future returns to land, which are often related to historical and current returns, which in turn depend on agricultural production, but also on government program payments.

Remotely sensed satellite imagery provides information that is potentially useful for predicting crop production levels before harvest. Such data have been used worldwide to predict crop yields (Das, Mishra, and Kalra; Groten; Maselli et al.; Rasmussen). In the United States, such data have been used to predict corn yields (Hayes and Decker; Lee) and wheat yields in the Midwest (Doraiswamy and Cook; D. L. Kastens et al.; J. H. Kastens et al.; Rudorff and Batista). The existence of a relationship between agricultural production (crop and forage yields) and land values indicates remote sensing might be useful in determining agricultural land values by assessing both crop and pasture productivity and the interaction of rainfall, irrigation, and water-holding capacity. However, remotely sensed imagery would not be expected to improve current hedonic land models if productivity were known and included in predictive models. Unfortunately, parcel-level production data are not readily available to policy makers and researchers.

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Consequently, using remotely sensed imagery as a proxy for production might be useful in estimating land values.

A common method of estimating land values has been to use hedonic pricing models. Rosen stated, "Hedonic prices are defined as the implicit prices of attributes and are revealed to economic agents from observed prices of differentiated products and the specific amount of characteristics associated with them. They constitute the empirical magnitude explained by the model" (p. 34). Therefore, the price is the market-clearing price for that attribute and is the point where buyers and sellers agree (King and Sinden). Although land is not a homogeneous factor of production, its attributes might be assumed to be.

Previous research provides guidance for specifying hedonic models in two areas: (*a*) attributes (land characteristics) that should be included, and (*b*) the appropriate functional form. The land characteristics found in previous investigations often can be broadly classified as geophysical (e.g., soil type, historical production) or socioeconomic (e.g., distance from town, road access, population, interest rates, and other macroeconomic indicators) (Downing and Gamble; Elad, Clifton, and Epperson; Pardew, Shane, and Yanagida; Stewart and Libby; Xu, Mittelhammer, and Barkley). A hedonic land value model is used in the present study to determine the ability of remotely sensed data to improve the explanation of parcel-by-parcel variation in land values.

There is little theoretical justification for choosing among functional forms. Thus previous researchers have used many different functional forms. For example, linear (Downing and Gamble), semi-log (King and Sinden), and double-log (Featherstone et al.; Pardew, Shane, and Yanagida) models have been employed to estimate land values. Some analyses have used the Box-Cox functional form (Elad, Clifton, and Epperson; Roka and Palmquist; Palmquist and Danielson). The Box-Cox functional form is a flexible functional form as it nests all first-order Taylor-series approximations to an unknown functional form including the three mentioned above. Therefore, our methodology includes a Box-Cox functional form to allow the data to determine the form of the hedonic model.

Background

Because we examine whether remotely sensed imagery may be useful in a hedonic model, it is helpful to define the remotely sensed data that will be used. Remote sensing involves collecting data about an object without coming into direct contact with it. Satellite imagery is one type of remote sensing: satellites orbit the earth, continuously collecting information on the intensity of reflected light of various frequencies. Satellites often collect surface data for a minimum of three frequency bands: red, green, and near-infrared (NIR).

Remotely sensed data have been employed for over 25 years to assess and monitor vegetation condition (D. L. Kastens et al.). In particular, red and NIR reflectance have been used to measure vegetation health and vigor, based on the inverse relationship between red reflectance and chlorophyll content, and the direct relationship between leaf structure and NIR reflectance. Vegetation index values, specifically the normalized difference vegetation index (NDVI), calculated from the red and NIR bands as [(NIR – red)/(NIR + red)], are often referred to as greenness values because they are strong indicators of vegetation condition and quantity (D. L. Kastens et al.).

Time-series analysis of vegetation index data has allowed scientists to examine global-scale phenological phenomena such as green-up (which occurs when an area's vegetation index breaks a 15% threshold of its historically determined range), duration of green period, onset of senescence (which occurs immediately after the maximum vegetation index value for the year is attained), as well as changes in biophysical variables such as leaf area index, biomass, and net primary productivity (Eastman and Fulk; Tucker et al.). Thus, a justification has been established for using remotely sensed data, specifically the vegetation index, to estimate crop and pasture production.

Conceptual Model and Data

A reduced-form conceptual model of farmland value can be expressed as:

(1) *Land Value = f(Geophysical Characteristics, Socioeconomic Characteristics),*

where the geophysical characteristics include information on historical production, soil traits, government payments, and conservation practices.[1] The socioeconomic characteristics include factors such as distance from town, road access, and recreational features.

A conceptual model of farmland value that includes remotely sensed information could be described as:

(2) *Land Value = f(Geophysical Characteristics, Socioeconomic Characteristics, Remotely Sensed Variables),*

where the remotely sensed variables include both geophysical variables (such as a vegetation index as a proxy for plant production) and socioeconomic variables (such as an urban or recreational effect).

The heuristic descriptions of land value in (1) and (2) are intentionally broad, as land investment typically is a long-term investment, driven by factors characterized as narrow and factors characterized as broad—both temporally and geographically. More specifically, although land investment is distinctly location specific, it could be that expectations at a broader level (e.g., regional) are more reliable indicators of the expectation for a desired location. Consequently, we consider a number of variables as possibly important for explaining land value, ones with decidedly different spatial scales.

The Kansas Applied Remote Sensing (KARS) program, through the University of Kansas, has developed a historical database of vegetation index values for the state of Kansas from 1990 through 1999. In addition to the vegetation index, a land cover database from KARS is used to add urban and recreational (i.e., lake) effects to the hedonic model. These historical remotely sensed data are matched with land sales data.

