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The Delaying Effect of Storage on Investment: Evidence from The Crude Oil Sector*

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Abstract

Our paper provides a theoretical framework able to represent with accuracy a consistent relationship between fixed capital investment, storage and the term structure of prices in a storable commodity market. It aims at understanding the interaction of storage capacity with irreversible investment decisions in mediating investment and commodity price dynamics. The results show that the presence of storage, while smoothing the spot price tends also to channel volatility into the future, thereby raising the options value of waiting and eventually delaying and making lumpier the investment in fixed capital. The time-varying expected price volatility related to the inventory levels is a new channel we identify to show why irreversible investment decisions in a storable commodity market capture more accurately both price and investment dynamics observed in the data as compared to an irreversible investment setting without storage capacity.

Keywords: Investment, Irreversibility, Storage, Commodity prices, Forward curve.

JEL classification: C51, C52, Q11.

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1 Introduction

The recurrent wild swings of commodity prices rekindle the need for relevant structural dynamic models, grounded in microeconomic foundations that feature forward-looking decisions of individual agents in response to unexpected shifts in the supply and demand fundamentals. As illustrated in the oil market where practitioners monitor closely inventory levels and rig counts, a metric often used to proxy the level of capital expenditure, models that capture investment and storage decisions are used to assess the prevailing supply and demand balances.

This paper seeks to shed light on an important channel that affects investment dynamics when it is costly and irreversible by placing at the fore the role of inventories. Dixit and Pindyck (1994) have laid out the theoretical groundwork for irreversible investment behavior under uncertainty and explained how irreversibility creates an option value of waiting when it comes to investing. Subsequently, models of fixed capital investment in the macroeconomic literature have been used to study the implications of sunk costs and the associated irreversibilities on the timing of firm investment decisions and subsequently on aggregate investment fluctuations. But this family of micro-founded macroeconomic models is silent in regards to the contribution of investment irreversibilities to commodity price dynamics.

On the other hand, the literature dedicated to the competitive commodity storage model in the tradition of Gustafson (1958) and variants of term structure models as in Schwartz and Smith (2000) and Routledge et al. (2000), have been interested mainly in replicating features of commodity prices while abstracting completely from implications for investment behavior. At the heart of these models is time-varying uncertainty which is either imposed exogenously or generated by constraints on investment, inventories, or from the exhaustibility of the commodity. While being successful at generating the defining dynamics of prices in commodity markets, the literature often does not provide much insights regarding the firm's decision to invest and how it is influenced by the embedded source of time-varying volatility. In this paper, we will show the extent to which the endogenous uncertainty resulting from the presence of storage capacity and constraints on investment affects the firm's decision to invest and the prevailing equilibrium spot and futures prices. This has yet to be explicitly modeled and studied.

In the theory of storage, price dynamics in a commodity market are dictated by supply and demand conditions.[1] Among the predictions derived from the equilibrium relationship between the prices and market fundamentals is that futures prices uncertainty can increase with storage to converge to the volatility of spot price at high inventory levels. The key insight being that the gap between the spot and futures price volatility is negatively correlated with inventories. At low inventory levels, spot prices react much more to supply and demand conditions to balance the market and therefore tend to be more volatile than and less correlated to futures prices. At higher inventory levels, as a result of the exploitation of storage arbitrage opportunities, spot and futures prices move together (i.e., are in lockstep variation) and are therefore equally as volatile. This prediction has been empirically validated early on by Fama and French (1988), Ng and Pirrong (1994) and Routledge et al. (2000) or more recently in Gorton et al. (2012). It is therefore this relationship, between futures price volatility and inventories that creates a delaying effect of storage on investment. Our paper thus studies the extent to which lockstep variation between

[1] Theory of storage refers to models linking the spread between spot price and future prices to inventory levels. Both the theory of storage in the tradition of Kaldor (1939), Working (1949) and Brennan (1958) or the one following Gustafson (1958) and Wright and Williams (1982) deliver similar predictions regarding the relationship between inventory levels and the wedge between spot and futures price volatilities.

both the spot and futures prices in the presence of storage capacity affects the investment dynamics.[2]

This article tries to connect and build upon three strands of the finance and economics literature as a mean to feature the essential structural forces driving the commodity price in the world market. In particular, it hones in on the interaction between storage and investment by showing that the presence of storage brings forth features in the investment and commodity price dynamics that cannot be accounted for when focusing only on investment. The crude oil market will be used as a guideline illustration since it embodies, fairly well, the occurrence of booms and busts cycles characterizing the prices behavior of most commodities on spot and futures markets.

The results shed light on (*i*) the importance of introducing storage in an irreversible investment model to generate the price and investment dynamics observed in the data, (*ii*) the key role of the storage arbitrage condition in dictating the impact of the irreversibility constraint on the timing of investment, i.e. causing a delaying effect of storage on investment, and (*iii*) the close relationship between the supply and demand with the term structure of forward curves. Nonetheless, if our theoretical framework manages to capture the core forces governing the observed dynamics in the crude oil market, it remains unable to generate sufficiently persistent volatility in the futures prices, a common issue of this type of dynamic commodity market models.

The article falls as follows: Section 2 summarizes the three main streams of literature along with the underlying theories from which are derived most empirical models of commodity markets. Section 3 combines theories and data so as to paint a consistent picture of crude oil market dynamics. The general framework with its associated theoretical insights are outlined in Section 4. Section 5 follows with simulations experiments and discusses the results. Section 6 concludes.

2 Complementary theories: A Literature Review

In the economics and finance literature, the majority of dynamic equilibrium models explaining the commodity price formation from supply and demand conditions draw on the investment and storage theories that provide the microeconomic foundations behind the agents' behavior. In addition, forward-looking decisions made by market participants are reflected in the term structure of commodity futures prices, which in turn affects the current actions. The feedback between future and current commodity prices call for looking also under the hood of future price formation and forward market dynamics mostly studied in the finance literature. Highlighting synergies and drawing insights from these theoretical developments is the purpose of this section.

2.1 Investment dynamics and constraints

Investment projects are more often than not met with large sunk costs, which make them partially if not mostly irreversible. The cost of adjusting the capital stock has drawn much attention in the investment literature. Evidence on the presence of nonconvex firm-level capital adjustment costs and investment irreversibilities is extensive. Doms and Dunne (1998) cast doubt on the smoothness of capital adjustment at the firm level using micro-data on firm-level fixed capital investment. Subsequently, Abel and Eberly (1994), Caballero and Engel (1999) and Cooper and Haltiwanger (2006) as well as

[2] Kellogg (2014) tests this theory on oil drilling investment in Texas and confirms that producers do indeed take into account time-varying volatility when making their investment decisions.

references therein, investigate the relevance of different adjustment costs for both firm-level and aggregate investment dynamics. They show that firm-level investment is indeed lumpy as they would like to invest more aggressively the wider the gap between the actual and targeted levels of capital. These features of investment point to microeconomic nonlinearities, i.e. both non-convex adjustment functions and investment irreversibility.

Nevertheless, lumpiness of investment at the firm-level has been found to disappear at the aggregate level. Thomas (2002) demonstrates how general equilibrium price effects wash out firm-level investment lumpiness and thus pegs nonconvexities in capital adjustment costs as inconsequential for business cycle analysis. More recently, Bachmann et al. (2013) and Bachmann and Ma (2016) nuance the debate on the relevance of micro-economic lumpiness for macroeconomics by pointing out that what can settle the issue is the relevant strengths of price and adjustment cost responses. The difference between firm-level models of aggregate investment and those of the "representative" firm is that the former are able to generate nonlinear aggregate investment. The presence of a hazard rate of adjustment that depends on the gap between the firm's level of capital and the desired one is the key to matching higher moments of aggregate investment. Therefore as in Caballero (1999), these microeconomic non-convexities generate an important "time-varying and history-dependent aggregate elasticity" of investment to shocks by allowing changes in the synchronization of firms' capital adjustments. Cooper and Haltiwanger (2006) investigate the relevance of different adjustment cost specification for investment at the firm, sector and aggregate levels. They found that investment whether at the firm or sector level is best described by a model which includes non-convex capital adjustment costs as in Abel and Eberly (1994) and Caballero and Engel (1999).

To assess the soundness of this short cut, models on aggregate investment turned to study microeconomic investment decisions. However, a more complete examination of the commodity markets dynamics calls for looking at the second pillar of the supply and demand fundamentals, and especially the speculative demand for storage.

2.2 The competitive storage model

In the literature focusing on the effect of storage in the commodity prices discovery, the competitive storage model with rational expectations and a non-negativity constraint on inventories in the tradition of Gustafson (1958) has been the workhorse of neoclassical studies on commodity price volatility. Deaton and Laroque (1992, 1996) pioneered its empirical validation by developing techniques to confront the observed price data with a version of the model with inelastic supply modeled by either i.i.d. or autocorrelated random shocks. Eventually they draw mixed conclusions. On one hand, it proves useful in reproducing most of the prominent features observed in commodity prices (persistence, nonlinearity, positive skewness and excess kurtosis) in particular as a result of the key non-negativity constraint on storage which creates two different price regimes. On the other hand, it appears unable to match the very high levels of serial correlations observed in commodity prices even after allowing for persistence in the supply disturbances. Most of the critiques have been addressed through several recent improvements on both the numerical (Cafiero et al., 2011) and empirical fronts (Cafiero et al., 2015; Guerra et al., 2015; Gouel and Legrand, 2017b).

However, so far existing econometrics studies conducted on the storage model, in relying on information taken from spot prices alone, have been limited to its crudest specification, that is without allowing for neither structural demand shifts nor a supply reaction to prices although versions of the model with elastic supply have existed for a long time.[3] Indeed, Wright and Williams (1982) demonstrate the implications of supply responsiveness in terms of the price behavior generated by the model. Interestingly, they show that storage has a destabilizing effect on the expected price and in turn on the planned production, but also that it is unclear whether planned production and storage act as substitutes or complements since the mean level of storage is left virtually unchanged. Therefore, from an investment standpoint accounting for storage should make it more volatile as its expected return is less certain. Put it another way, it is not so much the long-run average of investment but its dynamics and timing which might be primarily impacted by the introduction of storage into the framework.

Lastly, given the forward-looking nature of both types of models, investment, storage and their subsequent interactions might be mirrored in the term structure of commodity forward prices. This point has long been recognized by the financial economists.

2.3 The determinants of futures pricing

Although usually designed to model the risk premium of commodity futures contracts and assess the performance of trading strategies, valuable insights can be gained from the literature on commodity futures pricing which provide theoretical foundations to the behavior of prices on the futures market. Indeed, theories of the determinants of speculators' profit might help better understand the information which can be extracted from futures prices and more broadly from the term structure of the futures market.

For our case study, of particular interest is the literature built upon the theory of storage in the tradition of Kaldor (1939), Working (1949) and Brennan (1958), from which is derived a standard expression to link forward and spot prices according to the following storage arbitrage equation:

$$F_t(T) = S_t(1 + R_t(T)) + C_t(T). \tag{1}$$

$F_t(T)$ stands for the forward price as of time t for delivery at time T, S_t is the spot price, R_t is the interest cost of storage and C_t is the total net of interest cost of carry.[4] In this respect, abundant stocks can be regarded as a fair signal of a contango regime that is a negative spread or an upward sloping forward curve (Telser, 1958).

The thing is that models based on competitive storage alone cannot account for the longer term dynamics of the term structure since inventories only affect the short end of the forward curve. It turns out that the long term behavior of the term structure might be governed by elements such as the structure of production, new discoveries, investment dynamics, and prices of substitutes. Two avenues have emerged to address this shortcoming. One the one hand, reduced form models of futures prices as in Brennan and Schwartz (1985), Schwartz (1997), and Schwartz and Smith (2000), take spot prices and other factors which can influence futures prices as exogenous stochastic processes. On the other hand, Litzenberger

[3] Both the lack of reliability and availability of data on quantities mostly explain why they have been overlooked until now. Yet, a novel method to take to the data on prices and quantities richer model specifications is provided in Gouel and Legrand (2017a).

