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Functional Form and U.S. Agricultural Production Elasticities

C. Richard Shumway and Hongil Lim

Because so much agricultural policy analysis utilizes estimates of supply and demand elasticities, it is crucial to obtain the most reliable estimates possible. Where reliability cannot be adequately assessed, the sensitivity of elasticities to equally plausible a priori specifications should at least be ascertained. In this article, the sensitivity of U.S. output supply and input demand elasticities to choice of functional form is examined and tests are conducted to identify the preferred functional form. Considerable sensitivity is found to choice of functional form. Although most frequently used, the translog is generally the outlier and is the least preferred among the alternatives.

Key words: agriculture, dominance ordering, elasticities, functional form, production (U.S.).

Introduction

Several recent studies have reported output supply and input demand elasticity estimates for U.S. agriculture (Antle; Vasavada and Chambers; Shumway, Saez, and Gottret; Ball 1988; Huffman and Evenson). Some have focused on aggregate output measures, but several have included estimates of supply elasticities for multiple outputs. A summary of own-price output supply and input demand elasticities from five studies is reported in table 1. Elasticities are similar for a few categories with multiple estimates, such as all output and feed grain supplies. For far more categories, however, the elasticities vary widely among estimates. For example, the livestock supply elasticities range from .11 to 1.09, machinery demand from −1.27 to +.12, real estate demand from −.58 to −.02, labor demand from −.51 to +.02, hired labor demand from −1.50 to −.10, and energy demand from −.94 to −.25.

These five studies differ not only in functional form but also in data sources, observation period, estimation method, maintained theoretical structure, inputs treated as variable, and point at which elasticities are computed.[1] Two use the translog functional form and three the normalized quadratic. None uses the same data as any other. Although all are estimated using post-World War II data, the first observation varies from 1946 to 1951 and the last observation from 1974 to 1982. Most use aggregate time series data, but one uses data for cash grain farms in 42 states from six agricultural censuses. Estimation methods include seemingly unrelated regression (SUR), maximum likelihood, maximum likelihood followed by nonlinear least squares, and three-stage least squares (3SLS) followed by nonlinear least squares. Both static and dynamic models are included. The hypothesis that the restricted profit function is twice-continuously differentiable is main-

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Table 1. U.S. Output Supply and Input Demand Own-Price Elasticities

Item	Source				
	Ball (1988) Translog	Shumway, Saez, and Gottret (1988) Norm. Quad.[a]	Huffman and Evenson (1989) Norm. Quad.	Vasavada and Chambers (1986) Norm. Quad.[b]	Antle (1984) Translog
Output Supply:					
All Outputs				.54	.43
Livestock	1.09	.11/.13			
Fluid Milk	.64				
Grains	.84				
Food Grains		.31/.31			
Wheat			.97		
Feed Grains		.11/.12	.02		
Oilseeds	.43	.10/.12			
Soybeans			1.31		
Other Crops	1.11	.08/.15			
Input Demand:					
Machinery	−1.27	−.11/−.27	−.61	.12	−.25
Real Estate	−.58			−.03	−.18
Farm-Produced Durables	−1.16				
Labor			−.51	−.51	−.01
Hired Labor	−1.50	−.10/−.40			
Energy	−.94	−.26/−.28			−.25
Fertilizer			−1.20		
Fuel			−.72		
Materials		−.08/−.11		−.34	
Other Purchased Inputs	−2.90				

[a] First number from direct estimates for U.S.; second number from regional aggregation.
[b] Long-run elasticities.

tained in all five studies. The hypothesis that profit is linearly homogeneous in exogenous prices is maintained in all, but the convexity implication of profit maximization is maintained in only two of the five studies, those which use nonlinear least squares estimation methods. It is satisfied by the empirical estimates in three.

The purposes of this article are: (*a*) to determine the sensitivity of disaggregated output supply and input demand elasticities to choice of functional form in modeling a restricted profit function of U.S. agriculture, (*b*) to conduct a simple test for choice of functional form, and (*c*) to examine the impact of using the least preferred form for energy policy analysis. Three common functional forms are examined—translog, generalized Leontief, and normalized quadratic.