Parcel-specific land sales data were collected by members of the Kansas Society of Farm Managers and Rural Appraisers (KSFMRA) for the purpose of assisting its members in appraising agricultural real estate (see Featherstone et al. for more information

[1] For more information on the theoretical derivation of the reduced-form equation, see Rosen. The arbitrary classification is not particularly rigid. For example, though characteristics such as government payments might be better construed as socioeconomic, we maintain the classification of geophysical in order to keep those features that most impact the agricultural productivity in the same class.

about this data set). Although these data extend from 1977–1999, the vegetation index data set is only available from 1990–1999. Data reported by the KSFMRA include a subjective measure of land quality,[2] road access, amount of cropland, whether the land was financed with a contract, improvements, mineral rights, presence of irrigation, size of the parcel, and location in the state. The land value data were obtained with a legal description (township, range, and section numbers) and were converted to a geographical coordinate system (latitude, longitude) to make the land sales data compatible with the land cover and vegetation index data sets.[3]

Another important determinant of land values is the profitability of agriculture. This factor could be proxied by including a measure of crop prices. Because Kansas has several major crops, inclusion of only one price would not be appropriate and including all crop prices as separate variables may lead to multicollinearity, as crop prices are often highly correlated. Therefore, regional farm crop income is included as an explanatory variable for profitability. The crop income per region was obtained from various annual issues of the USDA/Kansas Department of Agriculture's *Kansas Farm Facts*, and was converted to 1999 constant dollars using a personal consumption expenditure (PCE) index reported by the Federal Reserve Bank of Kansas City.

Although crop income incorporates agricultural market changes into the hedonic model, a variable was needed to capitalize the income stream into land values. The real interest rate, an average yearly measure, was included to measure changes in the capitalization rate and was derived from the real estate interest rates and PCE-based inflation reported by the Federal Reserve Bank of Kansas City.

The vegetation index data set (NDVI) was created by KARS using data from the U.S. Geological Survey's Earth Resources Observations Systems for the years 1990–1999. Imagery from the Advanced Very High Resolution Radiometer (AVHRR) sensor was used because this sensor is well suited for monitoring crop response due to its temporal and spatial resolutions, and because AVHRR data are relatively inexpensive compared to other remotely sensed data (D. L. Kastens et al.).

Nearly cloud-free AVHRR vegetation index composites were created by saving the highest daily vegetation index value over a two-week period. NIR and red values used in the vegetation index, where the vegetation index is defined as [(NIR – red)/(NIR + red)], were provided in an eight-bit unsigned integer format, meaning each can take only integer values in the [0, 255] interval. This format would mathematically lead to a vegetation index value in the [-1, 1] interval. Thus, to maintain minimal data storage requirements (i.e., an eight-bit integer format), an affine transformation was adopted which first adds 1 to the vegetation index value, then multiplies it by 100, and finally rounds to the nearest integer. This procedure results in a final vegetation index value residing in the [0, 200] interval. The average vegetation index for the entire calendar year was then calculated and used for this analysis.

Figure 1 provides an image of the average vegetation index for the state of Kansas in 1999. Darker areas indicate higher vegetation index values, or "greener" land. As expected, the western region of the state, which receives less rainfall, is not as "green" as the eastern region.

[2] The land quality is determined by each KSFMRA agent, and is based on an agent's perception of the quality of land compared to the land surrounding it.

[3] Legal descriptions were converted to latitude and longitude using the LEO© software system (Kansas Geological Survey). For more information on this procedure, see appendix A.

Note: Lighter shading indicates lower vegetation index, darker indicates higher vegetation index.

Figure 1. Average vegetation index for the state of Kansas, 1999

The land cover data were created by KARS in 1995 (Whistler et al.). The land cover was divided into eight numeric categories: (1) Urban–Residential, (2) Urban–Commercial/Industrial, (3) Urban–Openland, (4) Cropland, (5) Grassland, (6) Woodland, (7) Water Bodies, and (8) Other Cover (sandbars, bare ground, etc.). Land cover is assumed to be fixed over the short run. Therefore, the 1995 land cover data from KARS are assumed to be representative of the entire 1990–1999 time period.

Empirical Model

The empirical model estimated is specified as follows:

$$
\begin{aligned}
(3)\qquad Land_i^{(\lambda_1)} = {} & a_0 + a_1 Con_i + a_2 Gov_i + a_3 Hwy_i + a_4 Gravel_i + a_5 Interest_i^{(\lambda)} \\
& + a_6 Income_i^{(\lambda)} + a_7 Imp_i + a_8 Irr_i + a_9 Min_i + a_{10} Qtr2_i \\
& + a_{11} Qtr3_i + a_{12} Qtr4_i + a_{13} Year_i + a_{14} Acres_i^{(\lambda)} + a_{15} Crop_i^{(\lambda)} \\
& + a_{16} HiQual_i + a_{17} LoQual_i + a_{18} C_i + a_{19} EC_i + a_{20} NC_i \\
& + a_{21} NW_i + a_{22} SC_i + a_{23} SE_i + a_{24} SW_i + a_{25} WC_i + a_{26} Urban_i^{(\lambda)} \\
& + a_{27} Rec_i^{(\lambda)} + a_{28} ENDVI_i^{(\lambda)} + e_i,
\end{aligned}
$$

where $Land_i$ is the 1999 constant dollar price per acre for observation (or parcel) i; $x^{(\lambda)}$ represents the Box-Cox transformation of variable x (for more information, see Greene); λ_1 and λ are the Box-Cox transformation parameters for the dependent variable ($Land_i$) and the independent variables (identified and defined in table 1), respectively; and e_i is a parcel-specific error term.