[4] $C_t(T) = Wf_t(T) + Rp_t(T) - Cy_t(T)$. $C_t(T)$, total net of interest cost of carry, which can possibly include transportation and warehousing fees Wf_t, risk premia Rp_t and convenience yield Cy_t. Following the literature of commodity storage model, we will assume a constant interest rate, zero convenience yield and risk neutral agents thereby leading to zero risk premia. Note also that under the risk-neutrality and constant interest rate assumptions the distinction between the futures and forward prices is irrelevant (Williams and Wright, 1991). Thus, the two terms will be used interchangeably in the rest of the paper.

and Rabinowitz (1995), Carlson et al. (2007) and Kogan et al. (2009), use a production economy with adjustment costs, to endogenously determine both the spot and futures prices. A common assumption in these models is the irreversibility of the investment or extraction decision. The presence of a nonconvex adjustment cost aligns this vein of the literature with the investment's one discussed above. The study of the impact of investment dynamics on prices builds the link between the literature related to investment project valuation and prices with the one focusing on investment dynamics at the different firm, sector and aggregate levels.

From an empirical perspective, the term structure of commodity futures prices displays patterns that set commodities apart from other assets. In Litzenberger and Rabinowitz (1995), Carlson et al. (2007) and Kogan et al. (2009), it is noted that commodity futures curves are often backwardated, implying that the spot price of a commodity enjoys a premium over the futures prices.[5] Litzenberger and Rabinowitz (1995) cast their analysis in an optimal exhaustible resource extraction framework using option pricing techniques which places the exhaustibility of the resource at the center of the analysis. The option value of waiting associated with the depletion of a finite resource in a stochastic environments necessitates, at least, that the spot price of the commodity exceeds the present value of the future price in order to produce. Carlson et al. (2007) and Kogan et al. (2009) take up a similar approach but instead of accounting for the non-renewability of the commodity, they exploit the irreversibility of investment along with potential adjustment costs in the production of oil. Put differently, as in Bertola and Caballero (1994) and Caballero and Engel (1999) these investment constraints are instrumental for the empirical success of sector-level investment models. This latter strategy allows not only to explain the occurrence of both backwardation and contango in commodity markets, but also to generate time-varying volatility of commodity spot and futures prices, with price volatility increasing in the degree of backwardation or contango. In the short-run, demand or supply shocks result in large fluctuations of the spot and short-maturity futures prices because current supply is rather inelastic. At intermediate horizons, the effect of the shocks are dampened by the producers' supply responses. Regarding the longer contract maturities, whether supply and demand shocks or the inventory levels, all have little effect over the long-run equilibrium price which is governed instead by longer run factors including the dynamics in investment, commodity production and fresh discoveries.

Interestingly, Routledge et al. (2000) focusing on storage instead of investment, also examine the relation between volatility and the slope of the forward curve. As Carlson et al. (2007) and Kogan et al. (2009), they find that volatility is higher when there are no inventories to smooth fluctuations in supply. This is the case of a backwardated forward curve. In contango, when the future price is higher than the spot price, inventory levels are positive and therefore can buffer the impact of shocks. Nonetheless, as posited in Fama and French (1988) and Routledge et al. (2000) when the inventories reach a certain level, spot and forward prices tied through the storage arbitrage equation (1) exhibit lockstep fluctuations so that the volatility of futures prices starts rising again. Therefore, it is possible when inventories are high that the near term contracts are less volatile than the subsequent contracts, in violation of the "Samuelson hypothesis" according to which the volatility of the forward prices is decreasing in the time to delivery, since the mean reverting spot and futures prices give way to a decline in uncertainty as the contract horizon lengthens. Actually, this explanations matching is stressed by Kogan et al. (2009), who states that the observed V-shape of the volatility of futures prices can be obtained either through an irreversible

[5] In fact this result really depends on the sample period covered as this higher frequency of backwardation holds only until the late 1990's after which the market has been found to be in contango much more often.

investment model with capacity constraints as is the case in their paper, or through a storage model provided that storage capacities have both lower and upper bounds.[6] In their conclusion they add that, if storage should be more relevant to explain the fluctuations of the short-end of the forward curve, only the investment and capital dynamics are likely to deal with the back-end movements.

All in all, to some rare exceptions the modeling of commodity market fundamentals takes roots either in the storage theory or in the literature related to the irreversibility of investment and capital adjustment costs. However, along with the developments of futures market, there has been a growing interest in attempting to account also for the term structure of forward prices in which are reflected the forward-looking decisions of the market's participants. Despite this common goal and apparent complementarities, both modeling approaches have been developed quite independently of one another.

With this in mind, we use Cooper and Haltiwanger (2006)'s work as a point of departure for our paper to model the commodity supplier's capital dynamics. By working within this framework we believe not only to stick to the current trend in the literature on firm (sector)-level investment, but also to echo Kogan et al. (2009)'s motivation for emphasizing the importance of the supply side of the commodity market in order to reproduce the most prominent features of commodity price. The key assumption in their model is that the representative commodity producer cannot resell his already installed capital leading to a time-varying elasticity of supply with respect to shocks. While Kogan et al. (2009) abstract from the storage dimension in the commodity market, we want to bring together the price smoothing features of storage and capital adjustment rigidities in the same framework, so as to shed light on the inner working of commodity markets and the accompanying price dynamics. Additionally, we build upon the recent developments in the literature of the storage model through a more complex specification of supply responsiveness and persistence in the shock processes, while taking a close look to the implied variations of quantities and prices both on the spot and forward markets.

3 Theory and Facts

In this section, bringing altogether the above three investment, storage and futures pricing streams of literature, we will try to sketch out a narrative for commodity booms and busts cycles. Then, we turn to the US oil market data to check for relevance and adequacy with the theory, thereby motivating and guiding the final modeling approach.

3.1 A tale of investment and storage

In capital intensive industries, production often requires substantial upfront fixed capital investments. In this respect and according to the classic hedging theories, at the time of investment producers will use the futures market to lock in future selling prices by taking short positions in futures (i.e., selling futures contracts) pushing the market in backwardation until the installed capital becomes productive and the output comes online. Once the supply starts flowing on to the physical market, the spot price falls so that now it is the turn of refiners and other industrial consumers, willing to secure their future purchasing prices, to hold long positions in the forward market, driving up the prices of futures contracts and sending the market in contango. Should an adverse demand shock hit, resulting in a demand for

[6] The upper-bound on inventories is even not necessary if you recall the "inventory effects" demonstrated in Fort and Quirk (1988) and Routledge et al. (2000)'s explanation for the observed increase in futures price volatility with the degree of contango.

consumption lower than expected right at the time investment decisions in the oil sector have been made, production capacities in the market turn out to outsize the optimal level required to satisfy the demand. Since operating costs for maintenance and pumping are small as compared to upstream investments, especially in the conventional oil sector, producers trying to maintain their market shares keep on tapping the oil. It becomes more profitable to build inventories and absorb the surplus supply while postponing investment. As the capital stock decays over time, the production capabilities shrink slowly and so does supply. Inventories are drawn down, prices start to rise, resulting in an environment of higher prices, which stimulate investment again. Eventually, the tightening market gradually moves back from contango to backwardation.

To summarize and combine these facts with the underlying aforementioned theories, at full carry, the forward market is in contango, spot and futures prices are moving one for one as they are tied down by the storage arbitrage equation. This lockstep relationship between futures and spot prices implies that future prices are as volatile as spot prices, and the uncertainty over future price weighs on investment decisions while creating speculative opportunities more supportive of storage.

In contrast, when the market is in backwardation, the spot price is high due to declining inventories and a tightening market. In this case, spot price volatility increases because at low levels of stocks, there is no slack in the market, and the spot price must adjust in respond to rising demand. Over time, agents know that production capacity will adjust to new demand conditions. Therefore, futures price volatility declines and even more so the farther away the time to delivery, a feature in commodity markets known as the *Samuelson effect*. These conditions favor capital investment, hence boosting future productive capability.[7]

All these characteristics are well epitomized by the oil industry whose data is also both available and reliable, thus offering a good field of experiment as recently demonstrated in Kellogg (2014).

3.2 The data

Following Kellogg (2014) and for reasons mostly related to timeliness in the publication of data, especially those on inventories, our study focuses on the US oil market. The data are monthly and span from January 1986 to December 2014.[8]

Quantities: The US production and inventory levels data, noted Q and S respectively, are taken from the Energy International Administration (EIA).[9] To remove the trend and seasonality patterns, we divide the production and inventory levels by the moving average over the previous twelve months as in Gorton et al. (2012).[10]

[7] In the opposite contango state of the market, violations of the Samuelson effect can occur (Fama and French, 1988; Routledge et al., 2000).

[8] The starting date is dictated both by the availability of the monthly spot price data given by the Energy International Administration (EIA) and the CFTC data related to the traders' positions in the forward market. Although longer monthly series of oil spot prices are available, for instance from the World Bank pinksheet or the International Monetary Fund (IMF), starting in 1986 should not be very contentious since the oil futures market has been first traded in 1983 and was at the beginning not sufficiently liquid to be really relevant.

[9] `https://www.eia.gov/petroleum/`. As in Knittel and Pindyck (2016), we choose commercial stocks, i.e. excluding the Strategic Petroleum Reserve (SPR).

[10] Following Gorton et al. (2012) again, they have been lagged one month to account for most likely reporting and revision delays.

Prices Monthly futures prices are constructed using daily observations of the New York Mercantile Exchange (NYMEX) light sweet crude oil contracts.[11] The monthly futures price is equated to the last daily price of a given month. Since futures contracts are traded everyday, prices update accordingly. Consequently, the last day of the month price reflects all the information available that month. Several contracts are traded on each day in our data. To categorize each contract according to the number of months to its maturity, we identify the maturity date for each contract according to its month of delivery; the date of expiry for each contract is preset. Following Kogan et al. (2009), we sort each contract according to months to delivery. We divide the number of days it has left to maturity by 30 and round off the result. For contracts with less than 15 days to maturity, we add a month. The selected contracts are those maturing in the next 1, 6, and 12 (hereafter denoted F_1, F_6, and F_{12}). While this market is liquid, especially for the short term maturity contracts, there are still missing values for daily prices. To address the sparsity of the data, we use the spread between two consecutive contracts to fill in the missing daily price.[12] The spot price P is the West Texas Intermediate (WTI) crude price taken from the EIA. Spot and futures prices are deflated by the US CPI following the common practice in the literature (Kilian, 2009; Hamilton, 2009; Knittel and Pindyck, 2016; Baumeister and Kilian, 2016a) and expressed in deviation from the same log-linear time trend.

The slope of the term structure (SL): it is obtained by taking the logarithm of the ratio between the first and the twelfth months forward contracts (SL= $\log(F_{12}/F_1)$). A negative (positive) value of the demeaned slope SL= $\tilde{\mathrm{SL}} - \bar{\mathrm{SL}}$ is indicative of a backwardated (contango) market.

Expected price volatility: Our baseline measure will be the one-year historic volatility. As built in Kellogg (2014), it is obtained by taking the standard deviation of the logarithm of the return on the futures price within a 1-year rolling window for the horizon of the forward contract considered.[13]

Global real economic activity index (GRA): this indicator capturing cyclical variations in global real economic activity is based on the Dry Cargo Bulk Freight Rates as constructed in Kilian (2009) and available from the author's personal website.[14] It is stationary by construction as the author also opts for a log-linear trend modeling to focus on the cyclical fluctuations solely. Though not free from drawbacks we prefer this measure to the less specific world GDP growth, such as the OECD +6 monthly GDP data used in Dvir and Rogoff (2014), to assess the global demand pressure for industrial commodities often cited as one of the primary driver of prices.

Investment rate (I/K): it is built following the perpetual inventory method described in Bloom (2009), that is $(I/K)_t = I_t/0.5\,(K_t + K_{t-1})$. Regarding the investment, we take exactly the same real capital and exploration expenditures in the US oil sector used in Baumeister and Kilian (2016b). These nominal

[11] https://www.quandl.com/collections/futures/cme-wti-crude-oil-futures.

