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Wheat Futures Trading Volume Forecasting and the Value of Extended Trading Hours

by

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Wheat Futures Trading Volume Forecasting and the Value of Extended Trading Hours

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Abstract

Electronic trading in modern commodity markets has extended trading hours, lowered barriers to listing new contracts, broadened participation internationally, and encouraged entry of new trader types, particularly algorithmic traders whose order execution is automated. This paper seeks to understand how these forces have shaped the amount and timing of trading activity using the world's multiple wheat futures markets as a laboratory. To do so, we extend existing volume forecasting models found in the literature on volume weighted average price (VWAP) order execution (e.g. Bialkowski, et al 2008 and Humphery-Jenner 2011) with applications beyond trading algorithm design. We consider a setting with multiple trading venues for related commodities, specifically the front-month Chicago Mercantile Exchange Soft Red Wheat and Paris Euronext Milling Wheat futures contracts. We compare a series of nested forecasting models to infer whether past trading history, intraday volume dynamics, cross market trading activity, and other information are useful predictors of trading activity. We assess the value of extended trading hours and the existence of alternative trading venues by testing whether trading volume is more predictable at particular times throughout the trading day.

Keywords: Trading hours, High-frequency data, Volume predictions.

JEL classification: C51, C52, Q11.

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1 Introduction

Electronic trading in modern commodity markets has extended trading hours, lowered barriers to listing new contracts, and encouraged entry of new trader types, particularly algorithmic traders whose order execution is automated. This paper seeks to understand how these forces have shaped the quantity and timing of trading activity using the world's multiple wheat futures markets as a laboratory. To do so, we begin with the idea that traders, all else equal, wish to trade when markets are active and liquid, thereby minimizing the transaction costs associated with the price impact of large buy or sell orders. Traders may also trade to adjust their risk exposure in response to price and volume activity in their own and other markets. The ability to anticipate or predict the intraday timing of trading activity intensity is therefore of great interest to many types of traders.

Figure 1 shows average trading volume for each minute of the trading day in two commodity futures markets. There is an obvious and substantial periodicity or seasonality in intraday volume, with a U-shaped intraday pattern from the open to close of the overnight and day trading sessions. Traders preference for liquidity may explain some of the observable clustering of trading volume, but there is trading activity throughout both sessions. Trading also spikes during periods of information arrival. For example, figure 1 shows a trading volume spike at 11:00 in both markets coinciding with the release of USDA crop reports.

This paper develops forecasting models for trading volume to better explain these observable patterns and to link them to important issues in commodities trading. If we can predict trading volume at particular times of day and using particular information, we may be able to explain the use of order execution strategies and the role of algorithmic trading, the setting of trading hours and why some commodity markets trade round-the-clock while others don't, and the growing role of global information flows in driving trading and price. Large hedgers or speculators seeking to minimize transactions costs may improve order execution through better volume forecasts. Position traders concerned about price risk during the usually inactive overnight trading session may improve understanding of trading dynamics during that period. While trading hours have expanded since the advent of electronic trading in the mid-1990s, trading hours have been reduced in some commodity markets, most notably in US cattle futures which abandoned overnight trading entirely. Intraday volume forecasts may also provide evidence of the utility of current trading hours.

For our analysis, we begin by applying the empirical methods from [Bialkowski et al. \(2008\)](#); [Humphery-Jenner \(2011\)](#) to intraday Paris and Chicago wheat futures trading data sourced from CQG DataFactory. The sample spans the period from 1 July 2015 to 30 April 2018. We divide trading days into intervals of equal length (commonly 15 minutes) and estimate forecast model parameters using a fixed-size rolling window. Our forecast models increase in complexity and informedness, beginning with a benchmark prediction given by historic intraday volume, as in figure 1. We incorporate own-market univariate dynamic forecasting procedures (e.g. ARMA) and extend earlier efforts using cross-market trading activity, USDA report release times, and other exogenous variables inside these forecast models to incorporate known periods of higher information flow (e.g. ARMAX). Importantly, considering a series of nested models makes comparable the models' estimated parameters and their resulting predictive

power.

Comparing results from multiple forecasts allow us investigate whether a model containing a specific information set improves predictive power at particular times of the trading day, such as at the open or close of trading in a given market, in the overnight relative to the daytime session, or around known information releases. From these comparisons, we can infer whether that information is valued by the market and whether it has value at particular times of the trading day. Most notably we can assess the value of extended trading hours and the existence of alternative trading venues. We test whether trading volume is more predictable in periods when markets trade side-by-side and when cross-market information is incorporated into our forecast. If both of these are true, it suggests that extended hours provide value to market participants and that inter-market arbitrage provides motivation for trader activity.

Broadly, we find evidence of persistence in volume after filtering out historic average intraday trading activity. This intraday persistence suggests there is idiosyncratic information contained in intraday trading activity useful in understanding why traders trade when they do. We find no evidence that cross-market arbitrage is an explanation for this idiosyncratic component including during overnight trading. Even though trading volume is correlated across markets and with USDA report releases, there is significant information contained in market-specific historical trading volume relevant to trading activity at a given time on a given day.

