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Relaxing the Assumptions of Minimum-Variance Hedging

Sergio H. Lence

The most important minimum-variance hedge-ratio assumptions are (*a*) that production is deterministic and (*b*) that all of the agent's wealth is invested in the cash position. Stochastic production greatly reduces optimal hedge ratios. An alternative investment greatly reduces opportunity costs of not hedging by "diluting" the cash position. Stochastic production and/or alternative investments render the costs associated with hedging relatively more important, yielding almost negligible net benefits of hedging. Hence, hedging costs typically dismissed in hedging models for being seemingly negligible are important determinants of hedging behavior.

Key words: futures, hedging, minimum-variance hedge, risk management

Introduction

Minimum-variance hedge ratios (MVHs) have a prominent place in the applied hedging literature and are one hedging strategy often recommended by extension economists. Benninga, Eldor, and Zilcha have shown that MVHs are consistent with expected-utility maximization under some relatively plausible conditions.[1] MVHs can be identified in a straightforward manner with simple statistics that can be estimated from observable cash and futures prices. These two MVH properties, together with the development of more sophisticated econometric techniques and the advances in computing capacity, have caused much of the recent hedging research to focus on the econometrics of MVH estimation (Baillie and Myers; Castelino; Fackler and McNew; Lence, Kimle, and Hayenga; Mathews and Holthausen; McNew and Fackler; Myers and Thompson; Viswanath). In contrast, the research community has paid virtually no attention to issues such as the potential value of better econometric techniques to estimate MVHs and the practical conditions that render MVHs a clearly suboptimal hedging strategy. Two exceptions are Tomek and Lence.

Lence focused on the value of more information about MVHs for individual agents. Somewhat surprisingly, Lence found that the value of "better" MVH estimates is negligible and that optimal hedge ratios are substantially different from MVHs when stan-

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[1] Traditionally, MVHs were justified by assuming that agents wanted to minimize the variance of their combined cash-futures positions. However, hardly anybody would seriously use such an argument to advocate MVHs now. The reason for this assertion is that variance minimization in the context of the paradigm of expected-utility, maximization is equivalent to infinite risk aversion. Infinite risk aversion is very unrealistic and unpalatable to most academicians; for example, there is an intrinsic contradiction in having infinitely risk-averse agents holding risky cash positions. Hence, the analysis by Benninga, Eldor, and Zilcha is the most accepted justification for adopting MVHs.

dard MVH assumptions are relaxed to accommodate more realism.[2] Furthermore, he showed that under realistic conditions the optimal hedging strategy was simply not to hedge. The findings by Lence are important because they suggest that a large proportion of the current research agenda on hedging is so narrowly concentrated on estimation problems that issues potentially more significant are being neglected. His findings are also relevant for applied risk management because they indicate that, in many commonly encountered practical situations, MVH recommendations are not only (very) suboptimal but that the best decision may be not to hedge at all.

From both a research and an applied standpoint, one important question is left unanswered by Lence: what are the most critical conditions for the consistency of MVHs with expected-utility maximization in real-world situations? Among other reasons, the answer to such a question is of interest from a positive perspective because it may help explain why hedging is more pervasive in some circumstances (e.g., grain dealers) than in others (e.g., farmers). An answer to that question also can show educators and practitioners when MVH recommendations are valid and when they are not applicable.

The foregoing discussion suggests that investigating the robustness of MVHs to each of the standard MVH assumptions is a relevant issue. Therefore, the objective of this article is to extend Lence's analysis by exploring the implications of relaxing the standard MVH restrictions individually and by assuming realistic values instead. This study uses the basic apparatus employed by Lence to facilitate comparison with his results and also to better complement his analysis. In particular, the robustness of MVHs to individual MVH assumptions is assessed by means of direct (as opposed to indirect) measures of economic importance.

Theoretical Model

The basic model consists of a risk-averse decision maker with a utility function ($U(\cdot)$, $U' > 0$, $U'' < 0$) whose argument is terminal wealth (W_1).[3] At the decision date ($t = 0$), the agent engages in an activity that produces $Q_1 \geq 0$ (possibly random) commodity units for sale at the terminal date ($t = 1$) at the random cash price P_1. At $t = 0$, he can also sell X commodity units in the futures market at price F_0, by agreeing to repurchase them at $t = 1$ at the random futures price F_1. The net cash flow from the futures transaction occurs at the terminal date, at which time the agent must pay a brokerage fee of b dollars per commodity unit. To be allowed to open a futures position, he must make an initial margin deposit equaling a fraction k_L of the original futures price multiplied by the contract size. The decision maker will receive $R_L L$ ($R_L > 1$) dollars at the terminal date if he invests $L \geq 0$ dollars in risk-free notes at the decision date; if so desired, risk-free notes can be used to satisfy the initial margin deposit. At $t = 0$, he can also borrow $B \geq 0$ dollars by promising to repay $R_B B$ ($R_B > R_L > 1$) dollars at $t = 1$. In addition,

[2] Throughout the article, "standard MVH assumptions" are the assumptions made by Benninga, Eldor, and Zilcha to obtain the equivalency between MVHs and the hedge ratios that maximize expected utility. These assumptions are discussed in the next section.

[3] A 1-period framework with two dates (period's beginning and period's end) is assumed here for consistency with the standard MVH model. As pointed out by an anonymous referee, however, this assumption is one of the most suspect parts of the standard MVH model because of its lack of realism.

the agent may invest $I \geq 0$ dollars in an alternative activity that yields a random return of R_I dollars at $t = 1$ per dollar invested at $t = 0$.[4]

At the decision date, the agent is hypothesized to choose X, B, L, and I to maximize the expected utility of terminal wealth.[5] Terminal wealth is given by:

(1) $$W_1 = P_1Q_1 + (F_0 - F_1)X - b|X| - R_BB + R_LL + R_II,$$

subject to the restrictions

(2) $W_0 + B \geq v[E(Q_1)]E(Q_1) + L + I,$ (budget constraint);

(3) $0 \leq B \leq k_Bv[E(Q_1)]E(Q_1),$

$k_B \geq 0,$ (borrowing constraint);

(4) $L \geq k_LF_0|X|, \quad k_L \geq 0,$ (initial margin deposit contstraint);

and

(5) $I \geq 0,$ (nonnegative investment constraint);

where W_0 is initial wealth, $E(\cdot)$ is the expectation operator, $v[E(Q_1)]$ ($v > 0\ \forall E(Q_1) > 0$) is the average cost of producing the expected output ($E(Q_1)$), and k_B is the maximum amount that the agent can borrow expressed as a proportion of his initial wealth invested in the cash position. Total cost of production ($v[E(Q_1)]E(Q_1)$) is assumed to be deterministic. Restriction (2) denotes the budget constraint at the decision date. Expression (3) states that borrowings cannot surpass the agent's borrowing capacity, which is proportional to the value of the cash position. Constraint (4) follows from the initial margin deposit requirement. Initial margin deposits are always met with risk-free notes rather than cash; otherwise, agents would lose a risk-free net return of ($R_L - 1 > 0$) dollars per dollar of initial margin deposit.

