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The Accuracy of Producers' Probability Beliefs: Evidence and Implications for Insurance Valuation

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The accuracy of producers' subjective probability beliefs is examined through a survey of large cash-grain farmers in Illinois. Findings reveal that their subjective probability beliefs about important weather variables are systematically miscalibrated. The nature and extent of differences between subjective probability beliefs and probabilities based on long-term historic weather data are shown empirically, and through fitted calibration functions. The economic significance of inaccurate subjective probability beliefs is established in the context of insurance valuation. The results demonstrate that significant errors in producers' risk assessments and insurance valuation arise as a consequence of producers' systematically inaccurate probability beliefs.

Key words: precipitation insurance valuation, probability beliefs, risk assessment

Introduction

The vast majority of the existing risk management literature is underpinned with the assumption that producers accurately understand and rationally respond to the risks they face. While it is generally understood and acknowledged that subjective beliefs form the basis for individual behavior under risk (Bessler; Machina and Schmeidler), relatively little research has been conducted to assess the accuracy of producers' beliefs, or the economic implications of inaccurate beliefs. This research explores the important, but frequently unexamined assumption that producers possess accurate probability beliefs when evaluating risky variables affecting their financial well-being.

Particular attention in agricultural risk management has been devoted to the development and evaluation of crop yield, and crop revenue insurance contracts. Numerous studies have carefully examined risks represented in the distributions of crop yields and prices, and have developed various insurance valuation models equipped to deal with the resulting specifications (Day; Gallagher; Goodwin and Ker; Ker and Goodwin; Nelson; Stokes). On the behavioral side, issues related to moral hazard and adverse selection have also been carefully assessed and incorporated into explanations of the performance of popular insurance products, and into empirical and theoretical studies of crop insurance demand (Coble et al.; Just, Calvin, and Quiggin; Smith and Goodwin; Skees and Reed).

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While the bulk of the applications in agriculture have understandably targeted the large array of Federal Crop Insurance Corporation (FCIC) products, there has also been a rapidly increasing interest in the use of weather derivatives as mechanisms to manage specific agricultural risks. To date, the weather derivative market has developed much more rapidly in energy applications, and in insurance for outdoor public events, but studies which parallel crop insurance methods to evaluate weather insurance are also beginning to appear in the literature (Martin, Barnett, and Coble; Dischel; Sakurai and Reardon; Turvey; Changnon and Changnon). In any case, in most agricultural risk analyses, the common assumption is that producers have accurate beliefs about risks faced.

In this study, a survey designed to elicit subjective probability beliefs about important weather variables influencing producers' well-being was administered to a set of relatively large grain producers. The recovered subjective probability beliefs are compared to historic weather event distributions in both empirical (nonparametric) and fitted form. Calibration functions are then estimated to provide insight into the extent and nature of the differences between the probability distributions based on historic data and individuals' subjective probability measures.[1]

Standard precipitation insurance contracts are evaluated to demonstrate the economic significance of the differences between producers' beliefs and the underlying distributions of interest. Weather variables are employed due to their ubiquity, relevance to crop farmers, impossibility of influence by farmers, and widely available existing information to condition decision makers' priors. Further, insurance on weather variables naturally limits adverse selection and moral hazard, and thus isolates the impacts of inaccurate priors in a relatively straightforward fashion.

Expectations of Climate Variables Survey

A personally administered survey instrument was used to recover complete probabilistic descriptions of producers' climate expectations.[2] The potential impact of the survey elicitation process on the recovered probabilities has been well recognized in the literature, and some guidance exists for developing useful measures of subjective probabilities (see Nelson and Bessler). Numerous approaches have been used in past studies, including voluntary assessments in self-assigned intervals (Kenyon), allocation of probabilities into fixed categories (Eales et al., citing procedures from Bessler and Moore), economic games (Fisher and Tanner), use of market indicators (O'Brien, Hayenga, and Babcock), and many others, both with and without direct compensation. It is generally agreed the respondents should have a motivation to complete the task (seriousness, or have compensation for success), and that proper scoring rules or other methods should be used to help ensure that respondents' stated beliefs correspond to their true beliefs, within the limits of the encoding measures.

[1] A probability assessment is termed "calibrated" if the proportion of realizations equals the probability assigned (Lichtenstein, Fischoff, and Phillips). The long-term precipitation frequencies are taken as the "true," and are used as a set of repeated outcomes of the same event against which subjective probabilities can be assessed. Dawid more fully develops the concept of calibration, and demonstrates its usefulness in evaluating competing sequential forecasts (as in Bessler and Kling), and in cases that do not require probability forecasts to be interpreted against repeated trials as well. Curtis, Ferrell, and Solomon apply the methods to assess probability distributions more generally, and to identify impacts of aggregation of forecasts. The term calibration is used here to describe the congruence between the producer subjective beliefs and the long-term precipitation frequencies.

[2] A copy of the complete survey instrument is available from the author upon request.

In an attempt to design as sound an instrument as possible, the survey was developed with the guidance of an expert in the field of eliciting and coding subjective beliefs about climate events, and with input and approval of the Survey Research Laboratory—a University of Illinois resource for organizing and pretesting survey work, and which serves as an arm's-length evaluator of the actual instruments used.[3]

Formally trained enumerators were employed, and a single training session with each respondent was used at the beginning of each interview to assess the ability of the respondents to present proper probability measures. Participants were required to pass a probability consistency test before proceeding with the survey. The following characteristics determined the selection of participants: (*a*) their cooperation with the Illinois Farm Business-Farm Management (FBFM) record keeping association; (*b*) their proximity to a single weather reporting station [i.e., to mitigate the potential effects of widely differing experiences, all participants were in a territory covered by a single National Oceanic and Atmospheric Administration (NOAA) weather reporting station]; (*c*) their business enterprises were all relatively large cash-grain operations; and (*d*) their demonstrated understanding of probability concepts.