2 Literature Review

The problem of forecasting trading activity is salient to the existing research on volume weighted average price (VWAP) trade execution, first introduced in [Berkowitz et al. \(1988\)](#). The VWAP is a benchmark for large order execution, whereby the trader attempts to match order execution to trading activity as a proxy for liquidity. From an order execution standpoint, the VWAP aims to improve order execution by giving the practitioner the average price for an order of a given size by matching the size of order execution to the volume of trading over a given time window. There is a robust literature considering trading volume and trade impact following the emergence of time-series models suitable for intraday empirical applications and the growing availability of high-frequency data (e.g. [Konishi, 2002](#); [Darrat et al., 2003](#); [Manganelli, 2005](#)).

The fundamental problem for an VWAP algorithm is to anticipate upcoming trading volume to allocate portions of a given order across time. This paper extends these earlier VWAP forecasting efforts along multiple dimensions with applications beyond trading algorithm design. First, we consider a setting with multiple trading venues for related commodities, specifically the front-month Paris Euronext Milling Wheat and Chicago Mercantile Exchange Soft Red Wheat futures contracts. These markets are geographically-differentiated with differing trading hours and substantial interest from both physical and financial traders ([Irwin and Sanders, 2011](#); [Cheng and Xiong, 2014](#)). As information flow is nearly continuous but trading occurs in limited hours, when markets are closed, traders cannot receive information from the market and adjust risk exposure in response. Indeed, as shown in [Janzen and Adjemian \(2017\)](#), price discovery is diffuse and trading activity in multiple differentiated but related

Average Volume by Minute of Trading Day, 2015–2018

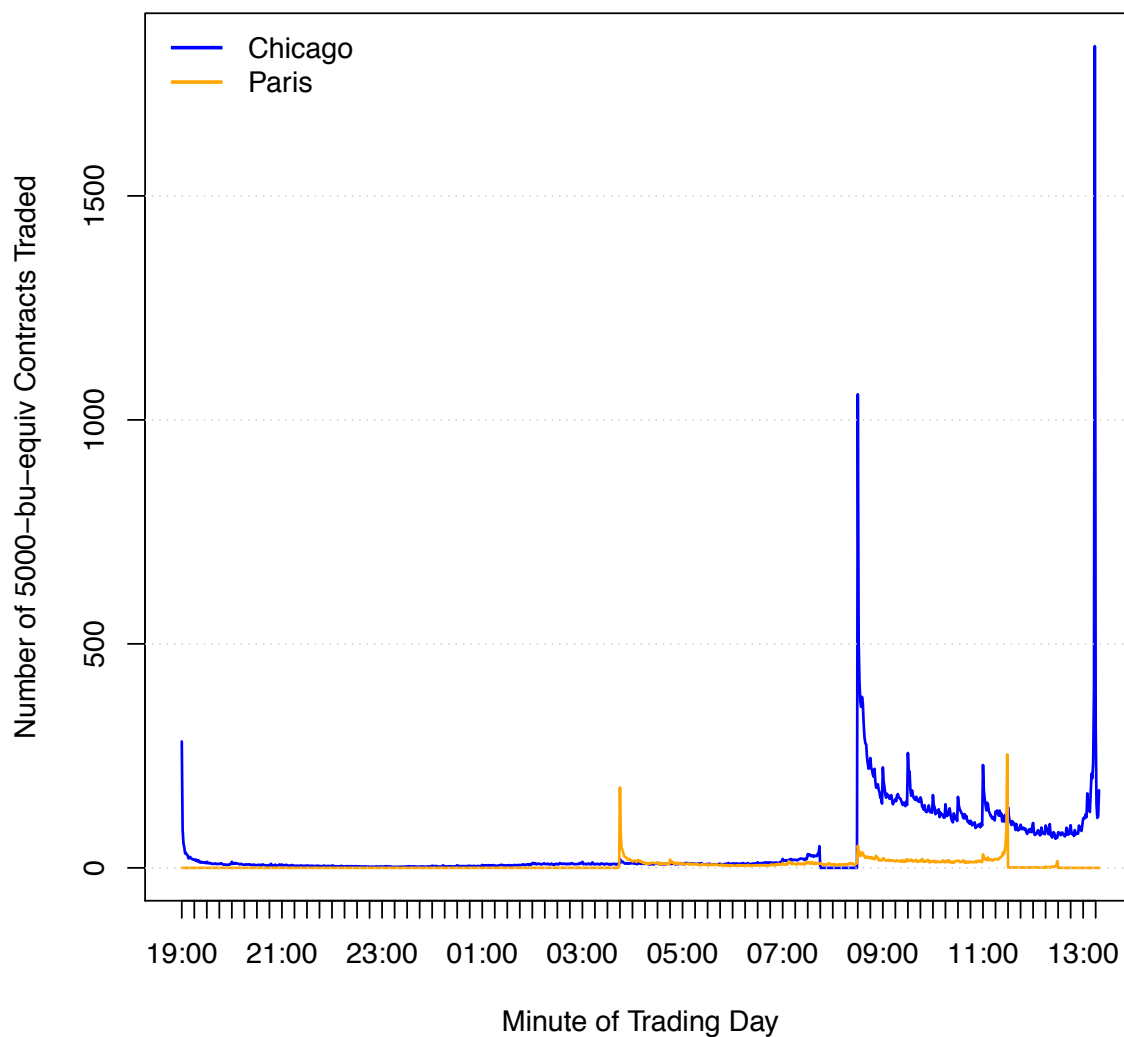


Figure 1: Average wheat futures contract trading volume for Chicago and Paris futures markets by minute of trading day.