The same procedures are used to estimate each functional form. Homogeneity, convexity, and symmetry are maintained in each estimation. Monotonicity is not maintained but is examined at each observation. Thus, all implications of price-taking, profit-maximizing behavior for a twice-continuously-differentiable technology are maintained or examined for each functional form.

Previous studies have documented that empirical estimates of structural and economic relationships can be sensitive to choice of functional form among possible second-order expansions (e.g., Swamy and Binswanger; Chalfant; Baffes and Vasavada; Howard and Shumway). However, none of the prior studies maintained curvature conditions in the comparison and none focused on the results that are most often used in U.S. agricultural policy analysis, i.e., the output supply and input demand elasticities.

All three functional forms are second-order Taylor series expansions, have exactly the same number of parameters requiring estimation, and provide equally plausible a priori approximations of a true but unknown functional form. However, all three do maintain important restrictions relevant to modeling production relationships. Even at the point

Table 2. Mean Revenue and Variable Cost Shares

Category	Mean Share
Outputs (Revenue):	
Livestock	.347
Fluid Milk	.130
Grains	.160
Oilseeds	.065
Other Crops	.298
Variable Inputs (Variable Cost):	
Durable Equipment	.147
Real Estate	.143
Farm-Produced Durables	.097
Hired Labor	.116
Energy	.063
Other Purchased Inputs	.434

of approximation, the generalized Leontief and normalized quadratic maintain quasi-homotheticity on the technology which implies that the input demands are strongly separable in input prices. The translog does not maintain this restriction, but it is less "separability flexible" than the other two functional forms (Pope and Hallam; Blackorby, Primont, and Russell).

Empirical Approach

Because of the quality and comprehensiveness of Ball's (1988) data, they were selected for use in this analysis. The data are an annual time series for the period 1948–79. They represent a comprehensive set of output and input quantity and price data organized into five output categories (livestock, fluid milk, grains, oilseeds, and other crops) and seven input categories (durable equipment, real estate, farm-produced durables, hired labor, energy, other purchased inputs, and self-employed labor). Their construction and input organization carefully follow the 1980 recommendations of an American Agricultural Economics Association (AAEA) Task Force. For details, see Ball (1985, 1988). Mean revenue or variable cost share of each category is reported in table 2.

Following Ball's specification, self-employed labor was treated as a fixed input. Because constant returns to scale were assumed in his construction of the data, estimation of all functional forms was conducted subject to this maintained hypothesis. With a single fixed input, constant returns to scale imply that the normalized profit function is linear homogeneous in the quantity of the fixed input. That is, with prices constant, a change in the quantity of the fixed input elicits an equal-proportion change in profit since variable inputs and outputs are voluntarily changed by the same proportion.

The system of first-derivative equations for each functional form was estimated using the same procedures and maintaining homogeneity, symmetry, and convexity in prices and constant returns to scale. To the extent possible, the procedures used by Ball were followed here. Disturbance terms were assumed to be additive, normally and identically distributed, with mean zero and a constant contemporaneous covariance matrix for each system. Each system was first estimated by iterative seemingly unrelated regression (ITSUR) while maintaining constant returns to scale and linear homogeneity and symmetry of the restricted profit function in prices. The estimates were iterated until the coefficient vector and covariance matrix stabilized. Then, using the covariance matrix from the ITSUR solution, convexity was maintained by the Cholesky factorization, and final estimation was carried out by constrained nonlinear least squares.

Two departures from Ball's approach were followed in our estimation procedures. First, Ball maintained linear homogeneity in prices by means of linear restrictions on the translog functional form. When linear restrictions are used to impose homogeneity on the quadratic, the functional form loses its second-order flexibility. Therefore, to permit comparable estimation procedures to be used for all functional forms and to also assure second-order flexibility of each, linear homogeneity in prices was maintained for all functional forms by normalization (division). The price of other purchased inputs was used as the numeraire (normalizing) price. It was chosen because its share equation was the one excluded from the system of first-derivative equations estimated by Ball.