Interviewers elicited producers' perceptions of the long-run probabilities of rainfall at various levels through a series of questions posed in both the cumulative distribution function (CDF) framework and inverse CDF framework. Numerous questions were recast throughout the survey to locate any changes in perceptions or misperceptions of the intent of questions. For example, if a respondent indicated the level of rainfall at which the 25th cumulative percentile occurred was 2 inches, the enumerator would later ask for the probability that 2 inches would be exceeded, to confirm the respondent replied in a manner consistent with the earlier answer. A pretest was administered to ensure comfort and adequate facility with probabilistic concepts, and internal checks were constructed to corroborate that respondents' probability measures were indeed consistent and representative of their beliefs.[4]

The survey included approximately 12 categories of variables affecting the producer's financial well-being, and took approximately one hour plus pretest time per respondent to administer. A total of 54 surveys were administered and processed into useable form.[5] Among the specific climate variables of interest included in the survey are April rainfall

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[3] Dr. Peter J. Lamb, currently Director of Cooperative Institute for Mesoscale Meteorological Studies and School of Meteorology, University of Oklahoma, provided survey design consultation and guidance.

[4] The pretest involved allocating probabilities to game-event outcomes. The enumerators then reviewed the tests with the respondents and discussed consistency requirements. For the weather questions, the enumerator asked for rainfall levels at fixed quantile values (14 total per farmer for the two distributions). A guide script for the enumerators included (example from April; July phrasing was similar):

> *We now want you to think about the amount of rainfall you would expect in a typical April in this location. What would you say the rainfall is with a 10% chance of occurrence—in other words, that 1 in 10 years would be at or below this level, and 9 of 10 would be above?* [___ inches].

Similar questions for the other percentiles were used. The enumerator was also instructed to record the answer from the 25% response on a later page in the survey and again ask the respondent for the probability associated with that level of rainfall, without any indication it was the same question in inverse form. The enumerator had the option to rephrase any previous questions if inconsistencies were found. The probabilities were required to be consistent (increasing CDF), and respondents were asked if they had any revisions to supply at the end of the test.

[5] While the sample is relatively small, these individuals were all commercial-scale farmers in proximity to a single weather reporting station and all were participants in a record keeping association, which signals they have high-quality financial information. Weather variables are of particular salience to such producers. Each producer provided considerable detail about his/her operation and beliefs. A larger sample would have necessitated loss of detail and would have required comparisons to data from more than one weather reporting station.

and July rainfall.[6] Higher April precipitation is considered by Illinois grain producers to be a negative event, as it tends to delay planting. Conversely, July precipitation is a positive event, as it tends to enhance crop growth and reproduction during a crucial phase of development. Precipitation levels in April and July were chosen because of their particular importance to grain farmers, and because the effects on the respondents are of opposite sign, thus generating a natural contrast for study of the accuracy of their probability beliefs.

Weather Variable Representations

A distributional representation is needed to summarize information from the historic weather data, and to provide a description of each producer's subjective probability beliefs. A distribution used extensively in various forms to model precipitation amounts is the Burr-12 distribution, also sometimes referred to as a three-parameter Kappa distribution in weather applications (Mielke; Mielke and Johnson; Tadikamalla). The Burr distribution covers the positive domain, may take on a wide range of skewness and kurtosis values, and can be used to fit almost any set of unimodal data (Tadikamalla).

The Burr distribution is highly flexible and contains the Pearson types IV, VI, and bell-shaped curves of type I, gamma, Weibull, normal, lognormal, exponential, and logistic distributions as special cases (Rodriguez; Tadikamalla). Because of this flexibility, it is widely accepted in the climate literature as a representation for precipitation levels, and was used to represent the historic distribution, and each producer's underlying subjective distribution.[7]

The Burr probability density function (PDF) and cumulative distribution function (CDF) for rainfall, Y, with parameters α, λ, and τ, are respectively:

$$(1) \qquad f(y) = \tau\lambda\alpha^{-1}(y/\alpha)^{\lambda-1}\left(1 + (y/\alpha)^{\lambda}\right)^{-(\tau+1)}, \qquad y, \alpha, \lambda, \tau > 0;$$

$$(2) \qquad F(y) = 1 - \left(1 + (y/\alpha)^{\lambda}\right)^{-\tau}.$$

Monthly data from the National Climatic Data Center on rainfall totals from 1900 to 2000 at the East Central Illinois weather reporting station were used to estimate the parameters of the underlying distributions of April and July rainfall using maximum-likelihood estimation. Goodness of fit of the estimated distributions was assessed using Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests of the differences between the empirical and fitted distributions. The results indicate an exceptionally good fit (i.e., failure to reject at any tabulated level of significance) for both April and July fitted

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[6] The survey was conducted during the summer of 1991 as part of a larger project examining producer beliefs. Producer subjective distributions were also recovered for commodity prices, temperature during pollination, winter precipitation, interest rates, and other variables affecting financial performance. Other researchers have also examined non-weather expectations. For example, Eales et al. assessed the congruence between producer and merchant expectations and distributions of commodity prices implied by the market. They found producers have accurate means but tend to have understated variances. Likewise, Pease et al. examined subjective beliefs about yield and found miscalibrated producers' expectations which could substantially affect insurance valuation. Kenyon concluded that producers have significantly miscalibrated beliefs with a tendency to overstate the probability of lower prices and understate the probability for large increases.

[7] Various related parameterizations have been presented in the literature, including Burr-3, Burr-12, Kappa, gamma, and Lomax versions. Martin, Barnett, and Coble use the gamma distribution to represent cumulative rainfall. Mielke demonstrates the favorable performance of the Burr over the gamma, but leaves other choices unranked. In this study, the Burr-12, Kappa-3, and Burr-3 parameterizations were each fitted, with negligible resulting differences. The results presented here are from the Burr-12 set of estimations only, as the other two were qualitatively identical.

distributions.[8] Parameters for each producer's subjective probability measures for both April and July rainfall were also estimated under the same parametric assumptions using nonlinear least squares between implied and tabulated response quantiles.