[12] The spread between two consecutive months, $\Delta_{1,2} = F_1 - F_2$, is constructed using daily prices. Then for all the days where the spread is missing, we fill it out with the closest available spread. If, for a given day, F_1 is missing, we fill it in using $F_2 + \Delta_{1,2}$. Once we obtained the constructed daily prices, we use the last price as the monthly price. If this price is still missing, we interpolate between the two closest available prices to construct the monthly price; i.e. $F_{1,t} = \frac{F_{1,t-1} + F_{1,t+1}}{2}$.

[13] When computed on the simulated series, the standard deviations are directly on the prices levels instead of the return as the latter are stationary by construction.

[14] http://www-personal.umich.edu/~lkilian/paperlinks.html.

figures are only available at a quarterly frequency from the Bureau of Economic Analysis (BEA), which provides also the Personal Consumption Expenditure (PCE) deflator they used to deflate the series. The data includes mining and oil field machinery along with investment in petroleum and natural gas structures. Then, as in Kellogg (2014), we generate a monthly investment series by assigning each quarterly reported value to the central month of each quarter, while the other month values are obtained through a linear approximation. Finally, to proxy the capital stock K, we use the Energy Information Administration (EIA) data on the number of development wells built on a monthly basis. Since development wells are drilled in a proven area of oil reservoir, they are investments in future productive capacity, contrary to the exploratory wells, which are omitted from this calculation.[15]

Speculative pressure (T): this metric aims at assessing the relative strength between the net short positions–the difference between short and long positions–of hedgers, assumed belonging to the group of "commercial traders", as compared to the net long ones held by the speculators or else the group of "non-commercial traders". It is based on the Working (1960)'s T-index according to which speculators are needed to balance the market for hedgers.[16] In other words, the speculative pressure increases with the number of short speculators relative to the total number of hedgers when the latter are net short. We obtain the historical positions of traders from the "Commitments of Traders Reports" published on the CFTC's website.[17]

We will now take look at the big picture these data portrayed and try to emphasize some of the key features.

3.3 Empirical features

Let us start with some descriptive statistics about the variables in consideration. The main moments are documented in table 1.

First, as can be seen, all the variables are fairly persistent, with 2nd-order autocorrelation coefficients all in excess of 0.4. In addition, the strong persistence of the investment rate, I, storage, S, but also the production, Q, are consistent with the existence of constraints on the supply-side in the oil industry previously underlined.[18] Adding to this the high levels of serial correlation displayed by the real economic activity, it comes as no surprise that the first and second-order autocorrelation coefficients of prices, respectively, lie well above 0.9 and 0.8 regardless of the contract horizons. This strengthens the widely-shared belief that it is both the supply constraints along with the global industrial demand which are among the essential drivers of the price behavior. In the end, this calls for bringing both supply and demand structural factors within the same model to paint a more complete picture of the oil price discovery and dynamics in the spot and futures markets. Regarding the slope of the term structure, $\tilde{SL}$, it is interesting to see that it is much less autocorrelated than the forward prices themselves (i.e., a cut by about one-third) reflecting quite frequent changes in the practitioners' expectations about long-run supply and demand.

[15] For the sake of comparison with the empirical studies of Cooper and Haltiwanger (2006) and Kogan et al. (2009), in the rest of the paper the investment rate will be denoted I_t.

[16] See Alquist and Gervais (2013)'s appendix for more detailed about the index's construction as well as the rationale behind it.

[17] `http://www.cftc.gov/files/dea/history/`. The CFTC contract code of the NYMEX light sweet crude oil product is 67651.

[18] For space reasons we did not report the higher-order autocorrelation coefficients but for the production and investment rate, all the first five lie above 0.4 and 0.6 respectively, while the storage ones end falling below 0.1.

Table 1: Descriptive statistics of the monthly detrended observables (1986:M1-2014:M12)

Variables	First-Order AC	2nd-Order AC	Coeff. of Variation	Skewness	Excess Kurtosis
Normalized inventory	0.80	0.56	0.05	0.23	0.62
Investment rate	0.96	0.90	0.03	1.59	6.93
Normalized production	0.31	0.45	0.04	-0.55	2.95
Slope of the term structure	0.64	0.42	0.09	-0.14	0.56
Hedging pressure	0.91	0.85	0.04	0.52	-0.70
Global real economic activity index	0.96	0.89	0.25	0.53	-0.16
Spot price	0.94	0.87	0.31	0.60	0.64
3-month futures price	0.93	0.86	0.31	0.55	0.45
6-month futures price	0.95	0.89	0.31	0.46	0.14
12-month futures price	0.93	0.89	0.32	0.47	0.25

Notes: The index of global real activity, the spot and futures prices are log-linearly detrended.

These changes in expectations affect the back end of the forward curve–e.g., within a year–and lead to a market balancing between both the backwardation and contango regimes.

Then, turning to the values of the coefficient of variation, the volatilities of the investment rate, storage and production are low and of the same order of magnitude, which is consistent with the rather sluggish and costly adjustments of the capital stock.[19] Furthermore, in comparison to the supply and demand variations, the much larger fluctuations exhibited by the spot and futures prices suggest very low supply and demand elasticities. Then again, the fact that the GRA index is almost as volatile as prices supports the view that short-run prices variations are primarily dictated by demand dynamics. The absence of any Samuelson effect, namely the decrease in the volatility of futures prices with the contracts horizon must be noted. This is in line with the Fama and French (1988) and Routledge et al. (2000)'s findings, who clearly demonstrate that when the market is in contango (i.e., about half of the time in our sample), inventories are high and may lead to violations of the Samuelson effect which otherwise is a phenomenon more significant in a backwardation regime.

Finally, unlike production, storage, investment rate and prices are positively skewed with fat right tails as indicated by the positive excess kurtosis coefficients. Together, these asymmetries in the investment and prices distribution illustrate (*i*) the importance of considering the nonnegativity constraints on both investment and storage,[20] and (*ii*) the prominent roles storage and investment might play in shaping the overall price dynamics, thereby supporting the relevance of studying the synergies between them.

Taking a closer look to the oil forward prices, we plot the main key features for two different maturities in Figure 1. Apart from the obvious succession of booms and busts episodes along with the clusters of volatility, the mean-reverting tendency and, especially for the nearest to delivery contract, the sharp spikes leading to the noted positively skewed distribution, two points deserve particular emphasis. First, the 18-month-ahead price seems to be less volatile when the market is in backwardation–e.g., when it lies below the front-month future price–confirming the fact that the Samuelson effect should be more pronounced when prices are backwardated. Second is to note that, although over the whole sample length,

[19] Production is even more volatile than the rate of investment as it is also subjected to random disturbances such as the disruptions related to climatic or geopolitical reasons.

[20] In the subsequent modeling we will consider investment as fully irreversible by setting a zero lower bound on I as in Kogan et al. (2009).

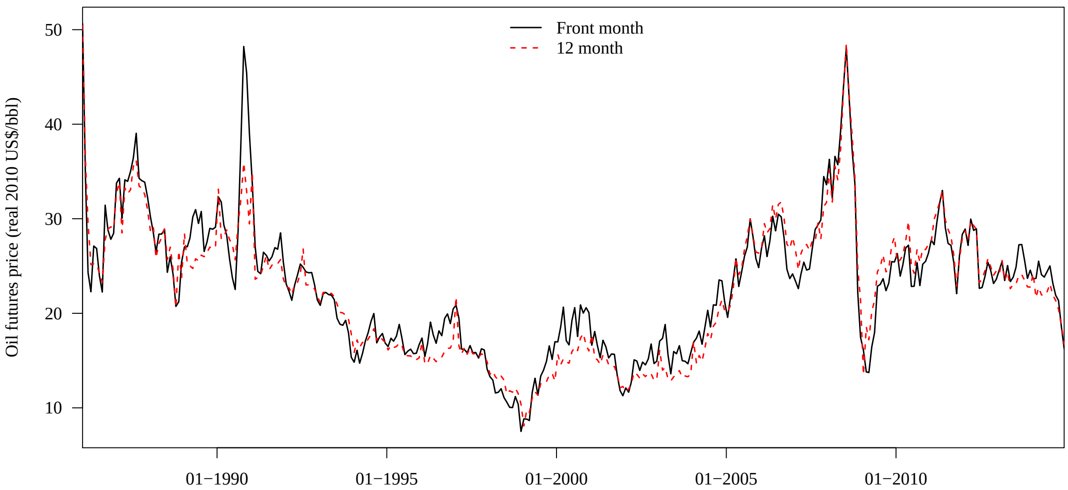

Figure 1: NYMEX Front-month and 12-Month Oil Futures Prices (real 2010 US$). The prices have been deflated by the US CPI before being log-linearly detrended.

the market is nearly as often in contango as in backwardation, both states are quite long-lasting. For instance, between the mid-1990's to the early 2000's, the market was in backwardation almost all the time before reverting into a sustained contango since 2005. This further supports the resilient supply hypothesis and the irreversible nature of investment inherent to capital intensive industries such as the oil sector, where excess capacities of production take long to be absorbed and thus result in a persistent glut with ballooned inventories, a depressed spot price and an upward-sloping forward curve. Applying the opposite reasoning when the supply falls short of the demand and the outcome is empty inventories, a rocketing spot price sending the market in backwardation for a while until a series of investment in new production capacities eventually lead fresh output to come online and restore the equilibrium.

The empirical analysis is completed by the computation of pairwise correlation coefficients displayed in table 2 and which appears quite consistent with the oil industry narrative delivered in subsection 3.1.[21] For one thing, storage is, as expected, negatively correlated with investment, global real economic activity

Table 2: Correlation coefficients of the monthly detrended observables (1986:M1-2014:M12)

Variables	S	I	Q	$\tilde{SL}$	T	GRA	P	F_3	F_6	F_{12}
S	–	-0.10	**0.12**	**0.37**	**0.15**	**−0.11**	–	–	–	–
I	-0.10	–	**−0.12**	**−0.17**	**−0.28**	–	**−0.57**	**−0.60**	**−0.61**	**−0.62**
Q	**0.12**	**−0.12**	–	–	**0.19**	–	–	–	–	–
$\tilde{SL}$	**0.37**	**−0.17**	–	–	**0.31**	–	–	–	**0.13**	**0.21**
T	**0.15**	**−0.28**	**0.19**	**0.31**	–	**0.38**	**0.39**	**0.44**	**0.48**	**0.49**
GRA	**−0.11**	–	–	–	**0.38**	–	**0.27**	**0.27**	**0.27**	**0.27**
P	–	**−0.57**	–	–	**0.39**	**0.27**	–	**0.97**	**0.96**	**0.94**
F_1	–	**−0.60**	–	–	**0.44**	**0.27**	**0.97**	–	**0.99**	**0.98**
F_6	–	**−0.61**	–	**0.13**	**0.48**	**0.27**	**0.96**	**0.99**	–	**0.99**
F_{12}	–	**−0.62**	–	**0.21**	**0.49**	**0.27**	**0.94**	**0.98**	**0.99**	–

Notes: All the coefficients are significant at 10% and in boldface are those significant at 5%.

[21] Only the correlation coefficients significant at the 10% percent level are given while in boldface are those significant at the 5% level.

and the spot price. On the other hand, it is positively correlated with production, the slope of the forward curve and the speculative pressure (T) arising from the traders' positions. In other words, storage increases when the market is in contango, namely when the slope of the forward curve rises with the contract horizon (SL>0). This is the outcome of an abundant production which, combined with a softening industrial demand, lead to a relatively low spot price environment and spur speculators to take long positions in the futures markets. Likewise, consistent with the aforementioned short narrative, it should come as no surprise that bursts of investment come along with producers willing to cover their future production by shorting futures contracts. This hedging pressure translates into a negative correlation between the investment rate and T.