Second, MINOS Version 5.0 (Murtagh and Saunders) was used both by us and by Ball to obtain parameter estimates consistent with the industry behaving as though it were a price-taking, profit-maximizing firm facing a constant-returns-to-scale aggregate production function. However, the specific software used to obtain a starting point and to perform the nonlinear least squares estimation differed. The reduced-gradient nonlinear programming procedure of Talpaz, Alexander, and Shumway was used here. Monotonicity in prices (an implication of price-taking, profit-maximizing behavior) was not maintained either in Ball's or our estimation, but it was checked for our estimates at each observation.

Ignoring the disturbance terms for ease of exposition, the first derivatives of the translog were the system of share equations:

$$(1) \qquad s_i = a_i + \sum_{j=1}^{n-1} a_{ij}\ln(p_j) + a_{it}t, \qquad i = 1, \ldots, n-1,$$

where s_i is the profit share ($p_i x_i/\pi$) of output or input i, p_i is the price of output or input i divided (normalized) by the price of other purchased inputs, x_i is the quantity of output or input i measured as a netput (i.e., positive for an output and negative for an input), π is normalized profit plus returns to self-employed labor, t is time, and the rest are parameters. For the generalized Leontief, the estimation equations were the system of netput supply equations:

$$(2) \qquad x_i/z = b_i(1/p_i)^{.5} + b_{ii} + \sum_{j=1}^{n-1} b_{ij}(p_j/p_i)^{.5} + b_{it}t, \qquad i = 1, \ldots, n-1, j \neq i,$$

where z is the quantity of the fixed input. For the normalized quadratic, they were:

$$(3) \qquad x_i/z = c_i + \sum_{j=1}^{n-1} c_{ij}p_j + c_{it}t, \qquad i = 1, \ldots, n-1.$$

The procedure used to test for choice of functional form was the nonnested dominance ordering test of Pollak and Wales. They have demonstrated that the likelihood values, appropriately adjusted by the Jacobian term when the dependent variable differs [as in (1) relative to (2) and (3)], provide an unambiguous ordering of preferred models when the number of estimated parameters in each model is the same. When one alternative is accepted and another rejected, it is always the model with the higher adjusted likelihood value that is accepted. This simple test does not necessarily imply a high probability of accepting one and rejecting the other. It is only asserted that if a composite model could be formulated such that each of the alternative models was nested within it, the probability would be zero of selecting by the likelihood ratio test the model with the lower likelihood function value and rejecting the one with the higher value.

Results

Elasticities were computed at the 1977 observation (the point of approximation) for each of the sets of estimates. They are reported for Ball's translog estimates and for our translog, generalized Leontief, and normalized quadratic estimates in table 3. Approximate standard errors are also reported based on first-order Taylor-series expansions of the elasticity equations (Miller, Capps, and Wells).[2]

Table 3. U.S. Output Supply and Input Demand Own-Price and Cross-Price Elasticities, Four Estimates