Results

Figure 1 depicts the subjective beliefs about precipitation levels for a selected set of five respondents with differing types of probability beliefs. As can be seen in the graph, different forms of incongruence between historic and subjective measures exist. For example, farmers #5 and #47 believed the density of April precipitation to be more spread out, and have a higher median than the true of 3.55 inches (these two examples represent the most common type of responses relative to April precipitation). The subjective probability measures for farmers #19 and #25 are generally shifted to a lower level than the true, but with somewhat longer right-hand tails. Respondent #44 displays overconfidence, and a slightly elevated central tendency.

Relative to July precipitation, respondent #25 has a higher median, while the others each have subjective beliefs with medians lower than the fitted underlying distribution of 3.42 inches. Respondent #47 displays extremely high pessimism with a highly overstated probability of zero or no rainfall. Exhibiting a median that is below the true and somewhat understated probabilities at the high range, respondent #44 represents a typical response for July rainfall. Respondent #5 has fairly accurate probability beliefs relative to July rainfall. For convenience in interpretation, the cumulative distribution functions are graphically displayed in figure 1 as well.

The five respondents depicted in the graphs are not meant to be representative of the entire sample, but were chosen simply to illustrate the nature of the information retrieved and to provide an understanding of the types of differences observed in the survey findings—both among producer responses and between individual producer beliefs and the historic measures.

Table 1 summarizes the farmer responses across the entire sample for both April and July precipitation. Several quantiles are tabulated under which the farmers' responses are summarized and compared to the actual precipitation values from the empirical distribution (the results are virtually identical when compared to the fitted distributions as well). For example, for April precipitation at the 25th percentile, the precipitation level corresponding to the actual distribution is 2.30 inches. In other words, there is a 75% chance of receiving at least 2.30 inches of precipitation in the month of April in this weather reporting district. Of the farmers surveyed, 63% expected more precipitation at the 25th percentile. The average of all responses at the 25th percentile of the distribution was 2.77 inches.

From table 1, note that the average of the expected precipitation is greater than actual precipitation experienced in history at all percentile levels, although by only a

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[8] Following a reviewer's suggestion to test for the robustness to sample period effects, the data were divided into 1900–1950 and 1951–2000, and examined for evidence of change. There was no statistically significant difference in the means or variances, and the overall results were virtually unchanged when using parameters from either subset for April. The results are qualitatively similar, but slightly stronger (more miscalibrated) when using only the latter half sample for July. While farmers' beliefs are likely conditioned by experience, the longest available data series was preferred to represent the underlying climate events. All reported results in this analysis use the full sample period.

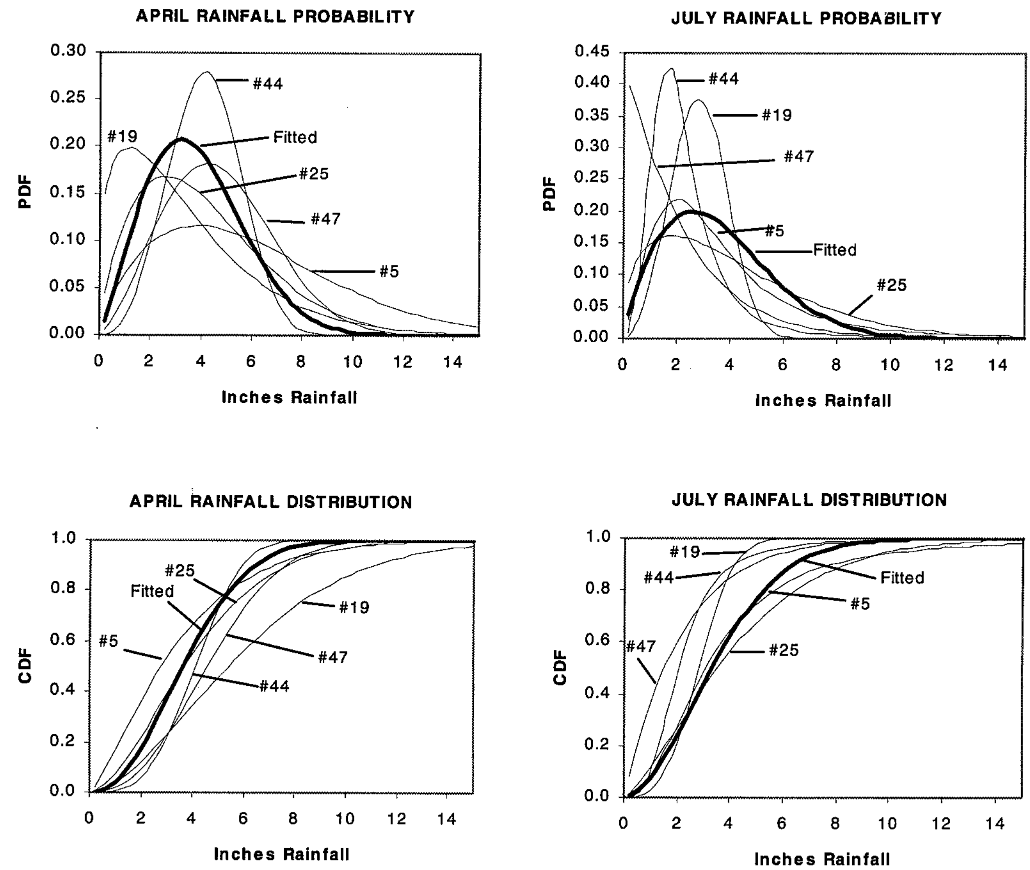

Figure 1. Fitted and producer probability measures for April and July rainfall

Table 1. Summary of Farmers' Subjective Probability Beliefs Relative to Actual Probabilities

