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Abstract

We test the microstructure invariance proposed by Kyle and Obizhaeva (2016) in the grain markets. Using the CME's intraday best-bid-offer data from 2008 to 2015, we find support for both trade size invariance and trading cost invariance at 1-minute, 5-minute, and 10-minute, although not in its original form. After rescaling the trading activity by spread cost per Benzaquen et al (2016), we find strong evidence for both hypotheses of invariance. The findings help understand the trading dynamics of grain commodities from both trading and regulatory perspectives. Specifically, we can derive the number of trades, trading cost, and illiquidity measure based on observable metrics, such as price, volume and historical volatility. These imputed measures can be further used to identify the systematic risks resulting from speculative transactions.

Keywords: Trading Invariance, Grain Futures, Market Microstructure

1 Introduction

It is common knowledge that trading activities, such as trading volume and the number of trades, are positively related to return volatility. But the exact functional relationship among them is not known until Kyle and Obizhaeva's (2016) microstructure invariance hypothesis. They assert through dimension analysis that risk transfer and transaction costs are identical in dollar terms across all assets. A testable hypothesis is that the arrival rate of bets is proportional to the 2/3 power of the risk transfer, i.e. product of expected dollar volume and return volatility. Their original empirical study validated the hypothesis using a proprietary portfolio transition dataset on stocks at the daily interval. We propose to test the trading invariance using a public dataset on agricultural commodities, namely corn, soybeans and wheat futures, at the intraday interval.

Understanding endogenous trading dynamics, such as bet sizing, bet arrival rate (number of bets per day), return volatility and liquidity (Kyle and Obizhaeva (2019)), is crucial to market participants as large orders and order splitting become an important aspect of the current electronic market. According to Haynes and Roberts (2015), automated trading was involved in 68.7%, 57.7% and 64.4% of corn, soybeans and wheat futures trading. The introduction of Globex electronic trading platform for grain futures in 2006 accelerated the financialization of agricultural commodities and facilitated large traders' participation by reducing their market impact and trading cost. Irwin and Sanders (2012) reported that open interests held by large/institutional traders accounted for about 90% in grain futures.

Kyle and Obizhaeva's microstructure invariance hypothesis provides a powerful framework for connecting such economic variables as trading volume, return volatility, number of transactions, and trade size. To the best of our knowledge, there has not been empirical test of intraday trading invariance for grain futures. The previous empirical research has been focused on stock portfolio transition by Kyle and Obizhaeva (2016), on intraday E-mini SPX futures from 2008 to 2011 by Andersen et al. (2016), and on 12 financial and commodity futures from 2012 to 2014 by Benzaquen et al. (2016). The first two studies confirmed the trading invariance within a single market, while the last one found that trading invariance did not hold true in this original form across different markets. Neither of the last two studies examined the invariance of the trading cost, as they focused only on the number of trades.

We aim to fill the gap by first testing the intraday trading size invariance across the three similar yet different electronic grain futures markets. We hypothesize, as with Benzaquen et al. (2016), that there is a need to modify the invariance form to accommodate the difference across the three commodities. We further contribute to the literature by testing the invariance of trading cost and by examining both trading invariances at different intraday intervals and over different sample periods.

We use a simple linear regression to test the invariances of trade size and trading cost, along the lines of Kyle and Obizhaeva (2016). We regress the number of trades and the relative bid-ask spread on trading activity (a multiplication of price, volume and volatility), as opposed to variance per transaction vs. the average number of trades. We consider both the trading activity in its original form as in Kyle and Obizhaeva (2016) and the modified form as in Benzaquen et al. (2016).

We will employ the intraday BBO (Best Bid Offer) data from 2008 to 2015 obtained from the CME Group. We aggregate the second-by-second data to 1-minute, 5-minute and 10-minute interval. we find support for both trade size invariance and trading cost invariance at 1-minute, 5-minute, and 10-minute, although not in its original form. After rescaling the trading activity by spread cost per Benzaquen et al (2016), we find strong evidence for both hypotheses of invariance. The finding helps understand the trading dynamics of grain commodities from both trading and regulatory perspectives. Specifically, we can derive the number of trades, trading cost, and illiquidity measure based on observable metrics, such as price, volume and historical volatility. These imputed measures can be further used to identify the systematic risks resulting from speculative transactions.

The rest of the paper is organized as follows. We first describe the microstructure invariance theory by Kyle and Obizhaeva (2016) in Section 2. We develop three hypotheses related to intraday market microstructure invariance in Section 3. We report the data and empirical results for the hypothesis tests in Section 4, and conclude in Section 5.