Note: Chicago trading hours during this period are 19:00 to 07:45 and 8:30 to 1:15 US Central Time. Paris trading hours are 03:45 to 11:30..

markets may be informative about the true value of the commodity. Both markets see more trading activity on the open and close of trading as the backlogged or anticipated information flow is processed by the market.

The VWAP literature identifies a number of important components of observed volume. In all cases, some measure of historic average volume forms a baseline forecast. Recent work attempts to augment historic average volume using dynamic models. For example, [Bialkowski et al. \(2008\)](#) decompose volume into (i) a (intraday) periodic or seasonal component describing volume for an average stock on an average day obtained from a static historical average, and (ii) a dynamic component which attempts to model unexpected changes in volume specific to each stock. [Bialkowski et al. \(2008\)](#) model the dynamic component using ARMA (1,1) or a SETAR processes. They find the latter better captures volume dynamics due to volatility clustering.

In a similar vein, [Humphery-Jenner \(2011\)](#) presents a Dynamic VWAP model (DVWAP) which aims at better incorporating intraday stock-specific random news through the use of contemporaneous market data. [Brownlees et al. \(2011\)](#) specify a Component Multiplicative Error Model (CMEM) in which the conditional expectation of the VWAP is expressed as the product of a daily, intraday seasonal, and intraday dynamic components. In general, existing empirical applications are confined to equities or derivatives traded in a single market.

3 Data and stylized facts

3.1 Data description

We consider intraday trading volume for the front-month Chicago Mercantile Exchange Soft Red Wheat and Paris Euronext Milling Wheat futures contracts, among the leading global wheat futures market in terms of price discovery ([Janzen and Adjemian, 2017](#)).¹ The sample spans the period from 1 July 2015 to 30 April 2018. We roll from the nearby to first-deferred contract on the 15th day of the month prior to the delivery month. This approximates the most actively traded contract in each market. Chicago and Paris both trade contracts for delivery in March, May, September, and December. Chicago also trades a July expiration, so for most of the year we compare trading in contracts for the same delivery period, with the exception of the period from mid-April to mid-June.

Trading occurs in Chicago in an overnight session between 19:00 and 7:45 and a day session between 8:30 and 13:20 US Central Time. Trading in Paris occurs between 3:45 and 11:30 US Central Time. In analysis of each market, we removed the periods without any trading activity. We also remove trading calendar holidays where one market is closed entirely. Hence, we consider every day where a given market is open without removing additional days due to low trading activity, such as the Friday after Thanksgiving.

To calculate trading volume, we divide the trading day into clock-time-based bins. In the literature we follow, bins of 5-20 minutes are common. Given the trading hours set by the exchanges in our context,

¹For brevity, in the rest of this article, we refer to these futures contracts by the city in which they were founded. For instance, the Chicago market stands for the Chicago Soft Red Winter Wheat futures market.

we can divide trading days into equal clock-time sized bins of 15 minutes. This creates 73 bins between 7:00 pm and 1:15 am, with the post settlement period in Chicago discarded from our analysis. Some of the 15-minute bins inside the 73-bin trading day are notable because we observe greater trading activity during these bins and because this trading activity may be related to trading in a lagged bin that does not immediately precede this bin. This may create serial correlation at atypical lags. Notable bins include:

- Bin 1 (19:00-19:14): Open of Chicago overnight session;
- Bin 36 (3:45-3:59): Open of Paris session;
- Bin 51 (7:30-7:44): Close of Chicago overnight session;
- Bin 55 (8:30-8:44): Open of Chicago day session;
- Bin 65 (11:00-11:14): Release of USDA WASDE and grain stocks reports;
- Bin 66 (11:15-11:29): Close of Paris session;
- Bin 73 (13:00-13:14): Close of Chicago;

Autocorrelation in trading volume may occur at atypical lags when trading in one of these bins is related to trading at a previous bin. For example, we may see abnormal partial autocorrelations at 55 lags if trading volume at the Chicago day open depends activity at the previous days close. Similarly, on USDA report days, we may observe unexpected autocorrelations at 10 and 65 lags if trading activity on the release of the report is related to trading at the open or prior close of Chicago trading.

