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Beef Supply Response Under Uncertainty: An Autoregressive Distributed Lag Model

Msafiri Mbaga and Barry T. Coyle

This is the first econometric study of dynamic beef supply response to incorporate risk aversion or, more specifically, price variance. Autoregressive distributed lag (ADL) models are estimated for cow-calf and feedlot operations using aggregate data for Alberta. In all cases, output price variance has a negative impact on output supply and investment. Moreover, the impacts of expected price on supply response are greater in magnitude and significance than in risk-neutral models.

Key words: autoregressive distributed lag model, beef supply response, dynamics, uncertainty

Introduction

It has long been recognized that dynamics play a particularly important role in beef production decisions. Cattle are simultaneously capital and consumption goods, so output supply response is closely connected to investment decisions (Yver; Jarvis; Rosen; Nerlove and Fornari). Given this close connection and a typical effective reproductive life of 8–10 years for beef cows, a dynamic model of output and investment decisions has a long horizon. Because uncertainty generally increases over a planning horizon and farmers are generally considered to be risk averse, price uncertainty and risk aversion play an important role in beef production decisions.

Empirical studies of beef production have focused on the modeling of dynamics and expected prices. These studies include models of adaptive expectations/partial supply response (Askari and Cummings 1976, 1977), polynomial distributed lags (Kulshreshtha), more general distributed lag and time-series models (Rucker, Burt, and LaFrance; Shonkwiler and Hinckley), and models explicitly derived from a dynamic optimization (Nerlove, Grether and Carvalho). Newer approaches are illustrated in recent econometric studies of beef supply response and the cattle cycle (Aadland and Bailey; Buhr and Kim; Chavas; Diebold, Ohanian and Berkowitz; Marsh 1994, 1999a,b; Mundlak and Huang; Nerlove and Fornari; Rosen, Murphy, and Scheinkman; Schmitz). These recent studies attest to the continued importance of improving models of beef supply response. However, it appears these studies have generally assumed risk neutrality by excluding the influence of uncertainty on decisions. One exception is Antonovitz and Green, who estimate static models of fed beef supply response incorporating price variance.

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Based on an extensive review of the literature, we believe this is the first econometric study of dynamic beef supply response that attempts to incorporate risk aversion or, more specifically, uncertainty as measured by output price variance. Here we specify an autoregressive distributed lag (ADL) model, which provides a general distributed lag structure without explicitly specifying a dynamic optimization. An ADL model is adopted because little is known about the specific forms of dynamic adjustment, and this approach can provide a relatively parsimonious approximation to a general dynamic process (Davidson and MacKinnon; Hendry, Pagan and Sargan). For example, Sarmiento and Allen argue that an error correction (or ADL) model is superior to the structural model adopted by Marsh in his 1994 study of U.S. beef slaughter supply response. Moreover dynamic optimization models with risk aversion are not yet developed. For example, risk aversion has been incorporated into static duality models of supply response (Coyle 1992, 1999), but not yet into dynamic duality models. The ADL methodology is applied here to the estimation of beef supply responses for cow-calf and feedlot operations using aggregate time-series data for Alberta.

Aside from beef, there are many econometric studies of agricultural supply response incorporating some measure of dynamics and uncertainty, but apparently the dynamic structure has been more restrictive than the ADL approach adopted here. These range from Nerlove partial adjustment models with price variance terms (see Brennan; Askari and Cummings 1976, 1977) to more recent GARCH lagged dependent variable models (Aradhyula and Holt; Holt and Aradhyula; Holt and Moschini; Holt). The most similar study may be that of Lin, who estimated a polynomial distributed lag model of wheat acreage response including a price risk variable similar to the current analysis.

Methodology

An ADL(m, n) dynamic model relating a dependent variable y to independent variable(s) q is specified as:

$$y_t = \alpha_0 + \sum_{i=1,\ldots,m} \alpha_i y_{t-i} + \sum_{i=0,\ldots,n} B_i q_{t-i} + e_t, \tag{1}$$

where $e_t \sim \text{IID}(0, \sigma^2)$. In our case, this depicts a dynamic supply response or investment demand equation conditional on price variables q. This model can be rewritten in different ways by linear transformation without changing the ability to explain data or interpret least squares estimates of coefficients. For example an ADL(1, 1) is equivalent to a standard error correction model (ECM), and (1) can be rewritten as a generalized ECM (Banerjee et al.). Thus, the choice between an ADL or ECM model is largely a matter of convenience in interpreting results. Here an ADL is adopted rather than an ECM approach because we are more interested in relating the model to more restrictive dynamic models, in particular polynomial distributed lag models, than in interpreting deviations from a hypothetical long-run equilibrium.

An important property of an ADL model with risk aversion is that it can be interpreted as a reduced form for a structural dynamic optimization model. This is similar to the case of ADL models under risk neutrality, and the argument can be sketched as follows. It is well known that under risk neutrality, dynamic optimization with quadratic costs of adjustment and a linear equation of motion (and static expectations) rationalizes the ECM or, equivalently, ADL(1, 1) model (Hendry and von Ungern-Sternberg; Salmon; Nickell).

Similarly, consider the following simple dynamic optimization problem with risk aversion:

$$\max_{\{I\}} \int_{t=0,\infty} U^*(Ep, \mathbf{w}, w^k, Vp, K_t, I_t) e^{-rt}\, dt \tag{2}$$

$$\text{s.t.: } \dot{K} = AK + BI, \quad K_0 = \bar{K},$$

where $U^*(\cdot)$ is the dual indirect utility function for a single-period maximization problem, e.g., a mean-variance problem,

$$\max_x Epf(\mathbf{x}, K, I) - \mathbf{wx} - w^k I - \alpha(\cdot)/2\, Vpf(\mathbf{x}, K, I)^2 = U^*(Ep, \mathbf{w}, w^k, Vp, K, I) \tag{3}$$

(Coyle 1999), or an expected utility-maximization problem. Ep and Vp are, respectively, the mean and variance for price p of output y; $\mathbf{w}$ is the price vector for variable inputs $\mathbf{x}$; w^k is the purchase (asset) price for capital K; I is gross investment; $y = f(\mathbf{x}, K, I)$ is a production function incorporating convex costs of adjustment; $\alpha(\cdot)$ is a nonlinear coefficient of risk-aversion function $\alpha(Epy - \mathbf{wx} - w^k I,\ Vpy^2)$; and r is an intertemporal discount rate—as in most dynamic models, the agent's utility function is assumed to be separable over time. The dynamic maximization hypothesis places second-order restrictions on the single-period dual $U^*(\cdot)$ with respect to K and I (Kamien and Schwartz); therefore, assuming $U^*(\cdot)$ is quadratic in (K, I) is consistent with this hypothesis. Then a quadratic $U^*(\cdot)$, linear equation of motion, and static expectations imply a linear decision rule for investment I (Anderson and Moore).[1] Furthermore, the closed-form solution of the Euler equation for the dynamic optimization (2) (and a standard terminal condition) imply an ECM or ADL(1, 1) model (1), where q includes price variance Vp as well as Ep.

Thus, an ADL model with risk aversion is consistent with a broad variety of dynamic optimization models. This provides a general parameterization of a reduced-form econometric model while avoiding inevitable misspecifications of a structural dynamic model. Data requirements for our reduced-form ADL model are described below.

