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Persistence in Commodity Prices

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This paper deals with the analysis of world commodity prices by examining 15 categories of commodity prices using fractional integration and including thus fractional points. We use data corresponding to the 1960–2018 period obtained from the World Bank, and the results indicate high degrees of persistence in the majority of the series, especially when using parametric methods. However, mean reversion is obtained in many cases when using semiparametric approaches. The possibility of structural breaks is also considered, and our results confirm the high degree of persistence in the data, which seems to have increased across time.

Key words: fractional integration, long memory, unit roots

Introduction

Using fractional integration, we measure persistence in commodity prices. Our data come from the World Bank's the Pink Sheet, which includes monthly prices since 1960 for commodities in three groups: energy, nonenergy (which includes agriculture products, fertilizers, metal, and minerals), and precious metals. The main goal of this paper is to determine whether the degree of persistence in commodity prices varies by commodity. We also test whether an exogenous shock in the series has a transitory or a permanent effect on the future behavior of the series.

Fractional integration is an appropriate technique to accomplish this goal because it is more flexible than the standard methods, which only consider integer degrees of differentiation (i.e., stationarity, $I(0)$, or nonstationarity, $I(0)$). Under the $I(0)$ specification, random shocks in the series will have transitory effects, while in the unit roots or $I(0)$ models, shocks will be more permanent in nature. In the context of fractional models, where the order of integration is in the range (0, 1), shocks will also have transitory—though long-lasting—effects if the order of integration is high and close to 1. This is important from a policy perspective. For example, when an exogenous shock affects a commodity price, if its order of integration is smaller than 1, the shock will have a transitory effect, not requiring strong measures from the authorities to recover its original long-term projection. On the other hand, if the order of integration is ≥ 1, then the shock will be more permanent and, if the shock is negative, strong actions would be necessary to recover the original level or trend. Our results indicate that most of the series examined display high degrees of persistence, with orders of integration that in some cases are equal to or higher than 1. Nevertheless, the results are heterogeneous across commodities, and mean reversion is found in many of the series, especially when semiparametric methods are used.

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Literature Review

The recent literature on commodity prices is extensive. Ng and Ruge-Murcia (2000) generalized the basic storage model to reproduce the degree of serial correlation observed in actual data. They incorporated agents that are willing to hold stocks more often than predicted by the basic model, including characteristics of the producing and trading mechanism to provide for incentives. The results, based on 12 commodities, show high levels of persistence in the series. Cashin, Liang, and McDermott (2000) studied the persistence of shocks to world commodity prices using monthly IMF data on primary commodities between 1957 and 1998. They focused on the duration of shocks to individual commodities and found that, on average, shocks are long-lasting with wide variability in persistence. Cashin and McDermott (2002) analyzed the behavior of real commodity prices over 1862–1999 using *The Economist*'s index of industrial commodity prices to examine long-run price trends and commodity-price cycles. Kellard and Wohar (2003) applied unit root methods to study the Prebisch–Singer hypothesis of a secular deterioration in relative primary commodity prices and the nature of their persistence for the 1900–1998 period. They found much less persistence in the series than previously reported, since 23 out of the 24 commodities examined could be classified as trend-stationary, $I(0)$, rather than as stochastic trends or $I(0)$ models.

Wang and Tomek (2007) argued that, according to price theory, commodity prices should be stationary, though most standard unit roots methods show that the series are nonstationary. They applied alternative unit root tests based on structural changes to some commodities and found no evidence in favor of unit roots, with results being very sensitive to the methodology used.

From a more historical perspective, Jacks, O'Rourke, and Williamson (2011) examined commodity price volatility since 1700, using greater volatility in commodity prices to try to explain the backwardness of poorer countries. Narayan and Liu (2011) employed Narayan and Popp's (2010) unit root tests, testing for persistence in ten metal commodity prices, and concluded that only shocks to gold, silver, platinum, aluminum, and copper are actually persistent. Labys (2017) investigated models and forecasting for primary commodity prices, extending the traditional linear modeling approach to nonlinear modeling. He proposed tests based on a battery of methods such as neural networks, correlation dimensions, Lyapunov exponents, fractional integration, and rescale range. His results indicate that price structure might be more complex than that based on simple AR(I)MA models.

Wets and Rios (2015) examined copper commodity prices and presented a new methodology for modeling commodity prices that allowed them to distinguish between short- and long-term analyses. This method also permits inflation to be considered in a nonlinear model. Oglend and Asche (2016) applied the Hylleberg et al. (1990) seasonal unit root test for cyclical nonstationarity in commodity prices. Their results show that longer cycles in many commodity prices are highly stochastic, so it is necessary to be careful when interpreting cycles using historical price movement.

Practically all of the papers cited above test for persistence by using unit root methods that simply consider integer degrees of differentiation. That is, 0 in case of stationarity, producing shocks with transitory effects, and 1 for nonstationarity and generating shocks with permanent effects. We depart from this strong restriction by allowing for fractional degrees of differentiation, thus permitting a higher degree of flexibility in the dynamic specification of the series and in the treatment of the shocks.

Our work extends previous literature. Jin and Frechette (2004) worked with agricultural futures prices in order to settle whether fractional orders of integration are suitable for modeling conditional volatility. Baillie et al. (2007) went a bit further and investigated the stochastic properties of various commodity prices; their results indicate that commodity futures returns series display self-similarity. Using 24 real commodity prices, Ghoshray (2013) showed that the persistence of shocks can change over time; this result helped policy makers introduce measures with more information. Finally, Chen et al. (2014) examined which factors determine commodity price persistence using Bai and Ng's (2004) PANIC method and 51 tradable commodities.

Methodology

We use techniques based on fractional integration (i.e., the series might require a fractional number of differences to render it stationary, $I(0)$). In other words, we say that a process $\{x_t, = 0, \pm 1, \pm 2, \ldots\}$ is integrated of order d and denoted as $x_t \approx I(d)$ if it can be represented as

$$(1) \qquad (1-B)^d x_t = u_t,\ t = 0,\ \pm 1,\ \ldots,$$

where B is the backshift operator ($Bx_t = x_{t-1}$), and u_t is supposed to be $I(0)$, defined for our purposes as a covariance stationary process in which the infinite sum of the autocovariances is finite. Thus, within this class, we can include the stationary autoregressive moving average (ARMA)-type of processes. In such a case, x_t is said to be a fractionally integrated ARMA (or ARFIMA) process.

The $I(d)$ processes were proposed by Granger (1980), Granger and Joyeux (1980), and Hosking (1981) based on the observation that many economic aggregates display estimates of the spectral density function that exploited at the 0 frequency (i.e., $f(0) \to \infty$; consistent thus with the $I(0)$ approach) but that, once the series was differenced, its spectrum was very close to 0 at the 0 frequency (i.e, $f(0) = 0$), which was a clear indication of overdifferentiation. Fractional models have since become popular in the analysis of economic and financial time series (see, e.g., Baillie, 1996; Gil-Alana and Robinson, 1997; Mayoral, 2006; Gil-Alana and Moreno, 2012; Haug, 2014; Abbritti et al., 2016).