3.2 Autocorrelation structure analysis

To understand the persistence of trading volume and the potential for autocorrelation at atypical lags, we generate autocorrelation and partial autocorrelation plots for observed trading volume and transformations of trading volume. Following reasoning in [Humphery-Jenner \(2011\)](#), measuring persistence in volume makes a strong case that dynamic forecasting models will provide better forecast performance relative to the static historic average volume forecast. In addition, if autocorrelation patterns obtained from observed volume in a given market (e.g. Chicago) differ from those in historical volume in the same market or those in a different market (e.g., Paris) then information may be gained from the latter to improve forecast accuracy in former.

Consider the observed and historical average autocorrelation profile plotted for two full trading days (i.e., 146 lags) in figure 2. These plots clearly show intraday seasonality with significant and steadily decaying positive persistence over the first 15 lags (i.e., during about 3 hours and 45 minutes). Serial correlation then turns negative for the following 9 hours (from bins 18 to 54), before again taking gradually positive values for the last 3 hours and 45 minutes. This alternation of positive and negative autocorrelation is a result of the typical U-shape behavior of trading activity during the day visible in figure 4, with spiking volumes at the open and close of the market, while trading activity is usually much

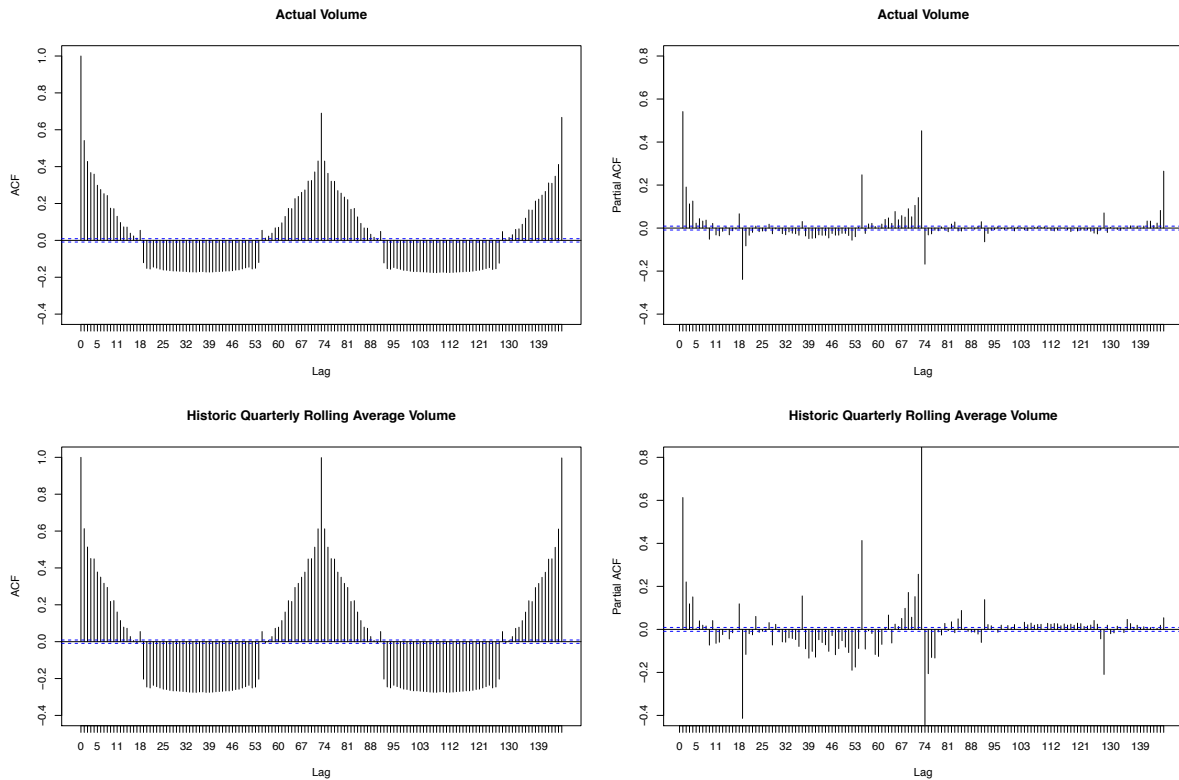


Figure 2: Autocorrelation and partial autocorrelation functions for observed and historic quarterly average trading volume.

more quiet during the rest of the day. Thus, this means that past trading volume can well help predict actual intraday volumes, and especially by capturing the typical seasonality.

Similarly, partial autocorrelation in observed volume is generally positive for recent lags and for the same time of day on previous days, suggesting for example, that volume at the open of trading is correlated with volume at the open on the previous day. Significant partial autocorrelation at lags of 65-75 also suggests trading in the day session may be more closely related to trading activity during the previous day session than during the intervening overnight trading session.

To consider the potential for forecast improvement, we consider autocorrelation in seasonally differenced trading volume, where we subtract historical quarterly average volume in a given trading bin from actual volume. These autocorrelations are shown in the top panels of figure 3. We also consider the difference between trading volume in another related market, the Paris wheat futures market, and Chicago trading volume. These differencing operations reduce serial correlation, suggesting both may help to forecast trading volume at particular times of day.