Data

Supply-response models were constructed for cow-calf and feedlot operations using biannual and quarterly data, respectively, for Alberta over the period 1976–1997. Data on replacement heifers on-farm are unavailable prior to 1976, and quarterly data are unavailable for cow-calf output and input. Cow-calf output at weaning is defined as the number of light feeder calves (400–500 lbs.) on-farm January 1 and July 1 in Alberta (Statistics Canada 1976–97b). This series closely approximates calf production over the year, as calves grow from birth to a weight of 400–500 pounds in six months on average. Biannual inventory figures would therefore capture cow-calf output. A similar series has been used as a measure of cow-calf output in the United States (Buhr and Kim). The output price is in Can$/cwt for Alberta light feeders (400–500 lbs.) (Agriculture and Agri-Food Canada). Input prices are a feed price index and hired labor wage index for Western Canada (Statistics Canada 1976–97a), and price (Can$/cwt) for Alberta replacement heifers (700 lbs.) (Agriculture and Agri-Food Canada).

[1] Nonstatic expectations over the plan are considered by Nickell.

Investment in the cow-calf operation is measured as the number of replacement heifers of all weights on-farm January 1 and July 1 in Alberta (Statistics Canada 1976–97b). Investment decisions presumably depend on size of breeding herd, output price, price of replacement heifers, and farm input prices. These variables are measured as the number of cows on-farm January 1 and July 1 (Statistics Canada 1976–97b), price (Can$/cwt) for Alberta feeder steers (700 lbs.) and price for replacement heifers (Agriculture and Agri-Food Canada), and feed price index and hired labor wage (Statistics Canada 1976–97a), respectively. Feedlot output is defined as the number of fed cattle slaughtered in Alberta plus net exports for slaughter from Alberta to the United States, and the output price is measured as the price (Can$/cwt) for Alberta feeder steers (> 900 lbs.) (Agriculture and Agri-Food Canada). Input prices are the feed price index, hired labor wage, and the price for Alberta feeder steers (700 lbs.).

Empirical Models

ADL models are expressed in terms of normalized prices as follows:

$$(4) \qquad y_t = \alpha_0 + \sum_{i=1,\dots,m} \alpha_i y_{t-i} + \sum_{i=0,\dots,n} \left(\beta_{1i} \frac{Ep_{t-i}}{w^0_{t-i}} + \beta_{2i} \frac{Vp_{t-i}}{w^{0\,2}_{t-i}} + B_{3i} \frac{\mathbf{w}_{t-i}}{w^0_{t-i}} \right) + e_t .$$

ADL models of this type will be specified to explain output supply (light feeder calves on-farm) and investment (replacement heifers on-farm) for cow-calf enterprises and output (fed cattle slaughter) for feedlot enterprises. Expectations (Ep, Vp) for a dynamic optimization problem such as (2) presumably are formed in the initial period of the plan. Convex adjustment costs imply an incomplete adjustment to long-run equilibrium, so the current level of y depends upon the history of expectations.

Here, w^0 is designated as the numeraire input price (the feed price index). This is related to the normalization implied by constant relative risk aversion (CRRA): assuming CRRA and utility maximization under risk, decisions about y given parameter values (Ep, $\mathbf{w}$, w^0, Vp, W_0) are unchanged under new values (λEp, $\lambda\mathbf{w}$, λw^0, $\lambda^2 Vp$, λW_0) for all $\lambda > 0$, including $\lambda \equiv 1/w^0$, where W_0 is initial wealth (Pope; Coyle 1999). CRRA is a common assumption in the empirical literature on asset pricing and is considered the benchmark case by Arrow. Because an adequate proxy for initial wealth specific to beef producers is unavailable, lags on normalized initial wealth are not included in the ADL.

This study shares a serious problem with almost all empirical studies of decisions under uncertainty, assuming either risk neutrality or risk aversion. In risk-neutral models, expected prices are proxied by ad hoc methods or by predictions from time-series models. These proxies are plugged into supply-response models which are commonly estimated by methods assuming implicitly that measurement errors are small. This two-step practice is followed even though standard instrumental variables (IV) methods can often be used to correct for linear measurement errors, leading to consistent estimates of both coefficients and standard errors (Pagan; Murphy and Topel). Similarly, supply response models under risk aversion use proxies for risk ignoring measurement errors. This generally leads to an underestimate of effects of risk on decisions (Pagan and Ullah). There is no general procedure for obtaining consistent estimators in such models, so in this sense standard two-step procedures are more defensible in risk-averse models than in risk-neutral models.[2]

In this study, expected output prices are proxied as a one-period lag on market prices, and variances of output prices are proxied as the weighted sum of squares of prediction errors $(p_t - p_{t-1})^2$ of the previous three years, with declining weights of 0.50, 0.33, and 0.17. In other words, expected price and price variance are calculated from ex ante information as $E_{t-1}p_t = p_{t-1}$ and $\text{var}(p_t) = 0.50(p_{t-1} - E_{t-2}p_{t-1})^2 + 0.33(p_{t-2} - E_{t-3}p_{t-2})^2 + 0.17(p_{t-3} - E_{t-4}p_{t-3})^2$. This particular formula for price variance has been used in other studies (Chavas and Holt; Coyle 1992, 1999).[3] Alternatively, expected prices and price variances were also calculated from GARCH models expressing market prices as a distributed lag of prices, and supply-response models were estimated using distributed lags or current levels of these measures.

There does not appear to be a correct procedure for choosing between these alternative proxies for risk, because there is not an estimation procedure that generally leads to consistent estimates of coefficients and standard errors for these alternative supply-response models (Pagan; Pagan and Ullah). Thus, we follow an ad hoc approach. Supply-response models using GARCH proxies for risk show lower t-ratios (the one exception is expected price in replacement heifer investment), lower R^2s, and greater autocorrelation of residuals (except for investment) than do supply-response models using a weighted sum of squares of prediction errors. We interpret these comparisons as more supportive of the sum-of-squares approach than of the GARCH approach to modeling risk.

A similar conclusion was reached by other studies of Western Canadian agriculture under risk aversion (Coyle 1992, 1999) that rejected proxies from ARIMA and GARCH models. Similarly, a study of crop price expectations for a group of Sasketchewan farmers concluded these reported expectations are less adequately explained as time-series forecasts (Sulewski, Spriggs, and Schoney). In a study of heterogeneous expectations within a dynamic U.S. beef supply model under risk neutrality, Chavas found that the largest minority of agents (47%) showed naive expectations (equal to the most recently observed price) rather than rational or quasi-rational expectations.[4]

Output quantity data, price ratios (Ep/w^0, $Vp/(w^0)^2$, $\mathbf{w}/w^0$), replacement heifers, and herd size were tested for unit roots by standard methods (Dickey-Fuller and Phillips-Perron, with and without allowing for trend stationarity in the alternative). In all cases, the unit root hypothesis was rejected at the 0.05 level. Because these tests are biased

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[2] If risk can be correctly specified as a GARCH model, then joint estimation of the GARCH and structural model can lead to consistent estimates of coefficients and standard errors (Pagan and Ullah; Holt and Aradhyula). However, this result depends critically upon the assumption that GARCH does not introduce specification errors. Regarding the moving-average proxy to risk which is emphasized in our study, Pagan and Ullah (p. 97) note such approaches inevitably lead to inconsistent estimators if price variance Vp_t is calculated using current price p_t. We have avoided this common problem. Pagan suggests an IV approach in the unlikely event of no autocorrelation underlying the proxy, but $\text{cov}(Vp_t, Vp_{t-4}) > 0$ in our case.