Table 1. Definitions of Variables

Variable	Definition
$Land_i$	Price per acre for observation (or parcel) i in constant 1999 dollars
Socioeconomic Variables:	
Con_i	Binary variable =1 if parcel i was sold by contract
Gov_i	Binary variable representing 1996 FAIR Act, =1 if year is ≥ 1996
$Gravel_i$	Binary variable representing gravel road access (dirt road is default) for parcel i
Hwy_i	Binary variable representing highway road access (dirt road is default) for parcel i
$Interest_i$	Real interest rate for the year in which parcel i was sold
$Income_i$	Farm income/acre for the region and year in which parcel i was sold (1999 constant $)
Imp_i	Binary variable =1 if there were improvements made to parcel i
Irr_i	Binary variable =1 if any of parcel i was irrigated
Min_i	Binary variable =1 if mineral rights were sold with parcel i
$Qtr2_i$	Binary variable =1 if parcel i is sold in 2nd quarter of year (1st quarter of year is default)
$Qtr3_i$	Binary variable =1 if parcel i is sold in 3rd quarter of year (1st quarter of year is default)
$Qtr4_i$	Binary variable =1 if parcel i is sold in 4th quarter of year (1st quarter of year is default)
$Year_i$	Year in which parcel i was sold
Geophysical Variables:	
$Acres_i$	Number of acres sold in parcel i
$Crop_i$	Percentage of parcel i that is cropland
$HiQual_i$	Binary variable =1 if parcel i is high quality land (medium quality is default)
$LoQual_i$	Binary variable =1 if parcel i is low quality land (medium quality is default)
	For the following 8 regional binary variables, the northeast region is the default.
C_i	Binary variable =1 if parcel i is located in the central region of Kansas
EC_i	Binary variable =1 if parcel i is located in the east central region of Kansas
NC_i	Binary variable =1 if parcel i is located in the north central region of Kansas
NW_i	Binary variable =1 if parcel i is located in the northwest region of Kansas
SC_i	Binary variable =1 if parcel i is located in the south central region of Kansas
SE_i	Binary variable =1 if parcel i is located in the southeast region of Kansas
SW_i	Binary variable =1 if parcel i is located in the southwest region of Kansas
WC_i	Binary variable =1 if parcel i is located in the west central region of Kansas
Remotely Sensed Variables:	
$Urban_i$	Urban variable, defined as the percentage of land within a 10-mile radius of parcel i that is classified as urban (categories 1–3) according to the land classification data set
Rec_i	Recreational variable, defined as the percentage of land within a 10-mile radius of parcel i that is classified as water bodies according to the land classification data set
$COVCROP_i$	Percentage of farmland within a 10-mile radius of parcel i that is cropland
$COVPAS_i$	Percentage of farmland within a 10-mile radius of parcel i that is pastureland
$ENDVI_i$	Expected vegetation index for parcel i, for a 10-mile radius—a derived value in the [0, 200] interval
$NDVICROP_{i,t}$	Vegetation index for cropland within a 10-mile radius for parcel i in year t in the [0, 200] interval
$NDVIPAS_{i,t}$	Vegetation index for pastureland within a 10-mile radius for parcel i in year t in the [0, 200] interval
$ANDVI_{i,t}$	Average vegetation index for parcel i, in year t, for a 10-mile radius—a derived value in the [0, 200] interval

The base model in (1) was obtained by excluding the remotely sensed variables ($Urban_i$, Rec_i, and $ENDVI_i$). The expanded model (2) includes the remotely sensed variables as depicted in (3).[4]

Several socioeconomic variables were included in the analysis. For instance, the size of the parcel may be important. Small parcels might be attractive to developers or nonfarmers, thus increasing the price received. On the other hand, farmers might prefer large parcels to improve machinery utilization, although access to finance could constrain the size of the parcels purchased. With development competing with farmers for land, road access is important. If a parcel had highway or gravel access, a developer or individual might be more willing to develop and the price of the land would be expected to be higher than for land with dirt road access. Land with mineral rights is expected to receive a higher price than land sold without mineral rights because of the potential development opportunity. A yearly trend was included to account for changes in technology that increase crop yields and decrease operating costs.

Geophysical variables were included as well. For example, if the farmland were improved, then it may receive a higher price than unimproved farmland. Cropland receives a premium over pastureland; thus knowing the percentage of land designated as cropland within the parcel should be important in explaining price. Likewise, the quality of the land sold is important, because high quality land is expected to bring a premium whereas low quality land will bring a discount relative to average quality land. Region-level farm income will positively affect the price of land sold as well. Similarly, changes in government farm programs should have an impact on land prices. Therefore, a binary variable was included to capture the effects of the 1996 farm bill.

The empirical model (3) contains three remotely sensed variables: $Urban_i$, Rec_i, and $ENDVI_i$. $Urban_i$ is the urban effect, which is the percentage of the land classified as urban (KARS land cover categories 1–3) within a defined radius encircling parcel i. This radius was defined as 10 miles.[5] An example of the $Urban_i$ variable can be seen in figure 2, which is an actual land classification map for Sedgwick County, Kansas. The hypothesized parcel i is denoted by a star. The $Urban_i$ value for parcel i was calculated by taking the percentage of the total points within the circle having an urban classification. In the example shown in figure 2, $Urban_i = 10.16\%$.