In sum, analysis of autocorrelation in intraday volume generally finds significant serial correlation in intraday volume and between volume and its historic lags. Thus, from a modeling point of view, the forecasting model we should employ can be a relatively parsimonious ARMA model on seasonally differenced data, in line with previous papers.

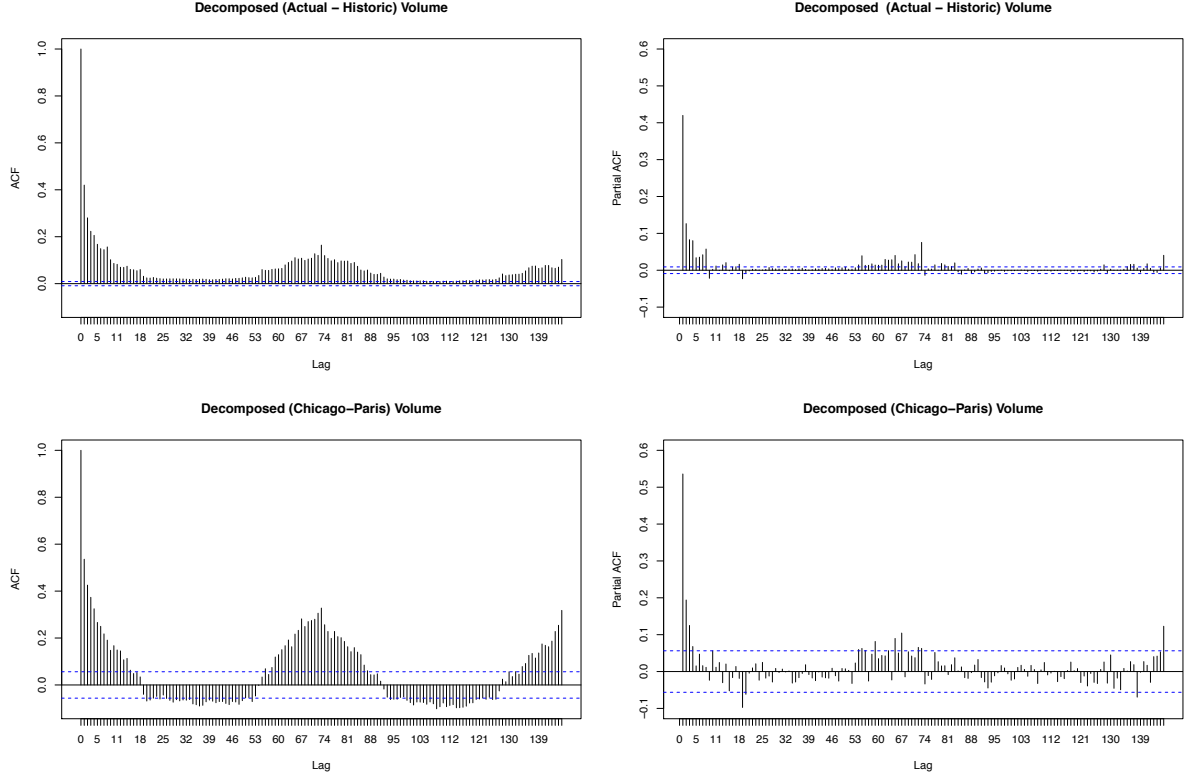


Figure 3: Autocorrelation and partial autocorrelation functions for Chicago deviations from historical trading volume and Chicago deviations from Paris historical trading volume.

4 Model

We start with the simple additive autoregressive structure of the trading volume model introduced by [Bialkowski et al. \(2008\)](#). They decompose volume V in period i of trading day t into seasonal historic and dynamic components denoted H and D respectively. We augment this simple model with additional information X , so the full forecasting model has three factors:

$$V_{i,t} = H + D + X. \quad (1)$$

Because of differences in context, we specify each of these components in ways that differ from previous applications to volume forecasts. In general, in our study of commodity rather than equity markets, we are unable to consider cross-section of equity trading activity. Our model specification follows the general rule that the forecast cannot incorporate information that is not available to traders at bin i on day t . For this reason we consider volume in log-levels, rather than trying to forecast the proportion of daily volume trading in bin i as in [Humphery-Jenner](#). Taking the logarithm of bin-specific volume accommodates scaling issues in the data, as volume in some bins is orders of magnitude larger than in others.

To model the historic or seasonal component H in volume, we consider the bin-specific moving average of trading volume in a given market over the past quarter (i.e., 62 trading days). This is a sufficiently long period to be representative of historical volume patterns, capturing the long-term

behavior and low-frequency movements.² Since we consider a single commodity, it is not possible to model the seasonal part by extracting a common factor across many markets as in [Bialkowski et al.](#). Instead we rely on our longer-run average to consistently estimate the intraday volume pattern

The dynamic component D is considered to be a stationary process in log-levels and modeled using low-order autoregressive processes similar to the existing literature. As in [Bialkowski et al.](#) and [Humphery-Jenner](#), our preliminary graphical analysis of autocorrelation and partial autocorrelation in volumes above suggests rapidly diminishing persistence that can be captured by a low-order ARIMA process. Following [Bialkowski et al.](#), we select a parsimonious ARMA(1,1).³

Finally, we include other covariates X that may provide additional information about the expected amount of trading volume. Variables in X include trading volume in other wheat futures markets in prior bins and indicators for irregular, non-seasonal periods of higher trading volume, for example the release of USDA crop reports. For example, volume in other markets must be lagged one bin because it is simultaneously determined with $V_{i,t}$ and so cannot be used for forecasting.

