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Risk and Return in Agriculture: Evidence from an Explicit-Factor Arbitrage Pricing Model

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This article develops and estimates an explicit-factor Arbitrage Pricing Theory (APT) model in an endeavor to uncover (*a*) the systematic risk properties of returns to agricultural assets, (*b*) the relationship between agricultural returns and returns on comparable-risk nonagricultural assets, and (*c*) the possible relevance of agriculture-related risks in general capital markets. The article concludes that: (*a*) farmer-held assets have exhibited significant systematic/factor risk over the 1963–82 estimation interval, but U.S. farmland has not exhibited such risk; (*b*) a grain-price index has been a priced factor in general capital markets; and (*c*) average returns on farmer-held assets have been significantly lower, and average returns on U.S. farmland significantly higher, than those on comparable-risk nonagricultural assets.

Key words: capital asset pricing, market factors, returns, systematic risk.

Introduction

Much research in agricultural finance employs portfolio and asset pricing theory to study the relationship between the risk and the rate of return to farm assets. Two related questions have motivated this inquiry: (*a*) To what "systematic" risks, risks that persist in diversified portfolios, are farm assets sensitive? (e.g., Barry; Irwin, Forster, and Sherrick; Arthur, Carter, and Abizadeh); and (*b*) Are mean returns to agricultural assets higher or lower than those required for comparable-systematic-risk nonagricultural assets? (e.g., Barry; Irwin, Forster, and Sherrick; Bjornson and Innes). From a normative point of view, this line of inquiry is relevant to policy discussions on the perceived "problem" of low returns to agriculture. From a positive point of view, this research sheds light on the risk-return characteristics of farm assets and, therefore, the prospective desirability of including these assets in an investment portfolio.[1] It also reveals the empirical merits of two competing hypotheses on the relationship between returns to agricultural and comparable-risk non-agricultural assets. The first hypothesis is that farmers accept lower returns than would investors in comparable-risk nonagricultural assets due to the lifestyle benefits of farming (e.g., Brewster); the second hypothesis is that investors require higher mean returns on agricultural assets, *ceteris paribus,* because these assets are illiquid (Barry) and permit their owners limited diversification opportunities (Bjornson and Innes).

Barry's article inaugurated this line of research by studying the Sharpe–Lintner–Mossin Capital Asset Pricing Model (CAPM), which predicts that the market portfolio is the only systematic factor relevant to asset pricing. By regressing excess agricultural returns (i.e., the difference between the agricultural returns and the risk-free rate of return) on a market portfolio proxy, Barry found that farm assets have exhibited little systematic/market risk

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and earned higher returns on average than the CAPM theory would have predicted. Irwin, Forster, and Sherrick extended the Barry analysis by adding an unanticipated inflation factor to the regression equation, as well as extending the estimation period and considering a broader-based market portfolio proxy; their results were consistent with Barry's, except that they found agriculture to exhibit a significant sensitivity to the inflation factor. The model of Irwin, Forster, and Sherrick essentially represented a two-factor version of the multi-systematic-factor alternative to the CAPM, namely the Arbitrage Pricing Theory (APT) of Ross and Connor.

Arthur, Carter, and Abizadeh considered a more general representation of the APT by constructing multiple factors from a principal component analysis of 24 assets' returns, finding further support for Barry's conclusion that farm assets are subject to little systematic risk. Bjornson and Innes also studied a principal-component-based APT model, but unlike Arthur, Carter, and Abizadeh, estimated their model using a broad-based subset of securities traded on major U.S. stock exchanges. The main purpose of the Bjornson–Innes article was to address the second question raised above, distinguishing between two types of agricultural asset returns: (*a*) returns to a farm operator's investment in his/her business, called "farm asset returns"; and (*b*) returns to an owner of Illinois farmland, called "landlord returns." Their results indicated that farm asset returns have been lower on average, and landlord returns higher on average, than returns to comparable-risk nonagricultural assets.

This article extends the foregoing research by studying agricultural returns in an APT model in which the systematic economic factors are explicitly specified (as in Chen, Roll, and Ross, and Ferson and Harvey), rather than implicitly specified by a principal component analysis (as in Arthur, Carter, and Abizadeh, and Bjornson and Innes). Following Bjornson and Innes, two types of agricultural asset returns are considered, those to farm-operator assets and those to farmland.

The approach taken in this research has a number of advantages over prior analyses. First, the study of an "explicit APT" model permits a much clearer identification of the relevant economic risks to which agricultural assets are sensitive than does an "implicit APT" (principal component based) model in which the factors represent complex combinations of underlying economic risks. In addition, this research allows for more than the two explicit factors considered by Irwin, Forster, and Sherrick, incorporating all factors that Chen, Roll, and Ross, and Ferson and Harvey have found to be significant in the return generating process for traded U.S. securities.