In (3), the remotely sensed variable Rec_i is the recreational effect, representing the percentage of land within a 10-mile radius encircling parcel i which is classified as water bodies.

Because a single vegetation index value was assigned to each 1.1 square kilometers across Kansas, using a single vegetation index value to determine the productivity of the land parcel could lead to biased and unstable results depending on whether a parcel was predominantly cropland or pasture. Instead, a circle containing several vegetation index values was used. The expected vegetation index ($ENDVI_{i,t}$) is the remote sensed imagery variable defined as:

[4] Each land parcel (i) was sold in a specific year (t); e.g., the land parcel was sold in 1993, 1994, or 1995, etc. If a land parcel was sold in more than one year (e.g., sold in 1993 and sold again in 1999), each sale was treated as a separate parcel i. The time-series data used in this estimation (i.e., interest rates, income, etc.) are associated with the year the parcel is sold (t). Therefore, t is not noted in (3).

[5] Models were estimated with radii of 4 to 15 miles (in one-mile increments). The results were robust to the increment change.

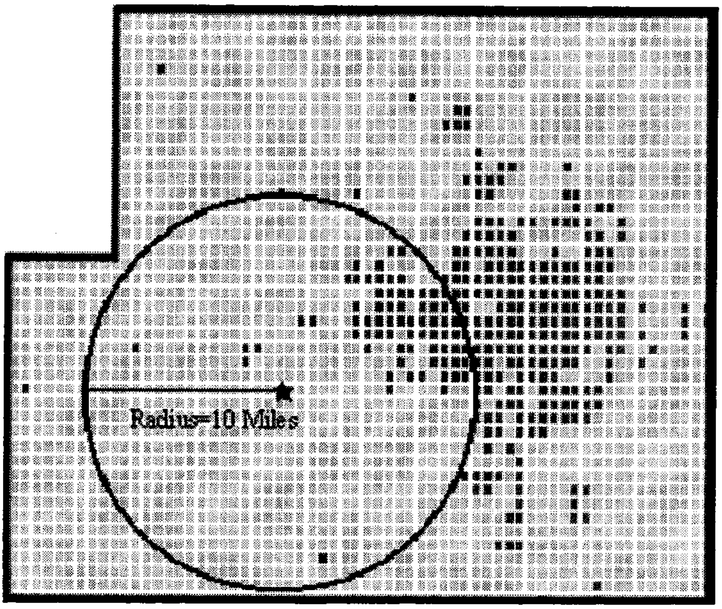

Figure 2. Sample urban land cover map for Sedgwick County, Kansas

$$(4) \qquad ENDVI_{i,t} = (ANDVI_{i,t-1} + ANDVI_{i,t-2} + ANDVI_{i,t-3})/3, \text{ and}$$

$$(5) \qquad ANDVI_{i,t} = (NDVICROP_{i,t} * COVCROP_i + NDVIPAS_{i,t} * COVPAS_i) * 100,$$

where $ANDVI_{i,t}$ is the actual vegetation index for parcel i in the year it was sold (t). $NDVICROP_{i,t}$ is the average vegetation index for cropland pixels within the defined radius of 10 miles for parcel i in year t, and $NDVIPAS_{i,t}$ is the average vegetation index for pastureland pixels within the defined radius of 10 miles for parcel i in year t. $COVCROP_i$ is the proportion of farmland pixels within the radius for parcel i designated as cropland, and $COVPAS_i$ is the corresponding proportion for pastureland. $COVCROP_i$ and $COVPAS_i$ were derived from the land cover data set. Current vegetation index values were not used because the majority of the land was sold before the cropping season began. In an attempt to better capture long-run productivity, a three-year average vegetation index, rather than a two- or one-year vegetation index variable, was used.[6]

The parameters obtained from (3) using the Box-Cox transformation are not readily interpretable because they are neither derivatives nor elasticities. Furthermore, for nonlinear functions, $E[f(x)] \neq f[E(x)]$. Thus, additional mathematics are required to numerically derive meaningful partial effects measures. Therefore, elasticities were calculated from the Box-Cox functional form (see appendix B) using:

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[6] In the model (3), the subscript t has been dropped because it is no longer needed.

(6) $$E_k = \frac{\left((\lambda_1 R + 1)^{(1/\lambda_1)-1} + \tfrac{1}{2}(1-\lambda_1)(1-2\lambda_1)(\lambda_1 R + 1)^{(1/\lambda_1)-3}\sigma_e^2\right) * a_k x_k^{\lambda}}{(\lambda_1 R + 1)^{1/\lambda_1} + \tfrac{1}{2}(1-\lambda_1)(\lambda_1 R + 1)^{(1/\lambda_1)-2}\sigma_e^2},$$

where x_k is a continuous variable, a_k is the parameter estimate associated with x_k, σ_e^2 is the variance of the error term from (3), λ_1 represents the Box-Cox transformation parameter for the dependent variable $Land_i$, d_m is a binary variable [referenced in equations (7) and (8)], λ represents the Box-Cox transformation parameter for the independent variables, and R is the right-hand side of (3) evaluated at any point of interest:

(7) $$R = a_0 + \sum_k a_k \left(\frac{x_k^{\lambda} - 1}{\lambda}\right) + \sum_m a_m d_m.$$

Because elasticities are not particularly meaningful for binary variables in nonlinear models, we calculate a "binary effect" instead:

(8) $$g_m = \left[\frac{\left(\lambda_1 R|_{d_m=1} + 1\right)^{1/\lambda_1} + \tfrac{1}{2}\left(1-\lambda_1\right)\left(\lambda_1 R|_{d_m=1} + 1\right)^{(1/\lambda_1)-2}\sigma_e^2}{\left(\lambda_1 R|_{d_m=0} + 1\right)^{1/\lambda_1} + \tfrac{1}{2}\left(1-\lambda_1\right)\left(\lambda_1 R|_{d_m=0} + 1\right)^{(1/\lambda_1)-2}\sigma_e^2} - 1\right],$$

which is the percentage change in land value expected from activating a binary variable while holding all else constant.