2 Market Microstructure Invariance

There are two approaches to develop market microstructure invariance: meta-model approach (Kyle and Obizhaeva (2016)) and dimensional analysis (Kyle and Obizhaeva (2017)). Both approaches produce similar results, although the latter makes less assumptions about microeconomic foundations in a theoretical model. We reproduce Kyle and Obizhaeva's (2016)) basic results here using the following notations, with dimensions expressed in the bracket. The easy-to-observe variables include:

$$\begin{aligned} Price &= P \quad [dollars/unit] \\ Trading\ Volume &= V \quad [units/day] \\ Returns\ Volatility &= \sigma \quad [day^{1/2}] \end{aligned}$$

and the following hard-to-measure variables:

$$\begin{aligned} Fundamental\ Value &= F \quad [dollars/unit] \\ Bet\ Size &= Q \quad [units] \\ Bet\ Number\ or\ Velocity &= \gamma \quad [/day] \\ Price\ Change/Bet &= \triangle P \quad [dollars/unit] \\ Price\ Impact &= \lambda \quad [dollars/unit^2] \\ Price\ Error &= \sqrt{var(log\frac{F}{P})} \quad [dimensionless] \\ Price\ Resiliency &= \rho \quad [dimensionless] \end{aligned}$$

Assuming a power function for price impact and transaction cost (denoted C) invariance, a meta-model can be written as follows:

$$\triangle P = \lambda Q^{\beta} \tag{1}$$

$$V = \gamma E[|Q|] \tag{2}$$

$$\sigma^2 = \gamma E[(\frac{\triangle P}{P})^2] \tag{3}$$

$$E[(\triangle P)^2] = \lambda^2 E[|Q|^{2\beta}] \tag{4}$$

$$C = \lambda E[|Q|^{1+\beta}] \tag{5}$$

$$m = \frac{E[|Q|]\sqrt{E[|Q|^{2\beta}]}}{E[|Q|^{1+\beta}]} \tag{6}$$

$$m_{\beta} = \frac{(E[|Q|])^{1+\beta}}{E[|Q|^{1+\beta}]} \tag{7}$$

where the last three variables are invariant across markets, therefore considered constant. The solution to key hard-to-observe variables, illiquidity $1/L$, expected bet value $E[|PQ|]$, number of bets γ, price impact in percentage $\frac{\triangle P}{P}$, and scaled bet value Z can be expressed respectively as follows:

$$\frac{1}{L} = \frac{C}{E[|PQ|]} = \Big(\frac{\sigma^2 C}{m^2 PV}\Big)^{1/3} \tag{8}$$

$$Z = \frac{|PQ|}{E[|PQ|]} \tag{9}$$

$$E[|PQ|] = C \cdot L \tag{10}$$

$$\gamma = \frac{1}{m^2}\sigma^2 L^2 \tag{11}$$

$$\frac{\triangle P}{P} = \frac{1}{L} m_\beta |\frac{Q}{E[|Q|]}|^\beta \tag{12}$$

where L and Z are dimensionless.

In order to solve the hard-to-observe variables, we first invoke the market microstructure invariance, hypothesizing that (1) the dollar distribution of the gains or losses from bets (dollar risk transfer) is the same across all markets when measured in units of business time:

$$I_B = P \cdot Q \cdot \frac{\sigma}{\gamma^{1/2}} \tag{13}$$

and (2) the dollar cost of executing bets is the same function of their risk transfers I_B across all markets: $C_B(I_B)$, where the subscript B stands for bet. So is the average (or unconditional mean) cost of executing bets $C_B = E[C_B(I_B)]$.

We then define trading activity W_B as dollar volume per bet adjusted for volatility. It is a measure of gross risk transfer:

$$W_B = \sigma \cdot P \cdot V = \sigma \cdot P \cdot E[|Q|] \cdot \gamma$$

Based on the two measures of risk transfer (I_B and W_B), we can express number of bets γ, relative bet size $\frac{Q}{V}$, and percentage cost of executing a bet $C(Q)$ as

$$\gamma = W_B^{2/3} \cdot \{E[|I_B|]\}^{-2/3} \tag{14}$$

$$\frac{Q}{V} = W_B^{-2/3} \cdot \{E[|I_B|]\}^{-1/3} \cdot I_B \tag{15}$$

$$E[|Q|] = W_B^{1/3} \cdot \frac{1}{P\sigma} \cdot \{E[|I_B|]\}^{2/3} \tag{16}$$

$$C(Q) = \frac{C_B(I_B)}{P|Q|} = \frac{1}{L} \cdot f(I_B) \tag{17}$$

where $f(I_B) = \frac{C_B(I_B)}{I_B}\frac{E[I_B]}{E[C_B(I_B)]}$ is invariance average price impact function. Equation (14) will become a basis of our test for invariance of risk transfer.

For different average impact functions $f(I)$, the cost function will take different forms, but has two components: bid-ask spread cost and market impact cost. For a linear model the average impaction function and the cost function are derived as

$$f(I) = [E|I|]^{-1/3}\kappa + [E|I|]^{-2/3}\lambda \cdot |I|$$
$$C(Q) = \sigma\Big[\kappa W^{-1/3} + \lambda E[W]^{1/3}\frac{|Q|}{E[V]}\Big]$$

For a square root model, the average impaction function and the cost function are derived as

$$f(I) = [E|I|]^{-1/3}\kappa + [E|I|]^{-1/2}\lambda \cdot |I|^{1/2}$$
$$C(Q) = \sigma\Big[\kappa W^{-1/3} + \lambda\sqrt{\frac{|Q|}{E[V]}}\Big]$$

In either case, the bid-ask spread cost scaled by volatility σ is proportional to trading activity $W^{-1/3}$. If the spread cost is measured in percentage, we have the linear relationship between spread cost and trading activity:

$$\frac{\frac{BAS}{P}}{\sigma} \propto W^{-1/3} \tag{18}$$

Equation (18) will become of our test for invariance of trading cost.