In the context of $I(d)$ models, three values of d play important roles. First, if $d = 0$, the process is said to have short memory; $d > 0$ has long memory, so named because of the strong degree of association between observations which are far apart in time. Another relevant value is 0.5: If $d < 0.5$, the process is still covariance (or second-order) stationary, unlike the case with $d \geq 0.5$, which is nonstationary, and becomes more nonstationary as we depart from 0.5 (it is nonstationary in the sense that the variance of the partial sums increases in magnitude with d). A final relevant value is 1: If $d < 1$, the series is mean reverting, with shocks disappearing in the long run; if $d \geq 1$, shocks persist indefinitely. In this context, if $d < 1$, the lower the value of d is, the faster the recovery after a shock is, allowing this type of process to treat shocks flexibly.

We estimate d using both parametric and semiparametric methods; in both cases, we use the Whittle function expressed in the frequency domain (Dahlhaus, 1989). In the parametric cases, we suppose that the d-differenced process is first white noise, then we also allow for weak autocorrelation. In the latter case, we use a nonparametric approach following Bloomfield (1973), which approximates highly parameterized ARMA models with very few parameters. In the semiparametric approach, we do not impose any structure on the $I(0)$ d-differenced process.

Data

We use commodity price data from the World Bank's Pink Sheet, which are reliable, updated regularly, and include prices of major agricultural commodities. The dataset includes monthly prices (reported in nominal U.S. dollars), indices (based on nominal U.S. dollars, 2010=100), and monthly, quarterly, and annual averages. We use monthly data because—although they could be more sensible to punctual shocks—quarterly data could hide persistence. The dataset includes data from seven commodity groups from 1960 through April 2019, divided into three groups: energy, nonenergy, and precious metals. Within energy, the World Bank includes prices for coal, crude oil, and natural gas. Nonenergy includes agriculture, raw materials, fertilizers, and metals and minerals. The final category is precious metals. For each commodity, the World Bank uses observations from strategic markets worldwide. Table 1 reports summary statistics for the commodity groups using a Laspeyres

Table 1. Descriptive Statistics from the World Bank Laspeyres Index

Series	Maximum	Minimum	Mean	Median	Std. Dev.
Energy	173.43	1.81	38.70	26.36	37.16
Nonenergy	128.11	17.86	52.53	48.34	25.94
Agriculture	129.92	20.77	57.33	55.14	25.48
Beverages	156.82	20.64	60.44	59.69	27.97
Food	132.36	19.79	58.40	55.50	26.83
Oils and meat	140.96	20.51	56.96	53.85	27.11
Grains	156.64	22.38	62.17	58.58	28.93
Other food	115.06	15.33	56.87	56.70	27.24
Raw materials	134.56	18.67	53.17	51.57	25.89
Timber	125.14	15.27	57.43	55.34	31.28
Other raw materials	159.29	18.77	48.52	46.53	22.13
Fertilizers	256.06	8.33	47.62	35.45	38.34
Metals and minerals	126.26	12.15	43.25	35.27	27.72
Base metals	127.17	13.22	46.75	38.65	28.29
Precious metals	153.29	3.27	37.41	27.99	35.10

Index from the World Bank.[1] The highest value of the indices corresponds to fertilizers, followed by energy and other raw materials, though the highest means are obtained in grains and beverages. Energy and fertilizers are the most volatile commodities. Using indices instead of actual prices allowed us to have a better understanding of the behavior of the whole market.

Figure A1 displays the time series plots; we observe that most prices have increased slightly over time. Figure A2 displays the periodograms of the series, and we observe here that the highest values occur in all cases at points that are close to the smallest (0) frequency, which is consistent with $I(d)$ processes where $d > 0$. Figure A3 presents the periodograms of the first-differenced data, and for some of the series (e.g., food, oils and meat, grains, other raw material, fertilizers, metals and minerals, and base metals) we observe values close to 0 at the 0 frequency, an indication of overdifferentiation. This motivates us to use fractionally integrated methods.

Empirical Results

We consider the following model:

$$y_t = \alpha + \beta t + x_t,\ (1 - B)^d x_t = u,\ t = 1, 2, \ldots, \tag{2}$$

where y_t is the time series we observe; α and β are unknown coefficients referring to the intercept and a liner time trend, respectively; and x_t is supposed to be $I(d)$, so that u_t is $I(0)$. We test the null hypothesis,

$$H_o : d = d_o, \tag{3}$$

in equation (2) for d_o values equal to -2, -1.99, $\ldots -0.01$, 0, 0.01, $\ldots$, 1.99, and 2 under different modeling assumptions for the $I(0)$ error term, u_t. For this purpose, we use a version of Robinson's (1994) Lagrange multiplier (LM) tests. This approach is convenient for several reasons: First, because it allows us to test any real value, including stationary ($d < 0.5$) but also nonstationary ($d \geq 0.5$) values; moreover, its limit distribution is standard $N(0,1)$, and this behavior holds independently of the inclusion of deterministic terms and/or the way of modeling the $I(0)$

[1] World Bank weights for energy are coal (4.6), crude oil (86.6), and natural gas (10.8). Weights for nonenergy are metals (31.6), agriculture (64.9), and fertilizers (3.6). Within nonenergy, agriculture is weighted as (i) beverages (8.4), (ii) food (40.0), and (iii) raw materials (16.5); food is weighted as (i) cereals (11.30), (ii) fats and oils (16.3), and (iii) other foods (12.4). Weights for precious metals are gold (77.8), silver (18.9), and platinum (3.3).