Following Lence, simulations are performed by using an equivalent formulation of the objective function stated in the preceding paragraph. The agent is postulated to choose $h \equiv X/E(Q_1)$, $s_B \equiv B/W_0$, $s_L \equiv L/W_0$, and $s_I \equiv I/W_0$ at the decision date to maximize the expected utility of terminal wealth ($E[U(W_1)]$). Quantity h is the hedge ratio; s_B is the proportion of initial wealth being borrowed; and s_L and s_I are the fractions of initial wealth invested in risk-free notes and in the alternative investment, respectively. Using these definitions, terminal wealth can be expressed as:

(6) $$\begin{aligned} W_1 &= W_0R \\ &= W_0\left\{[R_Pq + (1 - R_F - b/F_0)\frac{F_0}{v[E(Q_1)]}h - R_B]s_Q \right. \\ &\qquad \left. + (R_L - R_B)s_L + (R_I - R_B)s_I + R_B\right\}, \end{aligned}$$

subject to the restrictions

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[4] Investment I may also be interpreted as a portfolio of activities other than production and hedging.

[5] Following the typical framework of MVH models, it is assumed that the agent has already selected the optimal level of production. By proceeding in this way, we avoid having to define a specific production technology for the simulations performed in later sections.

(7) $s_Q + s_L + s_I - s_B = 1,$ (budget constraint);

(8) $1 \leq s_Q + s_L + s_I \leq 1 + k_B s_Q, \qquad k_B \geq 0,$

(borrowing constraint);

(9) $s_L \geq k_L \dfrac{F_0}{v[E(Q_1)]} h s_Q, \qquad k_L \geq 0,$

(initial margin deposit constraint);

and

(10) $s_I \geq 0,$ (nonnegative investment constraint);

where $R \equiv W_1/W_0$, $R_P \equiv P_1/v[E(Q_1)]$, $q \equiv Q_1/E(Q_1)$, $R_F \equiv F_1/F_0$, and $s_Q \equiv v[E(Q_1)] \cdot E(Q_1)/W_0$. Variable R_P is the ratio of terminal cash price to average production cost, q represents the units of terminal production per unit of expected production, R_F is the ratio of terminal to initial futures prices, and s_Q is the fraction of initial wealth invested in production. It is straightforward to show that expressions (6) through (10) are equivalent to expressions (1) through (5).

MVHs will usually be different from the hedge ratios that maximize the hypothesized objective function, that is, MVHs are generally inconsistent with expected-utility maximization. However, Benninga, Eldor, and Zilcha demonstrated that under certain conditions MVHs are also expected-utility, maximizing hedge ratios. The conditions assumed by Benninga, Eldor, and Zilcha are that (*a*) the agent is not allowed to borrow, lend, or invest in alternative activities ($s_B = s_L = s_I = 0$); (*b*) there are neither initial margin deposits nor futures brokerage fees ($k_L = 0$, $b = 0$); (*c*) production is deterministic ($Q_1 = E(Q_1)$); (*d*) random cash prices can be expressed as a linear function of futures prices plus an independent error term; and (*e*) current futures prices are unbiased ($F_0 = E(F_1)$). Throughout this article, the preceding set of assumptions is referred to as the standard MVH assumptions.

Of special interest for the present study of MVHs and alternative hedging strategies is the economic importance of the welfare losses resulting from placing suboptimal hedges instead of optimal hedges. Such economic importance is assessed by means of opportunity costs (*OC*s). As measured by net returns per invested dollar, *OC* is "the premium that the investor should require over his suboptimal return to bring his welfare to a level achieved by his optimal return" (Simaan, p. 579). *OC* is an amount determined with certainty at the decision date. Denoting by R_{opt} the return of the expected-utility maximizing strategy and by R_{alt} the return on a particular alternative investment, the *OC* of the latter is defined implicitly by

(11) $$E[U(W_0 R_{opt})] = E\{U[W_0(R_{alt} + OC)]\}.$$

Therefore, *OC* is the minimum certain net return that the agent requires to accept investing in the alternative strategy rather than in the optimum investment. For the purposes of this analysis, *OC*s have more desirable characteristics than other measures of welfare changes commonly used in hedging studies [e.g., percentage reduction in variance or in standard deviation, R^2 (Ederington)] or in the portfolio literature [e.g., percentage changes in expected utility (Pulley; Kroll, Levy, and Markowitz), regression analysis (Levy and

Markowitz 1977, 1979), percentage changes in certainty equivalents (Kallberg and Ziemba 1984)].

Numerical Simulations

Simulations are performed for decision makers with constant absolute risk aversion, that is, $U = -\exp(-AW_0R)$, with A being the coefficient of absolute risk aversion. Constant absolute risk aversion is helpful in numerical optimization because it yields expected utilities and *OC*s with closed-form solutions for most of the probability density functions used here (see appendix). Furthermore, optimal portfolios are almost identical for utilities with different functional forms but the same relative risk aversion (AW_0R) evaluated at $R = 1$ (Pulley; Kallberg and Ziemba 1979, 1984). Three levels of risk aversion are considered: low ($AW_0 = 1$), moderate ($AW_0 = 3$), and extremely high ($AW_0 = 10$) (Kallberg and Ziemba 1984). Here, the most realistic scenario is represented by (AW_0) = 3. Results for (AW_0) = 10 reflect implausibly high risk aversion (Levy and Markowitz 1979, 1977; Markowitz, Reid, and Tew). This latter scenario is presented for completeness rather than realism and should therefore be interpreted with care.