[3] As in Chavas and Holt, alternative weightings of squared prediction errors were considered in defining price variances (0.8, 0.15, 0.05; 0.7, 0.2, 0.1; 0.5, 0.3, 0.2; 0.33, 0.33, 0.33). This led to negligible changes in results for all final regression models.

[4] Another possibility is that price expectations can be proxied by futures prices. However, an earlier study concluded that Chicago futures prices forecast U.S. live cattle prices more poorly than do lagged cash prices (Martin and Garcia), and problems should be compounded in forecasts of Alberta prices. We confirmed this for Alberta live cattle prices (e.g., in regressions of cash price on lagged cash and futures prices, elasticities approximated 0.75 and 0.01, respectively). In addition, we estimated GARCH models explaining Alberta prices in terms of Chicago futures prices to obtain alternative estimates for mean and variance of feeder steer prices, but the corresponding feedlot output supply models showed greater diagnostic problems and less significant coefficient estimates than do the reported models.

in favor of the unit root hypothesis in the sense that they have low power (Kwiatkowski et al.), it is not necessary to transform data due to unit roots. Nevertheless, we also tested the null hypothesis of no unit root as suggested by Kwiatkowski et al. In all cases, the computed statistic $\hat{\eta}_\tau$ did not exceed the critical value at the 10% level, suggesting again there are no unit roots.

Two of the input price variables specified for the ADL models were found to be insignificant and were dropped from the models. Hired labor wage and replacement heifer price were jointly (and separately) insignificant for cow-calf output and investment equations, whereas hired labor wage was insignificant for feedlot output response. These results are not surprising because labor cost is a relatively small proportion of total costs for both cow-calf and feedlot sectors (labor costs are also relatively fixed in the short run), and investment in breeding stock is primarily internal to the firm (relatively few replacement heifers are purchased by cow calf producers). Based on our results, the ADL models for cow-calf and feedlot operations are specified as follows:

$$\text{(5a)} \qquad y_{It} = \alpha_0 + \sum_{i=1,\dots,m_1} \alpha_i y_{It-i} + \sum_{i=0,\dots,n_1} \left(\beta_{1i} \frac{Ep_{It-i}}{w_{t-i}} + \beta_{2i} \frac{Vp_{It-i}}{w_{t-i}^2} \right) + e_{1t},$$

$$\text{(5b)} \qquad I_t = \delta_0 + \sum_{i=1,\dots,m_2} \delta_i I_{t-i} + \sum_{i=0,\dots,n_2} \left(\varphi_{1i} \frac{Ep_{t-i}^I}{w_{t-i}} + \varphi_{2i} \frac{Vp_{t-i}^I}{w_{t-i}^2} \right) + \gamma C_{t-1} + e_{2t},$$

$$\text{(5c)} \qquad y_{IIt} = \theta_0 + \sum_{i=1,\dots,m_3} \theta_i y_{IIt-i} + \sum_{i=0,\dots,n_3} \left(\psi_{1i} \frac{Ep_{IIt-i}}{w_{t-i}} + \psi_{2i} \frac{Vp_{IIt-i}}{w_{t-i}^2} + \psi_{3i} \frac{w_{t-i}^c}{w_{t-i}} \right) + e_{3t},$$

where (y_I, Ep_I, Vp_I) are output supply, expected output price, and variance of output price for cow-calf operations; (y_{II}, Ep_{II}, Vp_{II}) are output supply, expected output price, and variance of output price for feedlot operations; w is a feed price index; and w^c is a price for beef input into feedlots. I is cow-calf gross investment as measured by replacement heifers, C is stock of beef cows, and (Ep^I, Vp^I) are mean and variance of price for beef purchased by feedlots. All variables in (5a)–(5c) are specified in logarithms, so coefficients can be interpreted as elasticities.[5]

Results for Cow-Calf Output Supply Response

Dynamics and uncertainty are presumably important in modeling cow-calf supply response due to long biological lags in production. Replacement heifers are typically bred at 15 to 27 months of age and give birth in another nine months, so the lag in births for the cow-calf operation is 24 to 36 months (with larger numbers bred on either end of this interval in order to maintain short calving seasons). Similarly, there is a biological lag

[5] This study reports single equation estimates of (5a)–(5c), but systems methods were also considered. First, equations (5a) and (5b) in their final forms (as in table 1, part A, and table 2, part B) were estimated as seemingly unrelated regressions (SUR). The Breusch-Pagan test statistic for a diagonal covariance matrix was $\chi^2_{[1]} = 2.5682$ (p-value = 0.109), so the hypothesis of zero contemporaneous covariance is not rejected at the 0.05 level. Second, data for equation (5c) (as in table 3, part A) were aggregated from quarterly to biannual, and all three equations were estimated jointly as SUR. The Breusch-Pagan test statistic was $\chi^2_{[3]} = 5.9361$ (p-value = 0.115); so again the hypothesis of zero contemporaneous covariance is not rejected at the 0.05 level.

of 24 to 36 months between the breeding of a replacement heifer and the production of an offspring ready for breeding.

A polynomial distributed lag (PDL) model of supply response was initially estimated. In principle, distributed lags can reflect either the formation of expectations or lags in supply response (although, as noted above, we rejected use of predicted values of ARIMA and GARCH models in second-stage supply-response models). Assuming that price expectations are to some extent measured by our proxies for (Ep, Vp), distributed lags are assumed to reflect lags in supply response. Then changes in prices do not influence output until after a biological lag of 24 to 36 months, i.e., an average of five periods using biannual data.[6]

A recommended approach to selecting the lag length and degree of polynomial is to (a) estimate unrestricted distributed lag models with long lag lengths and use simple nested tests for reducing the lag length, and (b) (given the selected lag length) use nested tests to select the degree of the polynomial (e.g., Davidson and MacKinnon, pp. 673–76; Sargan). For simplicity, the PDLs for Ep and Vp are assumed to have polynomials of the same degree and identical lag lengths. It is well known that test statistics must be interpreted with caution after such model selection or pretesting procedures. Thus, we do not compound the problem by testing for differences in lag structures between Ep and Vp. As long as the difference between specified and true lag length is less than the degree of the polynomial, biases are not necessarily introduced into a PDL estimator (Trivedi and Pagan; Hendry, Pagan, and Sargan).

A PDL(10, 4)—i.e., a 10-period lag length and fourth-degree polynomial—was selected following this procedure. Results were generally as anticipated: the sum of lagged coefficients for both expected price Ep and price variance Vp were significant and with anticipated signs, and the elasticity was larger for Ep (1.01) than for Vp (-0.06) (these estimates were obtained by an iterative Cochrane-Orcutt procedure). Static studies of agricultural production have also estimated considerably smaller elasticities of response for Vp than for Ep (e.g., Coyle 1992, 1999). On the other hand, there was substantial serial correlation in residuals, and a standard test for the common factor restrictions implied by an AR(1) model rejected these restrictions (Davidson and MacKinnon, p. 365). Thus, the PDL model appears to be seriously misspecified, and so results are not reported here.

The serial correlation due to misspecification in the PDL model suggests an ADL model is more appropriate. A serious criticism of the PDL approach is that the dependent variable depends on lagged values of the included independent variables but not on lagged values of the omitted variables reflected in the error term. Rather than respecifying the PDL model with a disturbance following an ARMA process, it is often more appropriate to specify an ADL model (Davidson and MacKinnon, pp. 679–84). By incorporating lagged dependent variables, an ADL model accommodates effects of lagged omitted variables while often maintaining a simple error structure and, as an approximation, shortening lags on included independent variables.