		Elasticity with Respect to the Price of										
Item	Model	Live-stock	Fluid Milk	Grains	Oil-seeds	Other Crops	Durable Equip-ment	Real Estate	Farm-Produced Durables	Hired Labor	Energy	Other Purchased Inputs
Output Supply:												
Livestock	Ball	1.09	.49	.48	.40	1.01	−.53	−.28	−.37	−.42	−.29	−1.59
	TL	1.26	.59	.73	.62	1.31	−.82	−.50	−.34	−.53	−.37	−1.96
		(.13)	(.05)	(.18)	(.09)	(.16)	(.10)	(.24)	(.11)	(.04)	(.02)	(.14)
	GL	.13	−.02	.00	.07	.08	−.04	.02	−.01	.02	.01	−.25
		(.05)	(.01)	(.04)	(.03)	(.06)	(.02)	(.01)	(.01)	(.01)	(.01)	(.09)
	NQ	.22	.02	.11	.17	.10	−.11	−.08	−.05	−.04	−.04	−.30
		(.06)	(.02)	(.05)	(.05)	(.07)	(.03)	(.03)	(.02)	(.02)	(.01)	(.13)
Fluid Milk	Ball	1.27	.64	.60	.48	1.17	−.56	−.32	−.32	−.55	−.41	−2.00
	TL	1.48	.69	.86	.73	1.53	−.95	−59	−.41	−.61	−.43	−2.30
		(.15)	(.05)	(.17)	(.09)	(.15)	(.09)	(.23)	(.10)	(.05)	(.03)	(.14)
	GL	−.06	.19	.08	−.08	.30	.09	.01	−.01	−.09	.06	−.50
		(.03)	(.06)	(.04)	(.03)	(.06)	(.02)	(.01)	(.01)	(.04)	(.02)	(.10)
	NQ	.04	.21	.16	.05	.26	.02	−.07	−.07	−.21	.02	−.42
		(.06)	(.06)	(.05)	(.05)	(.09)	(.04)	(.03)	(.02)	(.05)	(.02)	(.14)
Grains	Ball	.99	.49	.84	.41	.95	−.19	−.42	−.47	−.31	−.17	−2.12
	TL	1.30	.62	1.07	.76	1.39	−.63	−.76	−.64	−.40	−.27	−2.44
		(.22)	(.06)	(.14)	(.11)	(.20)	(.13)	(.24)	(.11)	(.05)	(.04)	(.22)
	GL	−.01	.06	.24	.15	.15	.12	−.06	−.03	−.03	.01	−.61
		(.07)	(.03)	(.15)	(.08)	(.11)	(.05)	(.03)	(.02)	(.03)	(.02)	(.24)
	NQ	.20	.11	.32	.31	.17	−.10	−.15	−.11	−.13	−.07	−.55
		(.10)	(.04)	(.13)	(.11)	(.12)	(.06)	(.05)	(.03)	(.04)	(.02)	(.27)
Oilseeds	Ball	1.12	.52	.55	.43	1.02	−.52	−.34	−.41	−.36	−.32	−1.69
	TL	1.48	.70	1.02	.89	1.65	−1.03	−.74	−.68	−.44	−.44	−2.39
		(.18)	(.05)	(.18)	(.10)	(.16)	(.11)	(.23)	(.11)	(.05)	(.03)	(.18)
	GL	.16	−.07	.20	.31	.13	.01	−.05	−.04	.12	−.04	−.73
		(.07)	(.03)	(.10)	(.10)	(.12)	(.05)	(.03)	(.02)	(.04)	(.02)	(.26)
	NQ	.41	.05	.42	.51	.16	−.22	−.21	−.15	−.09	−.13	−.73
		(.13)	(.05)	(.16)	(.17)	(.18)	(.09)	(.07)	(.04)	(.05)	(.03)	(.38)