	Percentile Level				
Description	10%	25%	50%	75%	90%
April Precipitation:					
Actual (inches)	1.40	2.30	3.55	4.98	6.39
Average farmer response (inches)	1.41	2.77	4.47	5.85	7.53
% of responses greater than actual	53.7	63.0	72.2	74.1	64.8
Standard deviation across respondents	0.56	1.02	1.24	1.45	2.07
July Precipitation:					
Actual (inches)	1.12	2.02	3.42	5.14	6.93
Average farmer response (inches)	0.81	1.79	3.03	4.65	6.18
% of responses greater than actual	16.7	22.2	25.9	42.6	38.9
Standard deviation across respondents	0.54	0.72	0.84	1.19	2.27

slight amount at the 10th percentile level. Clearly, the subjective probabilities elicited from this group of farmer respondents generally overweighted what they perceive as the negative event of excess April precipitation, with the fraction overstating the rainfall higher at levels generally considered less desirable. If the respondents had no systematic bias in their beliefs, then the percentage overstating the median might reasonably have been expected to be around 50%, but the miscalibration in the sample appears to be systematically toward overstated levels of precipitation. The standard deviation across responses at each quantile is also provided in table 1 to show the degree of agreement among respondents at each level.

The respondents' subjective probability beliefs about July precipitation follow a different, yet still pessimistic, pattern. In this case, more rainfall is considered to be a good event, and the respondents generally understate the likelihoods of occurrence. As observed in table 1, only 22% of the respondents overstated the quantity of rainfall at the 25th percentile of the actual distribution. In fact, at each percentile level, the farmers understated the incidence of precipitation, or equivalently, overstated the probability of what would be viewed as the negative event—lack of precipitation. As with April, the results are consistent whether the farmer responses are compared to the fitted or empirical distributions.

Individual Producer Calibration Tests

In addition to the information available in table 1 summarizing the entire set of respondents, it is useful to develop more descriptive measures of differences between individual producers' subjective beliefs and the fitted underlying distribution. And, in cases exhibiting significant differences, insight can be gained by more completely describing the nature and extent of the difference between subjective and actual distributions over different percentile levels or among differing events. For example, a producer may be very good at forecasting the likelihood of a low-rainfall event, but be poor at assigning probabilities to large-rainfall events. Or, the producer may have more accurate priors about April than July rainfall. Because risk management activities often focus only on ranges of adverse outcomes, an assessment of the congruence between historic events and subjective probability beliefs in specific regions of interest would be beneficial. To address these and related issues, calibration functions were estimated.

Calibration approaches were originally developed for identification of adjustment functions to apply to sequences of probability forecasts, based on differences between past forecasts and outcomes. In terms of probability beliefs, calibration describes the congruence between two different distributions (Curtis, Ferrell, and Solomon). Heuristically, the adjustment required to make the subjective beliefs correspond to the true distribution is termed the calibration function. Specifically, if the true distribution can be described as $\phi(x)$, and the estimated function is $F(x)$, then $K(F(x)) = \phi(x)$ implicitly defines a transformation, $K(\cdot)$, of F to generate estimates, $K(F(x))$, that are well calibrated. The function $K(\cdot)$ is called the calibration function. A parametric form can be chosen for the calibration function and estimated using standard methods, with the resulting shape of the estimated function used to interpret the nature of the miscalibration (Fackler and King).

For purposes of this study, the calibration function is based on the beta distribution with density:

$$K(x) = \frac{x^{p-1}(1-x)^{q-1}}{\beta(p, q)}, \tag{3}$$

where $\beta(p, q)$ is the beta function with parameters p and q. As noted in Fackler and King, the beta distribution is well known, flexible, and contains the uniform distribution as a special case when $p = q = 1$, implying perfect calibration. Regions of $K(\cdot)$ with slope greater than one correspond to regions of the subjective probability CDFs that need to have mass added, and regions of $K(\cdot)$ with slope less than one correspond to regions of the subjective distribution having too much mass. Other shapes of the fitted calibration curve similarly indicate the "reweighting" of the estimated distributions needed to correspond to those subsequently observed.

At least five general shapes for the calibration function emerge which summarize the nature of the miscalibration displayed by each individual. Figure 2 graphs the sample calibration functions corresponding to the following cases:

- CASE 1. Well calibrated or uniform ($p = q = 1$);
- CASE 2. Underconfidence or an overstatement of dispersion ($p > 1, q > 1$);
- CASE 3. Overconfidence or an understatement of dispersion ($p < 1, q < 1$);
- CASE 4. Understatement of location ($p > 1, q < 1$); and
- CASE 5. Overstatement of location ($p < 1, q > 1$).

Because the slope of the calibration function reflects the reweighting of the subjective distribution needed to make it correspond to the fitted distribution, the uniform case 1 is a straight line with slope 1 throughout, and therefore leaves the subjective beliefs unchanged. Case 2 is an "S"-shaped function that takes mass away from the tails (where the slope is less than one) and adds it to the interior region where the slope is greater than one. Case 3, by contrast, is a "reverse-S" shaped function which spreads the mass out by adding to the tails and reducing the central region where the calibration function slope is less than one. Case 4 is a "U"-shaped function, shifting mass to the right, and case 5 is an "inverted-U" shape, shifting mass to the left. The median is located correctly when $p = q$ (cases 1, 2, and 3 as shown in figure 2), but the calibration function can also cross the uniform from above or below at locations other than at $F(y) = 0.5$, indicating miscalibration in both location and dispersion.

Calibration functions were estimated for each participant's subjective distribution for both April and July rainfall using least squares between the recalibrated beliefs and the fitted distributions at each percentile level surveyed. Table 2 provides the summary of the results organized into two sections, with the upper panel reporting the parameter pairings from which general shapes can be inferred, and the lower panel giving more specific information about two attributes—median location and dispersion—that help in understanding the degree and nature of the miscalibration.