3 Hypothesis Development

The invariance that underlies Kyle and Obizhaeva (2016) is based on bets, not necessarily to intraday trades. As with Andersen et al. (2016), we can obtain intraday trading invariance by assuming that the average number of transactions per bet is invariant across assets and time, i.e. the proportional relationship between number of trades N and number of bets γ: $N \sim \gamma$. We can restate Equation 13 for any intraday time interval t.

$$I_t = P_t \cdot Q_t \cdot \frac{\sigma_t}{\gamma_t^{1/2}} \tag{19}$$

where the definition of all variables remain the same except that they are defined for the non-overlapping intraday interval, as opposed to daily interval. As such, we hypothesize that such invariance can broadly hold true to trades.

Hypothesis 1: number of trades N, adjusted for differences in trading activity W, are the same across different commodities.

Based on Equation 15, the invariance of bet size lies in $ln\Big(\frac{|Q|}{V} \cdot \big[W\big]^{2/3}\Big)$ or equivalently $ln\Big(N \cdot \big[W\big]^{2/3}\Big)$ per Equation 14. By substituting order size X for bet size Q, we can cast the trading invariance in the following testable regression format:

$$ln[\frac{V}{X}] = -ln[q] + \alpha_1 \cdot ln\big[W\big] + \epsilon \tag{20}$$

$$ln\big[N\big] = const + \alpha_1 \cdot ln\big[W\big] + \epsilon \tag{21}$$

where V is trading volume, X is order size, N is the number of trades (a proxy for the number of bet γ in Equation (11)), W is trading activity, q is median size of liquidity trade. The second equation is a simplification of the first equation, where the intercept *const* is a measure of bet number or risk transfer for a benchmark asset. The null hypothesis is that $\alpha_1 = 2/3$.

Hypothesis 2: trading cost as measured by quoted relative bid-ask spread, scaled by volatility, does not vary across difference commodities.

Quoted bid-ask spread in dollars S corresponds to the first component of the cost function $C(Q)$. The invariance hypothesis from Equation 18 implies the relative bid-ask spread scaled by volatility is proportional to trading activity.

$$ln[\frac{BAS}{P * \sigma}] = const + \alpha_2 \cdot ln\big[W\big] + \epsilon \tag{22}$$

where BAS is the quoted bid-ask spread, P is trade price, σ is volatility and W is trading activity. The intercept *const* can be interpreted as trading cost for a benchmark asset. The null hypothesis is that $\alpha_2 = -1/3$.

4 Data and Results

4.1 Data

The Trade and Quote data for corn, soybeans, and wheat futures are obtained from We obtain from the CME (Chicago Mercantile Exchange) Group. The CME intraday data is stamped to second. The time period ranges from January 2008 to May 2015. We use only the nearby futures contract for our empirical analysis.

As noted by Andersen et al. (2016), the CME group reports all contracts traded at a particular price as a single transaction, therefore treating the incoming order as a whole.
Table I presents summary statistics for the three commodities at 1-minute interval. Five variables are reported in the table: last trade price, number of trades, trading volume, number of contracts per trade, bid-ask spread in ticks. As with Andersen et al. (2016), we average these variables at the 1-minute interval and then across all sample days in order to minimize the sample noise. Average prices for corn, soybeans and wheat during the sample period are \$5.18/bushel, \$12.31/bushel, and \$6.56/bushel, respectively. It is clear from Table I that all variables, with the exception of price, show varying degree of skewness and kurtosis.

Table I: **Summary Statistics**

Summary statistics are based on the averages at the 1-minue interval over the whole sample period from 2008 to 2015.

Commodity	Variable	Obs	Mean	Std Dev	Min.	Med.	Max.	Skew.	Kurt.
Corn	P	225	5.18	0.01	5.15	5.18	5.21	-0.02	-0.08
Corn	N	225	74.44	70.28	42.60	58.02	980.96	10.10	125.36
Corn	V	225	212.65	202.11	116.30	162.84	2433.89	7.65	74.09
Corn	X	225	3.24	0.18	3.00	3.21	4.38	2.69	11.97
Corn	BAS	225	2.23	0.05	2.04	2.23	2.53	2.06	9.52
Soybeans	P	225	12.31	0.02	12.27	12.31	12.37	0.05	0.52
Soybeans	N	225	65.53	42.29	38.02	54.37	505.27	6.23	55.82
Soybeans	V	225	134.00	95.58	73.79	105.88	991.48	5.41	39.71
Soybeans	X	225	2.08	0.07	1.96	2.07	2.39	1.34	3.03
Soybeans	BAS	225	2.77	0.16	2.57	2.73	3.70	2.84	11.12
Wheat	P	225	6.56	0.01	6.52	6.56	6.59	-0.28	0.04
Wheat	N	225	47.10	46.50	25.81	36.01	626.70	9.31	109.85
Wheat	V	225	93.17	101.95	48.42	68.88	1324.25	8.86	98.84
Wheat	X	225	2.03	0.08	1.87	2.02	2.52	1.83	6.48
Wheat	BAS	225	2.54	0.11	2.40	2.51	3.45	4.14	26.94