Table 3. Continued

Item	Model	Elasticity with Respect to the Price of: Live-stock	Fluid Milk	Grains	Oil-seeds	Other Crops	Durable Equip-ment	Real Estate	Farm-Produced Durables	Hired Labor	Energy	Other Purchased Inputs
Other Crops	Ball	1.09	.49	.49	.39	1.11	−.61	−.28	−.32	−.47	−.22	−1.68
	TL	1.29	.61	.77	.68	1.53	−1.04	−.53	−.43	−.56	−.29	−2.04
		(.16)	(.05)	(.17)	(.09)	(.12)	(.09)	(.24)	(.10)	(.05)	(.03)	(.15)
	GL	.08	.12	.08	.05	.42	.01	.01	−.02	−.04	.07	−.79
		(.05)	(.03)	(.06)	(.05)	(.13)	(.04)	(.02)	(.02)	(.03)	(.02)	(.17)
	NQ	.10	.10	.09	.07	.24	−.04	−.04	−.03	−.08	.02	−.42
		(.07)	(.03)	(.07)	(.07)	(.15)	(.05)	(.03)	(.02)	(.04)	(.02)	(.21)
Input Demand:												
Durable Equipment	Ball	1.36	.55	.24	.47	1.45	−1.27	−.19	−.23	−.44	−.32	−1.61
	TL	1.68	.78	.73	.89	2.16	−1.92	−.57	−.36	−.69	−.49	−2.22
		(.16)	(.04)	(.18)	(.10)	(.13)	(.12)	(.23)	(.11)	(.04)	(.02)	(.15)
	GL	.07	−.08	−.14	−.01	−.03	−.11	.04	.01	.07	−.01	.19
		(.04)	(.02)	(.05)	(.04)	(.07)	(.04)	(.02)	(.01)	(.02)	(.01)	(.11)
	NQ	.23	−.02	.11	.19	.09	−.20	−.10	−.06	−.01	−.05	−.17
		(.06)	(.03)	(.07)	(.07)	(.10)	(.05)	(.03)	(.02)	(.03)	(.01)	(.17)
Real Estate	Ball	.86	.39	.64	.38	.81	−.24	−.58	−.62	−.25	−.20	−1.19
	TL	1.24	.59	1.06	.77	1.34	−.69	−.99	−.67	−.40	−.32	−1.94
		(.30)	(.06)	(.19)	(.10)	(.28)	(.14)	(.47)	(.12)	(.05)	(.03)	(.23)
	GL	−.04	−.01	.09	.05	−.02	.04	−.03	−.01	.00	−.01	−.06
		(.03)	(.01)	(.04)	(.03)	(.06)	(.02)	(.01)	(.01)	(.01)	(.01)	(.09)
	NQ	.19	.07	.21	.22	.10	−.13	−.13	−.09	−.11	−.06	−.28
		(.07)	(.03)	(.07)	(.07)	(.09)	(.04)	(.05)	(.02)	(.03)	(.01)	(.17)
Farm-Produced Durables	Ball	1.33	.46	.81	.53	1.07	−.32	−.71	−1.16	−.24	−.22	−1.54
	TL	1.24	.59	1.30	1.03	1.56	−.63	−.97	−1.31	−.34	−.30	−2.17
		(.23)	(.06)	(.19)	(.11)	(.17)	(.14)	(.24)	(.18)	(.06)	(.04)	(.18)
	GL	.02	.02	.06	.06	.08	.02	−.01	−.01	.01	.00	−.25
		(.02)	(.01)	(.04)	(.03)	(.06)	(.02)	(.01)	(.01)	(.02)	(.01)	(.09)
	NQ	.19	.10	.23	.23	.13	−.11	−.14	−.10	−.12	−.06	−.34
		(.06)	(.03)	(.06)	(.06)	(.09)	(.04)	(.03)	(.02)	(.03)	(.01)	(.15)