As shown in table 2, the most prominent recalibration needed for the April subjective distributions is to shift the mass to the left (inverted-U), and for July the most common fitted calibration function indicates the mass of the probability distributions needs to be shifted to the right (U-shaped). These shifts can occur in conjunction with either increases or decreases in dispersion, and thus it is also useful to tabulate the more general effects.

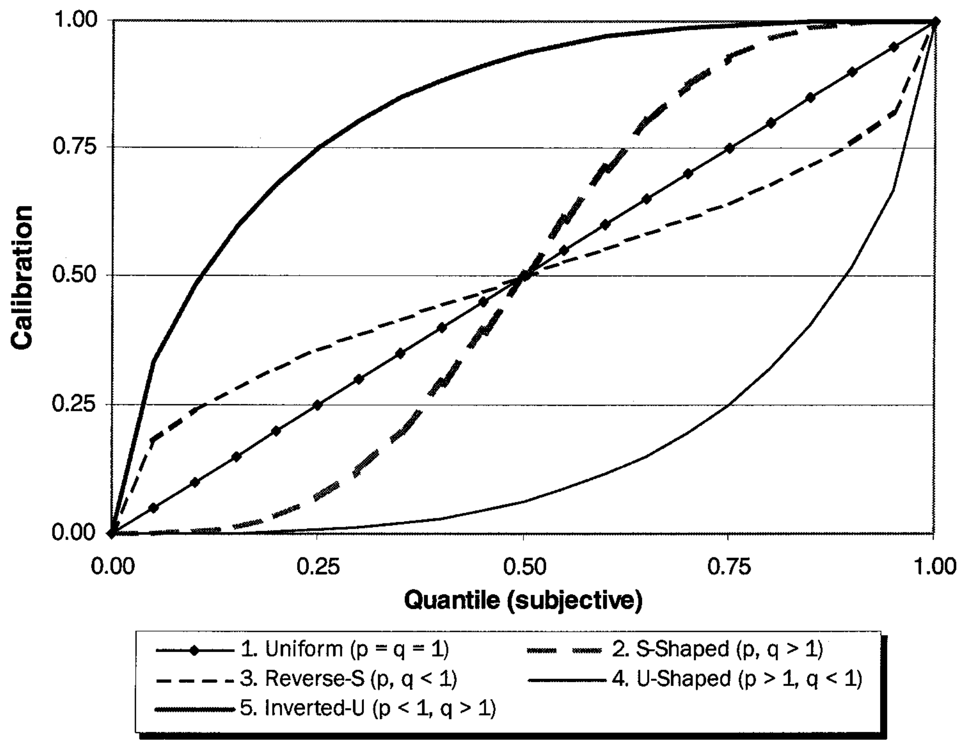

Figure 2. Calibration functions for five sample cases

Table 2. Summary of Calibration Functions

Fitted and Empirical Calibration Features	April Precipitation % of Farmers	July Precipitation % of Farmers
"*U*"-shaped calibration function	20.4	51.9
"Inverted-*U*" shaped calibration function	61.1	20.4
"*S*"-shaped calibration function	11.1	14.8
"Reverse-*S*" shaped calibration function	7.41	13.0
Median overstated and Dispersion overstated	57.4	20.4
Median overstated and Dispersion understated	14.8	5.6
Median understated and Dispersion overstated	7.4	16.7
Median understated and Dispersion understated	20.4	57.4

Note: Dispersion is considered overstated if the calibration function indicates that the probability in the interquartile range is understated by the producer (i.e., the slope of the calibration function is greater than one over the range). Dispersion measured by the standard deviation of fitted relative to true gives similar results.

The lower panel of table 2 provides evidence about combined attributes representing location and dispersion. The top two rows can be added together to obtain all the cases with median overstated (and can also be read from the 50th percentile column in table 1), while the lower two rows can be summed to obtain cases with the median understated. The first and third rows contain all the cases with dispersion overstated, while the second and fourth rows show the cases with dispersion understated. As observed from the table, of those occurrences where location is overstated, the dispersion tends to be overstated as well—both attributes which overstate risk. Of those understating the July location, the sample is more heavily weighted toward understatement of dispersion.

Because errors from the first-stage fitting of what is taken as the "true" distributions could affect the results, the calibration results must be interpreted with caution. As a check, empirical versions of the calibration functions were also constructed by plotting the response quantiles joined by linear line segments against the empirical quantiles from the data sets. The resulting Q–Q plots serve as nonparametric calibration functions. Although less smooth than the fitted versions, the results are qualitatively identical, providing additional support for the results.

In addition to the results for individual responses, calibration functions were also estimated for the simple average of all respondents. In the case of April, the resulting calibration function has an "inverted-U" shape, understating the location while overstating the dispersion. For July, the calibration function for the average response across producers displays a "U"-shape with a slightly understated dispersion.

It is apparent from both the tabulated survey results and the calibration tests that producers tended to overstate the amount of rainfall in April and understate the rainfall in July—i.e., both undesirable events are overweighted by producers. Further, the calibration tests reveal that dispersion in the subjective rainfall distributions has a tendency to be understated in the case of July rainfall and overstated in the case of April rainfall. Based on these findings, producers' beliefs are systematically what could be termed "pessimistic," rather than simply being misstated in a manner that applies regardless of the event being considered. Again, if the probability results were simply the manifestation of a "naive" mistake process, the types of mistakes would more likely have been consistent between the two events rather than displaying the upward bias in April and the downward bias in July probabilities.

Implications for Insurance Valuation

The impact of inaccurate priors depends both on the degree of difference from the underlying, and on the specific context in which the information is used. It could be the case that small inaccuracies have substantial consequences in risk management, or it could be the decision rules are such that the probability beliefs are relatively inconsequential and have little economic impact. To demonstrate the potential economic importance of having miscalibrated probability beliefs about weather variables, precipitation insurance is evaluated under each producer's fitted probability beliefs and compared to the actuarial value calculated under the distributions fitted to historic data. The differences can then be viewed as direct measures of the potential economic impact of the inaccurate prior beliefs.