Figures 1-5 show the average intraday trade price, number of trades, order size, volatility, and bid-ask spread for corn, soybeans and wheat, respectively. Except the price, all other metrics of trading activity show the well-known U-shape of intraday seasonality, namely higher activity at the beginning and the end of the trading day. On a relative basis, corn leads soybeans and wheat in terms of higher trading and liquidity based on the number of trades (Figure 2) and order size (Figure 3), and bid-ask spread (Figure 5).

4.2 Empirical Results: Baseline Case

We ran the two regressions to test for intraday trading invariance in the grain commodity markets:

$$ln[N] = const + \alpha_1 \cdot ln[W] + \epsilon \tag{23}$$

$$ln[\frac{BAS}{P * \sigma}] = const + \alpha_2 \cdot ln[W] + \epsilon \tag{24}$$

where N is the number of trades, W is the trading activity or dollar risk transfer, BAS is the bid-ask spread, P is the commodity price, and σ is volatility. The baseline case is the whole dataset sampled at the 1-minute interval. The first equation (23) examines the invariance of trade size whereas the second equation (24) tests for the invariance of trading cost. The intercept term *const* in the first regression can be interpreted as an average number of trades

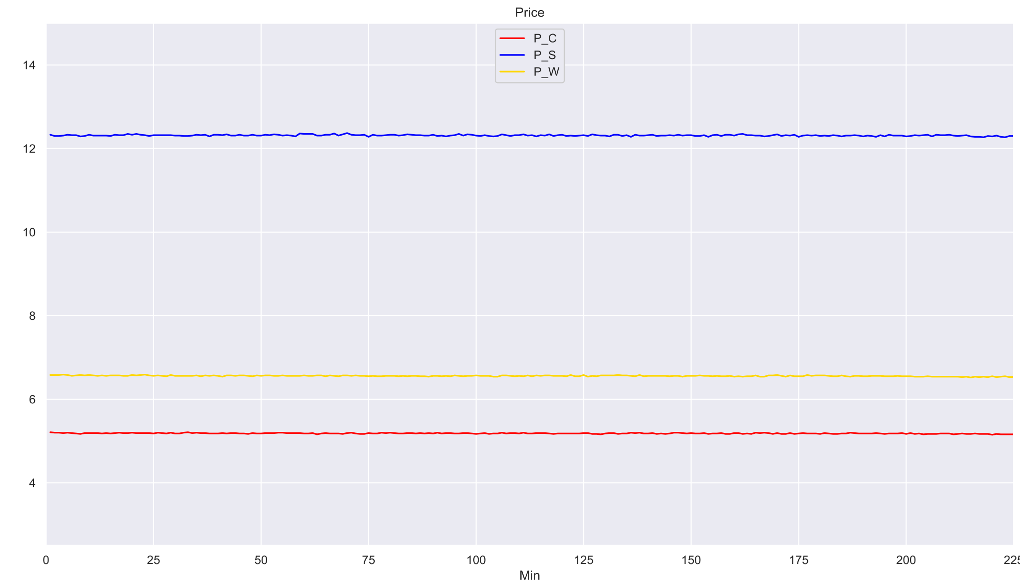

Figure 1: Price

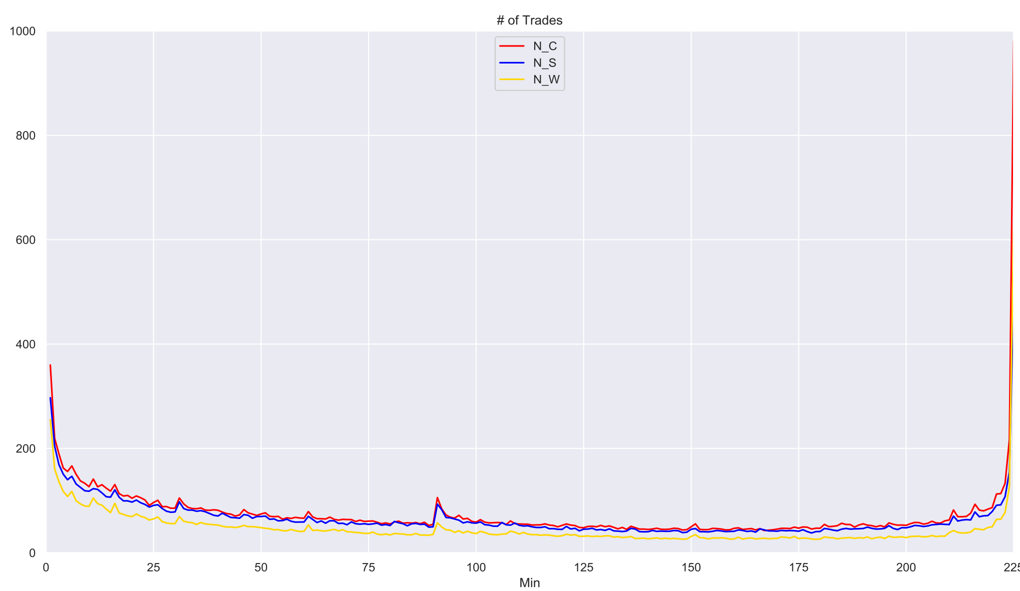

Figure 2: Number of Trades

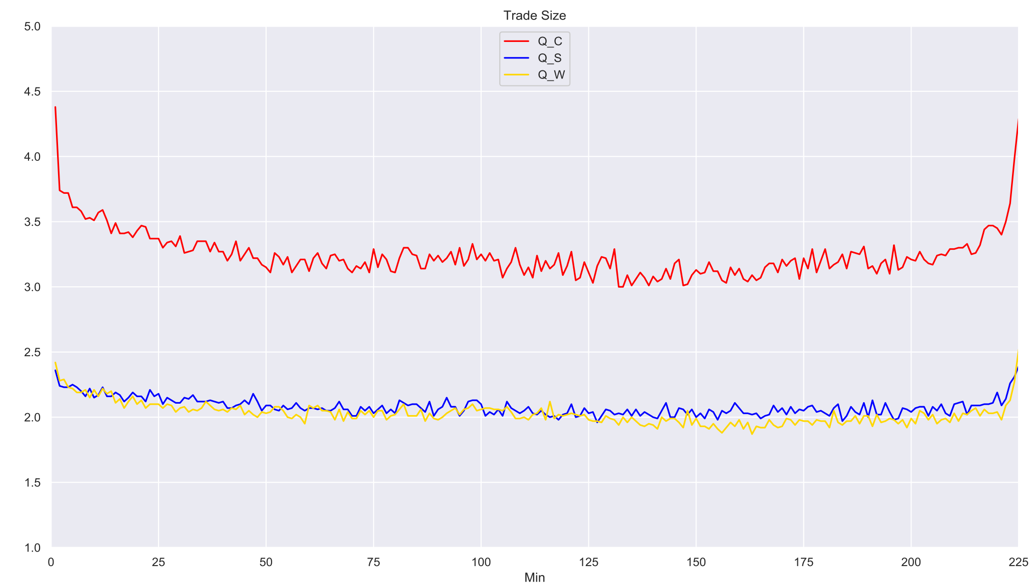

Figure 3: Order Size

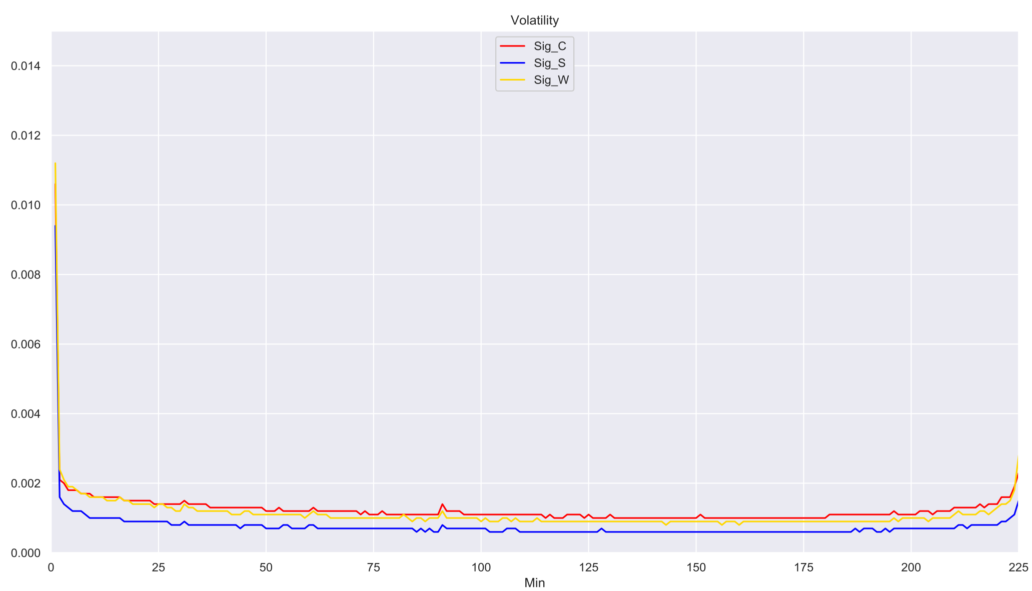

Figure 4: Volatility

Figure 5: Bid-Ask Spread

or an inverse measure of risk transfer. The intercept term *const* in the second regression can be interpreted as an average (benchmark) trading cost, conditional on trading activity. We also ran the same regressions with a scaled version of trading activity $W^* = \frac{W}{BAS*Q}$, where the denominator measures spread cost of trading following Benzaquen et al (2016). They argued that the trading invariance should be dimensionless as opposed to Kyle and Obizhaeva's (2016) dollar cost invariance.