An ADL(m, n) model is specified as follows:

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[6] The importance of biological lags was also checked by estimating various PDL and ADL models assuming that the distributed lags begin at zero rather than five periods. However, lags prior to five periods were almost always jointly insignificant.

$$(6) \qquad y_{It} = \alpha_0 + \sum_{i=1,\ldots,m} \alpha_i y_{It-i} + \sum_{i=5,\ldots,n+5} \left(\beta_{1i} \frac{Ep_{It-i}}{w_{t-i}} + \beta_{2i} \frac{Vp_{It-i}}{w_{t-i}^2} \right) + e_{1t}.$$

In order to select m and n, both were initially set at 5 and 10, respectively, and simple nested tests (F-tests and Schwarz criterion) were used to reduce the lag length. Models were estimated by OLS or by a grid search maximum-likelihood procedure if autocorrelation was detected. In this manner, an ADL(1, 5) model was selected, so that relative to the PDL model, the lag length on Ep and Vp is reduced from 10 to 5. In the selected ADL model, y_t depends on the lagged values $Ep_{t-5}, \ldots, Ep_{t-10}$ and $Vp_{t-5}, \ldots, Vp_{t-10}$ of Ep and Vp (earlier and later lags are insignificant). A time trend (allowing for technical change or trend stationarity) and a seasonal dummy were insignificant. An ADL(1, 5) model (6) may be inverted using a lag operator L:

$$(7) \qquad y_{It} = (1 + \alpha_1 + \alpha_1^2 + \ldots)\alpha_0 + (1 + \alpha_1 L + \alpha_1^2 L^2 + \ldots) \times \sum_{i=5,\ldots,10} \left(\beta_{1i} \frac{Ep_{It-i}}{w_{t-i}} + \beta_{2i} \frac{Vp_{It-i}}{w_{t-i}^2} \right) + (1 + \alpha_1 L + \alpha_1^2 L^2 + \ldots) e_{1t}$$

(Johnston and DiNardo, p. 244). In the inverted model, the lagged effect of (Ep, Vp) on y_I is represented by a combination of an infinite geometric lag and a finite distributed lag, and there is a geometric lag on e_1.

Part A of table 1 presents OLS estimates for the ADL(1, 5) model. As anticipated in a distributed lag model, a sum of lagged coefficients is more significant than most individual coefficients (Davidson and MacKinnon, pp. 673–74). The sum of lagged coefficients for Ep and Vp are significant and have the anticipated signs.[7,8] Since the model can be reparameterized so that a sum of lagged coefficients can be represented by a single coefficient (Davidson and MacKinnon, p. 674), the significance of a sum of lagged coefficients is evaluated as a simple t-test. The coefficient of the lagged dependent variable can be interpreted somewhat similarly to Nerlove partial response models; i.e., approximately 35% of the gap between current and steady-state output is closed in a single six-month period. The long-run impacts of Ep and Vp on output are similar to the sum of lag coefficients for the PDL model, and are calculated as 0.9275 = 0.3313/(1 – 0.6428) and –0.0479. Because the lagged dependent variable is significant, we conclude that this model does not reduce to a PDL model. Another study (Buhr and Kim) estimates elasticities of expected output price on U.S. calf crop output as 0.45 in the long run and 0.05 in the short run.[9]

[7] Results for the ADL model (table 1, part A) indicate that magnitudes and significance of coefficients for lagged Ep and Vp do not decline as the lag length increases (this pattern is not observed in table 1, part B, but apparently this model is misspecified due to the PDL restrictions). In contrast, estimates of ARIMA (and GARCH) models for Ep and Vp do show such a decline as lag length increases. These results suggest distributed lags in our models may primarily reflect lags in supply response rather than in expectations (results in table 3, part A also suggest this conclusion).

[8] Temporal risk implies that price uncertainty influences decisions under risk neutrality (e.g., Dixit and Pindyck). Therefore, significance of price variance does not necessarily imply rejection of risk neutrality. On the other hand, insignificance of price variance would imply rejection of risk aversion.

[9] Results for GARCH-based models of cow-calf output supply response can be illustrated as follows. Using estimates of Ep and Vp from an autoregressive GARCH(1, 1) output price equation, OLS estimates of an ADL(1, 5) similar to (6) and table 1, part A show: sum of lag coefficients for Ep and Vp are 0.2613 and –0.0387 with t-ratios 1.82 and 1.97, and Durbin-h statistic = –5.30. Thus, coefficient estimates are less significant than in table 1, part A, and the presence of autocorrelation suggests misspecification.

Table 1. Cow-Calf Output Supply Response

		[A] ADL(1,5): OLS		[B] ADL(1,5) + PDL(5,3): Auto(GS,ML)[a]	
Variable	Lag	Coefficient	*t*-Ratio[b]	Coefficient	*t*-Ratio[c]
y	1	0.6428	4.78	0.8065	10.27
Ep	5	0.1172	1.10	0.1042	2.05
	6	0.1884	1.83	0.0287	0.92
	7	-0.2172	2.05	0.0013	0.06
	8	0.3040	3.07	0.0040	0.29
	9	-0.2088	2.56	0.0190	0.80
	10	0.1476	2.62	0.0284	0.92
Vp	5	0.0016	0.26	-0.0017	0.49
	6	0.0186	2.78	-0.0072	2.39
	7	0.0089	1.16	-0.0035	1.56
	8	-0.0189	2.50	0.0025	1.10
	9	0.0262	3.77	0.0039	1.45
	10	-0.0163	2.88	-0.0059	1.79
Constant		5.03	2.64	2.71	2.44
R^2		0.9803		0.9755	
rho (GS,ML)		-0.23	1.37	-0.63	4.73
Durbin-*h* (OLS)		-1.59		-3.06	
Ramsey RESET $F(2,18)$		4.13		3.74	
Critical Value (0.01 level)		6.01		6.01	
Arch Heteroskedasticity $\chi^2_{[1]}$		3.92		0.89	
Critical Value (0.05 level)		3.84		3.84	
Sum of Lag Coefficients:					
Σ *Ep*		0.3313	2.52	0.1857	2.45
Σ *Vp*		-0.0171	3.16	-0.0120	3.96

[a] Auto(GS,ML) denotes Beach-MacKinnon grid search maximum-likelihood estimation under autocorrelation.
[b] *t*-Ratio with 20 degrees of freedom.
[c] *t*-Ratio with 23 degrees of freedom.

In contrast to the PDL model, there is no sign of autocorrelation. The Durbin-*h* statistic (asymptotically distributed as a standard normal under no autocorrelation) is insignificant, and a grid search maximum-likelihood procedure assuming an AR(1) yielded an insignificant value for the first-order autocorrelation coefficient, indicating the hypothesis of no autocorrelation is not rejected for AR(1).[10]

Standard diagnostic tests for various model misspecifications were considered. Using the Ramsey RESET test with various powers of predicted y_t, the hypothesis of a zero disturbance vector **e** was not rejected at the 0.01 level (see table 1, part A for the case of second and third powers). Sequential Chow tests for parameter constancy over all admissible sample partitions never rejected the null at the 0.01 level. Homoskedasticity was not rejected at approximately the 0.05 level using an ARCH test due to Engle. These results do not suggest any serious misspecification of the model.