Regarding the spot and forward prices, although slightly decaying over time, correlations across the whole set of maturities remain very high, showing actually that a great deal of the relevant information relating futures prices with demand and supply conditions is embedded in the shape of the forward curve. The closeness in the behavior of both the spot and front-month futures prices illustrates the expected convergence phenomenon occurring between physical and forward markets as the delivery date is approaching. Interestingly, the level but not the shape of the term structure seems to be in large part driven by the global economic activity which, through the speculative action of traders going long in the futures market, is reflected by the relatively high positive correlation coefficients between both the GRA and T indexes with the prices at all maturities. Put crudely, a strengthening industrial demand is associated with an overall rising financial pressure from speculators leading to an upward shift of the whole forward curve regardless of the contracts horizons. This sizable correlation justifies the introduction of a persistent demand factor as in Routledge et al. (2000) to account for such an underlying consumption growth engine which puts a lingering pressure on the supply side.

Nevertheless, further points deserve some comments, as one might be puzzled in view of the negative correlations between the investment rate and the spot and futures prices, or else the absence of significant relationships between storage and futures prices at all maturities. A possible explanation is that WTI futures contracts exchanged on the NYMEX might reflect also the global contemporaneous supply and demand conditions than solely those in the US. Another reason has to do with the typical lack of reliability of data on quantities in general and on inventories in particular. The really hard to square but still rather substantial floating storage is among the factors likely to make difficult the precise assessment of the actual inventory levels. The same is true for our monthly investment rate variable which remains an approximation of the actual E & P expenditures in the US oil sector. Still, albeit imperfect, the picture delivered by the correlations table remains on the whole coherent with the main forces at stake in the oil market functioning.

In light of these results, the subsequent modeling must be able to generate endogenously and alternatively backwardation and contango states. Conjointly, these states must be accompanied by lumpy and intermittent episodes of investment in complement to the storage activity, which is profitable whenever production capacities outstrip the industrial demand, especially during economic downturns. Indeed the relative abundance in the market goes hand-in-hand with (*i*) a bearish market poorly encouraging investment but rather much more inclined to support the accumulation of inventories by speculators willing to make the most of the resulting low spot price, (*ii*) a forward market in contango in response to the net-long positions of refiners hedging their future crude supply by buying forwards contracts, (*iii*) the strengthening of the futures prices volatility, since linked to the spot price through the arbitrage equation, thus leading to a more uncertain economic environment favorable to speculative storage while increasing the option value to delay investment. To what extent these stylized facts can be reproduced by embedding the speculative storage within the standard capital accumulation model with an irreversible constraint on investment is the topic further studied in the remainder of the paper.

4 Model

4.1 Model's equations

Commodity production There is an infinitely-lived representative commodity producer operating a stock of capital goods with a decreasing returns technology, $Q_t = AK_t^{\alpha_Q}\eta_t^Q$, with $\alpha_Q < 1$.[22] Production is affected by a multiplicative i.i.d shock η_t^Q. This disturbance can be emanating from supply disruptions such as unfavorable weather variations, labor strikes, or geopolitical events. The producer can invest in new capital goods each period, which are added to the existing productive stock next period. We assume that capital goods require one period for initial installation before they become productive and installation of the newly purchased capital goods is costly. Therefore the total cost of new capital is $\Phi(I_t, K_t) = p_K I_t + \phi(I_t, K_t)$, where the last term of the right hand side is a capital adjustment cost function.[23] Capital depreciates at rate δ_K. There is no secondary market for capital goods: once they are installed, they have no scrap value and thus investment is irreversible. This capital could be considered as industry specific and if there is an unfavorable industry-specific shock, there would be no party willing to purchase it. The irreversibility constraint on investment may lead the investment rate distribution to exhibit the same lumpiness, positive skewness and excess kurtosis as observed in the data.

Total beginning next period capital stock K_{t+1} equals to the undepreciated capital stock from last period plus the newly purchased capital in the previous period such that

$$K_{t+1} = (1-\delta_K)K_t + I_t. \tag{2}$$

The rational commodity producer is risk-neutral and price-taker of the spot price P_t. As of time t, since only $\eta_0^Q, ..., \eta_t^Q$ have been observed, P_t will be independent of future realizations of the shock. Thus, when at t the producer decides how much capital to purchase, he takes the current capital stock, K_t, current availability X_t, the current and future expected realizations of the demand shocks $\{Y_s\}_{s=0}^{\infty}$ so as to maximize his present value expected net profit

$$\mathrm{E}_t \sum_{k=0}^{\infty} \beta^t \left\{ P_{t+k} A K_{t+k}^{\alpha_Q} \eta_{t+k}^Q - p_K I_{t+k} - \phi(I_{t+k}, K_{t+k}) \right\}, \tag{3}$$

[22] It is widely documented that the rate of extraction of a given well is decreasing with the pressure function of the oil level remaining. As a result, it requires an ever-increasing amount of effort to keep production from declining. According to Hamilton (2009), over the decade from 1970 to 1980 the US tripled the number of wells without preventing the oil production from falling. Furthermore, if it is true that the recent emergence of new extraction techniques such as the horizontal drilling allowed for significant productivity gains and reductions in the production costs, it has to be noted that the lifetime of fracking wells is even shorter than the conventional ones so that the only way to keep the oil supply steady is to drill at a growing pace. Put it another way, an increasing amount of resources devoted to production can only offset the supply depletion from old wells. Finally, it is also hard to deny that, as oil will become scarcer, deeper wells located into always more remote areas or even offshore in deep water places will be needed so as to satisfy the raising global demand in the years to come. Together, these results lead us to prefer a production function with diminishing returns to scale.

[23] $\phi(I_t, K_t)$ can be a quadratic convex adjustment cost of the functional form $\phi\left(\frac{I_t}{K_t}\right)K_t$, where $\phi'(.) > 0$, $\phi''(.) \geq 0$, $\phi(\delta) = 0$, $\phi'(\delta) = 0$. The firm, thus, must pay and increasing and convex cost of investment.

subject to

$$
\begin{aligned}
K_{t+1} &= (1-\delta_K) K_t + I_t. \\
I_t &\geq 0.
\end{aligned} \tag{4}
$$

The Storage Demand In addition to producers and consumers, the commodity market comprises storers also assumed rational, risk-neutral and price-takers. Through the storage technology, the commodity is transferred from one period to another at a constant marginal cost k. Storers maximize the expected net profit from purchasing S_t of the commodity and selling it next period,

$$
\max_{S_t \geq 0} \quad [(1-\delta_S)\beta \operatorname{E}_t P_{t+1} - P_t - k] S_t, \tag{5}
$$

with δ_S the decay rate of inventories. The total availability of the commodity X_t is determined by the sum of the current supply of the commodity, Q_t, and the inventories inherited from the previous period net of the share lost due to decay, that is

$$
X_t = Q_t + (1-\delta_S) S_{t-1}. \tag{6}
$$

Market clearing The market demand for the commodity constitutes the consumption demand $D(P_t)$ and the speculative demand for storage S_t. In each period, the market clears for a spot price P_t when the availability equals total demand according to the equilibrium condition

$$
X_t = D(P_t) + S_t, \tag{7}
$$

with the linear demand function $D(P_t)$, assumed downward-sloping in P_t and subject to a multiplicative stochastic disturbance Y_t described by an autoregressive process.[24] Hence, the linear demand function takes the following form:

$$
D(P_t) = \bar{D}\left(1 + \frac{\alpha_D}{\bar{P}}(P_t - \bar{P})\right) Y_t, \tag{8}
$$

where $\bar{D}$ and $\bar{P}$ represent the steady-state values of final demand and the spot price respectively and α_D is the elasticity of demand.

4.2 The competitive equilibrium

Definition 1. A rational competitive equilibrium is given by the path of capital investment $\{I_t\}_{t=0}^{\infty}$, storage $\{S_t\}_{t=0}^{\infty}$ and prices $\{P_t\}_{t=0}^{\infty}$ such that: (i) the representative producer maximizes the present value of his expected net profits subject to the sequence of capital accumulation constraints in (4); (ii) storers maximize the present value of their net profits subject to the nonnegativity constraint on storage; and (iii) the market clears in every period.

From this point forward, we abstract from the presence of convex adjustment costs, ϕ, when solving the model. Even though the positive serial correlation of the observed investment rate might call for some

[24] The demand shock is an AR(1) process: $Y_t = \rho Y_{t-1} + \eta_t^Q$. Hence, the demand uncertainty is modeled as a persistent stochastic process akin to the cyclical fluctuations of the international business cycle.

kind of convex adjustment costs, we believe they matter less than the irreversibility constraint on I and so can be overlooked without changing neither the internal relationships among the variables nor the qualitative nature of the dynamics implied by the model.[25] Hence, for the sake of simplicity, $\phi(I_t, K_t)$ in equation (3) is set to zero in the rest of the paper. Taking the price of capital, p_K, as a constant and fixed at 1, it turns out that the producer's maximization problem given by the equations (3) and (4) delivers the following Euler equation:

$$\beta\,\mathrm{E}_t\left\{P_{t+1}\alpha_Q\left(AK_{t+1}^{\alpha_Q-1}\eta_{t+1}^{Q}\right)+(1-\delta_K)\right\}\leq 1,\ =1\text{ if } I_t>0. \tag{9}$$

The investment decision rests on the present value of the expected marginal return on capital, the left hand side of equation (9), being equal to its marginal cost. Should the right hand exceed the left, there is a large enough opportunity cost of investing today which makes the investment in the current period non-viable.

Likewise, regarding the speculative demand for storage, differentiating (5) with respect to $S_t \geq 0$ yields the following storer's optimality condition:

$$\beta(1-\delta_S)\,\mathrm{E}_t P_{t+1}-P_t-k\leq 0,\ =0\text{ if } S_t>0. \tag{10}$$

The storage decision hinges on whether the commodity price (net of the carrying costs) grows at the rate of interest. It is profitable to store only if the price growth rate equals the interest rate. Therefore, a market in contango is one in which the expected price would rise at the decay adjusted interest rate. Finally, as a result of the rational expectations assumption, both the investment and storage are forward-looking decisions resting on the expectations agents form regarding the sole next-period price F_1 without any consideration for F_2 and the subsequent forward prices.

4.3 Numerical Solution

The nonlinear nature of the decisions rules warrants the use of global numerical solution methods. We choose to employ a projection method to approximate the solution of our model. There are two endogenous state variables K_t, X_t, and one stochastic state variable Y_t. The decision variables are S_t and I_t, while P_t falls from the market clearing condition. P, in this set up, is regarded as an additional control variable and so are the future prices. To solve the model, the state space is discretized and the bounds are defined around the steady state values of the state variables. It thus consists in $[\underline{X},\overline{X}]\times[\underline{K},\overline{K}]\times[\underline{Y},\overline{Y}]$. The solution to the time iteration problem delivers three time-invariant policy functions $P=\mathscr{P}(X,K,Y)$, $I=\mathscr{I}(X,K,Y)$, $S=\mathscr{S}(X,K,Y)$. These policy functions solve the constrained system of equations (7) and (9)-(10), using the transition equations in (4) and (6) to bring the model forward.[26] The model solution yields nonlinear decision rules that are governed by endogenous thresholds, which in turn depend on the state variables. In order to assess the extent to which storage interacts with the irreversibility constraint on investment, we proceed by studying two models, identically calibrated and which differ

[25] The presence of convex adjustment costs will result in a zero-lower bound less often binding thereby leading to a smoother, more persistent and symmetric distribution rate of investment.

[26] The model is solved by implementing policy function iteration on an equally-spaced 3-D grid of state variables with 20*10*5 nodes. Between the points the policy functions are approximated using the cubic spline functions relying on the "Rational Expectations Complementarity Solver" (RECS) developed by Gouel (2013) and available from the following website `http://www.recs-solver.org//`. More details about the model resolution are available from the authors upon request.

only by whether there is the possibility for storage.

5 Simulations and Results

In this section, we will analyze the empirical relationships between storage, investment and the term structure of prices based on the simulation results of four models: a baseline investment model with and without an irreversibility constraint on investment. Both of them are, in turn, augmented or not with storage capacity. The former is no different from the model set out in the previous section except that we remove the possibility of stockpiling.