The parameter estimates for Equation 23 are reported in Table II, with the upper panel for trading activity W being the regressor and the lower panel for the scaled trading activity W^*. As we can see from the upper panel, the coefficient for lnW is 0.60, 0.57, and 0.60, respectively for corn, soybeans and wheat. All estimates are statistically significant at the 0.01 level. The fact that they are all similar to each other supports the invariance hypothesis of trade size. However, they are slightly less than the hypothesized value of 2/3 based on the original definition of trading activity W. The lower panel shows that the coefficient for lnW^*, the scaled version of trading activity W is 0.66, 0.64 and 0.66, respectively for corn, soybeans and wheat. The estimates based on W^* are all close to 2/3, clearly supporting the invariance of trade size as in Hypothesis 1. Another evidence for the invariance of trade size is from the intercept term, which is a proxy for the invariance I. The average (logarithm) number of trades ranges from 4.05 to 4.11 using W and from 5.16 to 5.41 using W^*. Using either measure of trading activity, the estimated invariance measure is comparable across three commodities, once again supporting Hypothesis 1.

The parameter estimates for Equation 24 are reported in Table III, with the upper panel for

Table II: **Parameter Estimates for H1: Trade Size Invariance**

Regression results are based on the averages at the 1-minue interval over the whole sample period from 2008 to 2015. The dependent variable is number of trades and the regressor lnW (lnW^*) is logarithm of (scaled) trading activity.

Commodity	Variable	Estimate	Std Err	t Value	$Pr > \|t\|$
Corn	const	4.11	0.01	675.91	<.0001
	ln W	0.60	0.01	65.39	<.0001
Soybeans	const	4.05	0.01	709.19	<.0001
	ln W	0.57	0.01	65.05	<.0001
Wheat	const	4.07	0.01	592.78	<.0001
	ln W	0.60	0.01	81.75	<.0001
Corn	const	5.41	0.02	250.9	<.0001
	ln W*	0.66	0.01	59.27	<.0001
Soybeans	const	5.16	0.02	283.36	<.0001
	ln W*	0.64	0.01	62.32	<.0001
Wheat	const	5.19	0.02	275.01	<.0001
	ln W*	0.66	0.01	81.57	<.0001

trading activity W being the regressor and the lower panel for the scaled trading activity W^*. The upper panel reports the coefficient for lnW is -0.31, -0.30, and -0.31, respectively for corn, soybeans and wheat. The fact that they are all similar to each other supports the invariance hypothesis of trading cost. However, these values are different from the hypothesized value of -1/3 based on the original definition of trading activity W. The empirical results support invariance of trading cost across commodities, but not in its original form. The lower panel shows that the coefficient for lnW^*, the scaled version of trading activity W is 0.34 for all three commodities. The estimates based on W^* are all close to -1/3, supporting the invariance of trading cost as in Hypothesis 2. As with the case of trade size, we also find supporting evidence of invariance based on the intercept terms, average trading cost for a benchmark asset.

4.3 Empirical Results: Different Time Interval and Subsamples

To check if the hypotheses fare with alternative intraday frequency and subsample periods, we run the regressions with data at 5-m and 10-m intervals, and with 2008-2011 subsample and 2012-2015 subsample. Table IV reports the parameter estimates for Hypothesis 1 using 5-minute and 10-minute data. The 5-minute and 10-minute coefficients for lnW are similar across the three commodities, despite being slightly smaller than the predicted value of 2/3. Furthermore, the lower the frequency the smaller the coefficient. This finding is consistent with Benzaquen et al. (2016), who attribute the underestimation at lower frequency to the bias induced by close-to-close volatility estimator. Therefore the 5-minute and 10-minute

Table III: **Parameter Estimates for H2: Trading Cost Invariance**

Regression results are based on the averages at the 1-minute interval over the whole sample period from 2008 to 2015. The dependent variable is logarithm of bid-ask spread scaled by volatility and the regressor lnW (lnW^*) is logarithm of (scaled) trading activity.

Commodity	Variable	Estimate	Std Err	t Value	$Pr > \|t\|$
Corn	const	5.93	0.01	1130.19	<.0001
	ln W	-0.31	0.01	-38.59	<.0001
Soybeans	const	5.76	0.01	1117.12	<.0001
	ln W	-0.30	0.01	-38.32	<.0001
Wheat	const	5.71	0.01	1009.86	<.0001
	ln W	-0.31	0.01	-51.72	<.0001
Corn	const	5.26	0.02	335.23	<.0001
	ln W*	-0.34	0.01	-42.14	<.0001
Soybeans	const	5.16	0.02	344.63	<.0001
	ln W*	-0.34	0.01	-40.62	<.0001
Wheat	const	5.13	0.01	344.98	<.0001
	ln W*	-0.34	0.01	-54.08	<.0001

results largely confirm our earlier finding with 1-minute data.