We denote with lowercase letters the variables expressed in equation (1) so that the actually estimated model is:

$$\begin{aligned} v_{i,t} &= h_{i,t} + d_{i,t} + x_{i,t} \\ &= \frac{1}{L} \sum_{l=1}^L v_{i,(t-l)} + \alpha + \sum_{k=1}^p \phi_p c_{i-k,t} + x_{i,t} + \sum_{j=1}^q \theta_j \varepsilon_{i-j,t}, \end{aligned} \quad (2)$$

with L the number of past trading days used to compute the historical rolling average, p and q respectively the number of autoregressive and moving average lags retained in the ARIMA process with intercept α . For each day t in our sample, we estimate equation (2) using data from the previous quarter. We use this model to generate one-bin-ahead forecast trading volume for each bin on day $t + 1$, allowing the dynamic component of the model to adjust forecast trading volume in bin i on that day to observed activity up to bin $i - 1$. These bin-ahead forecasts also incorporate information from observable covariates including cross-market trading volume up to bin $i - 1$.

5 Estimation Results

We generate forecasted trading volume for each bin on each trading day in our sample. Because the benchmark historic average bin-specific volume forecast requires one quarter of data to estimate, we cannot generate forecasts for the period prior to October 2015. Our main results are mean absolute forecast errors from the remaining days in our sample. In general, these results confirm those of the existing literature on intraday volume forecasting; dynamic forecasts that consider lagged intraday information are superior to static, historic average volume forecasts.

To understand the intraday variation in our forecasts, we plot mean absolute errors by trading bin in

²In the context of seasonally produced commodity such as wheat, the 20-day rolling window considered by [Bialkowski et al.](#) and [Humphery-Jenner](#) might be too short to consistently measure periodic elements of trading volume.

³We also considered automating model selection based on statistical AIC and BIC criteria, but this produced only small gains in forecast accuracy and greatly increased time required to estimate the models. Furthermore, such an automated procedure does not suit well a rolling regression setting since the selected model can vary from one subsample to another.

figure 4. We have hypothesized that some information may matter more for predicting (and therefore understanding) trading volume at different times of the trading day. We want to assess the degree to which trading volume is predictable at particular times of day. Additionally, we want to assess whether Paris trading volume may help predict Chicago trading volume during periods where both markets trade and whether knowing a USDA report is being released helps to predict trading volume around release times.

In general, figure 4 shows trading volume is more predictable around the open and close of trading and more predictable in the day session than the overnight session. Adding information about cross-market trading volume contributes little to improve forecast accuracy. Cross-market arbitrage may not be an important driver of volume, or such arbitrage may occur so quickly that it does not provide additional useful information at a 15-minute lag. Knowing the timing of USDA report releases does improve forecast accuracy, but only in the bin where the report is released. In the context of a dynamic model,

Note that the differences presented in figure 4 are differences in the logarithm of volume so they approximate a percentage change. Therefore, differences in forecast volume may be larger in levels during periods of extremely high volume such as the open and close, but these differences are small as a percentage of volume in that bin. In contrast, large forecast errors occur during the overnight in part because the denominator of these comparisons, actual bin-specific volume, is extremely small.