Results

Summary statistics for the variables used to estimate (3) are reported in table 2. The average price per acre of land sold in constant 1999 dollars was \$568/acre, with a range of \$19 to \$5,575/acre. The parcel size averaged 196 acres and ranged from 10 to 8,960 acres. On average, 67% of the land sold was cropland, and 0.82% and 0.46% of the land within a 10-mile radius of the tract sold was classified as urban and recreational, respectively. The average *ENDVI* value was 130, with a minimum value of 119 and a maximum of 178.

The data are spatial in nature; therefore, spatial autocorrelation might be present in the model. To determine if spatial autocorrelation existed, a Moran's I statistic was calculated using a first-power inverse distance weights matrix (for more information on Moran's I, see Hubert, Golledge, and Costanzo). With Moran's I values of 0.0415 and 0.0323 (with no spatial autocorrelation, the values would be expected to be 0.0001) for the base and expanded models, respectively, some degree of spatial autocorrelation is found.

However, "finding" autocorrelation does not necessarily imply it is appropriate to correct at all for spatial autocorrelation (Greene, p. 577). An autocorrelation correction typically imposes substantial, and perhaps undesirable, structure on the causal model. For example, completing the usual temporal autocorrelation correction means the modeler views y_t (dependent variable) to be a causal function of y_{t-1}, and the strength of that relationship is depicted by exactly the estimated autocorrelation parameter (the usual rho-hat). Additional predetermined impacts of lagged independent variables are also implied by the correction. Specifying and testing such rigid structure is often possible (see McGuirk and Spanos for additional insight), but would be especially cumbersome

Table 2. Summary Statistics for 8,178 Land Parcels, 1993–1999

Variable	Mean	Standard Deviation	Minimum	Maximum
Land ($/acre)[a]	567.50	348.71	19.00	5,575.00
Socioeconomic Variables:				
Con	0.03	0.18	0.00	1.00
Gov	0.60	0.49	0.00	1.00
Hwy	0.14	0.35	0.00	1.00
Gravel	0.57	0.50	0.00	1.00
Dirt	0.29	0.46	0.00	1.00
Interest (%)	7.25	0.56	6.07	8.01
Income ($/acre)[a]	173.65	122.63	92.16	1,957.01
Imp	0.10	0.30	0.00	1.00
Irr	0.07	0.25	0.00	1.00
Min	0.75	0.44	0.00	1.00
Qtr1	0.32	0.47	0.00	1.00
Qtr2	0.31	0.46	0.00	1.00
Qtr3	0.19	0.39	0.00	1.00
Qtr4	0.18	0.39	0.00	1.00
Year	1996.06	1.97	1993.00	1999.00
Geophysical Variables:				
Acres	196.23	248.90	10.00	8,960.00
Crop (%)	66.56	34.60	0.00	100.00
HiQual	0.19	0.39	0.00	1.00
MedQual	0.75	0.43	0.00	1.00
LoQual	0.06	0.25	0.00	1.00
C	0.18	0.39	0.00	1.00
EC	0.07	0.25	0.00	1.00
NC	0.16	0.36	0.00	1.00
NE	0.06	0.23	0.00	1.00
NW	0.10	0.30	0.00	1.00
SC	0.22	0.41	0.00	1.00
SE	0.07	0.26	0.00	1.00
SW	0.08	0.28	0.00	1.00
WC	0.06	0.23	0.00	1.00
Remotely Sensed Variables:				
Urban (%)	0.82	3.15	0.00	66.67
Rec (%)	0.46	2.14	0.00	50.00
ENDVI	129.86	4.69	119.30	177.56

[a] Dollar values are in constant 1999 dollars.

in the case of spatial autocorrelation correction, where numerous lagged variables require specific coefficient values. Thus, given the degree of spatial autocorrelation detected in the estimated models, autocorrelation correction was deemed inappropriate in these models.

Estimated parameters for the base and expanded models are reported in table 3, as well as the elasticities (continuous variables) and the binary effects (binary variables) calculated at the means of other variables. The models were estimated in SAS (SAS Institute, Inc.) using full information maximum likelihood. The elasticity and binary effect standard errors were calculated using the delta method (Greene, p. 278). The base model (with the remotely sensed variables $Urban_i$, Rec_i, and $ENDVI_i$ excluded) explained 31% of the in-sample variation in land prices, and the expanded model explained 33%.[7] The Box-Cox transformation shows the dependent variable is nearly transformed to a log form for both the base (-0.031) and expanded (-0.029) models. However, the independent variable transformation, at 0.296 and 0.493 for the base and expanded models, respectively, does not indicate a log or a linear transformation.

As observed from table 3, in the base model, all of the coefficients were statistically significant at the 5% level except quarter of the year the parcel was sold, parcels sold by contract, and interest rate. In the expanded model ($Urban_i$, Rec_i, and $ENDVI_i$ included), all of the coefficients were statistically significant at the 5% level except quarter of the year the parcel was sold, parcels sold by contract, and the government payments variable. Of course, the statistical significance of the majority of the variables could be due to the large number of observations (8,178).