The model is calibrated for grain storage (that is, the productive activity is grain storage). Hence, unless stated otherwise it is assumed that production is nonstochastic ($q = 1$) and that average production costs equal grain cash prices at the decision date ($v[E(Q_1)] = v(Q_1) = P_0$). It is further assumed that $F_0/v(Q_1) = F_0/P_0 = 1$; that the standard deviations of cash and futures price ratios are $SD(R_P) = SD(R_F) = 0.065$, $SD(R_P) = SD(R_F) = 0.085$, and $SD(R_P) = SD(R_F) = 0.130$ for one-month, one-quarter, and semiannual holding periods, respectively; that the correlation between cash and futures is $\rho_{PF} = 0.85$ for all three holding periods;[6] and that futures are unbiased ($E(R_F) = 1$).[7] The expected net return on the cash position is set equal to one standard deviation, that is, $E(R_P) = 1.065$, $E(R_P) = 1.085$, and $E(R_P) = 1.130$ for one-month, one-quarter, and semiannual holding periods, respectively.[8]

Simulations under the standard MVH scenario require that borrowings, lending, and alternative investments equal zero ($s_B = s_L = s_I = 0$, $s_Q = 1$) and that brokerage fees as well as initial margin deposits equal zero ($b = k_L = 0$). To uncover the effect of the standard MVH restrictions, simulations are conducted for scenarios in which at least one of such restrictions is relaxed to accommodate realistic values. The values selected to contrast with the standard MVH assumptions are as follows: (*a*) ratios of brokerage fees to current futures prices (b/F_0) equal to 0.005, 0.0025, and 0.00125;[9] (*b*) initial margin

[6] Results for $\rho_{PF} = 0.95$ are similar and are available upon request.

[7] Values for means, standard deviations, and correlation of R_P ($=P_1/P_0$) and R_F ($=F_1/F_0$) are based on corn and soybeans cash prices for North-Central Iowa ("Iowa State University Market News") and prices of the respective nearby futures contracts at the Chicago Board of Trade (*The Wall Street Journal*) on each month's last Thursday from 1983 through 1993. For both corn and soybeans, mean estimates of R_P and R_F are not significantly different from unity at either monthly, quarterly, or semiannual horizons. Standard deviation estimates of R_P and R_F for both commodities are about 0.065, 0.085, and 0.130 at monthly, quarterly, and semiannual horizons, respectively. The estimated correlation between R_P and R_F ranges from a low of $\rho_{PF} = 0.75$ (semiannual horizon for corn) to a high of $\rho_{PF} = 0.96$ (monthly horizon for soybeans).

[8] Although the sample estimate of $E(R_P)$ is 1, such value cannot be used because rational risk-averse agents would not store if they expected zero net returns from doing so.

[9] Brokerage fees of \$50 per 5,000 bushel contract yield b = \$0.01 per bushel or F_0 = \$2 per bushel if $b/F_0 = 0.005$, F_0 = \$4 per bushel if $b/F_0 = 0.0025$, and F_0 = \$8 per bushel if $b/F_0 = 0.00125$. Hence, the values of b/F_0 used in the simulations are representative of typical brokerage fees for agricultural commodities such as oats, barley, corn, wheat, and soybeans.

deposits of 5% and 10% of the net futures position's absolute value ($k_L = 0.05$ and $k_L = 0.10$);[10] (*c*) borrowings of up to 100% of the value of the cash position ($k_B = 1$); and (*d*) 50%, 25%, and 5% of the initial wealth invested in the cash position ($s_Q = 0.50$, $s_Q = 0.25$, and $s_Q = 0.05$).

The annual interest rate on the risk-free notes is 4% ($R_L = 1.04$), the annual interest rate on borrowed funds is 10% ($R_B = 1.10$), and the expected annual net return on the unrelated risky activity is 12% ($E(R_I) = 1.12$).[11] For consistency with R_P, the standard deviation of R_I is set at $SD(R_I) = 0.0095$ ($=1.12^{1/12} - 1$) for the monthly scenario, at $SD(R_I) = 0.0287$ ($=1.12^{1/4} - 1$) for the quarterly scenario, and at $SD(R_I) = 0.0583$ ($=1.12^{1/2} - 1$) for the semiannual scenario. Correlation coefficients between R_I and R_P, and R_I and R_F are postulated to be 0.30 ($\rho_{PI} = \rho_{FI} = 0.30$).[12]

To compare with the standard MVH setting which assumes deterministic production, simulations are also performed allowing for stochastic production. Stochastic production is represented by q following a beta distribution independent of R_P, R_F, and R_I, that is,

$$p(q) = \frac{1}{\text{Beta}(\alpha, \beta)} \frac{q^{\alpha-1}(q_{\max} - q)^{\beta-1}}{q_{\max}^{\alpha+\beta-1}}, \qquad 0 \le q \le q_{\max}, \tag{12}$$

where $p(q)$ is the pdf of q, Beta($\cdot$) is the beta function, and $\alpha > 0$, $\beta > 0$, and $q_{\max} > 0$ are parameters. Parameter $q_{\max}$ must satisfy the restriction $E(q) = 1$, which is implied by the definition of q ($\equiv Q_1/E(Q_1)$). By the properties of the beta pdf, such restriction is met by setting $q_{\max} = (\alpha + \beta)/\alpha$. High- and low-production variability scenarios are simulated by using the combinations of α and β values reported by Nelson and Preckel (p. 374) that yield the greatest ($\alpha = 4.5988$, $\beta = 4.4407$) and smallest ($\alpha = 6.5248$, $\beta = 2.1741$) variances for q.

Three alternative bivariate pdfs for R_P and R_F are used, namely, bivariate normal, bivariate gamma, and bivariate log-normal. The base scenario is represented by the bivariate normal pdf because this pdf is consistent with the standard MVH assumption of cash prices being a function of futures prices plus an independent error term. In the two alternative settings, R_P and R_F have the same means, variances, and first-product moments as in the base scenario, but it is assumed instead that they are either bivariate gamma distributed or bivariate log-normally distributed. The gamma pdf is used because it violates the standard MVH assumption of cash prices being a linear function of futures prices plus an independent error term. The gamma pdf is also probably more realistic than the normal pdf because it is positively skewed and gamma random variables are positive. In addition, the gamma pdf coupled with constant absolute risk aversion yields a closed-form for expected utility (Johnson and Kotz, p. 219). The log-normal pdf is a popular assumption about price behavior. But expected utility does not exist for constant absolute risk aversion and log-normally distributed prices (Kotz and Johnson, p. 134). Therefore, simulations for the log-normal pdf are performed assuming a quadratic utility function.

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[10] Marshall (p. 26) reports that initial margin deposits range generally from 5% to 10% of the contract's value.

[11] These figures were chosen because they are representative of actual values at the time this study was conducted.

[12] Correlations of $\rho_{PI} = \rho_{FI} = 0.30$ seem realistic in the light of the wide range of correlation estimates reported by previous studies for agriculture (e.g., Young and Barry; Boehlje and Trede; Musser and Stamoulis; Hazell).