Table 3. Continued

Item	Model	Elasticity with Respect to the Price of: Live-stock	Fluid Milk	Grains	Oil-seeds	Other Crops	Durable Equip-ment	Real Estate	Farm-Produced Durables	Hired Labor	Energy	Other Purchased Inputs
Hired Labor	Ball	1.63	.84	.57	.50	1.69	−.67	−.31	−.26	−1.50	−.38	−2.10
	TL	1.88	.88	.81	.67	2.03	−1.20	−.57	−.34	−1.59	−.43	−2.13
		(.15)	(.07)	(.17)	(.16)	(.18)	(.10)	(.22)	(.11)	(.09)	(.04)	(.16)
	GL	−.07	.13	.06	−.18	.14	.12	.00	.01	−.42	.20	.01
		(.05)	(.05)	(.06)	(.05)	(.11)	(.04)	(.02)	(.02)	(.06)	(.03)	(.14)
	NQ	.16	.30	.27	.14	.29	−.02	−.16	−.12	−.76	.10	−.19
		(.08)	(.08)	(.08)	(.08)	(.15)	(.06)	(.04)	(.03)	(.12)	(.03)	(.20)
Energy	Ball	1.47	.82	.41	.59	1.04	−.65	−.33	−.31	−.50	−.94	−1.59
	TL	1.76	.82	.72	.88	1.41	−1.14	−.61	−.40	−.57	−1.11	−1.77
		(.14)	(.06)	(.17)	(.09)	(.15)	(.09)	(.22)	(.11)	(.05)	(.05)	(.13)
	GL	−.03	−.11	−.02	.09	−.33	−.03	−.02	.00	.27	−.17	.36
		(.04)	(.04)	(.06)	(.04)	(.07)	(.03)	(.02)	(.01)	(.04)	(.03)	(.12)
	NQ	.20	−.04	.18	.26	−.07	−.13	−.11	−.08	.14	−.13	−.22
		(.05)	(.04)	(.06)	(.05)	(.07)	(.03)	(.03)	(.02)	(.05)	(.03)	(.12)
Other Purchased Inputs	Ball	1.41	.69	.91	.54	1.39	−.56	−.34	−.38	−.48	−.28	−2.90
	TL	1.63	.77	1.14	.83	1.72	−.90	−.65	−.50	−.49	−.31	−3.24
		(.16)	(.05)	(.18)	(.10)	(.16)	(.10)	(.23)	(.11)	(.05)	(.03)	(.42)
	GL	.21	.17	.28	.26	.67	.08	−.02	−.06	.00	.06	−1.65
		(.08)	(.03)	(.11)	(.09)	(.14)	(.05)	(.03)	(.02)	(.03)	(.02)	(.06)
	NQ	.25	.14	.26	.26	.36	−.07	−.09	−.08	−.04	−.04	−.94
		(.11)	(.05)	(.12)	(.13)	(.17)	(.07)	(.06)	(.03)	(.05)	(.02)	(.30)

Notes: The first elasticity in each set is from Ball's translog and the second through fourth are from our translog (TL), generalized Leontief (GL), and normalized quadratic (NQ) estimations, respectively. All are computed at the point of approximation, i.e., 1977. Approximate standard errors for our estimates are in parentheses below the estimates.

Some differences were noted due to estimation method. Ball's own-price elasticity estimates differed from our translog (TL) estimates by magnitudes ranging from .05 to .65, and four of his 11 estimates lay outside the 95% confidence interval of our estimates.

The largest differences, however, were due to functional form. Our TL and generalized Leontief (GL) own-price elasticity estimates differed by magnitudes of .50 to 1.81; all 11 estimates of each functional form lay outside the other functional form's 95% confidence interval. Our TL and normalized quadratic (NQ) estimates differed by magnitudes of .38 to 2.30; all 11 TL estimates lay outside the NQ confidence intervals, and 10 NQ estimates lay outside the TL confidence intervals. Differences between the GL and NQ estimates were considerably smaller and ranged from .02 to .71; five NQ and three GL estimates (all for input demands) lay outside the other functional form's confidence intervals.

Our TL own-price elasticity estimates were all larger in absolute magnitude than were Ball's TL estimates, which in turn were generally larger than either the GL or NQ estimates. Our finding that TL elasticities were larger than the GL or NQ elasticities is consistent with Diewert and Wales' observation when global curvature properties were maintained on each functional form. Our finding generalizes their result since curvature properties were maintained only locally for the TL and GL in this study. Eight of the 11 NQ estimates were larger than the GL estimates, but they generally differed by less than any other pair of estimates.

The rank-ordered magnitudes of the own-price elasticity estimates were positively correlated with the revenue and cost shares. For all functional forms, the demand elasticity was largest for other purchased inputs, the category that received nearly half the expenditure on all variable inputs. The correlation was stronger for inputs than for outputs, and for TL than for GL and NQ functional forms.