[10] A portmanteau Lagrange multiplier test of white noise against MA(1) (Harvey, p. 278) did not suggest an MA process.

As the five-period lag length (2.5 years) is longer than in many other reported ADL models, it is interesting to consider the effects of incorporating PDL restrictions into the ADL model. Given an ADL(1,5), a third-degree polynomial was selected for the distributed lags in *Ep* and *Vp*. Results are reported in table 1, part B. OLS led to serial correlation in the residuals, so this model was estimated by the Beach-MacKinnon grid search maximum-likelihood procedure for an AR(1) model (Beach and MacKinnon) as programmed in SHAZAM 8.0 (White).[11] Results for the sum of lagged coefficients are broadly similar to those reported in table 1, part A, but there is considerable variation for individual coefficients. A test of common factor restrictions implied by AR(1) rejected the AR(1) model (Davidson and MacKinnon, p. 365).

These results suggest that adding PDL restrictions to the ADL model does not substantially reduce standard errors of estimates but does lead to significant model misspecification. Consequently, the unrestricted ADL model is preferred to the ADL model with PDL restrictions.

The above models were also estimated without the price variance (*Vp*) terms, i.e., assuming risk neutrality. In contrast to the above results, the expected price (*Ep*) terms were insignificant. Deleting *Vp* from the models in table 1, parts A and B, the sums of lag coefficients and *t*-ratios in parentheses for *Ep* were 0.1058 (0.91) and –0.0189 (0.30), respectively. Assuming the risk-averse model is the true model, this result suggests that, by deleting *Vp*, these particular risk-neutral models lead to poor (substantially biased) estimates of the impact of expected price *Ep* on cow-calf output supply response.[12]

As the coefficients of *Vp* are relatively small in magnitude and the simple correlation between *Ep* and *Vp* is small, such a bias in the risk-neutral dynamic model may seem surprising. Nevertheless the source of the bias can be verified as follows. The ADL(1,5) model (6),

$$y_{It} = \alpha_0 + \alpha_1 y_{It-1} + \sum_{i=5,\ldots,10} \left(\beta_{1i} Ep^*_{It-i} + \beta_{2i} Vp^*_{It-i} \right) + e_{1t},$$

$$\text{with} \quad Ep^*_{It-i} \equiv \frac{Ep_{It-i}}{w_{t-i}} \quad \text{and} \quad Vp^*_{It-i} \equiv \frac{Vp_{It-i}}{w^2_{t-i}},$$

can be reparameterized as:

$$\text{(8)} \qquad y_{It} = \alpha_0 + \alpha_1 y_{It-1} + \gamma Ep^*_{It-5} + \sum_{i=5,\ldots,10} \beta_{2i} Vp^*_{It-i} + \sum_{i=6,\ldots,10} \beta_{1i} z_{it} + e_{1t},$$

$$z_{it} \equiv Ep^*_{It-i} - Ep^*_{It-5}, \quad i = 6, \ldots, 10,$$

where γ satisfies the restriction $\gamma = \Sigma_{i=5,\ldots,10}\, \beta_{1i}$ (Davidson and MacKinnon, p. 674); i.e., γ is the sum of lag coefficients for Ep^*_I in the ADL(1,5) model (6) including *Vp*. Thus, the effects on the sum of lag coefficients for *Ep* from omitting *Vp* can be calculated by

[11] In the presence of lagged dependent variables, the error sum of squares criterion ESS(β, ρ) [after the model is transformed for AR(1) errors] generally has multiple solutions, and estimates of the parameter covariance matrix conditional on an estimate of ρ (as in most applications of the Cochrane-Orcutt and Hildreth-Liu) are inconsistent (Betancourt and Kelejian; Davidson and MacKinnon, pp. 334–40). This suggests a combined grid search, nonlinear least squares or maximum-likelihood approach with β and ρ estimated jointly rather than sequentially. Nevertheless, similar results were obtained by an iterative Cochrane-Orcutt procedure.

[12] Similar results were obtained in a comparison of static risk-averse and risk-neutral models of Manitoba crop agriculture (Coyle 1992). Expected prices were generally insignificant in a normalized quadratic risk-neutral model, but were significant in a risk-averse model with price variances.

applying the standard omitted variable analysis (e.g., Greene, p. 334) to (8), i.e., by analyzing the biases on γ due to omitting Vp from (8). The appropriate auxiliary regressions for calculating partial correlations between Ep^*_{It-5} and omitted Vp^* are:

(9) $$Vp^*_{It-j} = \rho_{j0} + \rho_{j1} y_{It-1} + \rho_{jm} Ep^*_{It-5} + \sum_{i=6,\dots,10} \rho_{ji} z_{it} + v_{jt}, \quad j = 5, \dots, 10.$$

The expected bias on γ due to omitting Vp from the ADL(1,5) (8) is:

(10) $$\sum_{i=5,\dots,10} \beta_{2i}\, \rho^*_{jm},$$

where ρ^*_{jm} are OLS estimates of (9). Note from (9) and the definition of z that ρ_{jm} is the impact of a one-unit change in all Ep^*_{It-5} $\{i = 5, \dots, 10\}$ on Vp^*_{It-j} conditional on y_{It-1} (the ρ^*_{jm} vary from 15.26 to 10.12 with *t*-ratios greater than 3.0). Using estimates of β_{2i} from table 1, part A, the expected bias is estimated as –0.2235, which is similar to the change in estimated sum of lag coefficients for Ep in going from the risk-averse to the risk-neutral model (–0.2255).

Results for Replacement Heifer Investment Response

Cow-calf output measured as calves on-farm is essentially linearly related to the number of beef cows, so cow-calf output response is an accumulated impact of beef cow investment decisions. Nevertheless, it is of interest to model directly replacement heifer investment response, since this is not easily unscrambled from output supply response.

The investment decision is modeled as depending on expectations for output and input prices for the cow-calf enterprise. In addition, the current investment decision obviously depends upon the accumulated stock of beef cows. In principle, the rate of investment also depends on the firm's marginal rate of time preference or discount rate, which may be proxied loosely by a market interest rate. But it is difficult to measure changes in expected real interest rates. The effects of variable interest rates have not been incorporated into any previous econometric studies of beef production decisions or into any dynamic duality models, and we do not include such a variable here.

Given the current number of beef cows or equivalently heifers of the appropriate age on-farm, the immediate effect of an investment decision is on the allocation between replacement heifers and fed heifers. Then the changes in replacement heifers eventually lead to a change in herd size, which has a longer-run feedback effect on investment decisions. This suggests that, if we specify a dynamic investment equation as conditional on herd size—i.e., if we control for herd size (and hence control for the longer-run indirect feedback effects of herd size on investment decisions)—then lags in response may be shorter than otherwise.

Alternatively, an investment equation can be specified independently of number of beef cows in period t. This can be interpreted as a reduced-form investment equation incorporating effects of longer-run induced changes in herd size on investment. In this case, longer lags are likely: coefficients for zero or immediate lags may reflect allocation decisions between replacement and fed heifers, whereas longer lags reflect interactions between investment and herd size.