Table 2. Estimates of *d* under the Assumption of White Noise for u_t

Series	With No Terms 1	With an Intercept 2	With an Intercept and a Linear Trend 3
Energy	1.10 (1.05, 1.17)	**1.15 (1.09, 1.22)**	1.15 (1.09, 1.22)
Nonenergy	1.00 (0.96, 1.06)	**1.36 (1.29, 1.44)**	1.36 (1.29, 1.44)
Agriculture	1.00 (0.95, 1.06)	**1.36 (1.28, 1.44)**	1.36 (1.28, 1.44)
Beverages	1.01 (0.96, 1.06)	**1.22 (1.16, 1.29)**	1.22 (1.16, 1.29)
Food	1.01 (0.96, 1.07)	**1.32 (1.24, 1.41)**	1.32 (1.24, 1.41)
Oils and meat	1.01 (0.96, 1.08)	**1.29 (1.20, 1.39)**	1.29 (1.20, 1.39)
Grains	1.03 (0.97, 1.09)	**1.29 (1.22, 1.37)**	1.29 (1.22, 1.37)
Other food	1.01 (0.96, 1.07)	**1.12 (1.06, 1.20)**	1.12 (1.06, 1.20)
Raw materials	1.00 (0.95, 1.06)	**1.30 (1.24, 1.38)**	1.30 (1.24, 1.38)
Timber	1.01 (0.95, 1.07)	**1.24 (1.16, 1.32)**	1.24 (1.16, 1.32)
Other raw materials	1.01 (0.96, 1.07)	**1.37 (1.31, 1.45)**	1.37 (1.31, 1.45)
Fertilizers	1.02 (0.97, 1.08)	**1.06 (1.01, 1.12)**	1.06 (1.01, 1.12)
Metals and minerals	1.02 (0.97, 1.09)	**1.23 (1.17, 1.31)**	1.23 (1.17, 1.31)
Base metals	1.02 (0.97, 1.08)	**1.22 (1.15, 1.29)**	1.22 (1.15, 1.29)
Precious metals	1.09 (1.03, 1.15)	**1.17 (1.11, 1.24)**	1.17 (1.11, 1.24)

Notes: The estimated model is $y_t = \alpha + \beta t + x_t$, $(1-B)^d x_t = u_t$. We report the estimates of d with no deterministic terms (column 1), with an intercept (column 2), and with a constant and a linear trend (column 3). Values in bold are the selected cases according to these deterministic terms. Values in parentheses are 95% confidence bands for the values of d.

errors. Finally, it is the most efficient test in the Pitman sense against local departures from the null. Dealing with the assumptions for the error term, we first suppose that u_t is a white noise process; then, we also consider weak autocorrelation using Bloomfield's (1973 nonparametric approach, which approximates ARMA processes with very few parameters. This latter method is very appropriate in the context of Robinson's (1994) tests (see Gil-Alana, 2004). Finally, noting the monthly nature of the data examined, we also consider a seasonal (monthly) AR(1) process of the form:

$$u_t = \rho u_{t-12} + \varepsilon_t,\ t = 1, 2, \ldots, \tag{4}$$

where ε_t is a white noise process.

Table 2 displays the results for the case of white noise errors. In Table 3 we assume that the errors are autocorrelated throughout using Bloomfield's (1973) model, while in Table 4 we use the seasonal AR(1) process as described in equation (4). In all three cases we consider the three alternatives of (i) no deterministic terms in the model (i.e., assuming *a priori* that $\alpha = \beta = 0$ in equation 2); (ii) including a constant ($\beta = 0$ in equation 2), and (iii) with a constant and a linear trend (and thus the coefficients α and β are unknown and estimated from the data). Values marked in bold in the tables indicate the selected cases in relation with these deterministic terms. This is done noting that, under the null hypothesis in equation (3), the model in equation (2) can be rewritten as

$$(1-\beta)^{d_o} y_t = \alpha(1-B)^{d_o} 1 + \beta(1-B)^{d_o} t + u_t,\ t = 1, 2, \ldots, \tag{5}$$

where, based on the $I(0)$ nature of u_t, standard t-tests can be applied on α and β.

We start the analysis with the results based on white noise errors (Table 2). We fail to reject the null hypothesis of a 0 time-trend coefficient in all cases. Thus, we focus on the values of d based on the model with an intercept only. Amazingly, the values are significantly higher than 1 in all cases, with the values ranging from 1.06 (fertilizers) to 1.37 (other raw materials). Thus, according to this model, shocks will have permanent effects, and the differenced data still present a degree of association across time. Nevertheless, these results might be biased due to the absence of autocorrelation in the error term.

Table 3. Estimates of d under the Assumption of Autocorrelation (Bloomfield) for u_t

Series	With No Terms 1	With an Intercept 2	With an Intercept and a Linear Trend 3
Energy	1.01 (0.92, 1.09)	**1.02 (0.94, 1.13)**	1.02 (0.94, 1.13)
Nonenergy	0.98 (0.90, 1.08)	**1.12 (1.03, 1.23)**	1.12 (1.03, 1.23)
Agriculture	0.97 (0.90, 1.07)	**1.05 (0.99, 1.17)**	1.05 (0.99, 1.17)
Beverages	0.98 (0.91, 1.09)	**1.09 (1.01, 1.21)**	1.09 (1.01, 1.21)
Food	0.97 (0.90, 1.07)	**1.01 (0.92, 1.10)**	1.01 (0.92, 1.10)
Oils and meat	0.94 (0.87, 1.04)	0.87 (0.78, 0.97)	**0.87 (0.78, 0.97)**
Grains	1.00 (0.92, 1.10)	**1.07 (0.98, 1.20)**	1.07 (0.98, 1.20)
Other food	0.97 (0.90, 1.08)	0.96 (0.87, 1.06)	**0.96 (0.87, 1.06)**
Raw materials	0.98 (0.90, 1.07)	**1.15 (1.05, 1.26)**	1.15 (1.05, 1.26)
Timber	0.98 (0.90, 1.06)	1.00 (0.91, 1.10)	**1.00 (0.91, 1.10)**
Other raw materials	0.97 (0.89, 1.08)	**1.12 (1.01, 1.23)**	1.12 (1.01, 1.23)
Fertilizers	1.01 (0.93, 1.12)	**1.01 (0.93, 1.12)**	1.01 (0.93, 1.12)
Metals and minerals	0.97 (0.90, 1.08)	**1.03 (0.94, 1.16)**	1.03 (0.94, 1.16)
Base metals	0.97 (0.89, 1.08)	**1.05 (0.95, 1.15)**	1.05 (0.95, 1.15)
Precious metals	0.96 (0.90, 1.05)	0.96 (0.90, 1.03)	**0.96 (0.90, 1.03)**

Notes: The estimated model is $y_t = \alpha + \beta t + x_t,\ (1 - B)^d x_t = u_t$. We report the estimates of d with no deterministic terms (column 1), with an intercept (column 2), and with a constant and a linear trend (column 3). Values in bold are the selected cases according to these deterministic terms. Values in parentheses are 95% confidence bands for the values of d.