Discussion of Results

Results from the simulations are reported in tables 1 through 6. Table 1 shows the standard MVH setting. In this instance, MVHs are also expected-utility maximizing hedges. Given the parameters employed, MVHs are identical to the correlation coefficient between cash and futures prices (MVH $= \rho_{PF} = 0.85$). The *OC* of suboptimal hedging increases with the level of risk aversion. For moderate risk aversion ($AW_0 = 3$) the *OC* of not hedging is large: *OC* ranges from 3.17% through 5.64% of initial wealth per year (corresponding to one-month and one-quarter holding periods, respectively). *OC*s of suboptimal hedging strategies increase at an increasing rate with the distance from the optimal hedge ratio. For moderate risk aversion and a one-month (one-quarter) holding period, the *OC* of reducing the hedge ratio from 0.80 to 0.40 is 1.53% (0.87) of initial wealth per year, but further reducing the hedge ratio from 0.40 to 0 has an *OC* of 4.09% (2.29). It is important to note that, even though *OC*s of suboptimal hedge ratios can be very high when such ratios are substantially different from the optimum, *OC*s are almost negligible for hedge ratios not far from the optimum. For example, *OC*s of hedge ratios between $h = 0.80$ and $h = 0.90$ are less than 0.06 cents per dollar of initial wealth per year. This observation indicates that, even under the standard MVH assumptions, there seems to be little to lose by departing slightly from MVHs.

Table 2 reports *OC*s for the same scenario as table 1 but under the assumption of positive brokerage fees. First, brokerage fees cause optimal hedges to be smaller than MVHs, sometimes substantially so. For example, optimal hedge ratios for moderately risk-averse agents facing brokerage fees of only 0.25% of the initial futures price are $h = 0.6528$ for one-month holding period and $h = 0.7347$ for one-quarter holding-period, in contrast to MVH $= 0.85$ for either holding period. Second, as a result of brokerage fees, there is a noticeable reduction in the *OC* of not hedging. This finding implies that there is much less to be gained by switching from a zero hedge to an optimal hedge. For moderate risk aversion and zero hedging, brokerage fees of 0.25% of the initial futures price cause *OC*s to fall by 42%, 26%, and 11% for one-month, one-quarter, and semiannual holding periods, respectively.

Table 3 presents the standard MVH scenario of table 1 but relaxing the assumption of no initial margin deposits. Comparing tables 1 through 3 reveals that initial margin deposits reduce both optimal hedge ratios and *OC*s of not hedging.[13] But the effects of typical initial margin deposits are generally smaller than those of typical brokerage fees. Under moderate risk aversion, optimal hedge ratios lie between $h = 0.7842$ (one-quarter holding period and 10% margin) and $h = 0.8315$ (one-month holding period and 5% margin). For moderately risk-averse agents, 5% initial margin deposits reduce *OC*s of not hedging by 4%, 8%, and 7% for one-month, one-quarter, and semiannual holding periods, respectively.

Initial margin deposits have the largest relative impacts on the one-quarter holding period scenario. In contrast, brokerage fees exert the largest relative changes on the setting with one-month holding period. Therefore, no generalizations can be made as to which holding period scenario is the most affected by relaxing standard MVH assumptions.

[13] Blank provides an alternative analysis of hedging capital requirements with similar findings regarding optimal hedge ratios.

Table 1. Optimal Hedge Ratios and *OC*s under MVH Assumptions

Holding Period (months)	Risk Aversion	Optimal Hedge Ratio	Opportunity Cost of Hedging (annual net return as percentage of initial wealth)										
			$h = 0$	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$	$h = 0.6$	$h = 0.7$	$h = 0.8$	$h = 0.9$	$h = 1$
1	low	0.85	1.85	1.44	1.08	0.77	0.51	0.31	0.16	0.06	0.01	0.01	0.06
1	moderate	0.85	5.64	4.36	3.26	2.32	1.55	0.94	0.48	0.17	0.02	0.02	0.17
1	high	0.85	19.93	15.23	11.25	7.94	5.26	3.15	1.60	0.57	0.06	0.06	0.57
3	low	0.85	1.05	0.82	0.61	0.44	0.29	0.18	0.09	0.03	0.00[a]	0.00[a]	0.03
3	moderate	0.85	3.17	2.46	1.84	1.32	0.88	0.53	0.27	0.10	0.01	0.01	0.10
3	high	0.85	10.86	8.38	6.25	4.44	2.96	1.78	0.91	0.33	0.04	0.04	0.33
6	low	0.85	1.22	0.95	0.72	0.51	0.34	0.21	0.11	0.04	0.00[a]	0.00[a]	0.04
6	moderate	0.85	3.70	2.87	2.15	1.54	1.03	0.62	0.32	0.11	0.01	0.01	0.11
6	high	0.85	12.58	9.73	7.27	5.18	3.45	2.08	1.06	0.38	0.04	0.04	0.38

Note: Low, moderate, and high risk aversion are represented by $AW_0 = 1$, $AW_0 = 3$, and $AW_0 = 10$, respectively. Random variables R_P and R_F are bivariate normally distributed with $\rho_{PF} = 0.85$. One-month holding period assumes $E(R_P) = 1.065$ and $SD(R_P) = SD(R_F) = 0.065$; one-quarter holding period assumes $E(R_P) = 1.085$ and $SD(R_P) = SD(R_F) = 0.085$; and semiannual holding period assumes $E(R_P) = 1.130$ and $SD(R_P) = SD(R_F) = 0.130$. Values for other unspecified parameters correspond to the standard MVH assumptions (i.e., $s_Q = E(R_F) = 1$ and $b = s_L = k_L = k_B = 0$).

[a] Amount positive but smaller than 0.005%.

Table 2. Optimal Hedge Ratios and *OC*s under MVH Assumptions with Positive Brokerage Fees

Brokerage Fee (% of initial futures price)	Holding Period (months)	Risk Aversion	Optimal Hedge Ratio	Opportunity Cost of Hedging (annual net return as percentage of initial wealth)			
				$h = 0$	$h = 0.5$	$h = 0.85$[a]	$h = 1$
0.125	1	Low	0.5541	0.78	0.01	0.22	0.50
0.250	1	Low	0.2583	0.17	0.15	0.89	1.40
0.500	1	Low	0.0000	0.00	1.49	3.32	4.31
0.125	1	Moderate	0.7514	4.38	0.48	0.07	0.47
0.250	1	Moderate	0.6528	3.29	0.18	0.30	0.92
0.500	1	Moderate	0.4555	1.59	0.02	1.19	2.28
0.125	1	High	0.8204	18.46	2.63	0.02	0.82
0.250	1	High	0.7908	17.06	2.16	0.09	1.12
0.500	1	High	0.7317	14.45	1.37	0.36	1.84
0.125	3	Low	0.6770	0.66	0.04	0.04	0.15
0.250	3	Low	0.5040	0.37	0.00[b]	0.17	0.36
0.500	3	Low	0.1580	0.04	0.17	0.69	1.03
0.125	3	Moderate	0.7923	2.75	0.37	0.01	0.19
0.250	3	Moderate	0.7347	2.36	0.24	0.06	0.31
0.500	3	Moderate	0.6193	1.67	0.06	0.23	0.63
0.125	3	High	0.8327	10.40	1.61	0.00[b]	0.40
0.250	3	High	0.8154	9.96	1.44	0.02	0.49
0.500	3	High	0.7808	9.10	1.14	0.07	0.70
0.125	6	Low	0.7760	1.02	0.13	0.01	0.08
0.250	6	Low	0.7021	0.84	0.07	0.04	0.15
0.500	6	Low	0.5541	0.52	0.00[b]	0.15	0.34
0.125	6	Moderate	0.8253	3.48	0.54	0.00[b]	0.16
0.250	6	Moderate	0.8007	3.28	0.46	0.01	0.20
0.500	6	Moderate	0.7514	2.88	0.32	0.05	0.31
0.125	6	High	0.8426	12.36	1.99	0.00[b]	0.42
0.250	6	High	0.8352	12.14	1.91	0.00[b]	0.46
0.500	6	High	0.8204	11.70	1.74	0.02	0.55