A PDL model for investment conditional on beef cows can be specified as:

$$(11) \qquad I_t = \delta_0 + \sum_{i=0,\ldots,n} \left(\varphi_{1i} \frac{Ep_{t-i}}{w_{t-i}} + \varphi_{2i} \frac{Vp_{t-i}}{w_{t-i}^2} \right) + \gamma_1 C_{t-1} + \gamma_2 D_t + e_{2t},$$

where C is number of beef cows, D is a seasonal dummy variable ($D = 1$ for January–June, and 0 otherwise), and w is a feed price index. The output price p is the feeder input price to feedlots, which is proxied by the price (Can\$/cwt) for Alberta feeder steers (700 lbs.), and (Ep, Vp) are calculated as above. A hired labor wage and replacement heifer price were also considered, but these were insignificant. This is not surprising, because labor costs are a small proportion of cow-calf total costs, and relatively few replacement heifers are purchased. A time trend was also insignificant.

A PDL(8, 3) model (11) was selected. In contrast to the PDL cow-calf output supply response model, the lag process becomes insignificant after four years rather than seven years into the past. This difference in lag length is not surprising since (11) controls for the longer-run feedback effects of herd size on investment. OLS results are reported in table 2, part A (all variables are in logarithms). The sums of lag coefficients for both expected price Ep ($\Sigma_i\ \beta_{1i}$) and price variance Vp ($\Sigma_i\ \beta_{2i}$) are significant and with anticipated signs. The restrictions on the distributed lags implied by the PDL model are not rejected (an F-statistic of 0.7816, 10 and 15 df, probability = 0.646), and there is no serial correlation in the residuals. Nevertheless, a one-period lag in investment is significant when added to this model; i.e., this model is rejected for an ADL model (with PDL restrictions).

An ADL(m, n) model for investment conditional on beef cows is written as:

$$(12) \qquad I_t = \delta_0 + \sum_{i=1,\ldots,m} \delta_i I_{t-i} + \sum_{i=0,\ldots,n} \left(\varphi_{1i} \frac{Ep_{t-i}}{w_{t-i}} + \varphi_{2i} \frac{Vp_{t-i}}{w_{t-i}^2} \right) + \gamma_1 C_{t-1} + \gamma_2 D_t + e_{2t}.$$

OLS results for the selected ADL(1, 4) model are presented in table 2, part B. The sums of lag coefficients for Ep and Vp are significant with anticipated signs. Long-run elasticities for Ep and Vp (conditional on herd size) are 1.2589 and –0.0806, which are similar to long-run elasticities for the PDL model (0.9317, –0.0604). The Durbin-h statistic suggests there is no serial correlation. For comparison, OLS results for an ADL(1, 4) model with lags restricted to conform to a second-order poynomial are presented in table 2, part C. The polynomial restrictions are not rejected (an F-statistic of 0.6407, 4 and 26 df, probability = 0.638), and the Durbin-h statistic suggests no serial correlation. Results are similar to those reported in table 2, part B.

Diagnostic test results can be summarized as follows. Ramsey RESET tests did not reject hypotheses of zero disturbance vectors **e** at the 0.05 level. For the base model in table 2, part B, sequential Chow tests for parameter constancy only once rejected the hypothesis of constancy at the 0.01 level. Homoskedasticity was not rejected at the 0.10 level.[13]

[13] Results for GARCH-based models of replacement heifer investment response can be illustrated as follows. Using estimates of Ep and Vp from an autoregressive GARCH(1, 1) output price equation, OLS estimates of an ADL(1, 4) similar to (9) and table 2, part B show: sum of lag coefficients for Ep and Vp are 0.4354 and –0.0272 with t-ratios 4.62 and 1.29, and Durbin-h statistic = 2.06.

Table 2. Cow-Calf Investment (Replacement Heifers) Equation

		[A] PDL(8,3): OLS		[B] ADL(1,4): OLS		[C] ADL(1,4) + PDL(4,2): OLS	
Variable	Lag	Coefficient	*t*-Ratio [a]	Coefficient	*t*-Ratio [b]	Coefficient	*t*-Ratio [c]
y	1	—		0.6112	4.54	0.6318	5.37
Ep	0	0.4561	4.06	0.3809	2.18	0.4845	3.98
	1	0.2869	6.52	0.2783	1.27	0.0702	1.82
	2	0.1071	1.76	-0.2127	0.96	-0.1248	1.81
	3	0.0250	0.43	-0.1767	0.81	-0.1005	2.49
	4	-0.0109	0.28	0.2197	1.46	0.1431	1.32
	5	-0.0126	0.31	—		—	
	6	0.0084	0.18	—		—	
	7	0.0403	1:11	—		—	
	8	0.0714	0.75	—		—	
Vp	0	-0.0006	0.06	-0.0152	0.96	-0.0270	2.39
	1	-0.0076	1.73	-0.0180	0.96	-0.0015	0.30
	2	-0.0081	1.55	0.0105	0.51	0.0087	1.22
	3	-0.0049	0.99	0.0144	0.73	0.0037	0.83
	4	-0.0007	0.16	-0.0235	1.70	-0.0165	1.83
	5	0.0017	0.36	—		—	
	6	-0.0005	0.11	—		—	
	7	-0.0101	3.09	—		—	
	8	-0.0297	3.45	—		—	
C		1.853	12.87	0.9549	3.44	0.9291	3.93
Constant		-13.98	6.76	-8.583	3.18	-8.479	3.60
D		-0.4393	19.13	-0.5946	13.60	-0.5996	15.01
R^2		0.9633		0.9548		0.9503	
rho (GS, ML)		-0.16	0.97	-0.10	0.63	-0.24	1.56
Durbin-Watson		2.15		—		—	
Durbin-*h*		—		-0.167		-0.627	
Ramsey RESET $F(2,24)$		3.43		0.13		1.45	
Critical Value (0.01 level)		5.61		5.61		5.61	
Arch Heteroskedasticity $\chi^2_{[1]}$		5.07		1.12		0.53	
Critical Value (0.05 level)		3.84		3.84		3.84	
Sum of Lag Coefficients:							
$\Sigma\, Ep$		0.9317	4.14	0.4895	3.28	0.4724	3.41
$\Sigma\, Vp$		-0.0604	4.93	-0.0313	3.08	-0.0325	3.37

[a] *t*-Ratio with 25 degrees of freedom
[b] *t*-Ratio with 26 degrees of freedom.
[c] *t*-Ratio with 30 degrees of freedom.

In contrast, reduced-form investment models cannot be estimated directly with our data set. The PDL and ADL models excluding herd size are:

$$I_t = \delta_0 + \sum_{i=0,\ldots,n} \left(\varphi_{1i} \frac{Ep_{t-i}}{w_{t-i}} + \varphi_{2i} \frac{Vp_{t-i}}{w_{t-i}^2} \right) + \gamma_2 D_t + e_{2t}, \tag{13}$$

$$I_t = \delta_0 + \sum_{i=1,\ldots,m} \delta_i I_{t-i} + \sum_{i=0,\ldots,n} \left(\varphi_{1i} \frac{Ep_{t-i}}{w_{t-i}} + \varphi_{2i} \frac{Vp_{t-i}}{w_{t-i}^2} \right) + \gamma_2 D_t + e_{2t}.$$

Cow-calf output supply response results include lags to 14 and 10 periods for (Ep, Vp) in PDL and ADL models, respectively (see table 1 for ADL models). Consequently, the lag lengths n for (13) should exceed 14 and 10, respectively. However, the PDL model cannot be estimated with our data set, and there are insufficient degrees of freedom to obtain reasonable estimates of the ADL model.