Table 4. Estimates of d under the Assumption of a Seasonal (monthly) AR(1) for u_t

Series	With No Terms 1	With an Intercept 2	With an Intercept and a Linear Trend 3
Energy	1.10 (1.05, 1.17)	**1.15 (1.08, 1.22)**	1.15 (1.08, 1.22)
Nonenergy	1.00 (0.95, 1.06)	**1.36 (1.29, 1.44)**	1.36 (1.29, 1.44)
Agriculture	1.00 (0.95, 1.06)	**1.36 (1.28, 1.45)**	1.36 (1.28, 1.45)
Beverages	1.01 (0.96, 1.07)	**1.22 (1.16, 1.29)**	1.22 (1.16, 1.29)
Food	1.01 (0.96, 1.07)	**1.32 (1.24, 1.41)**	1.32 (1.24, 1.41)
Oils and meat	1.01 (0.96, 1.07)	**1.29 (1.20, 1.39)**	1.29 (1.20, 1.39)
Grains	1.02 (0.97, 1.08)	**1.29 (1.22, 1.37)**	1.29 (1.22, 1.37)
Other food	1.01 (0.96, 1.07)	**1.13 (1.06, 1.20)**	1.13 (1.06, 1.20)
Raw materials	1.00 (0.95, 1.06)	**1.31 (1.24, 1.38)**	1.31 (1.24, 1.38)
Timber	1.01 (0.95, 1.07)	**1.24 (1.17, 1.33)**	1.24 (1.17, 1.33)
Other raw materials	1.01 (0.96, 1.07)	**1.37 (1.30, 1.46)**	1.37 (1.30, 1.46)
Fertilizers	1.02 (0.97, 1.08)	**1.06 (1.00, 1.12)**	1.06 (1.00, 1.12)
Metals and minerals	1.02 (0.96, 1.09)	**1.23 (1.17, 1.31)**	1.23 (1.17, 1.31)
Base metals	1.02 (0.97, 1.08)	**1.22 (1.16, 1.29)**	1.22 (1.16, 1.29)
Precious metals	1.09 (1.03, 1.15)	**1.17 (1.10, 1.24)**	1.17 (1.10, 1.24)

Notes: The estimated model is $y_t = \alpha + \beta t + x_t,\ (1 - B)^d x_t = u_t$. We report the estimates of d with no deterministic terms (column 1), with an intercept (column 2), and with a constant and a linear trend (column 3). Values in bold are the selected cases according to these deterministic terms. Values in parentheses are 95% confidence bands for the values of d.

Table 5. Summary of the Estimates of *d* across Tables 2–4

	White Noise	Bloomfield	Month AR(1)
Energy	1.10 (1.05, 1.17)	**1.02 (0.94, 1.13)**	1.15 (1.08, 1.22)
Nonenergy	1.00 (0.95, 1.06)	1.12 (1.03, 1.23)	1.36 (1.29, 1.44)
Agriculture	1.00 (0.95, 1.06)	**1.05 (0.99, 1.17)**	1.36 (1.28, 1.45)
Beverages	1.01 (0.96, 1.07)	1.09 (1.01, 1.21)	1.22 (1.16, 1.29)
Food	1.01 (0.96, 1.07)	**1.01 (0.92, 1.10)**	1.32 (1.24, 1.41)
Oils and meat	1.01 (0.96, 1.07)	**0.87* (0.78, 0.97)**	1.29 (1.20, 1.39)
Grains	1.02 (0.97, 1.08)	**1.07 (0.98, 1.20)**	1.29 (1.22, 1.37)
Other food	1.01 (0.96, 1.07)	**0.96 (0.87, 1.06)**	1.13 (1.06, 1.20)
Raw materials	1.00 (0.95, 1.06)	1.15 (1.05, 1.26)	1.31 (1.24, 1.38)
Timber	1.01 (0.95, 1.07)	**1.00 (0.91, 1.10)**	1.24 (1.17, 1.33)
Other raw materials	1.01 (0.96, 1.07)	1.12 (1.01, 1.23)	1.37 (1.30, 1.46)
Fertilizers	1.02 (0.97, 1.08)	**1.01 (0.93, 1.12)**	1.06 (1.00, 1.12)
Metals and minerals	1.02 (0.96, 1.09)	**1.03 (0.94, 1.16)**	1.23 (1.17, 1.31)
Base metals	1.02 (0.97, 1.08)	**1.05 (0.95, 1.15)**	1.22 (1.16, 1.29)
Precious metals	1.09 (1.03, 1.15)	**0.96 (0.90, 1.03)**	1.17 (1.10, 1.24)

Notes: Values in bold provide evidence of $d \leq 1$. A single asterisk (*) indicates mean reversion.

Table 3 displays the results under the assumption of autocorrelation in ut. However, as mentioned above, rather than imposing a given parametric ARMA structure, we choose a nonparametric method that approximates ARMA structures with very few parameters (Bloomfield, 1973). It is nonparametric in the sense that the model has no explicit functional form and is simply determined by its spectral density function, which is given by

$$\text{f}(\lambda;\sigma^2) = \frac{\sigma^2}{2\pi}\exp(2\sum_{r=1}^{m}\tau_r\cos(\pi r)), \qquad (6)$$

where σ^2 is the variance of the error term and m indicates the last of the Fourier frequencies associated with the short-run parameters. Bloomfield (1973) showed that equation (6) approximates fairly well the spectrum of ARMA processes with a small number of parameters, and Gil-Alana (2004, 2008a) found this model very suitable in the context of fractional integration, particularly with Robinson's (1994) tests; moreover, this model is stationary across all values of τ, unlike what happens in the ARMA case, and produces autocorrelations, which also decay exponentially fast.

The first thing we observe in Table 3 is that the time trend is now statistically significant in several cases: oils and meat, other food, timber, and precious metals; for the rest of the series, the intercept seems to be sufficient to describe the deterministic terms. We also observe that the estimated values of d are now much smaller than they had been in the previous table. In fact, the $I(0)$ hypothesis cannot be rejected in a number of cases and we find evidence of mean reversion in oils and meat since the confidence interval for the values of d excludes the value of 1. Thus, according to this model, and for this series, the effect of an exogenous random shock will disappear in the long run, the series reverting to its original trend sometime in the future.

When we permit seasonal autoregressions (in Table 4), the values are above 1 in all cases and are in fact very similar to those reported in Table 2 for the case of white noise errors, which suggests that this component may not be relevant when explaining the data.

Table 5 summarizes the results in Tables 2–4. We observe evidence of unit roots only if the disturbances are autocorrelated throughout Bloomfield's (1973) model, and the only case of mean reversion takes place for oils and meat in the Bloomfield model. In all the other cases, the estimated values of d are significantly higher than 1. Thus, according to this method, the differenced series still display a long memory pattern, with a high degree of dependence between the observations.