Note: Low, moderate, and high risk aversion are represented by $AW_0 = 1$, $AW_0 = 3$, and $AW_0 = 10$, respectively. Random variables R_P and R_F are bivariate normally distributed with $\rho_{PF} = 0.85$. One-month holding period assumes $E(R_P) = 1.065$ and $SD(R_P) = SD(R_F) = 0.065$; one-quarter holding period assumes $E(R_P) = 1.085$ and $SD(R_P) = SD(R_F) = 0.085$; and semiannual holding period assumes $E(R_P) = 1.130$ and $SD(R_P) = SD(R_F) = 0.130$. Values for other unspecified parameters correspond to the standard MVH assumptions (i.e., $s_Q = E(R_F) = 1$ and $s_L = k_L = k_B = 0$).

[a] A hedge ratio of $h = 0.85$ is equal to the MVH.

[b] Amount positive but smaller than 0.005%.

In table 4, the MVH assumption of deterministic production has been dropped. Because production is not known with certainty at the time of decision making, the hedge ratio is the ratio of the futures position to the expected production level. Similar to the effect of brokerage fees and initial margin deposits, stochastic production reduces optimal hedges and decreases *OC*s of not hedging. In contrast to the scenarios with brokerage fees or initial margin deposits, however, optimal hedge ratios decrease with risk aversion

Table 3. Optimal Hedge Ratios and *OC*s under MVH Assumptions with Positive Initial Margin Deposits

Initial Margin Deposit (% net futures position's absolute value)	Holding Period (months)	Risk Aversion	Optimal Hedge Ratio	Opportunity Cost of Hedging (annual net return as percentage of initial wealth)			
				$h = 0$	$h = 0.5$	$h = 0.85$[a]	$h = 1$
5	1	Low	0.7944	1.61	0.22	0.01	0.11
10	1	Low	0.7388	1.39	0.14	0.03	0.17
5	1	Moderate	0.8315	5.39	0.84	0.00[b]	0.22
10	1	Moderate	0.8129	5.14	0.75	0.01	0.27
5	1	High	0.8444	19.65	3.05	0.00[b]	0.62
10	1	High	0.8389	19.37	2.95	0.00[b]	0.66
5	3	Low	0.7513	0.82	0.09	0.01	0.09
10	3	Low	0.6526	0.62	0.03	0.06	0.17
5	3	Moderate	0.8171	2.93	0.44	0.00[b]	0.14
10	3	Moderate	0.7842	2.69	0.35	0.02	0.20
5	3	High	0.8401	10.60	1.68	0.00[b]	0.37
10	3	High	0.8303	10.34	1.58	0.01	0.42
5	6	Low	0.7642	0.99	0.12	0.01	0.09
10	6	Low	0.6784	0.78	0.05	0.05	0.18
5	6	Moderate	0.8214	3.45	0.52	0.00[b]	0.16
10	6	Moderate	0.7928	3.21	0.44	0.02	0.22
5	6	High	0.8414	12.32	1.98	0.00[b]	0.42
10	6	High	0.8328	12.07	1.88	0.00[b]	0.47

Note: Low, moderate, and high risk aversion are represented by $AW_0 = 1$, $AW_0 = 3$, and $AW_0 = 10$, respectively. Random variables R_P and R_F are bivariate normally distributed with $\rho_{PF} = 0.85$. One-month holding period assumes $E(R_P) = 1.065$ and $SD(R_P) = SD(R_F) = 0.065$; one-quarter holding period assumes $E(R_P) = 1.085$ and $SD(R_P) = SD(R_F) = 0.085$; and semiannual holding period assumes $E(R_P) = 1.130$ and $SD(R_P) = SD(R_F) = 0.130$. Initial margin deposits are assumed to be met with borrowed funds. Values for other unspecified parameters correspond to the standard MVH assumptions (i.e., $s_Q = E(R_F) = 1$ and $b = 0$).

[a] A hedge ratio of $h = 0.85$ is equal to the MVH.

[b] Amount positive but smaller than 0.005%.

in the presence of stochastic production.[14] Also, for moderate and high risk aversion, the reduction in *OC*s due to stochastic production is much larger than that caused by either brokerage fees or initial margin deposits. For example, for moderate risk aversion and one-quarter holding period, *OC*s of not hedging fall by 81% (87) in the presence of low (high) production variability.

Table 5 summarizes the simulation results from gamma and log-normal pdfs. The top half of column 4 reveals that, albeit always smaller, the optimal hedge ratios for the gamma pdf are almost the same as the optimal hedge ratios for the normal pdf (which are identical to the MVHs). Optimal hedges for a gamma pdf and moderate risk aversion range from $h = 0.8484$ through $h = 0.8490$, which compares to MVHs of 0.85. The

[14] This observation implies that, in the presence of stochastic production, infinite risk aversion cannot be used to advocate MVHs.