5.1 Calibration

In calibrating the models, if the choices have been largely guided by the above empirical analysis, a particular attention has been also paid to their internal consistency as it is a connection between two strands of literature mostly developed independently of one another. The annualized calibrated parameter values are reported in table 3 and are selected so as to follow the standard calibration and, when available, estimation results found in the investment and storage literature. The share of capital α^K is set to 0.33

Table 3: Model Parameters

Parameter	Description	Value
α_K	Capital share	0.33
δ_K	Capital depreciation rate	0.1
p_K	Purchase price of capital	1
α_D	Demand elasticity	-0.1
k	Physical storage cost	0.05
δ_S	Storage decay rate	0.05
β	Discount factor	0.95
A	Scale parameter	β^{-1}
$\bar{D}$	Long-run average demand	24
$\bar{P}$	Long-run average price	0.65
ρ	Persistence of demand shock	0.85
σ_{η^Q}	Standard deviation of supply shocks	0.05
σ_{η^D}	Standard deviation of demand shocks	0.17

Notes: The table describes all the annualized parameter values characterizing the model and used in the simulations.

which is equivalent to a long-run supply elasticity of $\alpha^K / \left(1-\alpha^K\right) \approx 0.5$. Since no distinction is drawn between short and long-run demand elasticities, α^D is set to -0.1, a value in the range of estimates provided in Dahl (1993) and Cooper (2003) and also reflecting the lack of substitutes for such a basic product. It has to be said that these low values of supply and demand responsiveness have been further suggested by the substantial gap in volatility between the prices and quantities noted in Table 1. The depreciation rate of capital δ_K is set to 10% which is intermediate in the typical range of 8 and 12% used in the related literature (Kogan et al., 2009; Kung and Schmid, 2015). Likewise, the decay rate on storage δ_S is fixed at 5% to be consistent with the 3% used in Routledge et al. (2000). We decided to also account for a physical storage cost, k, unlike most of the empirical studies of the storage model, which specify either a proportional cost as in Deaton and Laroque (1996) or a constant marginal storage cost following

Cafiero et al. (2011), but never both at the same time. The selected value–i.e., k=8% of the long-run average price–lies within the range of the rather low estimated values on a set of 13 different commodities ($\approx$ 5% Gouel and Legrand (2017b)) and the 10% value provided by the World Bank and FAO studies in the case of grains storage in the Middle-Eastern and North African countries World Bank and FAO (2012, figure 2-4). Regarding the demand shock Y, its role in the model's internal mechanics close in spirit to the one played by the income shock variable embedded in the storage framework developed in Routledge et al. (2000), Dvir and Rogoff (2009, 2014) and Bachmann and Ma (2016). Hence, for the persistence value of the demand shock ρ, then again we choose an intermediate value between those they used which range from 0.65 to 0.95. Lastly, the volatilites of supply and demand shocks are chosen so as to match the observed coefficient of variation of production (table 1) and consumption.[27]

Before running simulations, it is yet interesting to highlight the key theoretical implications which can be drawn from the set of restrictions introduced in the modeling. For this, we restrict to both the irreversible investment models allowing or not for the possibility of stockpiling.

5.2 Theoretical insights

States variables and regimes thresholds: The nonnegativity constraints yield nonlinear decision rules for storage and investment, which are governed by trigger levels for availability and capital, respectively. These thresholds levels are functions of the other state variables and can be written as $X_t^* = X^*(K_t, Y_t)$ and $K_t^* = K^*(X_t, Y_t)$. K_t^* represents the capital stock above which it is optimal to delay investment and let the capital stock depreciates. For different levels of X_t, it is determined from the optimality condition (9) of the producer's problem when investment is null and the inequality holds. When capital is abundant (e.g., above K_t^*) the marginal product of capital is too low to make investment profitable. At the same time X_t^* is the availability level under which the spot price is too high relative to the expected price to justify holding any inventories. When the availability of the commodity in the market is large, the target level of capital is lower. This is mainly driven by the lower prices resulting from the large amount of commodity in the market. Similarly, a large capital stock entails that the price at which storage becomes profitable is low leading in a high availability threshold as opposed to when the capital stock is small. Given the properties of the expected price and spot price functions, the relationships between K_t^* with X_t and Y_t as well as between X_t^* with K_t and Y_t are summarized in the two observations below.

Observation 1. *X_t^* is a nondecreasing function of K_t.*

Observation 2. *The desired level of capital K_t^* is*

1. *strictly decreasing in X_t and strictly increasing in Y_t when $S_t > 0$;*
2. *increasing with Y_t and invariant to X_t when $S_t = 0$.*

First, looking at the impact of capital on the availability threshold X^* under which stocks are empty, observation 1 simply shows that a greater capital level is associated with a higher expected supply weighing on any increase of the expected price.

[27] The market demand is known to be much more volatile than production as it is both a function of the wild fluctuations of the global real economic activity and the more stable demand for immediate consumption whose coefficient of variation, equal to 0.0521, is very close the production one. This comes as no surprise as it only consists of the difference between the production and the stock variation, which explains why the latter descriptive statistics have not been documented in table 1. The selected value is thus a mix between these two extreme values.

Then, switching to the effects of availability on the target capital level K^*, two distinct regimes must be explored. With this in mind, the first statement in observation 2 corresponds to the case when inventory building is profitable, that is $S_t > 0$. The interpretation is straightforward recalling from the arbitrage equation (10) that $\mathrm{E}_t P_{t+1} = \beta (1-\delta_S)^{-1} (\mathscr{P}(X_t, K_t, Y_t) + k)$, which implies that variations in spot and expected prices are coupled. Hence, since the expected price is a decreasing function of the availability in the market, when the commodity is abundant and storage is positive the expected price is low. This in turn reduces the net marginal benefit from investment and in the end lowers the desired level of capital. The result points to the first source of divergence between the two models with and without storage. Put simply, storage increases the value of postponing investment by lowering the cut off value above which it is optimal to delay capital expenditures. In addition, and perhaps even more importantly, by inducing conditional heteroskedasticity in prices the storage mechanism implies a higher uncertainty around the expected price which both supports speculation and heightens the option value of deferring investment. Through these two channels, the presence of storage has a postponing effect on investment. The outcome is a gradual shrinkage of the productive capacity accompanied by withdrawals of inventories eventually making the occurrence of a stockout more and more likely.

Finally, once the supply falls short of the demand, stocks are sold out–e.g., $S_t = 0$–and from (10) $\mathrm{E}_t P_{t+1} = \mathrm{E}_t \left[D^{-1} (Q_{t+1}, Y_{t+1}) \right]$. In other words, the expected price is no longer a function of the spot price. The stockout regime reverts to the model of irreversible investment without inventory building capacity, in which the desired capital level is a function of the realized supply-side shock, the steady state values of the price and consumption demand of the commodity. The realized shock Y_t raises the trigger value of capital. To borrow the terminology from the option pricing literature, the value of waiting to invest is lower when the realized value of the disturbance is high. Indeed, given the assumed persistence of shocks to the consumption demand for the commodity, the whole conditional distribution $F(Y_{t+1}|Y_t)$ shifts to the right giving greater probability to larger Y_{t+1} values, thereby rendering the opportunity to invest and to terminate the option more attractive than waiting. Then again, this investment incentive is further strengthened by what can be called the "uncertainty effect". The reason is that, if the high spot price deters inventory speculation, it also provides a much kinder environment for investing since the expected price, now decoupled from the very volatile spot price, is more stable and even smoothed out according to the Samuelson effect. The volatility of the expected price is now constant since the expected price itself is a function the capital stock and the expected demand shock.[28]

All in all, the combination of persistent demand shocks and time-variations in both the desired level of capital and uncertainty of future price are the key reasons behind the certain substituability, albeit imperfect, between storage and investment.

Together, both zero-lower bounds on storage and investment split the state space into four regions demarcated by the capital and availability thresholds determined by the current supply and demand conditions. They are described in the following observation:

[28] The constant volatility of the expected price in a stockout regime is discussed in Deaton and Laroque (1992, 1996).

Observation 3. *The competitive equilibrium can be characterized by four regimes:*

1. $I_t = 0$, $S_t = 0$, *if* $K_t \geq K_t^*$ *and* $X_t \leq X_t^*$;
2. $I_t = 0$, $S_t > 0$, *if* $K_t \geq K_t^*$ *and* $X_t > X_t^*$;
3. $I_t > 0$, $S_t = 0$, *if* $K_t < K_t^*$ *and* $X_t \leq X_t^*$;
4. $I_t > 0$, $S_t > 0$, *if* $K_t < K_t^*$ *and* $X_t > X_t^*$.

If both $I = 0$ and $S = 0$, the expected price is only a function of expected production, which is constant AK_t^{α}. This is a regime where the economy has overcapacities of production and it is optimal to defer investment. Furthermore, in spite of a relatively low spot price, it is not profitable to store neither because of the large productive capacity. Indeed, as stated in observation 1 the very high capital level weighs on the expected price, preventing the latter from rising sufficiently so as to cover the purchasing and carrying cost of inventories.

If only $I = 0$, there is still an excess of capital but not enough to preclude expected price from increasing so that it becomes profitable to stockpile (e.g., $S > 0$). So long as the expected commodity price grows at a higher rate than the rate of interest (net of decay), it is profitable to store.

If now $S = 0$ but $I > 0$, the market is tight resulting in a high spot price. Futures market is backwardated and selling inventories today is optimal. Although the availability is too scarce to allow storage, the associated environment of high prices encourages investment to reach the optimal level of capital K^*.

Finally, as mentioned previously the investment-storage substituability is only partial and it might be the case that it is optimal to invest and store jointly so that $I > 0$, $S > 0$. It has to be noted that, in this regime, both the S and I values are much lower than those reached when one of the constraint is binding. The market equilibrium can enter this regime for instance if, despite a capital level lying below the desired K^*, a high positive value of the supply shock η^Q yields an availability in excess of the threshold X^*. The latter scenario is even more likely if supplemented by a persistent low demand state Y which will lower X^* while raising K^*. The outcome is a relatively slack market in which inventories are used to absorb excess supply. Yet, and contrary to the first regime when $I = 0$ and $S = 0$, here the stock of capital is not large and so (*i*) is not pressing down the expected price increase needed to cover the carrying cost of inventories, (*ii*) it is still optimal to invest and bring the capital stock to the target K^*. Moreover, the quantities S at stake being lower than they are when investment is stuck at its zero-lower bound, they have too limited an impact on the expected price volatility to make very attractive the option value of waiting.

Let us move now to the characterization of the policy functions prevailing in the various market regimes.