Table 6. Estimates of *d* on the Growth Rate Series

Series/Bandwidth	$T^{0.3} \approx 7$	$T^{0.4} \approx 13$	$T^{0.5} \approx 27$	$T^{0.6} \approx 51$	$T^{0.7} \approx 99$
Energy	0.327	0.095	0.016	0.011	0.025
Nonenergy	0.077	−0.137	−0.061	0.054	0.132
Agriculture	0.303	−0.034	−0.027	0.025	0.067
Beverages	−0.263	0.323	−0.119	0.062	0.056
Food	−0.092	**−0.291**	−0.045	−0.011	0.013
Oils and meat	−0.345	**−0.312**	**−0.213**	**−0.150**	**−0.096**
Grains	−0.264	**−0.423**	−0.088	0.073	0.078
Other food	−0.267	**−0.399**	−0.151	0.068	−0.013
Raw materials	0.162	0.204	−0.125	0.085	0.159
Timber	0.185	0.088	**−0.170**	−0.014	−0.005
Other raw materials	0.222	0.066	**−0.186**	−0.019	0.067
Fertilizers	−0.479	**−0.291**	−0.151	0.030	0.051
Metals and minerals	−0.491	**−0.457**	**−0.246**	−0.039	0.118
Base metals	−0.500	**−0.500**	**−0.288**	−0.059	0.116
Precious metals	0.016	−0.071	0.147	0.291	0.043
95% confidence band for $I(0)$	±0.310	±0.228	±0.161	±0.115	±0.082

Notes: Reported values are the estimates of d based on the "local" Whittle estimate (Robinson, 1995). Values in bold provide evidence of mean reversion.

As an alternative approach, we also estimate the order of integration with a semiparametric "local" (in the sense that it only uses a set of frequencies degenerating to 0) Whittle approach (Robinson, 1995). We could adopt other semiparametric methods, also based on the Whittle method in the frequency domain (e.g., Shimotsu and Phillips, 2005; Abadir, Distaso, and Giraitis, 2007, etc.), but they require additional user-chosen parameters. The results might be very sensitive to the choice of these numbers. In this respect, the approach employed here relies only on a single bandwidth. We choose this number to be equal to T^m, where T is the sample size, and $m = 0.3$, 0.4, 0.5, 0.6, and 0.7. Table 6 reports the results.

Based on the results reported in Table 5, we believe the series to be nonstationary. Thus, for the estimation of d carried out in Table 6 (and based on the semiparametric "local" Whittle method), we perform the analysis based on the first-differenced data, adding the value 1 to recover the estimate of d on the commodity prices. We observe in the table that—contrary to the previous method, there are several cases where mean reversion takes place. Thus, for example, this happens for oils and meat across all bandwidth numbers, consistent with the parametric result reported above in the case of Bloomfield disturbances; however, we also observe a few more cases of mean reversion for other commodities: metals and minerals and base metals for $m = 0.3$, 0.4, and 0.5, fertilizers with $m = 0.3$ and 0.4, food, grain, and other food with $m = 0.4$, and timber and other raw material with $m = 0.5$. Thus, according to these semiparametric results strong evidence against mean reversion and permanency of the shocks is only obtained in five out of the 12 series examined: energy, nonenergy, agriculture, beverages, raw materials, and precious metals. For these five series, strong actions will be required to recover the original trends. For the other series (metals and minerals, base metals, fertilizers, food, grain, other food, timber, other raw material, and particularly oils and meat), shocks will have transitory effects that disappear by themselves in the long run. Therefore, these results have also implications in terms of policy actions. When an exogenous shock unexpectedly increases commmodity prices, policy responses should depend on the commodity and its degree of persistence. Strong policy responses should follow exogenous shocks to commodity prices for energy, non-energy, agriculture, beverages, raw materials, and precious metals, but other commodity prices may experience mean reversion, making such strong actions unnecessary since the series will likely recover by itself the long run.

Table 7. Evidence of Structural Breaks in the Data

Series	No. of Breaks	Break Dates
Energy	3	1973M04; 1982M02 and 2004M03
Nonenergy	4	1973M03; 1987M12; 1997M03 and 2006M01
Agriculture	3	1973M02; 1993M05 and 2007M02
Beverages	4	1974M01; 1988M07; 1999M02 and 2007M12
Food	4	1973M01; 1984M10; 1993M11 and 2007M02
Oils and meat	4	1973M01; 1985M05; 1994M03 and 2007M04
Grains	4	1973M01; 1984M10; 1998M01 and 2006M11
Other food	3	1972M12; 1988M01 and 2005M12
Raw materials	3	1973M07; 1989M05 and 2006M05
Timber	4	1970M03; 197M901; 1989M06 and 2006M05
Other raw materials	4	1973M07; 1987M05; 1997M07 and 2006M05
Fertilizers	4	1974M01; 1982M11; 1995M01 and 2006M12
Metals and minerals	3	1973M06; 1987M07 and 2005M02
Base metals	3	1973M06; 1987M07 and 2005M08
Precious metals	3	1973M02; 1982M01 and 2006M04

Notes: Results are based on Bai and Perron (2003) and Gil-Alana (2008a) for detecting multiple breaks.

As a robustness check and based on the above results, we examine whether the high degree of persistence obtained in the series might be a consequence of structural breaks that we have not considered. For this purpose, we use methods from both Bai and Perron (2003) and Gil-Alana (2008b), obtaining almost identical results in both cases. Table 7 displays the number of breaks for each series and the estimated break dates, which occur in all cases at specific dates, corresponding to around 1973, 1987 (or earlier, in 1982, in some cases), between 1993 and 1997, and finally around 2006–2007. All of these dates have connections with specific historical issues. In 1973, cutting off OPEC countries' supply led to the first oil price crisis. In 1987, a short but deep stock crisis in the New York market dragged down European and Asian markets. Between 1993 and 1997, different currencies in Asia, America, and Europe experienced problems; finally, the worst global financial crisis since 1930 hit the world's economies around 2006–2007.

Tables 8 and 9 report the estimated coefficients of d for each subsample in each series under the assumptions of uncorrelated and autocorrelated errors. Starting with the case of white noise errors, we observe that all the values are in the $I(0)$ interval or are significantly higher than 1. In fact, the lowest values of d generally occur in the first (in most cases) or third subsamples, observing an increase in the degree of dependence during the latter subsamples. Nevertheless, evidence of mean reversion is not found in any subsample for any series.

Allowing autocorrelated disturbances, the estimates of d are generally smaller and some evidence of mean reversion is now obtained in the case of metals and minerals and base metals in the first subsamples, precious metals and food in the second subsamples, energy and other food in the third subsample; fertilizers in the fourth subsample, and oils and meat in all four subsamples. For the remaining cases, the $I(0)$ hypothesis cannot be rejected, implying lack of mean reversion.

The general conclusion obtained in this work is that commodity prices display a high degree of persistence, implying that random shocks produce permanent changes in many of the series. This result is obtained when using parametric methods; however, more heterogeneous results are obtained when we use semiparametric approaches, and there is some evidence of mean reversion in some cases (food, oils and meat, grains, other food, timber, other raw material, fertilizers, metals and minerals, and base metals).