Table 4. Optimal Hedge Ratios and *OC*s under MVH Assumptions with Stochastic Production

Production Variability	Holding Period (months)	Risk Aversion	Optimal Hedge Ratio	Opportunity Cost of Hedging (annual net return as percentage of initial wealth)			
				$h = 0$	$h = 0.5$	$h = 0.85$[a]	$h = 1$
Low	1	Low	0.8169	0.14	0.02	0.00[b]	0.01
High	1	Low	0.7635	0.12	0.02	0.00[b]	0.01
Low	1	Moderate	0.7390	0.35	0.04	0.01	0.04
High	1	Moderate	0.6060	0.23	0.01	0.04	0.10
Low	1	High	0.4489	0.43	0.01	0.34	0.65
High	1	High	0.3041	0.20	0.08	0.63	1.03
Low	3	Low	0.8162	0.24	0.04	0.00[b]	0.01
High	3	Low	0.7620	0.21	0.02	0.00[b]	0.02
Low	3	Moderate	0.7369	0.59	0.06	0.01	0.08
High	3	Moderate	0.6025	0.40	0.01	0.07	0.17
Low	3	High	0.4445	0.73	0.01	0.60	1.14
High	3	High	0.3007	0.33	0.14	1.10	1.79
Low	6	Low	0.8148	0.56	0.08	0.00[b]	0.03
High	6	Low	0.7586	0.49	0.06	0.01	0.05
Low	6	Moderate	0.7322	1.37	0.14	0.04	0.18
High	6	Moderate	0.5951	0.90	0.02	0.17	0.42
Low	6	High	0.4366	1.69	0.04	1.51	2.80
High	6	High	0.2945	0.76	0.37	2.69	4.33

Note: Low, moderate, and high risk aversion are represented by $AW_0 = 1$, $AW_0 = 3$, and $AW_0 = 10$, respectively. Random variables R_P and R_F are bivariate normally distributed with $\rho_{PF} = 0.85$. One-month holding period assumes $E(R_P) = 1.065$ and $SD(R_P) = SD(R_F) = 0.065$; one-quarter holding period assumes $E(R_P) = 1.085$ and $SD(R_P) = SD(R_F) = 0.085$; and semiannual holding period assumes $E(R_P) = 1.130$ and $SD(R_P) = SD(R_F) = 0.130$. Random variable q is independently distributed as beta, with parameters $\alpha = 6.5248$ (4.5988) and $\beta = 2.1741$ (4.4407) for the low (high) production variability scenario. Values for other unspecified parameters correspond to the standard MVH assumptions (i.e., $s_Q = E(R_F) = 1$ and $b = s_L = k_L = k_B = 0$).

[a] A hedge ratio of $h = 0.85$ is equal to the MVH.

[b] Amount positive but smaller than 0.005%.

similarity between the optimal hedge ratios from gamma and normal pdfs is even more evident when *OC*s are considered. For moderate risk aversion, the maximum absolute difference between gamma and normal *OC*s is merely 0.12% (=3.70 − 3.58) of initial wealth per year, achieved at zero hedging and a semiannual holding period. *OC* difference between gamma and normal scenarios become noticeable only under very suboptimal hedges and extreme risk aversion. The maximum absolute difference in *OC*s between gamma and normal pdfs is 1.26 cents per dollar (=12.58 − 11.32) of initial wealth per year (attained at zero hedging, a semiannual holding period, and extreme risk aversion).

The bottom half of table 5 reports the results from simulations using a log-normal pdf. Although optimal hedge ratios under log-normally distributed prices are generally different from MVHs, here they are identical because a quadratic utility function is being used. Therefore, caution should be exercised when comparing the log-normal figures with those of the other simulations. Most of the comments made about the normal pdf results in table 1 apply to the log-normal pdf results as well. The major difference between the two sets of simulations is that *OC*s are almost always smaller under the log-

Table 5. Optimal Hedge Ratios and *OC*s under MVH Assumptions with Alternative Probability Density Functions for R_P and R_F

Probability Density Function	Holding Period (months)	Risk Aversion	Optimal Hedge Ratio	Opportunity Cost of Hedging (annual net return as percentage of initial wealth)			
				$h = 0$	$h = 0.5$	$h = 0.85$[a]	$h = 1$
Gamma[c]	1	Low	0.8497	1.84	0.31	0.00[b]	0.06
Gamma[c]	1	Moderate	0.8490	5.58	0.93	0.00[b]	0.17
Gamma[c]	1	High	0.8467	19.27	3.06	0.00[b]	0.60
Gamma[c]	3	Low	0.8496	1.04	0.18	0.00[b]	0.03
Gamma[c]	3	Moderate	0.8487	3.12	0.52	0.00[b]	0.10
Gamma[c]	3	High	0.8453	10.30	1.71	0.00[b]	0.35
Gamma[c]	6	Low	0.8495	1.21	0.21	0.00[b]	0.04
Gamma[c]	6	Moderate	0.8484	3.58	0.61	0.00[b]	0.12
Gamma[c]	6	High	0.8442	11.32	1.94	0.00[b]	0.42
Log-normal[d]	1	Low	0.8500	0.16	0.03	0.00	0.00[b]
Log-normal[d]	1	Moderate	0.8500	0.58	0.10	0.00	0.02
Log-normal[d]	1	High	0.8500	3.04	0.84	0.00	0.14
Log-normal[d]	3	Low	0.8500	0.29	0.05	0.00	0.01
Log-normal[d]	3	Moderate	0.8500	1.07	0.18	0.00	0.03
Log-normal[d]	3	High	0.8500	5.88	1.83	0.00	0.71
Log-normal[d]	6	Low	0.8500	0.70	0.12	0.00	0.02
Log-normal[d]	6	Moderate	0.8500	3.26	0.52	0.00	0.09
Log-normal[d]	6	High[e]	0.8500	10.72	4.23	0.00	1.80

Note: Low, moderate, and high risk aversion are represented by $AW_0 = 1$, $AW_0 = 3$, and $AW_0 = 10$, respectively. Random variables R_P and R_F are bivariate normally distributed with $\rho_{PF} = 0.85$. One-month holding period assumes $E(R_P) = 1.065$ and $SD(R_P) = SD(R_F) = 0.065$; one-quarter holding period assumes $E(R_P) = 1.085$ and $SD(R_P) = SD(R_F) = 0.085$; and semiannual holding period assumes $E(R_P) = 1.130$ and $SD(R_P) = SD(R_F) = 0.130$. Values for other unspecified parameters correspond to the standard MVH assumptions (i.e., $s_Q = E(R_F) = 1$ and $b = s_L = k_L = k_B = 0$).

[a] A hedge ratio of $h = 0.85$ is equal to the MVH.

[b] Amount positive but smaller than 0.005%.

[c] Random variables R_p and R_F follow a bivarate gamma pdf.

[d] Random variables R_p and R_F follow a bivariate log-normal pdf. Utility function is hypothesized to be quadratic.

[e] A value of $AW_0 = 7.5$ is used because *OC*s are not positive real numbers when $AW_0 > 7.5$.

normal pdf than under the normal pdf. In summary, the findings from the simulations employing gamma and log-normal pdfs suggest that the effect of relaxing the assumption of normally distributed prices is negligible.