The most common forms of precipitation insurance can be valued in a manner analogous to standard option pricing approaches. Numerous precipitation guarantee valuation models have been reported elsewhere in the literature to take advantage of specific attributes of producer demand, but most are developed in terms of the expected loss functions (Martin, Barnett, and Coble; Turvey; Aquila Energy Derivatives Group; Dischel).

Typically, an insured event, such as cumulative precipitation in a specified interval of time, is offered for insurance at various trigger points or strike prices, and at a fixed liability for each unit of excess or deficit. In the current context, rainfall totals measured at a single weather reporting station during the months of April and July are the insured events. The indemnity triggers, often termed strikes or k, could be offered at either producer-selected levels or at standardized increments, for example, at 2.5 inches, 3.0 inches, 3.5 inches, and so on.

As is typical, the insurance contract is written to pay a constant (λ) times the amount by which the insured event exceeds the trigger (k), and make no payments if the trigger is not exceeded. The scale of λ is chosen to make the contract magnitude meaningful to the users, and in the case of rainfall insurance, multiples of $1,000 are commonly used. The strike prices are set to provide a meaningful "menu" to appeal to producers with differing needs. For instance, a producer with a large machinery base and light soils may consider excess rainfall less of a problem than a producer who needs more workable field days to put in a crop. The first farmer might prefer a relatively high strike compared to the latter farmer, to more nearly mimic the points at which each begins to suffer economic losses due to excess rainfall.

The indemnity payoff function for excess rainfall can be written as $\max\{0, y - k\} * \lambda$, where y is the realized rainfall total. Given a probability density $f(y)$ governing the rainfall outcome y, the expected (actuarial) value, V_r, of the excess rainfall insurance contract is specified as:

$$V_r = \lambda * \int_k^{\infty} (y - k) f(y)\, dy. \tag{4}$$

Similarly, July-drought insurance is evaluated which pays λ per inch of rainfall deficit to k during the month of July, with a resulting indemnity function of $\max\{0, k - y\} * \lambda$. The actuarial value, V_d, of such a contract can thus be found by evaluating:

$$V_d = \lambda * \int_0^{k} (k - y) f(y)\, dy. \tag{5}$$

The values of insurance against excess April rainfall were calculated using equation (4) across strike prices from 2 inches to 10.5 inches in half-inch increments, and using $\lambda = \$1{,}000$. At each strike, the valuation equation was applied using the fitted rainfall distribution for $f(y)$, and then repeated using each producer's subjective beliefs to describe the probability density $f(y)$. The result is one valuation relationship for each farmer, and the actuarial values at each strike against which they can be compared.[9]

Table 3 presents the complete results of the actuarial calculations and producer valuation results for insurance against excess rainfall in April. Columns A and B give the strike price or level of rainfall insured against, and the associated probability of triggering the insurance under the actual rainfall distribution. Column C contains actuarially fair values of insurance (expected costs) which range from approximately $1,895 at a 2 inch strike price, down to only $1.39 per $1,000/inch coverage at a strike of 10.5 inches. For example, as seen from table 3, the actuarially fair payments to a policy holder who insures at a strike price of 5 inches would be $347.93. Column D reports the average across all respondents of their perceived probability of triggering insurance payments at that strike. Comparison to corresponding entries in column B provides a direct indication of the mistakes in risk assessment arising from miscalibrated beliefs.

[9] The insurance values were also calculated under the empirical rainfall distributions as well. The results were slightly "lumpier" than those reported under the fitted distributions, but are qualitatively unchanged in all respects.

Table 3. April Excess Rainfall Insurance: Actuarial Values and Producer Valuation Summary (λ = $1,000)

[A] Strike (inches)	[B] Probability Rain > k	[C] Actuarial Insurance ($)	[D] Subjective Probability Rain > k	[E] Average Value to Producer ($)	[F] Average Percent Misvalued (%)	[G] Percent Respondents Who Overvalue (%)	[H] Average Value Given Overstated ($)	[I] Self-Selected Percent Overvalued (%)
2.0	0.806	1,895.31	0.859	2,525.93	33	74	2,925.52	54
2.5	0.712	1,515.31	0.787	2,126.21	40	74	2,506.79	65
3.0	0.611	1,184.37	0.706	1,766.73	49	74	2,121.68	79
3.5	0.509	904.36	0.621	1,451.01	60	72	1,798.29	99
4.0	0.412	674.19	0.535	1,179.97	75	72	1,490.14	121
4.5	0.324	490.42	0.451	951.79	94	72	1,221.84	149
5.0	0.248	347.93	0.373	762.54	119	67	1,047.63	201
5.5	0.184	240.63	0.302	607.45	152	65	866.74	260
6.0	0.132	162.17	0.239	481.68	197	69	670.33	313
6.5	0.092	106.47	0.186	380.55	257	69	536.43	404
7.0	0.063	68.07	0.141	299.73	340	69	426.85	527
7.5	0.041	42.37	0.105	235.41	456	69	337.81	697
8.0	0.026	25.67	0.077	184.32	618	67	272.70	962
8.5	0.016	15.13	0.055	143.79	850	67	213.74	1,312
9.0	0.010	8.68	0.038	111.65	1,186	67	166.54	1,818
9.5	0.006	4.84	0.026	86.16	1,679	67	128.83	2,560
10.0	0.003	2.63	0.018	65.97	2,410	65	101.53	3,763
10.5	0.002	1.39	0.012	50.00	3,505	63	79.26	5,616

Column E in table 3 lists the average implied values of insurance at each strike. Interestingly, this group of producers, on average, overvalued the risk-costs associated with rainfall at every level tabulated. The difference at the actuarially fair point is due solely to misperceptions of the risks faced (column F), in this case resulting in perceived values of insurance which exceed the actuarial values by $631 (33%) at the 2 inch strike, to $611 (40%) at the 2.5 inch strike, and so on to the point where the overstatement is nearly 35 times the actual value at a strike of 10.5 inches. While the dollar value of the error declines with the strike, the percentage overstatement explodes as the actuarial value approaches zero. Under either case, respondents clearly overestimate the risks associated with what is perceived to be the negative event of excess April rainfall.