Allowing for structural breaks, we observe that all series display three or four breaks, mainly around the years 1973, 1987, 1997, and 2007. If the errors are uncorrelated, then no evidence of mean reversion will be obtained in any subsample; however, if autocorrelation is permitted, then breaks will occur in some subperiods at the beginning of the samples for series such as food, energy,

Table 8. Estimates of *d* across Subsamples Based on White Noise Errors

Series	Subsample 1	Subsample 2	Subsample 3	Subsample 4	Subsample 5
Energy	1.01 (0.94, 1.10)	1.00 (0.88, 1.15)	1.20 (1.07, 1.36)	1.27 (1.14, 1.44)	–
Nonenergy	1.40 (1.25, 1.58)	1.46 (1.31, 1.67)	1.08 (0.95, 1.27)	1.29 (1.17, 1.48)	1.39 (1.25, 1.56)
Agriculture	1.37 (1.25, 1.52)	1.37 (1.21, 1.57)	1.28 (1.13, 1.37)	1.41 (1.27, 1.59)	–
Beverages	1.11 (1.08, 1.27)	1.30 (1.19, 1.44)	1.22 (1.09, 1.39)	1.02 (0.92, 1.17)	1.24 (1.08, 1.46)
Food	1.19 (1.01, 1.43)	1.39 (1.19, 1.67)	1.11 (0.99, 1.29)	1.18 (1.06, 1.35)	1.41 (1.27, 1.59)
Oils and meat	1.23 (1.06, 1.46)	1.32 (1.10, 1.61)	1.14 (0.99, 1.37)	1.18 (1.05, 1.38)	1.35 (1.20, 1.54)
Grains	1.25 (1.12, 1.42)	1.32 (1.18, 1.51)	1.29 (1.16, 1.47)	1.18 (1.02, 1.42)	1.29 (1.16, 1.46)
Other food	1.09 (0.97, 1.26)	1.21 (1.10, 1.36)	0.88 (0.79, 1.01)	1.19 (1.04, 1.37)	–
Raw materials	1.36 (1.23, 1.53)	1.31 (1.19, 1.47)	1.28 (1.18, 1.42)	1.30 (1.18, 1.45)	–
Timber	1.22 (1.09, 1.38)	1.23 (1.05, 1.47)	1.22 (1.04, 1.46)	1.28 (1.18, 1.41)	1.20 (1.02, 1.36)
Other raw materials	1.24 (1.13, 1.38)	1.43 (1.29, 1.64)	1.46 (1.25, 1.71)	1.24 (1.12, 1.41)	1.38 (1.24, 1.57)
Fertilizers	1.03 (0.95, 1.12)	0.93 (0.82, 1.09)	0.98 (0.88, 1.10)	1.14 (1.00, 1.37)	1.23 (1.11, 1.38)
Metals and minerals	1.15 (1.01, 1.35)	1.24 (1.11, 1.42)	1.14 (1.05, 1.25)	1.35 (1.21, 1.51)	–
Base metals	1.17 (1.02, 1.35)	1.21 (1.09, 1.38)	1.13 (1.04, 1.25)	1.30 (1.17, 1.47)	–
Precious metals	1.21 (1.10, 1.36)	1.25 (1.06, 1.54)	1.03 (0.95, 1.14)	1.09 (0.99, 1.23)	–

Notes: Values in parenthesis refer to the 95% confidence bands for the values of d.

other food, fertilizers, metals and minerals, base metals, and precious metals. For oils and meat, mean reversion takes place in all subsamples except the last. Generally, we observe higher degrees of persistence in the final periods of the sample, implying that, in the last few years, more policy actions should be adopted in the event of shocks in the series to recover their original long-term projections.

Concluding Comments

We have examined in this paper the stochastic properties of a group of commodity prices by looking at the orders of integration of the series from a fractionally integrated viewpoint. This approach is more general than others, which are based exclusively on integer degrees of differentiation and only consider the cases of stationarity, $I(0)$, and nonstationarity, $I(0)$. Our preliminary results show that the data are very sensitive to the methodology used and the assumptions made regarding the error term. Thus, using parametric methods and imposing white noise or seasonal autoregressions, all orders of integration are significantly higher than 1, implying high degrees of dependence even in the differenced data; however, under Bloomfield's (1973) nonparametric approach, which permits weak autocorrelation for the error term, the unit root cannot be rejected in a number of cases and

Table 9. Estimates of *d* across Subsamples Based on Autocorrelated Errors

Series	Subsample 1	Subsample 2	Subsample 3	Subsample 4	Subsample 5
Energy	1.02 (0.93, 1.14)	0.94 (0.56, 1.30)	**0.81***** **(0.69, 0.98)**	1.04 (0.85, 1.38)	–
Nonenergy	0.76 (0.48, 1.48)	1.03 (0.80, 1.25)	0.94 (0.77, 1.18)	1.13 (0.98, 1.33)	1.16 (0.92, 1.56)
Agriculture	1.18 (0.95, 1.42)	0.84 (0.69, 1.04)	1.12 (0.99, 1.31)	1.22 (0.93, 1.62)	–
Beverages	0.91 (0.77, 1.14)	1.15 (0.99, 1.33)	1.14 (0.87, 1.57)	0.97 (0.82, 1.19)	0.84 (0.63, 1.13)
Food	0.72 (0.48, 1.19)	**0.75***** **(0.54, 0.97)**	0.95 (0.76, 1.18)	0.96 (0.80, 1.16)	1.14 (0.90, 1.56)
Oils and meat	**0.76***** **(0.59, 0.98)**	**0.55***** **(0.31, 0.87)**	**0.79***** **(0.57, 0.93)**	**0.81***** **(0.68, 0.99)**	1.01 (0.70, 1.36)
Grains	1.12 (0.70, 1.47)	0.98 (0.77, 1.23)	1.01 (0.82, 1.29)	0.88 (0.64, 1.19)	1.09 (0.88, 1.43)
Other food	0.84 (0.66, 1.09)	1.02 (0.82, 1.24)	**0.74***** **(0.63, 0.91)**	0.86 (0.67, 1.20)	–
Raw materials	1.08 (0.89, 1.33)	1.04 (0.86, 1.29)	1.16 (1.02, 1.44)	1.23 (1.01, 1.51)	–
Timber	1.19 (0.91, 1.60)	0.86 (0.65, 1.22)	0.80 (0.57, 1.09)	1.21 (1.00, 1.44)	1.02 (0.80, 1.28)
Other raw materials	1.25 (0.99, 1.53)	0.99 (0.78, 1.21)	0.92 (0.72, 1.34)	1.06 (0.88, 1.29)	1.13 (0.91, 1.46)
Fertilizers	1.07 (0.94, 1.20)	0.88 (0.70, 1.12)	1.07 (0.83, 1.35)	0.79*** (0.66, 1.29)	1.12 (0.94, 1.37)
Metals and minerals	**0.71***** **(0.57, 0.96)**	0.99 (0.80, 1.23)	1.09 (0.94, 1.27)	1.05 (0.85, 1.34)	–
Base metals	**0.76***** **(0.60, 0.94)**	0.97 (0.79, 1.18)	1.07 (0.93, 1.24)	1.06 (0.80, 1.38)	–
Precious metals	1.04 (0.86, 1.25)	**0.76***** **(0.60, 0.95)**	0.89 (0.79, 1.02)	1.01 (0.86, 1.17)	–