The results from slackening the assumption that all the initial wealth is invested in the cash position are shown in table 6. From looking at the effects of dropping each of the standard MVH assumptions individually, it is clear that letting decision makers have a large alternative investment has the greatest impact on *OC*s. Because of the presence of a large alternative investment (e.g., other crop or livestock enterprises, or mutual funds), *OC*s decrease so much that the *OC*s of not hedging often become almost negligible. By reducing the cash position to 5% of the initial wealth, the *OC* of not hedging drops by 99% (98, 96) to only 0.05 cents per dollar (0.08, 0.15) of initial wealth per year for moderate risk aversion and one-month (one-quarter, semiannual) holding period.

The total impact of the alternative investment on *OC*s can be conceptualized as the

Table 6. Optimal Hedge Ratios and *OC*s under MVH Assumptions with an Alternative Investment

Cash Position's Share of Initial Wealth (%)	Holding Period (months)	Risk Aversion	Optimal Hedge Ratio	Opportunity Cost of Hedging (annual net return as percentage of initial wealth)			
				$h = 0$	$h = 0.5$	$h = 0.85$[a]	$h = 1$
50	1	Low	0.8938	0.51	0.10	0.00[b]	0.01
25	1	Low	0.9814	0.15	0.04	0.00[b]	0.00[b]
5	1	Low	1.6821	0.02	0.01	0.00[b]	0.00[b]
50	1	Moderate	0.8938	1.53	0.30	0.00[b]	0.02
25	1	Moderate	0.9814	0.46	0.11	0.01	0.00[b]
5	1	Moderate	1.6821	0.05	0.03	0.01	0.01
50	1	High	0.8938	5.18	0.99	0.01	0.07
25	1	High	0.9814	1.54	0.37	0.03	0.00[b]
5	1	High	1.6821	0.18	0.09	0.04	0.03
50	3	Low	0.9514	0.33	0.07	0.00[b]	0.00[b]
25	3	Low	1.1543	0.12	0.04	0.01	0.00[b]
5	3	Low	2.7771	0.03	0.02	0.01	0.01
50	3	Moderate	0.9514	0.98	0.22	0.01	0.00[b]
25	3	Moderate	1.1543	0.36	0.12	0.02	0.01
5	3	Moderate	2.7771	0.08	0.06	0.04	0.03
50	3	High	0.9514	3.31	0.74	0.04	0.01
25	3	High	1.1543	1.21	0.39	0.08	0.02
5	3	High	2.7771	0.28	0.19	0.13	0.11
50	6	Low	0.9845	0.41	0.10	0.01	0.00[b]
25	6	Low	1.2536	0.17	0.06	0.02	0.01
5	6	Low	3.4062	0.05	0.04	0.03	0.02
50	6	Moderate	0.9845	1.23	0.30	0.02	0.00[b]
25	6	Moderate	1.2536	0.50	0.18	0.05	0.02
5	6	Moderate	3.4062	0.15	0.11	0.08	0.07
50	6	High	0.9845	4.14	0.99	0.08	0.00[b]
25	6	High	1.2536	1.67	0.60	0.17	0.07
5	6	High	3.4062	0.49	0.36	0.28	0.24

Note: Low, moderate, and high risk aversion are represented by $AW_0 = 1$, $AW_0 = 3$, and $AW_0 = 10$, respectively. Random variables R_P, R_F, and R_I are trivariate normally distributed with $\rho_{PF} = 0.85$ and $\rho_{PI} = \rho_{FI} = 0.30$. One-month holding period assumes $E(R_P) = 1.065$, $SD(R_P) = SD(R_F) = 0.065$, and $SD(R_I) = 0.0095$; one-quarter holding period assumes $E(R_P) = 1.085$, $SD(R_P) = SD(R_F) = 0.085$, and $SD(R_I) = 0.0287$; and semiannual holding period assumes $E(R_P) = 1.130$, $SD(R_P) = SD(R_F) = 0.130$, and $SD(R_I) = 0.0583$. Values for other unspecified parameters correspond to the standard MVH assumptions (i.e., $E(R_F) = 1$ and $b = s_L = k_L = k_B = 0$).

[a] A hedge ratio of $h = 0.85$ is equal to the MVH.

[b] Amount positive but smaller than 0.005%.

sum of two separate effects, namely, a "diversification" effect and a "dilution" effect. The diversification effect is attributable to the risk reduction achieved by having returns from the cash position less than perfectly associated with returns from the alternative investment. In contrast, the dilution effect is due to the fact that, because the cash position in the presence of an alternative investment is only a fraction of the initial wealth, the *OC* of suboptimal hedging is also a fraction of the *OC* found when all initial wealth was invested in the cash position. In the present simulations, the diversification effect can be measured as the *OC* obtained when $\rho_{PI} \to 1$ minus the *OC* calculated with $\rho_{PI} = 0.30$; the dilution effect is simply the difference between the total effect and the diversification effect. Contrary to what intuition might suggest, it can be shown theoretically that all of the reduction in *OC*s from the alternative investment is caused by dilution as opposed to diversification (see appendix).

Interestingly and somewhat paradoxically, table 6 shows that optimal hedge ratios in the presence of an alternative investment are larger than MVHs. Optimal hedge ratios for diversified agents with $s_Q = 0.25$ are 0.9814, 1.1543, and 1.2536 for one-month, one-quarter, and semiannual holding periods, respectively, in contrast to MVHs of 0.85 for all three holding periods. The reason why diversification actually increases optimal hedge ratios is that in this example the returns on the alternative investment are assumed to be positively correlated with the returns on futures ($\rho_{FI} = 0.3$). Hence, futures contracts can be used to reduce total risk because they partly offset the riskiness of the alternative investment. Proof of this assertion is that optimal hedge ratios increase as the cash position's share of initial wealth decreases. Had it been assumed a negative (zero) correlation between returns on futures and returns on the alternative investment, the optimal hedge ratio in the presence of an alternative investment would have been lower than (equal to) the MVH.[15]

Relaxing standard MVH assumptions one at a time allows us to conclude that all of the constraints investigated thus far tend to reduce the *OC*s of not hedging when slackened individually. Our results suggest that, in terms of affecting the *OC*s of not hedging, the most important MVH restrictions are that production is deterministic and that there are no alternative (large) investments. In particular, the dilution due to a large alternative investment seems to have the greatest impact on *OC*s. Our analysis also indicates that relaxing standard MVH constraints one at a time yields optimal hedge ratios smaller than MVHs, except for the scenario with a positively correlated alternative investment. In this instance, optimal hedge ratios are actually greater than MVHs because of the alternative investment's positive correlation with futures.