For the base model, land sold with mineral rights (*Min*) received a price premium of 6.8% compared to land sold without mineral rights. The time trend (*Year*) indicates a 6.1% increase in real land values per year after accounting for other effects. The 1996 farm bill (*Gov*) resulted in a decrease in land price of 5.5% compared to the previous government program regime. Land parcels with paved and gravel road access received a price premium of 17.7% and 6.9%, respectively, compared to those with only dirt road access. Land scored as high quality received a price premium of 27.5%, whereas poor quality land received a discount of 25% relative to average quality land. The quarter of the year in which the land was sold had no statistically significant impact on the price received. The interest rate effect was negative, reflecting the tendency for land prices to decrease as real interest rates increase; however, this variable was not statistically significant in the base model. The northeast region (default region) received the highest price per acre of all land sold, which is not surprising given the northeast region borders the western corn belt. Land sold in the western regions had the largest price discounts compared to the northeast, an expected result given the lower rainfall in those areas of the state.

For the expanded hedonic model, the urban and recreational effects are both positive and statistically significant. However, a one percentage point increase in the urban effect increased the price per acre of land sold by only 0.034%. Holding other variables at their means, the model-predicted land price for $Urban_i$ = 0.82% (mean value) is \$550.09/acre, and the model-predicted land price for the figure 2 (Sedgwick County) $Urban_i$ = 10.16% is \$650.96/acre. The resulting change in land price from the $Urban_i$ mean value of 0.82% to the figure 2 $Urban_i$ value of 10.16% is approximately 18.34%.

The variable used to proxy the productivity of the land ($ENDVI_i$) is positive and statistically significant. Comparing a model prediction of land price when $ENDVI_i = 130$ (which has a model-predicted land price of \$550.09/acre) with one using $ENDVI_i = 134$

[7] The R^2 values were calculated as the squared correlation between the predicted and actual land values per acre.

Table 3. Base and Expanded Regression Model Results for 8,178 Land Parcels, 1993–1999

	Base Model			Expanded Model		
Variable	Parameter Estimate	Elasticity/ Binary Effect	Standard Error	Parameter Estimate	Elasticity/ Binary Effect	Standard Error
Intercept	-91.613*			-43.652*		
Socioeconomic Variables:						
Con	0.030	0.038	0.029	0.030	0.036	0.030
Gov	-0.046*	-0.055*	0.018	-0.014	-0.017	0.018
Hwy	0.134*	0.177*	0.017	0.118*	0.153*	0.017
Gravel	0.055*	0.069*	0.012	0.042*	0.052*	0.012
Interest	-0.074	-0.162	0.087	0.056*	0.179*	0.087
Income	0.009*	0.052*	0.015	0.002*	0.031*	0.012
Imp	0.204*	0.282*	0.019	0.211*	0.290*	0.018
Irr	0.319*	0.478*	0.024	0.315*	0.463*	0.023
Min	0.054*	0.068*	0.012	0.049*	0.061*	0.012
Qtr2	0.003	0.004	0.011	-0.002	-0.002	0.011
Qtr3	-0.016	-0.019	0.013	-0.021*	-0.025*	0.012
Qtr4	-0.008	-0.010	0.013	-0.012	-0.015	0.013
Year	0.049*	0.061*	0.005	0.023*	0.028*	0.004
Geophysical Variables:						
Acres	-0.034*	-0.197*	0.006	-0.009*	-0.144*	0.006
Crop	0.024*	0.100*	0.006	0.017*	0.157*	0.007
HiQual	0.199*	0.275*	0.016	0.188*	0.255*	0.015
LoQual	-0.238*	-0.250*	0.012	-0.222*	-0.234*	0.012
C	-0.228*	-0.241*	0.018	-0.212*	-0.225*	0.018
EC	-0.065*	-0.076*	0.023	-0.085*	-0.097*	0.022
NC	-0.259*	-0.269*	0.017	-0.232*	-0.242*	0.018
NW	-0.319*	-0.321*	0.018	-0.246*	-0.255*	0.020
SC	-0.228*	-0.242*	0.017	-0.229*	-0.240*	0.017
SE	-0.131*	-0.145*	0.023	-0.163*	-0.177*	0.021
SW	-0.424*	-0.401*	0.016	-0.358*	-0.348*	0.018
WC	-0.402*	-0.385*	0.018	-0.315*	-0.314*	0.020
Remotely Sensed Variables:						
Urban				0.031*	0.034*	0.002
Rec				0.008*	0.006*	0.002
ENDVI				0.169*	2.247*	0.113
λ	0.296*			0.493*		
λ_1	-0.031*			-0.029*		
R^2	0.305			0.329		

Notes: A single asterisk (*) denotes parameter estimate is significantly different from zero at the 5% level. The reported standard errors are elasticity and binary effect standard errors, not parameter standard errors.

(with a model-predicted land price of \$590.33/acre) suggests a one standard deviation increase (table 2) in the NDVI-based vegetation index would cause land price to increase by 7.31%.[8]

The parameter estimates of the expanded model had the same signs as the base model except for the real interest rate and *Qtr2*. Interest rate is positive and statistically significant, a surprising result given it is expected to be negative. However, with only seven unique (annual) interest rate observations, the unexpected result is likely a small sample issue that is hard to disentangle without parcel-specific interest rate information. Interestingly, the real annual growth in land values expected by the expanded model (*Year*) was 2.8%, which seems more appropriate than the 6.1% predicted by the base model given Kansas land values and inflation rates over the 1990s. Of course, real growth (*Year*) is typically due to change in productivity or nonagricultural demand. In that regard, including proxies for nonagricultural demand for agricultural land should provide an improved description of real growth. Thus, if agricultural productivity and nonagricultural demand have been increasing throughout the 1990s, then it should not be surprising to find the expanded model has a reduced estimate for growth relative to the base model.