Column G of table 3 lists the percentage of respondents whose implied values, given their subjective probability distributions, are greater than the value under the fitted distribution. Across the sample, roughly 70% of the respondents overvalued the insurance. Because the different perceptions of risks result in different implied values, it is reasonable to expect different responses to the availability of such insurance. For instance, it could be reasonable to assume that only those producers who perceived themselves to have a positive expected payoff to insurance would buy, and at the strike price for which the positive expected payoff were greatest. This form of self-selection may be viewed as favorable adverse selection to the producers, but is really just a result of having inaccurate probability beliefs.[10]

[10] The discussion is presented in terms of actuarial values only without the additional value the producer would be willing to pay as a risk premium if risk averse. Likewise, insurance loading costs are not considered. From an insurance provider's perspective, the positive misperceptions of value by producers provide a greater potential to add profit loadings to insurance contracts or cover greater actual expense loadings, and should stimulate the supply of such insurance relative to a case in which producers had accurate beliefs.

Table 4. July Rainfall Deficit Insurance: Actuarial Values and Producer Valuation Summary (λ = $1,000)

[A] Strike (inches)	[B] Probability Rain < *k*	[C] Actuarial Insurance ($)	[D] Subjective Probability Rain < *k*	[E] Average Value to Producer ($)	[F] Average Percent Misvalued (%)	[G] Percent Respondents Who Overvalue (%)	[H] Average Value Given Overstated ($)	[I] Self-Selected Percent Overvalued (%)
0.50	0.026	4.91	0.055	12.67	158	57	20.17	311
0.75	0.052	14.53	0.092	30.98	113	59	46.66	221
1.00	0.083	31.23	0.134	59.10	89	59	86.49	177
1.25	0.119	56.31	0.179	98.04	74	63	135.01	140
1.50	0.158	90.79	0.226	148.58	64	63	200.46	121
1.75	0.200	135.47	0.276	211.29	56	65	275.56	103
2.00	0.244	190.93	0.326	286.53	50	67	362.82	90
2.25	0.289	257.57	0.377	374.44	45	67	468.04	82
2.50	0.335	335.63	0.427	474.99	42	67	586.41	75
2.75	0.381	425.17	0.476	587.96	38	69	709.89	67
3.00	0.427	526.14	0.524	712.98	36	69	852.34	62
3.25	0.471	638.35	0.569	849.53	33	69	1,006.20	58
3.50	0.514	761.52	0.611	996.97	31	70	1,160.71	52
3.75	0.556	895.30	0.650	1,154.61	29	70	1,334.72	49
4.00	0.596	1,039.25	0.686	1,321.68	27	72	1,505.16	45
4.25	0.633	1,192.88	0.719	1,497.39	26	72	1,695.14	42
4.50	0.669	1,355.67	0.749	1,680.94	24	72	1,892.45	40
4.75	0.702	1,527.07	0.776	1,871.59	23	72	2,095.90	37
5.00	0.733	1,706.50	0.800	2,068.59	21	72	2,304.73	35

Nonetheless, assuming only producers whose implied values exceed the actuarial values actually purchase the insurance gives even more striking results. Column H in table 3 tabulates the averages of the perceived values at each strike for the subset of producers whose implied insurance values are greater than the actuarial value. As observed, the dollar value overstatement is greatest at the lower strikes, and declines as the probability interval evaluated in the insurance decreases. The percentage overstatement in value (column I) is near 100% at 3.5 inches, a strike situated nearly at the mean of the fitted distribution.

Table 4 presents comparable results for July drought insurance, with λ = $1,000. The table is constructed across strikes from 0.5 to 5 inches in half-inch increments. The probability range covered in this interval is from approximately 1% likelihood, or a 1-in-100 years drought event, to 5 inches, covering the outcomes of nearly three-quarters of all years. Actuarially fair insurance at a strike of 3.25 inches has a value of approximately $638 (column C).

The producers again substantially overstate the probability of needing the insurance (triggering payment), and overvalue the risks of drought across all farmers at every strike tabulated, with the greatest percentage overvaluation occurring at the extreme low range of the outcome distribution. The percentage of respondents who overvalue the insurance (table 4, column G) is not as great as was the case with April excess rainfall insurance, but still exceeds 50% across the entire range of outcomes. As with April insurance, percentage and value of the differences between the producers' valuations and the actuarial valuations are very large (columns H and I). Again, the results demonstrate that inaccurate probability beliefs of the nature possessed by the producers in this sample can have a significant impact on the evaluation of risk.