Policy functions' behaviors X^* threshold point implies that the price function is nonlinear and follows a two regime equilibrium which, from equation (10), can be written as:

$$P_t = \mathscr{P}(X_t, K_t, Y_t) = \begin{cases} \beta(1-\delta_S)\,\mathrm{E}_t\{P_{t+1}\} - k & \text{if } X_t > X_t^* \\ D^{-1}(X_t) & \text{otherwise.} \end{cases} \tag{11}$$

The rule can be summarized in the following functional form:

$$\mathscr{P}(X_t, K_t, Y_t) = \max\left[\beta(1-\delta_S)\,\mathrm{E}_t\, P_{t+1} - k, D^{-1}(X_t)\right]. \tag{12}$$

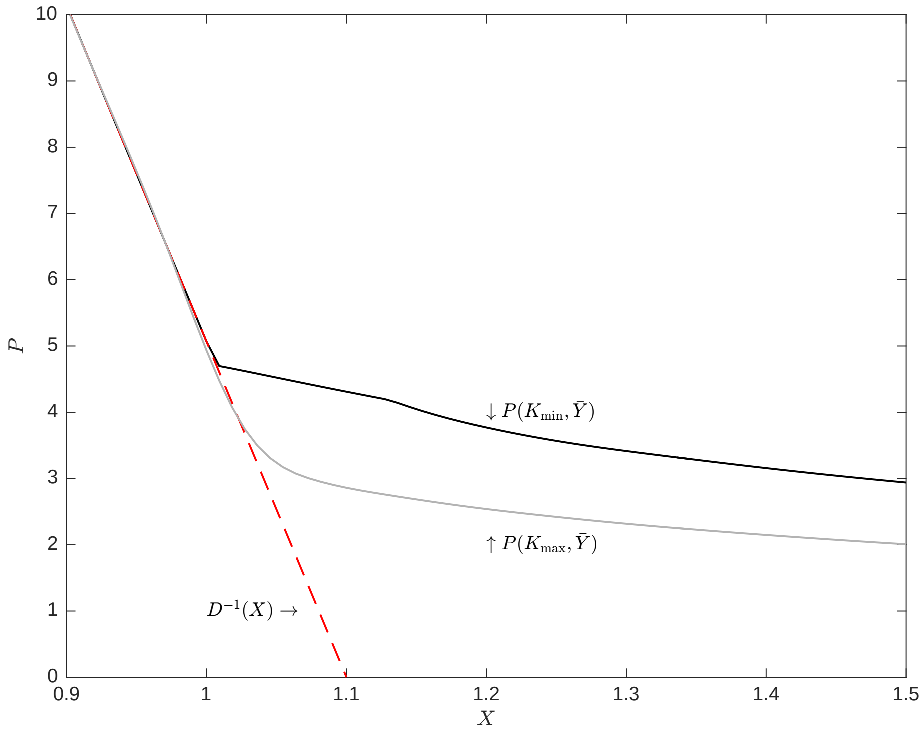

Figure 2: Price Policy Function

Notes: Two price functions, obtained for the highest and the lowest capital levels, are plotted against different levels of availability X. Additionally, the inverse demand function that prevails when inventories are null is plotted in the dashed line.

Figure 2 depicts the two regimes of the price policy function with respect to the availability and for two extreme values of K. The abscissa of the kink is X^* below which inventories are zero. As a result, the spot price is only function of the consumption demand assumed linear. In line with observation 1, a high level of capital is associated with a right shift of X^*. When there is a large productive capacity, the spot price is lower for all levels of availability and storage becomes optimal at a higher availability level. Characterizing $\mathscr{P}(X_t, K_t, Y_t)$ in terms of the quadrants shaped by X_t^* and K_t^* gives:

Observation 4. *The equilibrium price function $\mathscr{P}(X_t, K_t, Y_t)$ relates to different levels of capital and availability in the following manner :*

1. *$\mathscr{P}(X_t, K_t, Y_t)$ is strictly decreasing in X_t;*
2. *$\mathscr{P}(X_t, K_t, Y_t)$ is nonincreasing in K_t.*

The first part of the observation is obvious and does not deserve additional comments. Regarding the second point, the associated explanation is actually rather intuitive. Knowing that when inventories are carried over, the price function is linked to the behavior of the expected price along the capital stock dimension. Since the equilibrium of the model is described in terms of a desired level of capital K^*, the expected price is also a function of this threshold. Specifically, when the capital stock lies below K^*, investment is positive and brings back the next period capital stock to its optimal level. In this regime, the expected price is disconnected from the level of capital and so is the current spot price.

On the other hand, when the capital stock is above K^*, there is no investment and the next period capital stock equals the current capital stock less depreciation. Since a high level of capital entails a high expected supply, it turns out that the expected price and the spot price are decreasing in the capital stock whenever its current value exceeds K^*. Consequently, whether K is below or above K^*, the spot price is either independent or a decreasing function of the capital stock.

The same characterization can be derived for the other two investment and storage policy functions.

Observation 5. *The equilibrium investment and storage functions, $I_t = \mathscr{I}(X_t, K_t, Y_t)$ and $S_t = \mathscr{S}(X_t, K_t, Y_t)$, relate to availability, capital and demand shock in the following manner :*

1. *$\mathscr{I}(X_t, K_t, Y_t)$ is nonincreasing in X_t and K_t and increasing in Y_t ;*
2. *$\mathscr{S}(X_t, K_t, Y_t)$ is nondecreasing in X_t, nonincreasing in K_t, and decreasing in Y_t.*

The observation is further illustrated in Figure 3 exhibiting the investment and storage functions as they relate to the state variables, X, K, and Y. The substituability between investment and storage as well

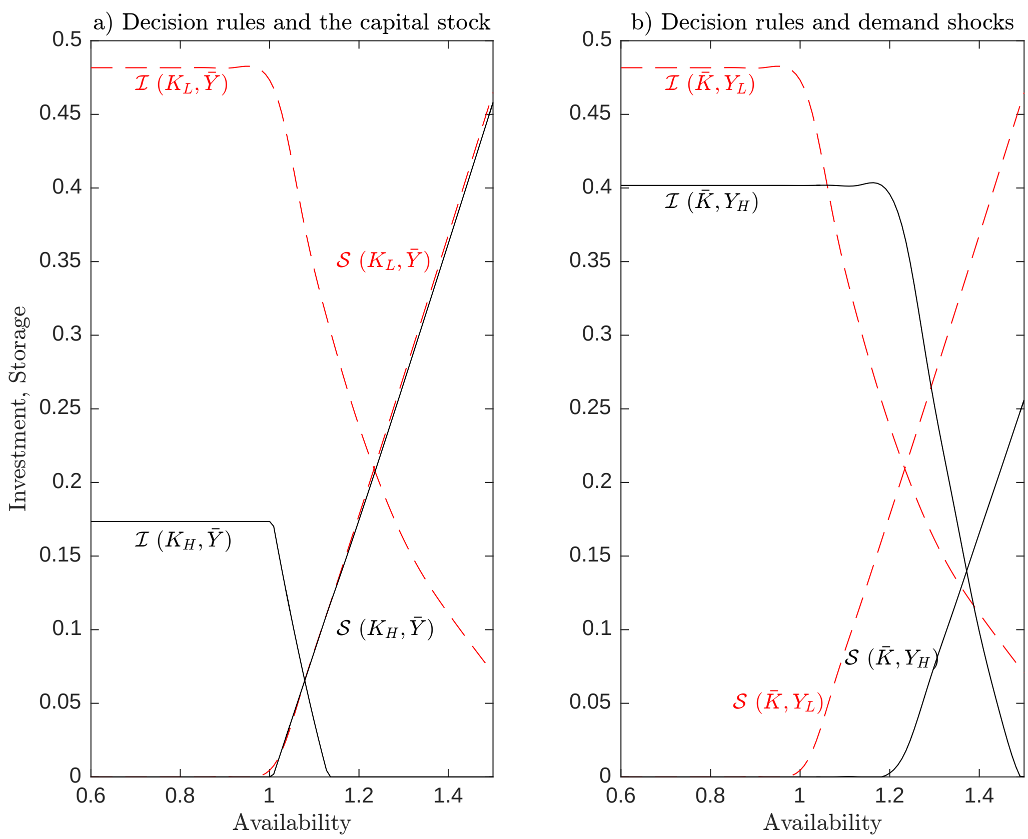

Figure 3: Investment and Storage Policy Functions

Notes: The investment and storage functions for the highest and lowest value of capital and demand shock are plotted against different levels of availability. Panel a) represents the two nonlinear functions in terms of the two extreme values of capital. Panel b) represents the functions for the two extreme realizations of the demand shock.

as the nonlinearities embedded in this model are well represented by these functions and the different kinks at the thresholds, X^* and K^*, can be identified.

As can be seen on the left panel, when $X < X^*$, I is constant with respect to availability and decreasing with K. The reason is that the investment decision is driven by the expected price. Below X^*, the relationship between the expected and spot price is severed. In a stockout market, the expected price only depends on the next period stock of capital along with the expected state of the demand governed by the autocorrelated shock Y. Investment follows suit: for a given fixed capital stock lying below K^*, optimal investment is constant and equals the gap between K and its target K^* so as to restore the optimality. Therefore, the higher K the lower I. In sum, whether K exceeds its target, I is either null or decreasing in K.

On the other hand, when the commodity is abundant in the market so that X lies above X^*, I is decreasing in X. In this regime, as the storage arbitrage condition holds, the expected price is indeed a function of the spot price which is decreasing with the availability. Moreover, given that each period the market availability equals the sum of past inventories with the realized production, an increase in the productive capacity is sluggish whenever stockpiling is optimal. It must be added that the rise in the option value of waiting stems from the higher uncertainty inherent in the transmission of spot price volatility to the expected price. It turns out that the bigger the quantity in store, the greater the delaying effect on investment which, consequently, is a nonincreasing function of availability for any given level of capital.

For its part, storage is nondecreasing in X and Y. Storage is profitable only when the commodity is sufficiently abundant in the market so as to depress the spot price. Investment and storage are negatively correlated, with inventories standing high when the market is flooded. Since excess supply is persistent, as the constant gradual shrinkage of the capital stock is slow, storage becomes more important as an adjustment channel while the productive capacity adjusts. Therefore, storage does not vary much with the capital stock.

As already mentioned and illustrated in the panel b) of figure 3, investment and, interestingly storage too, are really reactive to demand shocks. Investment response is of the same order of magnitude. All that means is that the persistent demand shock entails shifts in the whole demand curve affecting the expected price level and, in turn, the equilibrium investment and storage schedules. As a result, a higher demand pushes upward the expected price and so both the X^* and K^* thresholds.

Finally, the risk-neutrality assumption and constant interest rate allow us to equate the future price to the expected price at n maturity, such that,

$$F_{t,t+n}(X_t, K_t, Y_t) = \mathrm{E}_t\, P_{t+n}. \tag{13}$$

Figure 4 displays the futures curve for 12 maturities along several dimensions with the zero-maturity price as spot price. The top panels represent the futures curves when the realized demand shock is the highest while the bottom panels stand for the futures curves when the demand shock is at its lowest. Specifically, the futures price in panels a) and b) are displayed for different levels of incoming inventories keeping the capital stock at its average level. Panels c) and d) plot futures curves for different capital levels for the mean previous inventory level.

In all states of the world, futures prices converge to a state-invariant long-term equilibrium price, namely the unconditional mean of the spot price. The reversion to the unconditional mean is established in Routledge et al. (2000) and rests on the fact that storage has no impact on the long term price recalling that, in steady state, inventories are zero. Incoming inventories dampen the effect of the positive demand

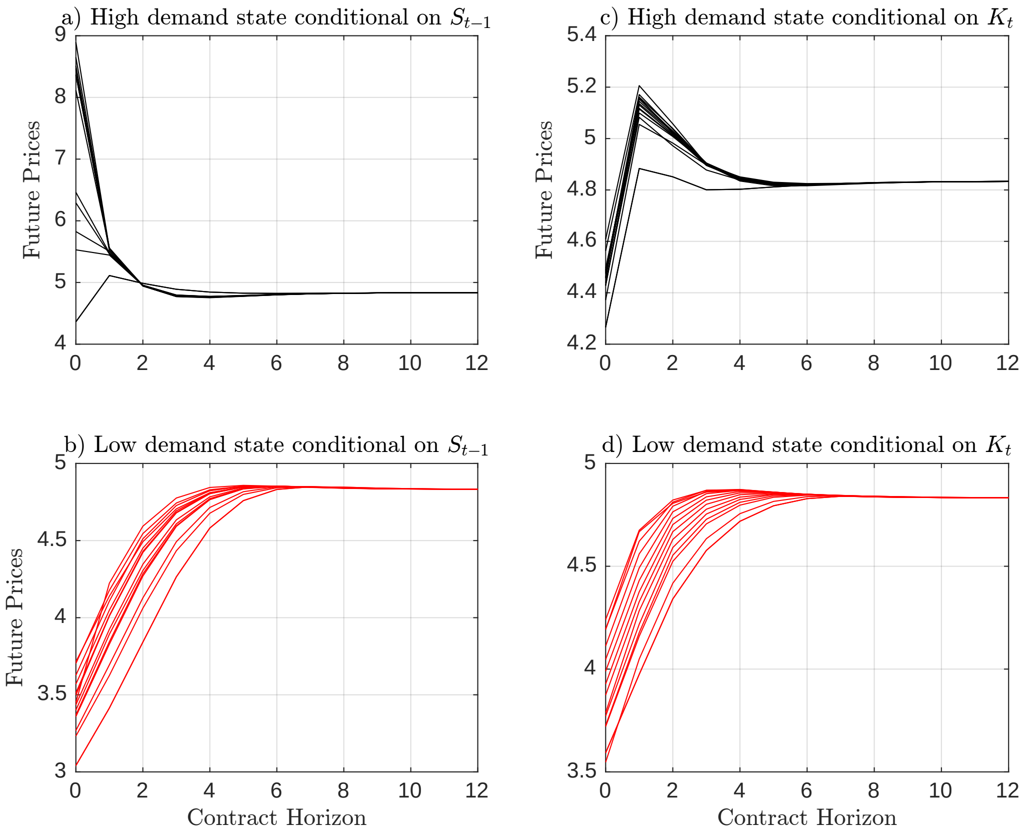

Figure 4: Futures Curves

Notes: The futures prices are plotted for each of the 12 contract maturity, with the zero maturity contract as the spot price. Panels a) and b) represent the futures curves for different levels of past inventory at the average capital level, when the demand shock is at its highest and lowest, respectively. Similarly, panels c) and d) show the futures curves for different levels of capital at the average income storage level, when the demand shock is at its highest and lowest, respectively.

shock on prices, the lowest spot price curve is thus associated with a high level of inherited inventories. Nevertheless futures curves are in backwardation when the demand shock is favorable. The degree of backwardation is highest when the quantities in store are the lowest if not zero.