The magnitude of the region effects decreased in the expanded model relative to the base model. This finding is probably explained by the likelihood that regional effects are due to weather and productivity differences, and the remotely sensed data captured and quantified this impact in the expanded model. A Lagrange multiplier (LM) test was calculated to determine if, collectively, the expected vegetation index and the urban and recreational effects were statistically different from zero. The calculated LM-test statistic value was 401.46 and had a p-value of less than 0.0001. Therefore, remotely sensed and land cover data add to the explanatory power of this hedonic land value model.

In addition to the expanded hedonic model, several other models were estimated to determine if remotely sensed imagery might be capturing and quantifying information closely related to other variables. The expanded model in (3) was estimated without the regional variables, resulting in an *ENDVI* elasticity of 3.03 and an R^2 of 0.30. This model was similar in explanatory power to the base model reported in table 2, again indicating remotely sensed imagery information might substitute for regional binary variables. The expanded model in (3) was also estimated without the land quality variables. The *ENDVI* elasticity was 2.32 and the R^2 was 0.29, suggesting land quality could not be entirely proxied with remote sensing imagery.

Conclusions

Hedonic land value models have been used since the 1920s. However, the best estimator of agricultural land price variation is probably the underlying productivity of the land parcel sold. Unfortunately, reliable productivity data are not always readily available to researchers and policy makers. Therefore, a proxy of land productivity was developed. Because remotely sensed images have been successfully used to estimate corn and

[8] We found no remote sensing studies that quantified the relationship between vegetation index and crop yields. The studies only reported the correlation between vegetation index and yield. Nonetheless, the literature routinely documents substantial positive correlations between vegetation index and yield, implying our results are qualitatively consistent with the literature.

wheat production in the Midwest, it follows that these images might be useful for predicting land productivity. A remotely sensed variable obtained from the Kansas Applied Remote Sensing (KARS) program was added to a hedonic land pricing model to examine the change in the estimation power of the hedonic model. In addition, land cover data from KARS were used to estimate an urban effect and a recreational effect. The urban effect variable was the percentage of land within a radius of 10 miles of the parcel which was classified for urban use. The recreational effect variable was the percentage of land within a radius of 10 miles of the parcel which was classified as water bodies.

Two models—a base hedonic model with no remotely sensed variables and an expanded hedonic model, which included the remotely sensed productivity variable and urban and recreational effects—were estimated using Kansas land value data. Except for interest rate in the base model, which was insignificant, in both models, all variables with the exception of a few binary variables statistically affected the price per acre of land at the 5% significance level. In the expanded hedonic model, the urban and recreational effects were statistically significant but economically small when compared to the data means and standard deviations. Consequently, these characteristics typically would not have a large impact on the price per acre on most land sold in Kansas. The remotely sensed productivity variable was significant and large, indicating that knowing the "greenness" of the land increased the estimation power of the hedonic pricing model. Therefore, remotely sensed data do add information to hedonic pricing models for agricultural land in Kansas.

While remotely sensed data add information to a hedonic pricing model, the question arises as to whether the costs associated with working with remotely sensed data are worth the benefits of the information added to the model. Working with remotely sensed data requires extensive time and computing capabilities (see appendix A). Based on the results, the benefits of using remotely sensed data are marginal: a gain of only 2% in accuracy (R^2 change from 31% to 33%), if only the model fit is considered. However, the skills learned can easily be applied in other research where the benefits might be larger. Therefore, the potential benefits of including remotely sensed data information will likely vary depending on the particular situation.

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Appendix A: Geo-Referencing Data Sets

Initially the three data sets used in this analysis (land value, land cover, and vegetation index) were not analytically compatible because they had different spatial references. The land value data were spatially referenced by legal descriptions. The land cover data were spatially referenced by latitude/longitude and had a centroid value every 1.1 km across the state of Kansas (approximately 10 million observations). The vegetation index data also were referenced by latitude/longitude with a centroid value every 1.1 km, only these centroid points did not match those of the land cover data. Further, the vegetation index data set had unique values for each of 10 years, though vegetation index centroid locations were constant across time.

To mitigate these differences, several steps were taken to allow the three data sets to be used in the empirical models. First, the legal descriptions of the land value data were converted to a centroid latitude/longitude using the LEO© software system, developed by the Kansas Geological Survey. This procedure had the effect of mapping each land parcel (approximately 9,000 observations) to a single unique point in space (with a latitude value and a longitude value), allowing this data set to serve as the reference data with each observation uniquely numbered.

Using MATLAB© (The Math Works), for each point in the reference data set, a numerical search routine was performed on the land cover and vegetation index data sets in turn. Specifically, the distance to each point in these data sets (from the reference point) was first calculated, followed by trapping all points where distances were less than or equal to some assigned radius of interest R (here, 10 miles). As an example, text figure 2 depicts the location and the associated land cover classification value for land cover points within a 10-mile search radius of a single point in the reference data set.

Values of trapped land cover data were then paired to the nearest trapped vegetation index value. The land cover values and their associated vegetation index values were then aggregated for use in the final estimation. In the data set used, each reference point had eight land use percentages (one for each land use type), and eight vegetation index values (one associated with each land use) for 1990–1999.