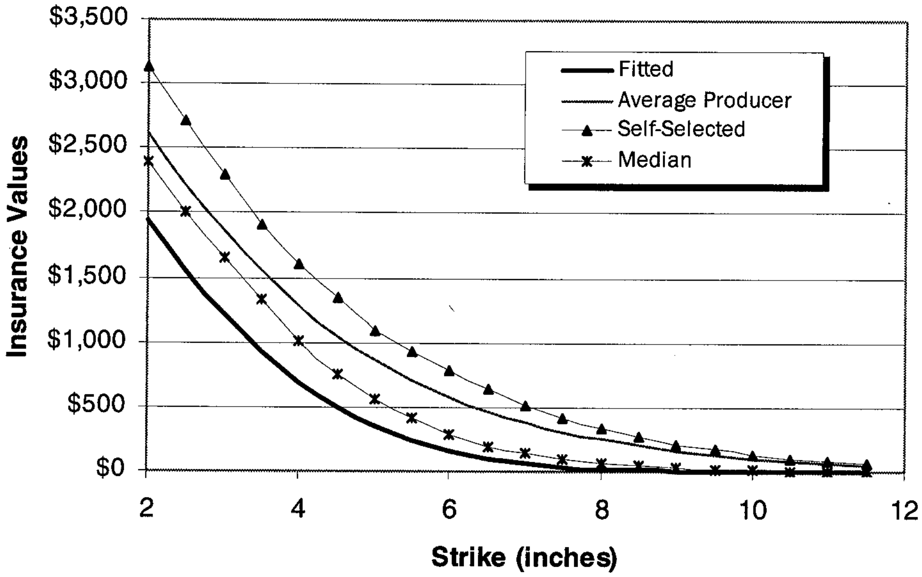

Figure 3. April excess rainfall insurance values under fitted and producer probability distributions

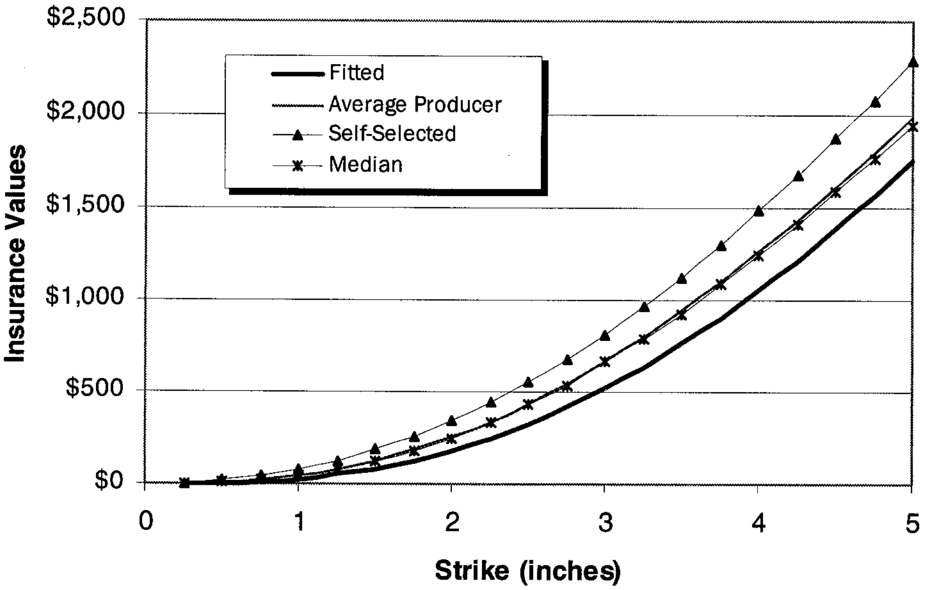

Figure 4. July drought insurance values under fitted and producer probability distributions

Figures 3 and 4 summarize the results for the actuarial value, average across all farmers, median across all farmers, and the average across farmers who would self-select insurance based on having overstated expected values of insurance for April and July, respectively. The figures take on the familiar shapes of traditional option or insurance values, as expected. These results are found independently of the calibration tests, but extend the findings by converting the differences to measures having economic interpretation as well—the value of insurance at different strike prices. Although only summary statistics are shown, it is worth noting that the valuation relationships for the individuals vary greatly, with the majority falling well above the actuarial level, and a few that either fall below or cross the actuarial relationship from below. Based on findings derived either from the averages or from the individual results, clearly the producers in this study sample substantially overstated the value of this type of insurance due to their miscalibrated beliefs about adverse outcomes.

Summary and Conclusions

Much effort has been devoted to evaluation of production insurance of various forms and on other risk-management tools. However, relatively little attention has been paid to what could be called the maintained hypothesis of this line of reasoning—i.e., that subjective beliefs held by the decision makers are accurate. The results from this study indicate producers hold systematically inaccurate beliefs about weather variables having important impacts on their financial well-being. The differences between subjective priors and the underlying weather event distributions are highly varied, but display the tendency across respondents to overstate likelihoods for negative events, and thus understate the incidence of positive events. Despite the wide differences in beliefs, they commonly lead to substantial overvaluation of both excess rainfall insurance during planting, and drought insurance during a critical phase of crop development.

The results, of course, are subject to limitations of the data, but nonetheless are important because they challenge acceptance of the assumption that producers accurately understand, and therefore can rationally respond to, production risks faced. The implications for precipitation insurance are direct: inaccurate subjective beliefs can lead to substantial overstatement of the value of insurance, and there could be significant self-selection of participation due solely to differences in producers' perceptions of the risks faced.

More generally, the results suggest those designing new insurance and risk management tools should include the potential effects of inaccurate risk assessments by users on demand for new products. And, interestingly, in cases where inaccurate beliefs would lead to underusage of insurance, it may be more effective to educate potential users about the actual risks faced than to subsidize the products enough to make them appear attractive to farmers with miscalibrated beliefs. This point may be especially relevant to the design of crop yield insurance programs, where there is evidence showing farmers expect yields that are too high relative to the true, and consequently understate the probabilities of very low yields.

Future research should examine a similar question with regard to producers' perceptions of other risky variables, with particular attention paid to producers' beliefs about yield and revenue risks, and the impact of potential inaccuracies on the demand for yield and revenue insurance products. Other extensions could likewise investigate the role of beliefs about risk in input usage and marketing behavior, to identify just two other instances where the assumption of the accuracy of producers' beliefs may merit further examination. One thing is clear from these results: the assumption that producers possess accurate understanding of the risks they face should not be accepted without further scrutiny of the potential types of miscalibrations of beliefs which might exist, and the potential effects on their assessments and responses to risk.

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