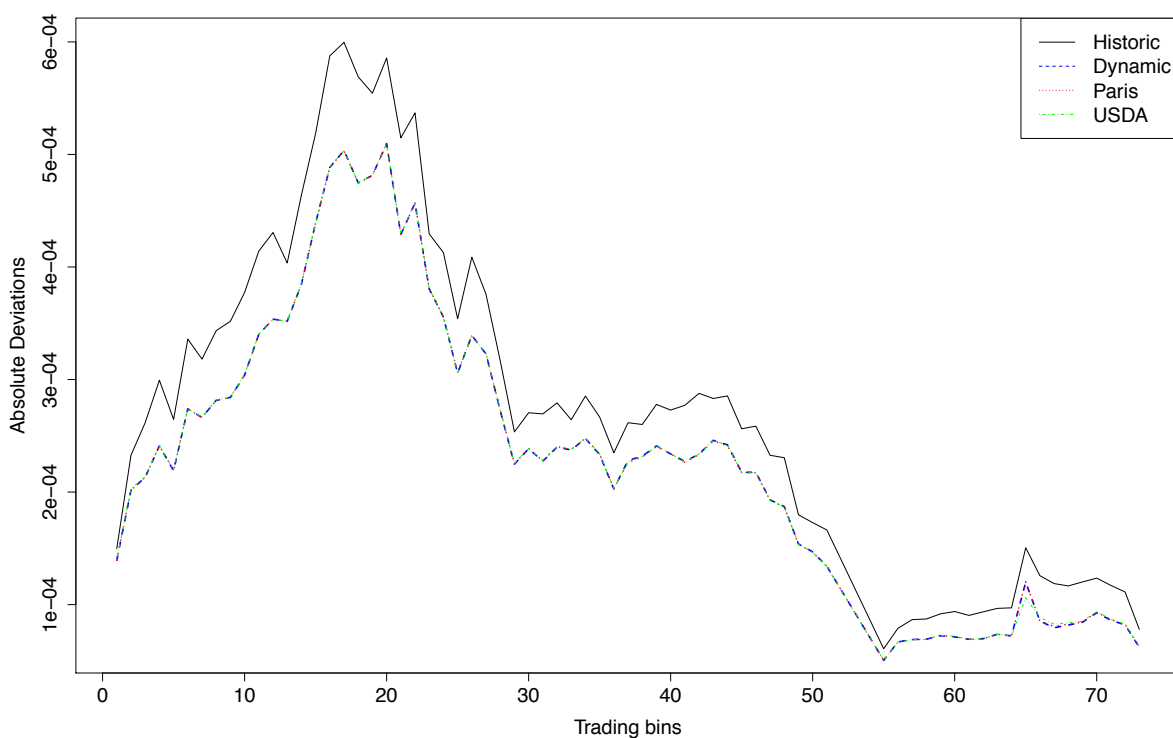


Figure 4: Mean absolute error in forecasts of Chicago wheat futures trading volume by bin of trading day

Note: Each line plots the mean absolute deviation from observed volume through the trading day for four forecast models that incorporate increasing amounts of information to generate bin-ahead forecasts. The benchmark 'Historic' forecast uses only historic average bin-specific volume. 'Dynamic' adds intraday lagged volume. 'Paris' adds lagged cross-market volume. 'USDA' adds information about the timing (but not content) of USDA crop report releases.

To consider the robustness of these results, we generate similar forecasts using separately estimated night and day session forecast models and for Paris trading volume where Chicago volume is used as the lagged cross-market volume covariate. These results are found in figures 5 and 6, respectively. These results are qualitatively similar to those found in figure 4. Dynamic forecast models generate more accurate forecasts with other information providing little additional improvement in forecast accuracy.

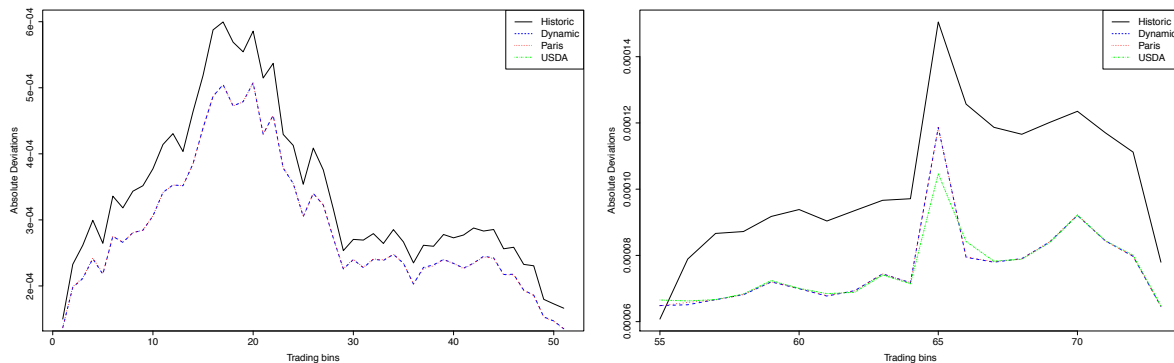


Figure 5: Mean absolute error in forecasts of Chicago wheat futures trading volume by bin of trading day for separately estimated day and night session models

Note: Each line plots the mean absolute deviation from observed volume through the trading day for four forecast models that incorporate increasing amounts of information to generate bin-ahead forecasts. These forecasts are generated using separately estimated forecast models for day and night sessions.

6 Conclusion

We extend existing volume forecasting models in order to better understand known intraday patterns in commodity futures trading volume, including the U-shaped pattern in daytime trading activity and the relative lack of trading in overnight markets. We find some evidence of persistence in volume but no evidence for cross-market arbitrage as an explanation for overnight trading. Even though trading volume is correlated across markets and with USDA report releases, there is significant information contained in market-specific historical trading volume relevant to trading activity at a given time on a given day.

This result relies on the assumption maintained throughout that information enters the forecasting model linearly and that bin-specific comparisons of future work should incorporate cross-market and other information in a non-linear, non-constant and timely manner. It should also explore the nature of the scaling problem in trading volume data. Null results in this context may simply speak to the difficulty in extracting useful signal from data with substantial intraday variability. When volume can change from minute to minute by orders of magnitude, forecasting is difficult.