Appendix B: Derivation of Elasticities and Binary Effects

Elasticities are derived by first taking the expectation of the function and by then taking the derivative. In most cases, however, taking the expectation eliminates the error term. When the dependent variable is nonlinear, this is not the case, and thus the elasticities cannot be calculated directly from text equation (3) because $E(y)^{\lambda_1} \neq E(y^{\lambda_1})$. Therefore, a different approach must be taken to derive the elasticities. Text equation (3) can be rewritten generally as:

$$\text{(A1)} \qquad Land = \left[\left(\left(a_0 + \sum_k a_k\left(\frac{x_k^\lambda - 1}{\lambda}\right) + \sum_m a_m d_m + e\right) + 1\right) * \lambda_1\right]^{1/\lambda_1},$$

where x_k represents a continuous variable, d_m represents a dummy variable, a_0, a_k, a_m, λ, and λ_1 are parameters to be estimated, and e is an error term. The first step in deriving the elasticities is to take the expectation. This yields:

$$\text{(A2)} \qquad E[Land] = E\left[\left(\left(a_0 + \sum_k a_k\left(\frac{x_k^\lambda - 1}{\lambda}\right) + \sum_m a_m d_m + e\right) * \lambda_1 + 1\right)^{1/\lambda_1}\right].$$

Equation (A2) can be rewritten as:

$$\text{(A3)} \qquad E[Land] = E\left[\left(\lambda_1(R + e) + 1\right)^{1/\lambda_1}\right],$$

where

$$\text{(A4)} \qquad R = a_0 + \sum_k a_k\left(\frac{x_k^\lambda - 1}{\lambda}\right) + \sum_m a_m d_m.$$

Considering the term within the expectation operator in the right-hand side of (A3) to be a function of e, $f(e)$, we consider a second-order Taylor-series expansion of $f(e)$ around 0. This results in:

$$\text{(A5)} \qquad E[Land] = E\left[\left(\lambda_1 R + 1\right)^{1/\lambda_1} + \left(\lambda_1 R + 1\right)^{(1/\lambda_1)-1} e + \frac{\lambda_1}{2}\left(\frac{1}{\lambda_1} - 1\right)\left(\lambda_1 R + 1\right)^{(1/\lambda_1)-2} e^2\right],$$

which, after noting that $E(e) = 0$, can be reduced to:

$$\text{(A6)} \qquad E[Land] = \left(\lambda_1 R + 1\right)^{1/\lambda_1} + \tfrac{1}{2}\left(1 - \lambda_1\right)\left(\lambda_1 R + 1\right)^{(1/\lambda_1)-2}\sigma_e^2,$$

where σ_e^2 is the variance of the error term from (A1).

The next step is to obtain the derivative of the expected value of *Land* with respect to x_k, which is calculated as:

$$\text{(A7)} \quad \frac{dE[Land]}{dx_k} = \frac{dE[Land]}{dR}\frac{dR}{dx_k} = \left(\left(\lambda_1 R + 1\right)^{(1/\lambda_1)-1} + \tfrac{1}{2}\left(1 - \lambda_1\right)\left(1 - 2\lambda_1\right)\left(\lambda_1 R + 1\right)^{(1/\lambda_1)-3}\sigma_e^2\right) * a_k x_k^{\lambda-1}.$$

Equation (A5) can then be used to derive the elasticity of the expected value of *Land* with respect to x_k, as follows:

$$\text{(A8)} \quad E_k = \frac{dE[Land]}{dx_k} * \frac{x_k}{E[Land]} = \frac{\left(\left(\lambda_1 R + 1\right)^{(1/\lambda_1)-1} + \tfrac{1}{2}\left(1 - \lambda_1\right)\left(1 - 2\lambda_1\right)\left(\lambda_1 R + 1\right)^{(1/\lambda_1)-3}\sigma_e^2\right) * a_k x_k^{\lambda}}{\left(\lambda_1 R + 1\right)^{1/\lambda_1} + \tfrac{1}{2}\left(1 - \lambda_1\right)\left(\lambda_1 R + 1\right)^{(1/\lambda_1)-2}\sigma_e^2}.$$

Since the derivatives of the dependent variable *Land* with respect to the binary variables do not exist, it is necessary to determine the effect of these variables on y in a different framework, i.e., by obtaining a binary effect. The binary effect is $(Land_1 - Land_0)/Land_0$, where $Land_1$ and $Land_0$ are the model-predicted values of the *Land* when the binary variable of interest is equal to one and zero, respectively. The binary effects are obtained by using (A6). The binary effect of the binary d_m on *Land*, or g_m, is specified as:

$$\text{(A9)} \qquad g_m = \left[\frac{\left(\lambda_1 R|_{d_m=1} + 1\right)^{1/\lambda_1} + \tfrac{1}{2}\left(1 - \lambda_1\right)\left(\lambda_1 R|_{d_m=1} + 1\right)^{(1/\lambda_1)-2}\sigma_e^2}{\left(\lambda_1 R|_{d_m=0} + 1\right)^{1/\lambda_1} + \tfrac{1}{2}\left(1 - \lambda_1\right)\left(\lambda_1 R|_{d_m=0} + 1\right)^{(1/\lambda_1)-2}\sigma_e^2} - 1\right],$$

where $R|_{d_m=1}$ is (A4) when $d_m = 1$, and $R|_{d_m=0}$ is (A4) when $d_m = 0$. Finally, elasticities and binary effects represented by (A8) or (A9) might be evaluated at each observation, at the means, or at any other point of interest.