The mean of the absolute own-price elasticities for the GL and NQ were both .34. These means compared to 1.42 for our TL and 1.13 for Ball's TL estimates, and to .17, .18, .25, and .76 for the other four studies reported in table 1. Except for Ball's and our TL estimates, these and prior short-run elasticity estimates tend to be quite inelastic. Even with a year to respond to changes in expected prices, agricultural producers collectively appear to respond slowly. Asset fixity, specialized equipment and skills, uncertainty, and cost of information all combine to dampen the speed of adjustment.

Examination of the cross-price elasticities in table 3 reveals that only our TL estimates supported Ball's finding of gross complementarity in all outputs and inputs.[3] For the NQ, gross complementarity was also found among all outputs. For the generalized Leontief, the hypothesis of gross complementarity was supported for seven of the 10 output pairs. Thus, largely consistent with Ball's finding, we found little evidence from these data of short-run joint production in inputs due to constraining allocatable inputs. Ball's finding of input gross complementarity, however, was not supported for one input pair by the NQ estimates and for nine of the 15 input pairs by the GL estimates.

Monotonicity of our estimated profit functions in prices was checked by examining the sign of each predicted dependent variable at each observation. It was violated by the NQ for two output equations at the first observation, by the GL for two output equations at the first observation and one at the second observation, and by our TL for one input equation at the second observation and by another input equation at the second, fifth, and sixth observations. However, monotonicity was not rejected at the .01 level of significance for either the NQ or the GL. It was rejected for our translog.

Likelihood values for the three functional forms are reported in table 4. The likelihood value of the TL has been adjusted by the appropriate Jacobian term since its dependent (share) variables are functionally related to the dependent (quantity ratio) variables of the other two functional forms. The likelihood value is highest for the GL and lowest for the TL. The likelihood value for the NQ is just slightly lower than that for the GL and is much higher than the value for the TL. These results suggest that the GL is the preferred functional form and is followed closely by the NQ.

Our findings compare favorably with Ornelas' recent conclusions based on an alternative testing procedure. Using higher-level aggregates of these same data and nesting all three

Table 4. Log-Likelihood Function Values

Functional Form	Log-Likelihood Value
Translog	−204.10[a]
Generalized Leontief	−84.20
Normalized Quadratic	−85.68

[a] Adjusted by the Jacobian of the vector of dependent share variables in equation (1) with respect to the vector of quantity ratios in (2) and (3).

of our functional forms within a Box–Cox specification (Appelbaum; Berndt and Khaled), Ornelas conducted a likelihood ratio test (LRT) on each functional form as a more restricted version (or nested hypothesis) of the Box–Cox restricted profit function. He failed to reject the NQ at the 5% level of significance (LRT = 4.65 with a critical value of 5.99).[4] The GL was barely rejected at the 5% level (LRT = 6.35). The TL was soundly rejected even at the 1% level of significance (LRT = 16.50 with a critical value of 9.21). Thus, both sets of tests document least empirical support from these data for the TL as a valid specification of the restricted profit function. In both cases, the test statistics for the GL and NQ are much closer than are the test statistics for the TL and any other functional form.

The corroborated finding that both the GL and NQ are preferred to the TL for this data set takes on added practical importance since the absolute magnitudes of elasticities for these two functional forms are similar. Although we do not know the true functional form, if any, we find least support for the most popular form currently used by applied economists and econometricians. One practical question is: How serious an error would have been made in policy analysis if elasticities from the TL had been used instead of those from one of our preferred functional forms—the GL or NQ? This question is examined briefly here with respect to energy policy.