Table V reports the parameter estimates for Hypothesis 2 using 5-minute and 10-minute data. Again, the 5-minute and 10-minute coefficients for lnW are similar across the three commodities, despite being slightly larger in absolute value than the predicted value of -1/3. Furthermore, the lower the frequency the smaller the coefficient. Therefore the 5-minute and 10-minute results largely confirm our earlier finding with 1-minute data.

Lastly, we examine how the invariance relationship evolves over time. Table VI reports the parameter estimates for Hypothesis 1 using two subsamples. The upper panel employs data from 2008 to 2011, while the lower panel uses data from 2012 to 2015. We find that the invariance of trade size holds within each subsample. The difference between the two subsamples is the larger intercept in more recent years, i.e. lower risk transferred on average. The lower risk transfer in the second subsample is consistent with the statistical facts that there is an increase in the number of trades per minute and an decrease in the order size for all three commodities.

Table VII reports the parameter estimates for Hypothesis 2 using two subsamples. The upper panel employs data from 2008 to 2011, while the lower panel uses data from 2012 to 2015. Again, we confirm the invariance of trading cost for both subsamples across the three commodities. The difference between the two subsamples is the slightly larger intercept in more recent years, i.e. higher benchmark trading cost on average, which seems to be inconsistent with rising liquidity over the years. Since the difference is relatively small,

Table IV: **Parameter Estimates for H1: 5-minute and 10-minute intervals**

Regression results are based on the averages at the 5-minute and the 10-minute intervals over the whole sample period from 2008 to 2015. The dependent variable is number of trades and the regressor lnW (lnW^*) is logarithm of (scaled) trading activity.

Commodity	Variable	Estimate	Std Err	t Value	$Pr > \|t\|$
Corn	const	4.28	0.06	68.87	<.0001
5-m	ln W	0.58	0.02	24.14	<.0001
Soybeans	const	4.33	0.06	77.07	<.0001
5-m	ln W	0.53	0.02	23.20	<.0001
Wheat	const	4.22	0.04	111.40	<.0001
5-m	ln W	0.58	0.02	29.77	<.0001
Corn	const	5.46	0.02	243.51	<.0001
5-m	ln W*	0.61	0.03	21.70	<.0001
Soybeans	const	5.30	0.02	256.68	<.0001
5-m	ln W*	0.58	0.03	22.42	<.0001
Wheat	const	5.21	0.01	370.91	<.0001
5-m	ln W*	0.64	0.02	31.76	<.0001
Corn	const	4.46	0.13	33.29	<.0001
10-m	ln W	0.55	0.04	14.77	<.0001
Soybeans	const	4.57	0.11	41.98	<.0001
10-m	ln W	0.49	0.03	15.46	<.0001
Wheat	const	4.37	0.09	51.17	<.0001
10-m	ln W	0.55	0.03	18.91	<.0001
Corn	const	5.59	0.07	78.02	<.0001
10-m	ln W*	0.56	0.05	12.46	<.0001
Soybeans	const	5.48	0.06	94.46	<.0001
10-m	ln W*	0.53	0.04	14.04	<.0001
Wheat	const	5.27	0.04	124.50	<.0001
10-m	ln W*	0.62	0.03	18.37	<.0001

we further test for the significance of a time trend in the regression (Equation 24) using the whole sample. We find the statistically insignificant coefficient for the trend factor, hence no consistent support for increase in trading cost over time.

5 Conclusion

We test the microstructure invariance proposed by Kyle and Obizhaeva (2016) in the grain futures markets. Using the CME's intraday best-bid-offer data from 2008 to 2015, we find support for both trade size invariance and trading cost invariance at the 1-minute inter-

Table V: **Parameter Estimates for H2: 5-minute and 10-minute intervals**

Regression results are based on the averages at the 1-minute interval over the whole sample period from 2008 to 2015. The dependent variable is logarithm of bid-ask spread scaled by volatility and the regressor lnW (lnW^*) is logarithm of (scaled) trading activity.

Commodity	Variable	Estimate	Std Err	t Value	$Pr > \|t\|$
Corn	const	6.00	0.05	112.46	<.0001
5-m	ln W	-0.35	0.02	-17.24	<.0001
Soybeans	const	5.88	0.05	114.35	<.0001
5-m	ln W	-0.34	0.02	-16.51	<.0001
Wheat	const	5.68	0.03	201.09	<.0001
5-m	ln W	-0.31	0.01	-21.70	<.0001
Corn	const	5.29	0.02	328.53	<.0001
5-m	ln W*	-0.38	0.02	-18.87	<.0001
Soybeans	const	5.24	0.02	300.08	<.0001
5-m	ln W*	-0.38	0.02	-17.45	<.0001
Wheat	const	5.14	0.01	453.64	<.0001
5-m	ln W*	-0.35	0.02	-21.29	<.0001
Corn	const	6.19	0.12	51.48	<.0001
10-m	ln W	-0.40	0.03	-11.91	<.0001
Soybeans	const	6.09	0.11	53.78	<.0001
10-m	ln W	-0.39	0.03	-11.85	<.0001
Wheat	const	5.70	0.07	76.71	<.0001
10-m	ln W	-0.32	0.03	-12.64	<.0001
Corn	const	5.40	0.05	111.82	<.0001
10-m	ln W*	-0.42	0.03	-13.87	<.0001
Soybeans	const	5.38	0.05	109.96	<.0001
10-m	ln W*	-0.43	0.03	-13.61	<.0001
Wheat	const	5.18	0.03	149.27	<.0001
10-m	ln W*	-0.36	0.03	-13.12	<.0001