Most futures curves are monotonous except when inventories are very high. In this case, the front-month price is above the spot price and the subsequent future prices decline. This goes to show that when there is an abundance of the commodity in the market, a favorable demand shock is not enough to alleviate the supply glut. Inventories affect the front-end of the forward curve in line with observations in Kogan et al. (2009). A favorable demand shock, often accompanied by either low or zero outgoing inventories, raises both the spot price and the subsequent forward prices. Given that Y is persistent, the distribution of the shocks shifts rightward and therefore the expected value of Y would be higher. There is a clear ordering of prices along the demand realizations regardless of the other state variables X_t and K_t. On the other hand, a negative demand shock results in an upward sloping futures curve for all possible values of incoming inventories, the lowest futures curve is associated with the highest level of past inventories or equivalently the highest level of availability. The term structure along the capital stock dimension is hump-shaped for the high values of Y. Since the inherited inventories are the same, the difference in the spot price comes from difference in the production capacity at the time the shock hits. In the presence of positive storage, a very favorable demand shock is more often than not associated with stockouts. Therefore next period price rises above the spot price today which is still kept low since there are positive incoming inventories.

All the above observations on the resulting decision rules for I and S and the price function have provided a general picture into the inner workings of an irreversible investment model with storage capacity. To go further in the empirical analysis, we simulate at a monthly frequency 1,000 sequences of length 500 for each state and decision variables of the model and, discarding the first hundred and fifty, we end up with a total 350,000 simulations across which to compute the different moments investigated. Regarding the futures prices, for liquidity and availability reasons we consider only the first twelve maturities.[29] As in Bloom (2009) the simulated investment variable has been divided by $0.5\,(K_t + K_{t-1})$ so as to be directly comparable to the observed investment rate variable I. The slope of the forward curve is defined as

$$\mathrm{SL}_t = \log\left(\frac{F_{t,12}}{F_{t,1}}\right). \tag{14}$$

As of time t, the forward curve is classified as being in contango (backwardation) when the demeaned slope $\tilde{\mathrm{SL}} = \mathrm{SL}_t - \bar{\mathrm{SL}}$ is positive (negative).

5.3 Discussion

We will study the influence of storage on the dynamics of prices and investment, when investment is irreversible. We will discuss simulation results of the full model, denoted as ModelIRREVS, as well as the three other models where we either remove the irreversibility constraint on investment or that of storage, or both. ModelREVS is an identical model to our full baseline model except that investment is fully reversible. ModelIRREV is an irreversible investment model with no storage capacity. Lastly, ModelREV is a model of reversible investment with no storage capacity. All of them are parametrized according to the calibration table 3. Using these four iterations of the model will allow us to isolate the interaction of storage capacity with the irreversibility constraint on investment; the crux of this paper. These results are displayed in Table 4.

The starting point is the unconstrained investment model with no inventory capacity. In this model, the demand for the commodity is only for consumption purposes and the only source of supply in the commodity market is the quantity produced. There is no capacity to absorb excess supply through storage. The only room for maneuver to counter unanticipated transitory supply or demand shocks is through changes in the production capacity. In this model, price and investment dynamics are far from what we observe in the data. When investment is unconstrained (ModelREV), the investment rate is negatively autocorrelated and barely positively skewed, while the spot price is negatively skewed, much more volatile and as half as persistent as its 3-month future counterpart. Looking at the ModelREVS's results, introducing storage without constraining investment does not really improve the dynamic properties of the induced prices and investment rate variables. On the whole, storage capacity smooths out the spot price, renders it positively skewed as well as more persistent and increases the lumpiness of investment along with the volatility of the expected future price. The presence of storage therefore stabilizes current

[29] It has already been shown in the previous section that future prices tend to converge to a long-run future price. What is more, although available the forward contracts beyond a year of maturities are too thinly traded to be really insightful. We can thus consider the 12-month forward contract to be F_∞ as in Routledge et al. (2000).

Table 4: Comparison of true with simulated data features

	Price	3-month future price	Investment	Storage
Volatility				
ModelIRREVS	0.27	0.07	0.02	0.11
ModelREVS	0.16	0.02	0.07	0.05
ModelIRREV	0.41	0.03	0.01	
ModelREV	0.33	0.01	0.04	
Data	0.31	0.31	0.03	0.05
Persistence				
ModelIRREVS	0.76	0.77	0.22	0.93
ModelREVS	0.54	0.75	-0.22	0.61
ModelIRREV	0.60	0.64	0.32	
ModelREV	0.31	0.77	-0.16	
Data	0.94	0.92	0.96	0.82
Skewness				
ModelIRREVS	0.55	-0.81	4.63	1.03
ModelREVS	0.88	-0.04	0.21	0.56
ModelIRREV	-0.07	-0.59	1.71	
ModelREV	-0.07	-0.01	0.19	
Data	0.59	0.66	1.59	0.23
Backwardation & Inaction rate (%)	$\tilde{SL} < 0$		$I = 0$	$S = 0$
ModelIRREVS	0.50		32.16	17.47
ModelREVS	0.53			10.09
ModelIRREV	0.54		24.08	
ModelREV	0.50			
Data	0.49			

Notes: The table shows moments of simulated prices, storage and investment rate time series from four models: ModelIRREVS is the full model with irreversible investment and storage; ModelIREVS is a model with reversible investment and storage; ModelIRREV is a model with irreversible investment and no storage capacity; and ModelRev is a model with reversible investment and no storage capacity. The corresponding models from the oil industry data are included in the last row of each subtable.

prices at the detriment of the stability of future production and prices.

Still, the impact of storage on the dynamics of the model change substantially with inclusion of a constraint on investment. The smoothing effect of storage on current price volatility is weakened when investment cannot adjust as easily. Indeed, comparing the spot price volatility from ModelREVS to ModelREV models, storage halves price volatility without constraint on investment. When investment becomes irreversible, price volatility drops by only a third (ModelIRREV vs. ModelIRREVS).

Investment irreversibility generates more persistence in spot and futures prices as well as the investment rate. Interestingly, storage capacity strengthens by about 20% this lingering effect of irreversibility on both the spot and future prices, bringing the values of their respective autocorrelation coefficients closer to those observed in the data.[30] What is more, the investment rate is twice as volatile, which is more in line with the true value, with an inaction rate increasing by a quarter. The stocks accumulation, as a substitute for capital investment increasing future supply, is a less costly source of additional supply

[30]Regarding the storage model literature, accounting for the dynamics of capital accumulation helps answering the puzzling lack of prices autocorrelation the model can explain which has first been noted in Deaton and Laroque (1992, 1996).

when investment is irreversible. In other words, the opportunity cost of investment, on average, increases in the presence of storage. Finally, when investment is irreversible, storage is also more persistent and stockouts are more frequent because it is relied upon more as a source of supply of the commodity than when investment can be costlessly adjusted. Finally, the four models are able to replicate the observed duration of time spent in backwardation.

Overall, as documented in Table 4, a model with investment as the sole variable of adjustment to demand and supply fluctuations does not generate the correct spot and futures prices stylized facts in terms of volatility, persistence and skewness. Thus, when it comes to modeling the dynamics of commodity prices it is important to allow for inventory building in combination with irreversible investment. Fluctuations in commodity prices are mainly driven by both these types of investment; neither storage nor fixed capital investment taken alone is enough. In addition, investment dynamics is also impacted by the presence of inventories as a second channel for transferring the commodity supply intertemporally. In particular, the rate of inaction in capital investment is higher when storage is possible. This can be explained by the presence of two substitutable ways to meet economic fluctuations.

Storage is used to mitigate the effects of a supply glut on the commodity price and therefore prevents the latter from dropping too quickly. At the same time, while storage is profitable, the production capacity or the targeted level of capital is decreasing in storage. In section 5.2, we explained that the storage arbitrage condition creates a tight link between the spot and expected futures prices. One implication of this condition is that storage, which occurs at lower spot prices, ties the next-period prices to this low level of price. As a result of the storage arbitrage, the expected futures prices are pinned down to a lower price regime, leading to a lower marginal profitability of investment and ultimately a lower investment rate. Additionally, the speculation through storage renders the expected futures prices more volatile. Because we are in an environment where the decision to invest is impacted by both uncertainty and sunk costs, the presence of storage through this volatility channel increases the value of postponing investment and of waiting to receive more information before committing to an investment project of this sort. As a result, in response to the higher uncertainty surrounding the expected price whenever inventories are carried over, producers tend to wait, thereby making investment even more intermittent and pronounced. In other words, we invest less often but more aggressively. It turns that both the rate of inaction and the positive spike rate are higher when the possibility to store the commodity does exist. Since investment is irreversible, it is more optimal to wait for more information, defer investment, and instead use storage to smooth fluctuations in the commodity market. In this set up, there is a temporary lack of investment which is caught up by a sharp increase when inventories are run down completely. When outgoing inventories are either running low or empty, the spot price is higher and decoupled from the expected futures prices, the market is thus in backwardation, and it is no longer optimal to delay investment. It is precisely the role of the volatility transmission from the spot to futures prices along the forward curve brought by storage that we would like to emphasize is the upcoming discussion focusing on both the irreversible investment models with and without storage (i.e., ModelIRREVS and ModelIRREV).

First is to remember that, as mentioned earlier, the postponing effect is intimately linked to the degree of uncertainty. The storage arbitrage condition, by connecting both the expected to the spot prices transfers the volatility of the latter into the future. This very condition results in an increased uncertainty around the expected price. When the expected price volatility rises, the anticipated marginal profitability of investment falls accordingly and the producer prefers to invest less or even nothing as to avoid being constrained in the future.

While the presence of storage has a strong smoothing effect on the spot price of the commodity, it is destabilizing for both investment and the expected price. Figure 5 illustrates the average expected marginal profitability of investment along with the spot and expected 3 and 12-month futures prices volatilities for both models conditional on the slope of the forward curve.