Consider, for example, that the U.S. chooses to impose an ad valorem tax that would effectively increase the price of all forms of energy used by American farmers by 50%. What would be the short-run (one-year) impact of that tax on energy usage, on other input demands, and on output combinations? If the point elasticities developed in this study could be extended realistically for such a large price change and if potential short-run compensating changes in other prices are ignored, the estimated impacts in percentage terms would be the elasticities multiplied by 50.[5] The implied impacts from each of the functional forms are reported in table 5. Estimated impacts from both the GL and NQ

Table 5. Estimated Percentage Impacts of 50% Ad Valorem Tax on 1977 Energy Prices

Item	Our Translog	Generalized Leontief	Normalized Quadratic
Output Supply:			
Livestock	−18.5	.5	−2.0
Fluid Milk	−21.5	3.0	1.0
Grains	−13.5	.5	−3.5
Oilseeds	−22.0	−2.0	−6.5
Other Crops	−14.5	3.5	1.0
Input Demand:			
Durable Equipment	−24.5	−.5	−2.5
Real Estate	−16.0	−.5	−3.0
Farm-Produced Durables	−15.0	0	−3.0
Hired Labor	−21.5	10.0	5.0
Energy	−55.5	−8.5	−6.5
Other Purchased Inputs	−16.5	3.0	−2.0

are included in the table for comparison purposes since the nonnested test support for the GL is only slightly greater than for the NQ, and the nested test support is higher for the NQ. If either the GL or NQ is the true functional form, policy decisions based on the TL would anticipate much larger changes (14–56%) in commodity and input markets from the tax than would actually be realized (0–10%). In addition, some of the largest anticipated changes (e.g., hired labor) would be in the opposite direction from realizations.

Conclusions

Attempting to narrowly bound estimates of output supply and input demand elasticity for a given category remains an exceedingly difficult task. Even using the same data, holding the point of evaluation constant, and using alternative functional forms with the same number of free parameters to be estimated, the implied elasticities can vary widely. In this profit function examination of U.S. elasticities using three functional forms—translog, generalized Leontief, and normalized quadratic—at least one of the three own-price elasticity estimates computed at the point of approximation (1977) differed from the others by a magnitude of at least .5 for all output and input categories. Some varied by as much as 2.3.

The translog estimates were always the farthest from alternative functional form estimates. In addition, all of the translog own-price elasticity estimates lay outside the 95% confidence intervals of both alternative functional forms, while the normalized quadratic and generalized Leontief estimates frequently lay within each other's confidence intervals.

A simple nonnested testing procedure documented that the most frequently used of the three functional forms, the translog, was the least preferred choice for these data. This finding corroborated the results of an earlier nested test using aggregates of the data employed in this study. If the translog functional form had mistakenly been selected as the form of choice for policy analysis when one of the alternatives was the true form, output and input partial-equilibrium responses to an energy tax would have been grossly overestimated. In some very important cases, the predicted response would have even been in the wrong direction.

In many cases such as this one, a choice of "best" functional form cannot be made on theoretical grounds. When the methods and data are adequate, formal empirical hypotheses may be tested to help narrow the range of viable alternatives. When they are not, a range of relevant alternatives should at least be explored to determine how sensitive the most important empirical results are to the alternatives.

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Notes

[1] There may exist a large number of well-defined output supply and input demand elasticities which differ by length of run, by which inputs are treated as fixed, and by equilibrium conditions. This article focuses on partial-equilibrium elasticities which treat all the prices as exogenous, except those for the fixed factor, self-employed labor.

[2] The standard errors reported in table 3 are conditional on the realization of the estimated covariance matrix from the first stage of the estimation procedure (i.e., before the constrained optimization).

[3] Ball (1988) cites Sakai in asserting that all normal outputs and inputs exhibit gross complementarity. That is an implication of long-run equilibrium. However, short-run constraints on allocatable fixed inputs (such as family labor in this study), along with decreasing returns to size (Leathers), can induce competitive short-run relationships among normal outputs produced by the same firm. Constraints can also induce competitive short-run relationships among normal inputs.

[4] The square-rooted quadratic functional form was also not rejected by Ornelas at the 5% significance level.

[5] In addition to the fact that these elasticities are point estimates, they are also partial-equilibrium estimates. General-equilibrium elasticities that allow for induced price adjustments in nonpesticide markets would be more useful for this policy analysis, but they cannot be derived without knowledge of the appropriate input supply and output demand relationships.

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