Further insight can be gained by dropping more than one MVH constraint at a time. The major findings of such an exercise can be summarized as follows.[16] First, typical brokerage fees may greatly reduce optimal hedge ratios and *OC*s of zero hedging for agents holding an alternative investment. The reason for this effect is that the *OC*s of hedging suboptimally are much lower in the portfolio context. Therefore, typical brokerage fees are relatively far more important when agents hold an alternative investment.[17] Second, when coupled with an alternative investment, typical initial margin deposits have an impact qualitatively similar to, but quantitatively smaller than, that of typical brokerage fees. This finding is not surprising, given the individual effect of brokerage fees and initial margin deposits in the scenario without an alternative investment (tables 2 and 3). Third, as expected from the individual effect of stochastic production (table 4), random production in the presence of an alternative investment reduces both optimal hedges and *OC*s of not hedging. Fourth, compared with having an alternative investment without borrowings (table 6), allowing for debt either increases or leaves

[15] This means that an alternative investment generally renders MVHs invalid even under the old justification of infinite risk aversion.

[16] Specific results are not reported because of space limitations, but they are available from the author upon request.

[17] In other words, brokerage fees seem small in absolute terms but become large relative to the *OC*s of not hedging, which yields an almost negligible net benefit from hedging.

unchanged both optimal hedges and *OC*s of not hedging. This effect occurs because agents borrow to enlarge their holdings of the alternative investment. These added holdings increase the agents' risk exposure, which can be partly offset by additional hedging.[18] Extremely risk-averse agents, however, find it optimal not to borrow under some circumstances; in these instances the optimal hedge and the *OC*s are the same as if borrowings were not allowed.

Conclusions

Relaxing the standard MVH assumptions one at a time reveals that the most important of such assumptions are that (*a*) production is deterministic and that (*b*) all of the agent's initial wealth is invested in the cash position. Stochastic production causes important reductions in both optimal hedges and *OC*s of not hedging, particularly so for higher levels of risk aversion. Allowing decision makers to hold an alternative investment results in optimal hedge ratios that are larger (smaller) than MVHs if the alternative investment is positively (negatively) correlated with futures. At the same time, however, an alternative investment greatly reduces the *OC*s of not hedging, mostly because the alternative investment "dilutes" the cash position, that is, the latter becomes less important to the agent in economic terms.

By greatly decreasing the *OC*s of zero hedging, allowing for an alternative investment or for stochastic production has far-reaching implications for hedging behavior. The reduction in the *OC*s of zero hedging renders the costs associated with hedging (e.g., brokerage fees) much more important in relative terms, thus yielding the net benefits of hedging almost negligible. This means that the costs associated with hedging, typically ignored in hedging models because of their seemingly negligible size, are important determinants of hedging behavior when agents hold only a fraction of their wealth as the cash position or when production is random.

Our analysis may help explain why most farmers do not hedge. Most farmers are involved in a set of activities that have imperfectly correlated returns and usually face great production variability. In addition, hedging costs for typical farmers are relatively high because they not only include brokerage fees and initial margin deposits but also other costs more difficult to measure but equally relevant. Examples of such costs are the opportunity cost of the time employed to follow the futures market, the additional complexity of tax returns when entering futures transactions, and the possibility of facing margin calls. Under this scenario, our model predicts that the costs of hedging quite likely outweigh its benefits, thus rendering hedging unattractive.

Our findings may also help explain why specialized firms such as grain elevators find it more attractive to hedge than not to hedge. Such firms hold most of their wealth in the form of inventories, and these are characterized by little randomness. At the same time, their "true" hedging cost is only slightly above brokerage fees and margin requirements because of their specialized resources. In addition, they usually face smaller brokerage fees (for example, they can negotiate fees as a result of their large volume of operations). Under such conditions, our model predicts that in most likelihood a substantial portion of the cash position will be hedged.

[18] Recall that returns on futures are positively correlated with returns on the alternative investment.

In addition to being helpful at explaining actual hedging behavior, our analysis is useful in that it points out limitations of MVH recommendations. In particular, it shows that MVHs are likely to be far from optimum when agents hold alternative investments along with the cash position, or when production is stochastic. Furthermore, in such contexts optimal hedges are extremely sensitive to the costs associated with hedging. In fact, seemingly low hedging costs may well cause expected-utility, maximizing hedge ratios to be zero.

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Appendix: Closed-Form Solution for *OC*s

For constant absolute risk aversion, OC is given by

$$OC = \frac{1}{AW_0}\ln\left\{\frac{E[-\exp(-AW_0R_{alt})]}{E[-\exp(-AW_0R_{opt})]}\right\}. \tag{A1}$$

By assuming additionally that q is deterministic, $s_L = k_L = k_B = 0$, and R_P, R_F, and R_I follow a trivariate normal pdf, the expression for expected utility becomes

$$\begin{aligned}(\text{A2})\quad E[-\exp(-AW_0R)] = -&\exp\{[E(R_P) + (1 - E(R_F))hF_0/v(Q_1)]g_Q + E(R_I)g_I\}\\ &\exp\{[(\sigma_P^2 - \sigma_F^2h^2F_0^2/v(Q_1)^2)g_Q^2 + \sigma_I^2g_I^2]/2\}\\ &\exp\{[\rho_{PI}\sigma_P\sigma_Ig_I - (\rho_{PF}\sigma_Pg_Q + \rho_{FI}\sigma_Ig_I)\sigma_FhF_0/v(Q_1)]g_Q\},\end{aligned}$$

where $g_Q \equiv -AW_0s_Q$, $g_I \equiv -AW_0s_I$, and $\sigma_i \equiv SD(R_i)(i = P, F, I)$. After some manipulation, substituting (A2) into (A1) yields

$$\begin{aligned}(\text{A3})\quad OC = &\frac{1}{2}\left[\sigma_F\frac{F_0}{v(Q_1)}AW_0s_Q\right]^2(h_{alt}^2 + h_{opt}^2)\\ &- [1 - E(R_F) + (\rho_{PF}\sigma_Ps_Q + \rho_{FI}\sigma_Is_I)AW_0\sigma_F]\frac{F_0}{v(Q_1)}AW_0s_Qh_{alt},\end{aligned}$$

where h_{opt} is the optimum hedge ratio and h_{alt} is any alternative hedge ratio. The optimum hedge ratio is obtained by setting the derivative of (A2) with respect to h equal to zero and solving the resulting expression for h, which yields

$$h_{opt} = \left[\frac{1 - E(R_F)}{AW_0\sigma_F^2s_Q} + \rho_{PF}\frac{\sigma_P}{\sigma_F} + \rho_{FI}\frac{\sigma_I}{\sigma_F}\frac{s_I}{s_Q}\right]\frac{v(Q_1)}{F_0}. \tag{A4}$$

It is clear from (A3) and (A4) that both OC and h_{opt} are independent of ρ_{PI}. Therefore, the diversification effect (which is measured as the OC obtained when $\rho_{PI} \to 1$ minus the OC calculated with $\rho_{PI} = 0.30$) is also independent of ρ_{PI}.