References

- Berkowitz, S. A., Logue, D. E. and Noser, E. A. (1988). The total cost of transactions on the NYSE. *The Journal of Finance*, 43(1), 97–112.
- Bialkowski, J., Darolles, S. and Fol, G. L. (2008). Improving VWAP strategies: A dynamic volume approach. *Journal of Banking & Finance*, 32(9), 1709 – 1722.

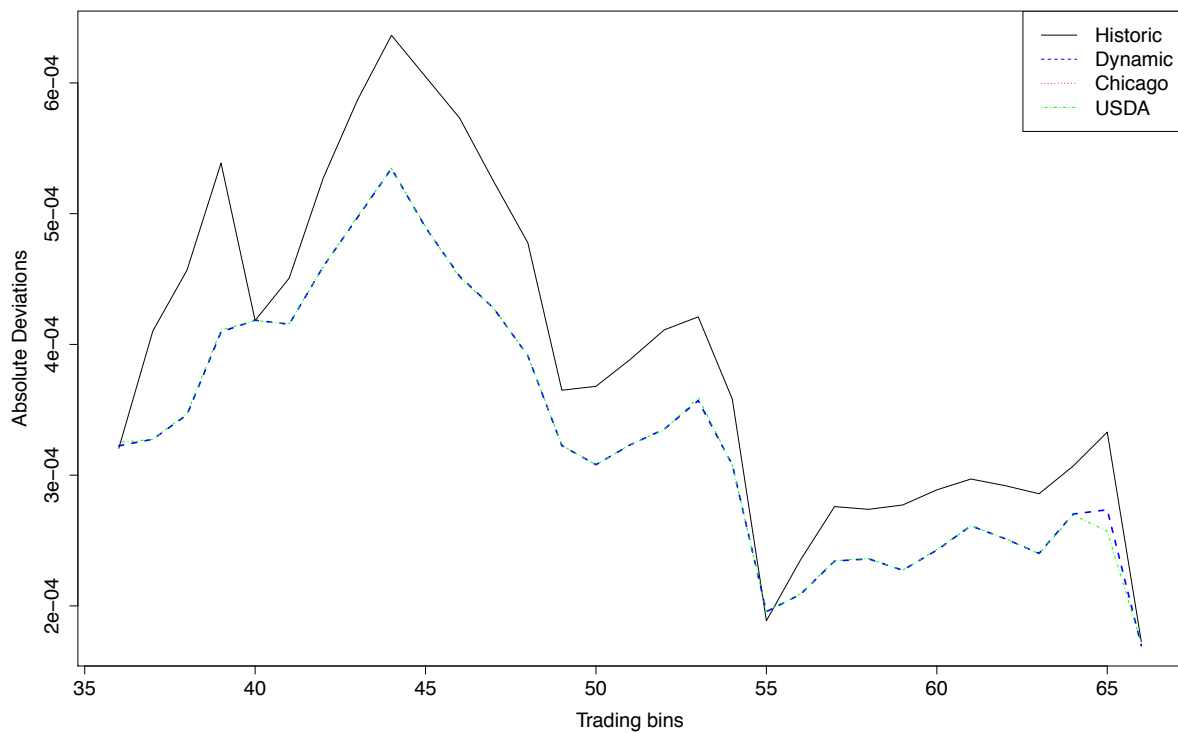


Figure 6: Mean absolute error in forecasts of Paris wheat futures trading volume by bin of trading day
Note: Each line plots the mean absolute deviation from observed volume through the trading day for four forecast models that incorporate increasing amounts of information to generate bin-ahead forecasts. The benchmark ‘Historic’ forecast uses only historic average bin-specific volume. ‘Dynamic’ adds intraday lagged volume. ‘Chicago’ adds lagged cross-market volume. ‘USDA’ adds information about the timing (but not content) of USDA crop report releases.

- Brownlees, C. T., Cipollini, F. and Gallo, G. M. (2011). Intra-daily volume modeling and prediction for algorithmic trading. *Journal of Financial Econometrics*, 9(3), 489–518.
- Cheng, I.-H. and Xiong, W. (2014). Financialization of commodity markets. *Annual Review of Financial Economics*, 6(1), 419–441.
- Darrat, A. F., Rahman, S. and Zhong, M. (2003). Intraday trading volume and return volatility of the DJIA stocks: A note. *Journal of Banking & Finance*, 27(10), 2035 – 2043.
- Humphery-Jenner, M. L. (2011). Optimal VWAP trading under noisy conditions. *Journal of Banking & Finance*, 35(9), 2319 – 2329.
- Irwin, S. and Sanders, D. R. (2011). Index funds, financialization, and commodity futures markets. *Applied Economic Perspectives and Policy*, 33(1), 1–31.
- Janzen, J. P. and Adjemian, M. K. (2017). Estimating the location of world wheat price discovery. *American Journal of Agricultural Economics*, 99(5), 1188–1207.
- Konishi, H. (2002). Optimal slice of a VWAP trade. *Journal of Financial Markets*, 5(2), 197 – 221.
- Manganelli, S. (2005). Duration, volume and volatility impact of trades. *Journal of Financial Markets*, 8(4), 377 – 399.