val, although not in its original form. After rescaling the trading activity by spread cost per Benzaquen et al. (2016), we find stronger evidence for both hypotheses of invariance. The results also hold for 5-minute, 10-minute intervals, and for the two subsamples pre- and post-2011. The confirmation of the microstructure invariance provides a concrete foundation for our understanding of the relationship between trade size (number of trades) and trading activity, and of the relationship between trading cost and trading activity. More specifically, we can measure and monitor with confidence the hard-to-measure variables, such as dollar value of risk transfer (gain/loss) and trading cost (impact) based on easy-to-observe variables, e.g. price, volume, and historical volatility. These imputed measures can be further used to identify the systematic risks resulting from speculative transactions.

Table VI: **Parameter Estimates for H1 with Subsamples**

Regression results are based on the averages at the 1-minute interval over the two subsample period from 2008 to 2011 (denoted "0811") and from 2012 to 2015 (denoted "1215") . The dependent variable is number of trades and the regressor lnW (lnW^*) is logarithm of (scaled) trading activity.

Commodity	Variable	Estimate	Std Err	t Value	$Pr > \|t\|$
Corn	const	3.42	0.00	750.48	<.0001
0811	ln W	0.62	0.01	104.37	<.0001
Soybeans	const	3.57	0.00	733.79	<.0001
0811	ln W	0.62	0.01	94.30	<.0001
Wheat	const	3.57	0.00	788.89	<.0001
0811	ln W	0.62	0.01	118.40	<.0001
Corn	const	5.00	0.02	272.59	<.0001
0811	ln W*	0.67	0.01	78.96	<.0001
Soybeans	const	4.95	0.01	331.54	<.0001
0811	ln W*	0.69	0.01	87.77	<.0001
Wheat	const	4.96	0.02	316.23	<.0001
0811	ln W*	0.69	0.01	108.49	<.0001
Corn	const	4.67	0.01	826.79	<.0001
1215	ln W	0.67	0.01	68.35	<.0001
Soybeans	const	4.52	0.01	686.73	<.0001
1215	ln W	0.60	0.01	55.92	<.0001
Wheat	const	4.61	0.01	499.34	<.0001
1215	ln W	0.64	0.01	77.72	<.0001
Corn	const	5.52	0.01	380.82	<.0001
1215	ln W*	0.70	0.01	69.08	<.0001
Soybeans	const	5.33	0.02	290.10	<.0001
1215	ln W*	0.64	0.01	54.83	<.0001
Wheat	const	5.36	0.02	305.66	<.0001
1215	ln W*	0.66	0.01	80.14	<.0001

Table VII: **Parameter Estimates for H2 with subsamples**

Regression results are based on the averages at the 1-minute interval over the two subsample period from 2008 to 2011 (denoted "0811") and from 2012 to 2015 (denoted "1215") . The dependent variable is logarithm of bid-ask spread scaled by volatility and the regressor lnW (lnW^*) is logarithm of (scaled) trading activity.

Commodity	Variable	Estimate	Std Err	t Value	$Pr > \|t\|$
Corn	const	5.89	0.01	929.52	<.0001
0811	ln W	-0.31	0.01	-37.54	<.0001
Soybeans	const	5.68	0.01	1059.39	<.0001
0811	ln W	-0.28	0.01	-38.66	<.0001
Wheat	const	5.65	0.00	1145.69	<.0001
0811	ln W	-0.29	0.01	-51.27	<.0001
Corn	const	5.08	0.02	300.90	<.0001
0811	ln W*	-0.35	0.01	-43.88	<.0001
Soybeans	const	5.04	0.01	359.26	<.0001
0811	ln W	-0.32	0.01	-42.83	<.0001
Wheat	const	4.99	0.01	351.23	<.0001
0811	ln W*	-0.33	0.01	-57.08	<.0001
Corn	const	5.98	0.01	1127.12	<.0001
1215	ln W	-0.30	0.01	-32.78	<.0001
Soybeans	const	5.87	0.01	845.90	<.0001
1215	ln W	-0.33	0.01	-29.55	<.0001
Wheat	const	5.81	0.01	664.09	<.0001
1215	ln W	-0.33	0.01	-42.68	<.0001
Corn	const	5.60	0.01	413.86	<.0001
1215	ln W*	-0.32	0.01	-33.49	<.0001
Soybeans	const	5.41	0.02	298.21	<.0001
1215	ln W*	-0.36	0.01	-31.34	<.0001
Wheat	const	5.42	0.02	318.86	<.0001
1215	ln W*	-0.34	0.01	-43.13	<.0001

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