Notes: Values in parenthesis refer to the 95% confidence bands for the values of d. Triple asterisks (***) indicate evidence of mean reversion (i.e., $d < 1$) at the 5% level. Values in bold refer to significant evidence of mean reversion at 5% level.

a small degree of mean reversion is found in the case of oils and meat. On the other hand, using a semiparametric "local" Whittle method, where no functional form is imposed on the error term, mean reversion is found in approximately half of the series examined: food, oils and meat, grains, other food, timber, other raw material, fertilizers, metals and minerals, and base metals. Thus, shocks affecting these series will disappear by themselves in the long run and no strong actions should be required in these cases.

Accounting for the possibility of structural breaks and using Bai and Perron's (2003) and Alana's (2008b) methods, we find the presence of multiple (3 or 4) breaks, most of them occurring during 1973 (first oil price crisis), 1987 (New York market crisis), 1993/97 (Asian crisis) and 2007 (world financial crisis). Once more, under autocorrelation, we observe mean reversion in various of the subsamples, especially for oils and meat, and the order of integration generally increases across the subsamples. As time goes by, in the event of an exogenous shock, strong actions will be required to recover the original levels of the data.

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Appendix

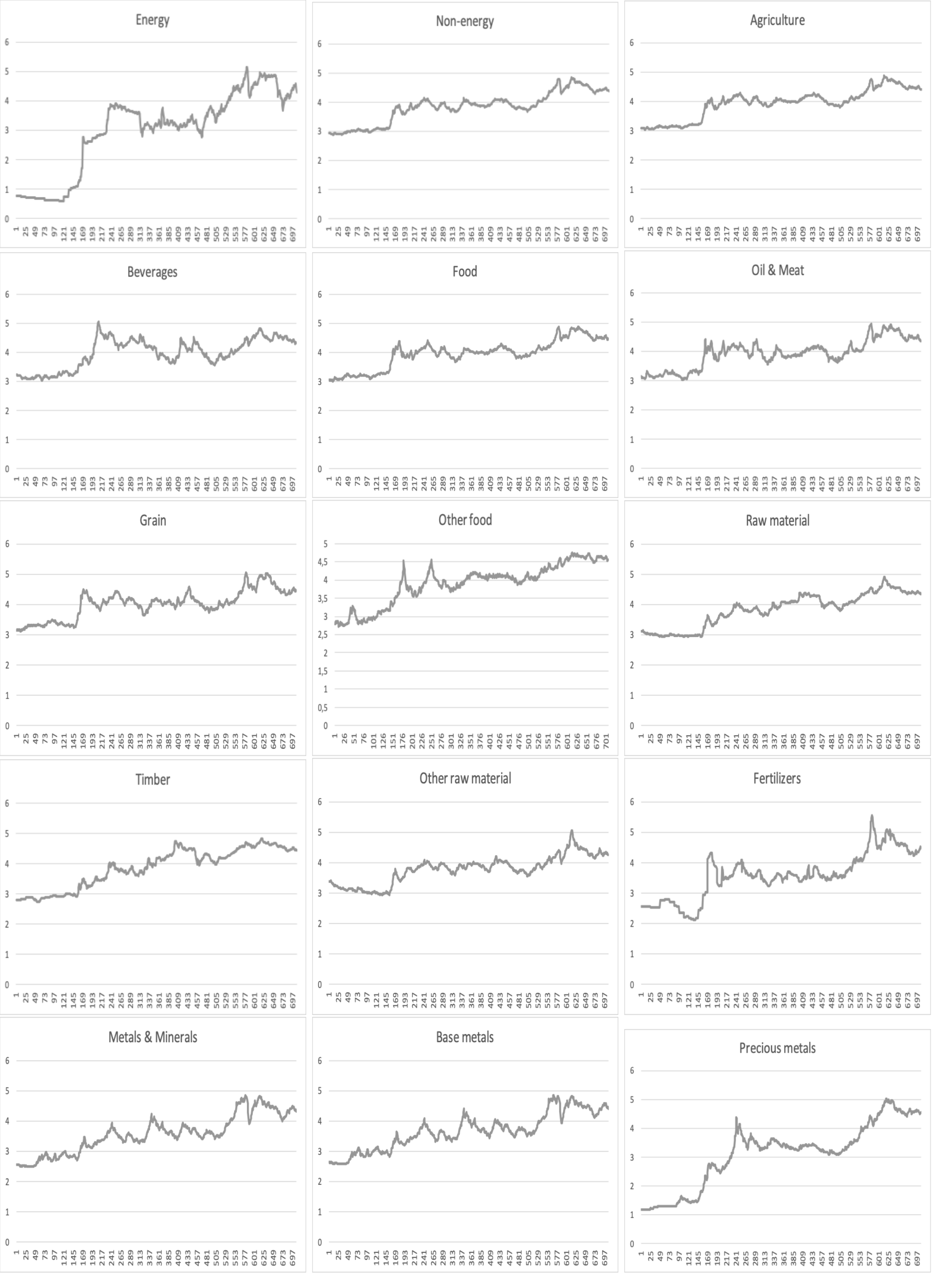

Figure A1. Time Series Plots

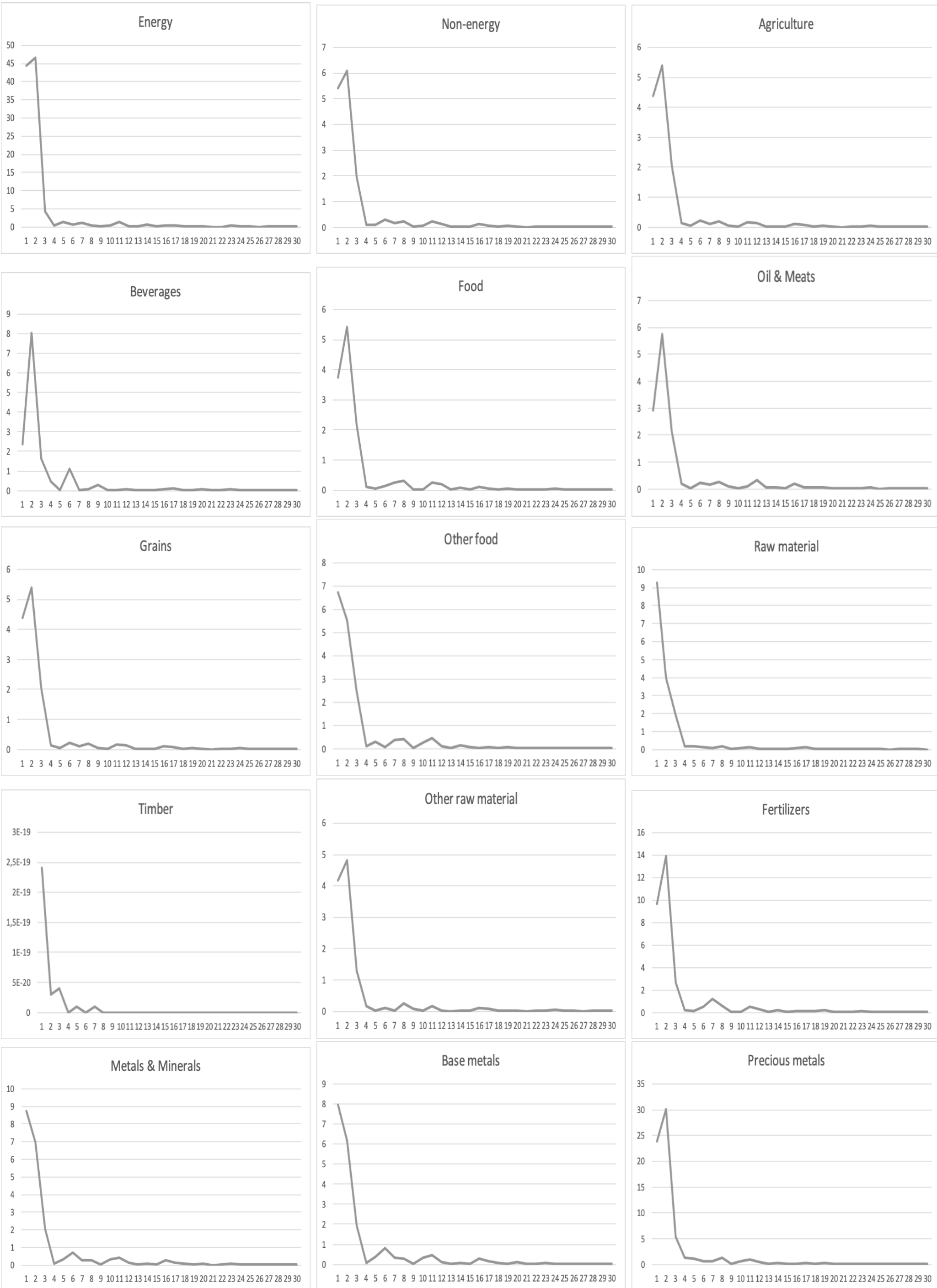

Figure A2. Periodograms of the Original Time Series

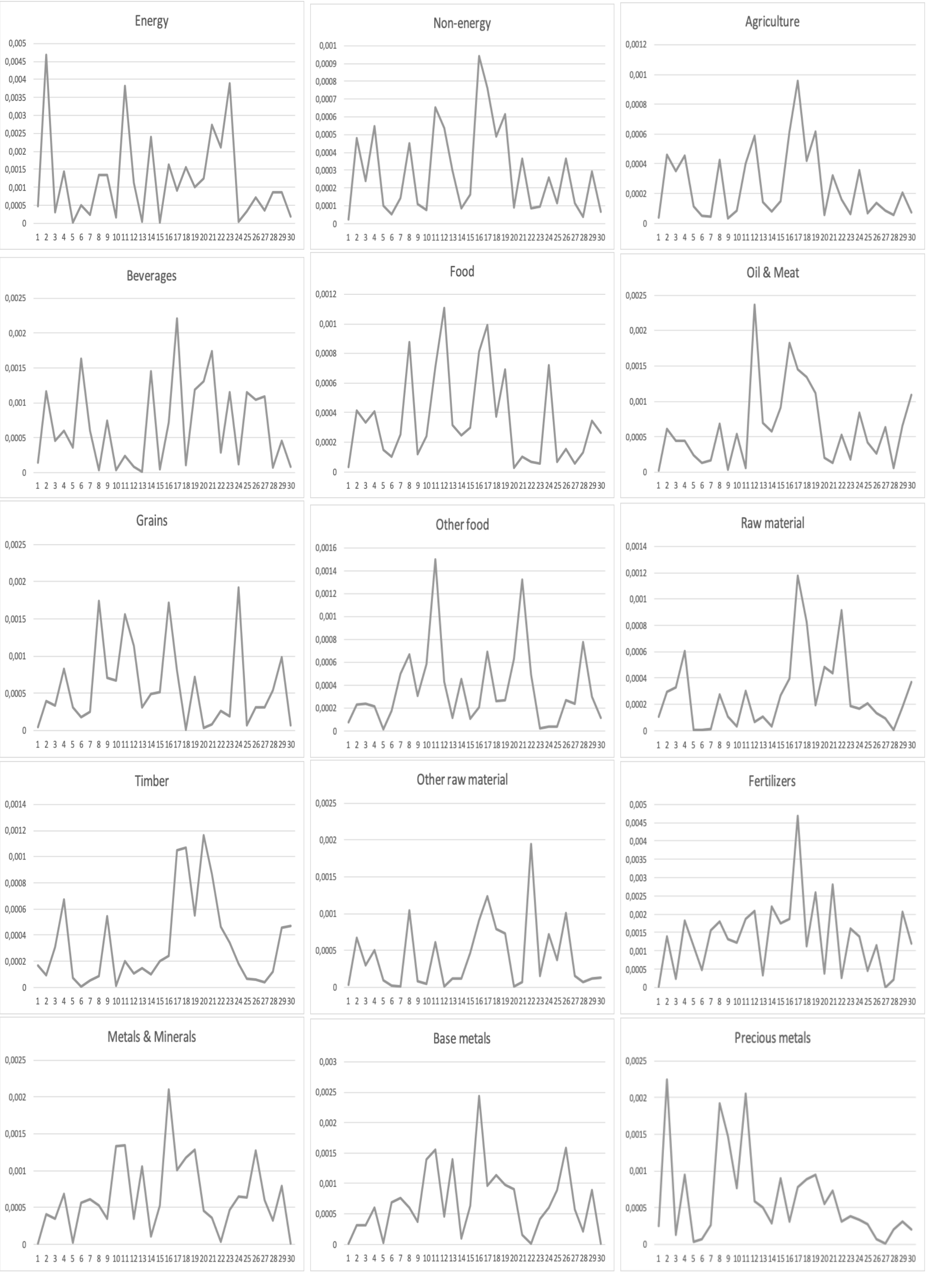

Figure A3. Periodograms of the First-Differenced Data