The above models were also estimated without the price variance (Vp) terms, i.e., assuming risk neutrality. In contrast to the cow-calf supply response models, estimates of the expected price (Ep) terms in the risk-neutral models were similar to those reported in table 2. Deleting Vp from the models in table 2, parts A, B, and C, the sums of lag coefficients (and t-ratios) for Ep were 0.6741 (1.88), 0.4340 (2.68), and 0.4504 (2.85), respectively, suggesting risk-neutral models may provide adequate estimates of the impact of expected price Ep on cow-calf investment response (although these models are misspecified by deleting Vp).

Long-Run Equilibrium Impacts on Investment for Cow-Calf Model

Estimates of long-run equilibrium impacts of Ep and Vp on reduced-form investment (13) can be obtained from estimates of the calf output model and investment model conditional on beef cows. ADL model (12) of investment conditional on beef cows and results in table 2, part B imply the following relation between long-run equilibrium levels I^{**}, Ep^{**}, Vp^{**}, C^{**} in logarithms (abstracting from the seasonal dummy):

$$(14)\qquad I^{**}(1-\delta_1) = \sum_i \varphi_{1i} Ep^{**} + \sum_i \varphi_{2i} Vp^{**} + \gamma_1 C^{**} + \delta_0 \Rightarrow$$

$$I^{**} = 1.259 Ep^{**} - 0.081 Vp^{**} + 2.455 C^{**} - 35.85.$$

Similarly, ADL model (6) of cow-calf output (calves on-farm) and results in table 1, part A imply the following long-run equilibrium relation:

$$(15)\qquad y_I^{**}(1-\delta_1) = \sum_i \varphi_{1i} Ep_I^{**} + \sum_i \varphi_{2i} Vp_I^{**} + \delta_0 \Rightarrow$$

$$y_I^{**} = 0.928 Ep_I^{**} - 0.048 Vp_I^{**} + 13.97,$$

where y_I^{**} is long-run equilibrium level of calves on-farm. Then impacts can be calculated assuming a relation between prices p and p_I. For example, assuming these prices move together, the long-run equilibrium impacts of (Ep, Vp) on reduced-form investment can be calculated in elasticities as follows (assuming an elasticity of 1.0 for beef cows with respect to calves, i.e., weaning rate does not change with herd size):

$$(16)\qquad \frac{\partial I^{**}}{\partial Ep^{**}} = 1.259 + 2.455(1.0)0.928 = 3.357,$$

$$\frac{\partial I^{**}}{\partial Vp^{**}} = -0.081 + 2.455(1.0)(-0.048) = -0.199.$$

These are more than double the estimated long-run elasticities conditional on herd size (1.259 and −0.081, respectively). Feedback effects of changes in herd size on investment more than double the calculated long-run elasticities. Another study (Buhr and Kim) estimates elasticities of expected output price on U.S. beef cow inventory as 1.11 in the long run.

Results for Feedlot Output Supply Response

Output supply response models were also estimated in a similar manner for Alberta feedlots. After weaning (typically at seven months), calves may be backgrounded (fed on-farm to a higher weight) or sold to feedlots with a grain feed ration resulting in feeding periods between 6 and 10 months (two to three quarters) before slaughter. Alternatively, the producer can hold back calves and place them on pasture until sold as heavy yearlings to feedlots in the following year. This involves a feeding period of 14 to 20 weeks, depending on the ration.

A PDL model for feedlot supply response was estimated first. Using quarterly data, the lag in explanatory variables (Ep/w, Vp/w^2, w^c/w) was initially assumed to begin in two (alternatively zero) periods, but surprisingly, coefficients for lags of less than five periods were almost always jointly insignificant for various PDL and ADL models considered. Since lag lengths appeared to be quite long, the lag in explanatory variables was respecified as beginning in five periods. Then the following PDL(16,5) model was selected:

$$(17) \qquad y_{IIt} = \theta_0 + \sum_{i=5,\dots,20} \left(\psi_{1i} \frac{Ep_{IIt-i}}{w_{t-i}} + \psi_{2i} \frac{Vp_{IIt-i}}{w_{t-i}^2} + \psi_{3i} \frac{w_{t-i}^c}{w_{t-i}} \right) + \sum_{i=1,\dots,3} \vartheta_i D_{it} + e_{3t},$$

where D_1, D_2, and D_3 are quarterly dummies. There was significant serial correlation [a 1.29 Durbin-Watson statistic for the OLS model and a significant 0.47 estimate of rho for the maximum-likelihood AR(1) model], and a test of common factor restrictions rejected the AR(1) model. Therefore, the PDL model appears to be misspecified, and in turn, detailed results are not presented here. The Beach-MacKinnon procedure for estimating (17) led to the following estimates of the sums of lag coefficients for Ep, Vp, and w^c: 4.524, −0.4506, and −1.526, with t-ratios of 2.19, 2.83, and 1.26, respectively.

An ADL(m,n) model is specified as:

$$(18) \qquad y_{IIt} = \theta_0 + \sum_{i=1,\dots,m} \theta_i y_{IIt-i} + \sum_{i=5,\dots,n+5} \left(\psi_{1i} \frac{Ep_{IIt-i}}{w_{t-i}} + \psi_{2i} \frac{Vp_{IIt-i}}{w_{t-i}^2} + \psi_{3i} \frac{w_{t-i}^c}{w_{t-i}} \right) + \sum_{i=1,\dots,3} \vartheta_i D_{it} + e_{3t},$$

and an ADL(1,13) was selected. This implies lags of up to 4½ years, which is surprisingly similar to the cow-calf output supply response model. Most individual coefficients are insignificant (presumably due in part to multicolinearity), but the long length of the lag is illustrated by the relative significance of the coefficient for the last period lag in Ep, and the sum of lag coefficients for Ep and Vp are more significant than individual coefficients (table 3, part A). It is not clear if there is serial correlation in the model: the

Durbin-h statistic (2.145) implies zero autocorrelation is rejected at the 0.05 level, but the Beach-MacKinnon procedure estimated rho as 0.19 and insignificant (a t-ratio of 1.62). Nevertheless, results are similar for both OLS and a grid search maximum-likelihood procedure for AR(1).

OLS results are reported in table 3, part A. The sums of lag coefficients for Ep and Vp are again significant and with anticipated signs, whereas the sum of lags for the feeder input/feed price ratio is again less significant. As a comparison, AR(1) estimates of the sums of lag coefficients for Ep, Vp, w^c are 2.2969, –0.2216, and –0.9640, with t-ratios of 3.47, 4.07, and 2.58, respectively.[14] The estimated coefficient of the lagged dependent variable suggests that approximately 28% of the gap between current and steady-state output is closed in a single three-month period. In this sense, speed of adjustment may be somewhat faster for feedlots than for cow-calf operations, as anticipated. The estimated long-run impact elasticities for Ep and Vp on feedlot output are 7.45 and –0.71, respectively, in contrast to 4.52 and –0.45 in the PDL feedlot model. Many studies have reported elasticities of cattle slaughter with respect to output price. For example, the long-run elasticity is estimated as 3.24 (Marsh 1994, for the U.S.), 0.90 (Buhr and Kim, for the U.S.), and 1.30 (Kulshreshtha, for Western Canada).

Diagnostic test results for the model in table 3, part A can be summarized as follows. Ramsey RESET tests did not reject the hypothesis of zero disturbance vector **e** at the 0.01 level, and almost the 0.05 level. Sequential Chow tests for constancy did not reject the hypothesis of constancy at the 0.01 level, and homoskedasticity was not rejected at the 0.10 level.