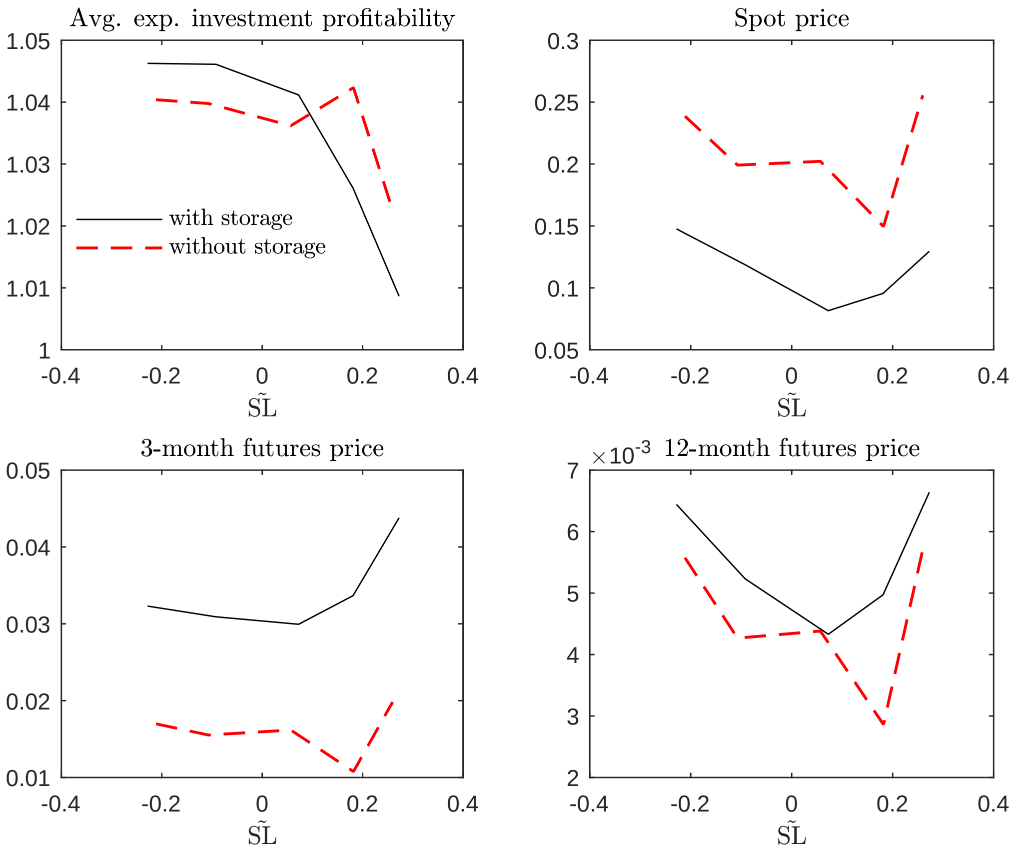

Figure 5: Conditional mean profitability of I and volatility of P, F_3, and F_{12}

Notes: The corresponding conditional average and standard deviations values from both models with and without storage are plotted against the slope of the forward curve, $\tilde{SL}$. These moments are computed for each of the 20th percentiles of the distribution of the simulated time series $\{I_t\}_{t=0}^{T}$ and $\{P_t\}_{t=0}^{T}$.

As discussed at length in Kogan et al. (2009), the conditional volatility of the spot and futures prices is V-shaped as we move along the slope of the forward curve: it is higher the more backwardated market, indicative of a supply shortage, dips when the market is moderately satiated and spikes again during a supply glut. Furthermore, introducing storage to an irreversible investment model cuts by almost one third the standard deviation of the spot price as documented in Table 4. Also noteworthy is that, on the contrary, with storage the expected futures prices whether at 3 or 12-month to maturities are always more volatile. This heightened volatility eventually affects the expected marginal return on investment which depends not only on the expected price but also, and given the irreversible nature of the investment, on the degree of future uncertainty according to the real options theory. If uncertainty is higher, increasing the variance of expected cash flows, the producers will want to avoid being constrained and thus will invest less today. The presence of storage thus generates higher uncertainty for instances of a market switching from a backwardated to a contango regime. Indeed, since the shape of the futures curve is positively correlated with storage, the higher the slope–e.g., the more inventories there are–the higher the options value of waiting and delaying investment. This is clearly what can be observed in the top left-hand panel of figure 5 with the collapse in the mean expected marginal profitability of investment. More importantly, this fall almost exactly parallels the strong rise in volatility of the 3-month and 12-month futures prices plotted in the bottom panels.

All in all, the increased endogenous instability of futures prices gives way to a more volatile investment rate compared to the crude irreversible investment model. What is more, this region of higher uncertainty would be accompanied by lower investment rates. Interestingly, it turns that the delaying effect of storage also helps explain why the volatility rises as the market turns into backwardation, that is as the slope of the forward curve takes on negative values. Indeed, the low excess capacity of production starts to translate into higher uncertainty because up until now, the producers have delayed investment more than they would have if they were no storage capacities. When outgoing inventories are running low, the spot price becomes more affected by both disturbances in supply and the persistent demand shock. Since the production capacity adjusts sluggishly, coupled with low outgoing inventory levels, the expected futures prices will be more volatile than under the stark model overlooking storage.

Correspondingly, we draw a comparison of the observed relationships between future uncertainty, investment, and storage with those obtained from our simulated variables in Figure 6. The similarities

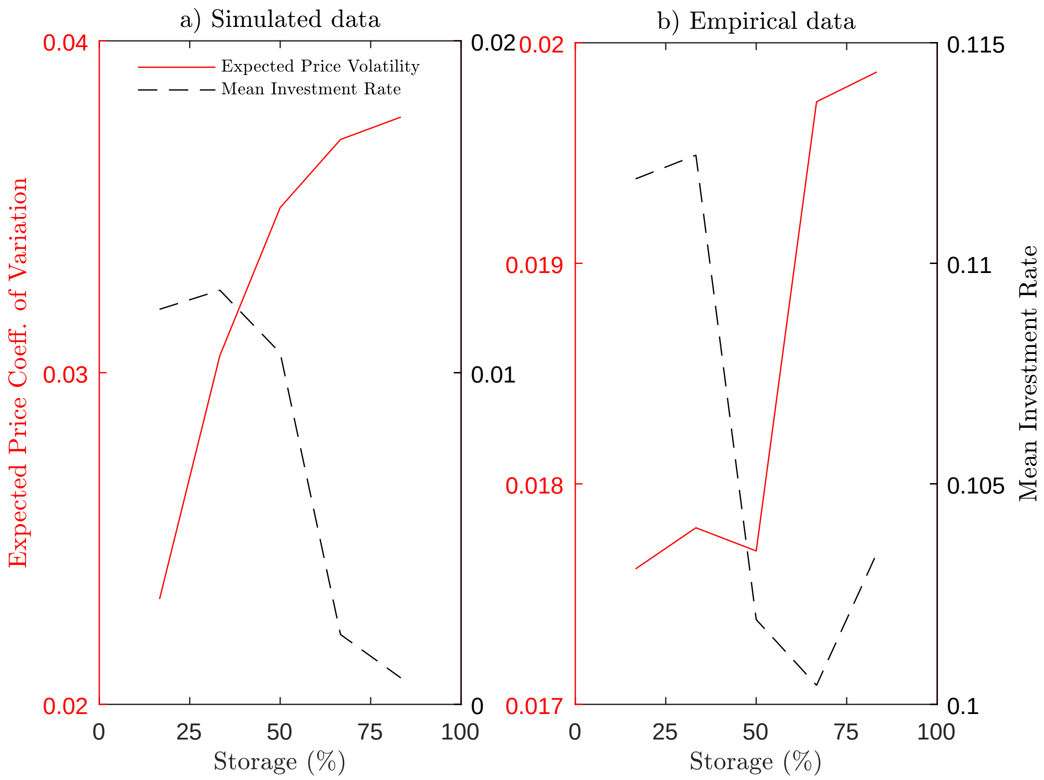

Figure 6: Conditional volatility of F_3 and mean investment rate

Notes: The left scale stands for the mean of the simulated investment rate while the right one represents the coefficient of variation of the simulated time series for the 3-month price, F_3. They both are computed for each 20th percentile of the storage levels distribution. Panel a) represents represents the conditional volatility of the expected price and the mean investment rate for each percentile of the storage distribution based on the simulated data, whereas Panel b) represents its empirical counterpart.

are noteworthy. Indeed, the investment rate is decreasing in the level of inventories, corroborating the relationship between the expected marginal profitability of investment and the slope of the forward curve in Figure 5. Furthermore, storage has a destabilizing effect on the forward price as measured here by the volatility of the 3-month futures price, as storage mostly affects the front-end of the forward curve (Kogan et al., 2009). The elevated levels of uncertainty when the storage mechanism is at play explains also why in this regime the optimal investment rate is lower. Finally, the volatility of the forward price starts to plateaued when the supply glut is at its highest meaning that the storage smoothing effect on the front-end of the forward curve dominates its destabilizing effect.

All in all, uncertainty about the future, when the producer is constrained and cannot resell his capital in bad states of the world, makes him more prudent when deciding to invest. This precautionary effect of the nonnegativity constraint on investment is most present when the market is in contango or weakly backwardated. The effect of the irreversibility constraint on investment is strengthened when storage is allowed for. This is a novel insight and certainly extends our understanding of a firm's investment decision when it can decide between two types of investment: fixed capital investment and inventory investment. Additionally, the endogenous time-varying volatility that is brought about by storage through the variations in tandem between the spot and the forward price is key in generating the delaying effect of storage on the investment rate.

6 Conclusion

Storage and investment are the two main economic mechanisms serving as the theoretical bedrock of most of the dynamic commodity models. In addition, the recent development of liquid futures market offers a valuable way to test empirically these forward-looking theories by looking at the market's reaction to various shocks and see if they match the model's predictions. Even from the strict perspective of the operators in the global market of crude oil, the two most scrutinized metrics are the inventory levels and drilling counts publicly published on a weekly basis. They are believed to mirror the prevailing supply and demand balance, where a glut is associated with ballooned inventories and diminishing capital expenditures. This is why we believe that it is worth studying investment and storage decisions jointly to account for fluctuations in prices, a point noted but thus far neither explored in the dynamics of capital accumulation nor competitive storage model literature.

This paper aims at filling this gap by building upon three strands of the economics and finance literature (e.g., capital accumulation, storage and futures pricing) to lay out a partial equilibrium-framework placing investment and storage at the forefront of the economic decisions dictating the dynamics of a commodity market. The simulations results obtained on four versions of the model depending whether or not investment is irreversible and storage is acknowledged clearly support the importance of considering both economic mechanisms to best account for the observed formation and fluctuations of both the spot and expected futures prices. They demonstrate that, at the margin, investment is less profitable whenever storage is possible. Indeed, not only carrying inventories will weigh on the expected price and in turn the marginal benefit of investing today, but also will raise future uncertainty and thus the options value of waiting and postponing investment. Put another way, operators invest less often but more aggressively. The deferred investment translate into lower capital stock and hence a mitigated production capacity. As a result, the commodity becomes scarcer, storing is more costly, thereby leading to an upward turn in the spot price and a tighter market (i.e., backwardated). The supply and demand tension is eventually alleviated with a renewed increase in the production capacity through capital investment. Such a narrative of the cyclical emergence of booms and busts in the oil industry broadly proves to match the data fairly well. The key insight which emerges out of the confluence of inventory and fixed capital investment is the implications of lockstep variations in the spot and forward prices on the investment decision. This tight link between the two prices translates into higher future uncertainty, rendering the nonnegativity constraint even more penalizing. Ultimately, storage reinforces the irreversibility of investment.

In terms of extensions, following Bachmann and Ma (2016) it might be worthwhile to study the effects of capital adjustment costs of various nature–fix, convex and non-convex–in the tradition of Cooper and Haltiwanger (2006) since they are possibly significant in capital intensive sectors like the oil industry. Among the expected effects of interest are a higher persistence in production translated into the volatility levels of forward prices which are dying-off too quickly in our current modeling as compared to the levels observed, although it is a shared drawback among many dynamic economic models of this kind. Attractive also should be considering the new frictions subsequent to the introduction of a second but not predetermined factor of production (for e.g., labor known to be the very first expendable in times of turmoil in the oil market) and the resulting effects on the dynamics implied by the model. Finally, and perhaps more challenging given the computational issues at stake, it would be to push the empirical analysis even further in estimating the model key parameters in place of the current calibration.

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