Given the long lag length, the ADL(1,13) model was also estimated under PDL restrictions. However, in contrast to other PDL models, a high-order polynomial of degree 8 was accepted. This PDL(13,8) places relatively few restrictions on the 13-period distributed lag, but these restrictions led to greater serial correlation than in the ADL model [as indicated by a Durbin-h statistic of –3.79 for the OLS model and the maximum-likelihood estimate of rho for the AR(1) model]. Beach-MacKinnon estimates for an AR(1) model are reported in table 3, part B. A test of common factor restrictions rejected the AR(1) model, so the PDL restrictions apparently misspecify the ADL model.

The above models were also estimated without the price variance (Vp) terms, i.e., assuming risk neutrality. In contrast to the above results, the expected price (Ep) terms were insignificant. Deleting Vp from the models in table 3, parts A and B, the sums of lag coefficients (and t-ratios) for Ep were –0.820 (0.20) and –0.072 (0.25), respectively. This finding suggests that, at least in this data set, by deleting Vp, these risk-neutral models substantially underestimate the impact of expected price Ep on feedlot output supply response. This bias was verified by estimating expected bias similarly to the cow-calf supply equation as in (8)–(10).

[14] Results for GARCH-based models of feedlot output supply response can be illustrated as follows. Using estimates of Ep and Vp from an autoregressive GARCH(1, 1) output price equation, OLS estimates of an ADL(1, 13) similar to (15) and table 3, part A show: sum of lag coefficients for Ep and Vp are 1.3050 and –0.0381 with t-ratios 1.57 and 1.63, and Durbin-h statistic = –1.60. Thus, coefficient estimates are less significant than in table 3, part A, and autocorrelation suggests misspecification.

Table 3. Feedlot Output Supply Response

		[A] ADL(1,13): OLS		[B] ADL(1,13) + PDL(13,8): Auto(GS,ML)[a]	
Variable	Lag	Coefficient	t-Ratio[b]	Coefficient	t-Ratio[c]
y	1	0.7213	6.93	0.7685	11.88
Ep	5	-0.0263	0.11	-0.1187	0.82
	6	0.0753	0.29	0.2044	1.28
	7	0.1838	0.75	0.1849	1.44
	8	0.2143	0.84	0.0112	0.11
	9	-0.1710	0.68	-0.0419	0.37
	10	0.3305	1.22	0.0697	0.80
	11	0.0851	0.28	0.2127	2.10
	12	0.0401	0.14	0.2477	2.41
	13	0.6312	2.14	0.1571	1.50
	14	0.0714	0.24	0.0479	0.38
	15	-0.0611	0.21	0.0369	0.30
	16	0.1834	0.59	0.1088	0.69
	17	0.0471	0.17	0.1251	0.76
	18	0.4731	2.05	0.2437	1.74
Vp	5	-0.0358	1.38	-0.0053	0.32
	6	-0.0004	0.01	-0.0336	2.46
	7	-0.0225	0.80	-0.0076	0.96
	8	-0.0154	0.66	0.0042	0.62
	9	0.0109	0.52	-0.0045	0.77
	10	-0.0228	1.13	-0.0165	3.01
	11	-0.0327	1.51	-0.0184	2.86
	12	-0.0012	0.06	-0.0116	1.77
	13	-0.0243	1.17	-0.0079	1.28
	14	-0.0074	0.34	-0.0156	2.17
	15	-0.0625	2.58	-0.0254	2.77
	16	0.0304	1.27	-0.0107	1.17
	17	0.0022	0.09	0.0305	2.37
	18	-0.0148	0.83	-0.0213	1.97
w^c	5	-0.5107	2.28	-0.3235	2.48
	6	0.7486	2.49	0.3775	2.01
	7	-0.4005	1.37	-0.1321	1.22
	8	0.0463	0.15	-0.0343	0.34
	9	0.0038	0.01	0.1581	1.59
	10	0.1499	0.49	0.0481	0.50
	11	-0.1195	0.42	-0.1954	2.08
	12	-0.3236	1.14	-0.2338	2.68
	13	-0.1055	0.38	-0.0096	0.13
	14	0.1870	0.66	0.1752	1.86
	15	-0.1090	0.38	0.0156	0.16
	16	0.0193	0.07	-0.3031	2.44
	17	-0.5008	2.12	-0.1405	1.02
	18	0.0342	0.17	0.0125	0.09

(continued . . .)

Table 3. Continued

	[A] ADL(1,13): OLS		[B] ADL(1,13) + PDL(13,8): Auto(GS,ML)[a]	
Variable	Coefficient	t-Ratio[b]	Coefficient	t-Ratio[c]
Constant	2.721	2.22	2.349	3.13
D1	0.1516	3.35	0.0968	2.82
D2	0.2177	4.76	0.1954	7.26
D3	0.1206	2.92	0.0587	1.95
R^2	0.9671		0.9359	
rho (GS,ML)	0.19	1.62	-0.45	4.21
Durbin-h (OLS)	2.145		-3.796	
Ramsey RESET $F(2,21)$	3.51		13.23	
Critical Value (0.01 level)	5.78		5.78	
Arch Heteroskedasticity $\chi^2_{[1]}$	1.57		3.48	
Critical Value (0.05 level)	3.84		3.84	
Sum of Lag Coefficients:				
$\Sigma\, Ep$	2.0768	1.97	1.4894	2.26
$\Sigma\, Vp$	-0.1962	2.18	-0.1432	2.52
$\Sigma\, w^c$	-0.8805	1.48	-0.5853	1.61

[a] Auto(GS, ML) denotes Beach-MacKinnon grid search maximum-likelihood estimation under autocorrelation.
[b] t-Ratio with 23 degrees of freedom.
[c] t-Ratio with 37 degrees of freedom.

Conclusion

This study has estimated dynamic models of beef supply response for cow-calf and feedlot operations in Alberta, allowing for price uncertainty and risk aversion. We believe this is the first study of dynamic beef supply response to incorporate price uncertainty or, more specifically, output price variance. As in several other studies of Western Canadian agriculture, expected output price and price variance are more effectively modeled as simple lags and weighted sums of squared prediction errors rather than as rational expectations or GARCH models. ADL and PDL models are estimated assuming distributed lags for variance of output price as well as for expected output price.

ADL models are estimated for cow-calf output (calves) and investment (replacement heifers) and for feedlot slaughter output. In all three cases, the sum of lagged coefficients for output price variance is negative and significant, as anticipated. The elasticity is much smaller than for the (positive) sum of lagged coefficients for expected price, as expected. The distributed lags for PDL models extend back 5–7 years. Because the selected ADL models show shorter but still substantial lags in explanatory variables, PDL restrictions are also considered for distributed lags in the ADL models. However, these PDL restrictions introduce substantial serial correlation in residuals, suggesting these restrictions misspecify the distributed lags in the ADL models.

In both cow-calf and feedlot output supply equations, the impacts of expected price on supply response are substantially greater in magnitude and significance than in risk-neutral models. This result suggests that, even if we are only concerned about measuring the impacts of expected prices on supply, it can be important to incorporate price uncertainty and risk aversion into econometric models. Moreover, it is important for policy to measure impacts of price uncertainty as well as impacts of expected prices on supply (e.g., Hennessy). Consequently, it is important for policy to incorporate price uncertainty and risk aversion into econometric models in order to measure the impacts of both expected price and price variance on beef supply response.

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