Second, in order to compare mean returns to agricultural and comparable-risk nonagricultural assets in an "explicit APT" or CAPM model, it is not sufficient to evaluate intercepts in regressions of excess agricultural asset returns on the explicit market factors as done in Barry and in Irwin, Forster, and Sherrick. In the CAPM, for example, it is well known that these intercepts tend to be significantly positive for assets with little systematic risk (i.e., assets with low betas); thus, Barry's finding of a significant positive agricultural return intercept, together with a low agricultural beta, is consistent with the so-called "empirical CAPM" and does not necessarily imply that the agricultural asset evaluated in his article has earned returns that are higher on average than comparable-risk/same-beta nonagricultural assets.[2] Similarly, in a multi-explicit-factor APT model, a positive intercept may reflect a systematic capital market phenomenon rather than an anomalous excess return for the particular asset under evaluation, in this case the agricultural asset. In order to compare agricultural asset returns with comparable-risk nonagricultural asset returns, it is necessary first to estimate the relationship that prevails in capital markets between assets' returns and assets' systematic risk measures, which here are the assets' sensitivities to (betas for) the explicit market factors; from this "Security Market Plane" estimation, we can deduce the expected return that is required for a nonagricultural asset with the same systematic risk properties (i.e., the same betas) as the agricultural asset of interest. We then can compare this required expected return estimate to the observed average agricultural return to deduce any significant differences.

Third, the Security Market Plane estimation reveals which economic factors earn significant risk premia and, hence, are "priced" in capital markets. (If a factor is priced, then an asset which is sensitive to that factor must earn an expected return premium as compensation for this factor risk.) Our analysis thus permits us to test for pricing of agriculture-related factors in general capital markets. The analysis of Arthur, Carter, and Abizadeh suggests two natural candidates for agriculture-related factors in that two of their four rotated factors reflect, respectively, grain price and meat price risk (see their table 4); however, the Arthur, Carter, and Abizadeh analysis was unable to test for pricing of factors in general capital markets because the set of assets analyzed therein was dominated by agriculture-related indices and, hence, was not representative of general capital markets. In what follows, we test for pricing of both a grain price and a meat price factor.

Fourth, this study incorporates some important generalizations of maintained model hypotheses in prior work, while providing evidence on stationarity of agricultural asset sensitivities to market factors and the robustness of results to alternative estimation periods. In the closely-related Bjornson and Innes study, for example, we imposed stationarity in all nonagricultural assets' factor sensitivities (betas) over the entire 22-year CAPM and 24-year APT estimation periods; in the CAPM analysis, stationarity was also imposed on the Security Market Line (market risk premium) parameters. Neither of these stationarity restrictions is particularly plausible, and neither is imposed in what follows.[3] In addition, Bjornson and Innes considered only a single estimation period for each of their analyses (CAPM and APT) and did not address questions of stationarity in the agricultural asset betas, subjects which are a focus of attention here.

Finally, from a methodological point of view, this research is the first (to our knowledge) to develop and implement a multi-explicit-factor APT estimation procedure which directly accounts for the "errors-in-variables" problem implicit in any Security Market Plane estimation. This technical advance and its advantages over prior attempts to overcome the "errors-in-variables" problem are described in more depth below.

The balance of the article is organized as follows. First, we present the model and empirical methods to be applied. Next, the data are described. Finally, results are presented and discussed.

The Model

Theory

We build our empirical analysis in this article on standard capital asset pricing theory. This theory is driven by the observation that covariability between asset returns leads to risk in a diversified portfolio; in contrast, asset-specific return variability does not contribute to portfolio risk and, hence, investors do not require a higher expected return on an asset to compensate for this "idiosyncratic/unsystematic" risk. Required expected returns on assets, and resulting equilibrium asset prices, are thus determined by an asset's sensitivities to economywide risks, or "common factors," factors which jointly determine the asset return covariation which is relevant in diversified portfolios.

Formally, we follow Connor by considering an insurable factor economy in which the economy's N assets follow the factor model,

$$(1) \qquad R_{it} - R_{ft} \equiv r_{it} = E(r_{it}) + f'_t B_{it} + \epsilon_{it}, \; i = 1, \ldots, N.$$

R_{it} and R_{ft} are logs of (one plus) time t rates of return on asset i and the risk-free asset, respectively; r_{it} is the corresponding excess log return on asset i in period t, which has the expectation $E(r_{it})$; f_t is the random K-vector of systematic economic factors, where K is the number of factors; B_{it} is the $(K \times 1)$-vector of asset i's factor sensitivities (loadings); and ϵ_{it} is the idiosyncratic return on asset i.[4] N-vectors of time t idiosyncratic returns and excess log returns (hereafter called simply "returns") will be denoted by ϵ_t and r_t, respectively; similarly, B_t will be used to denote the $(N \times K)$ matrix of factor sensitivities.

In this factor economy, it is assumed that the ϵ_t are serially uncorrelated, with $E(\epsilon_t | f_t)$

$= 0$ and $E(\epsilon_t \epsilon_t') = \Sigma$, while $E(f_t) = 0$. The following relation then holds in equilibrium (Connor, theorem 3):

$$E(r_t) = B_t \gamma_t, \tag{2}$$

where γ_t is a K-vector of nonstochastic factor risk premia at time t; in other words, the kth element of γ_t represents the expected excess return on an asset which has unit sensitivity to the kth common factor and zero sensitivity to the other $(K - 1)$ factors.

Equation (2) indicates that the only risk which is priced in equilibrium is systematic or factor-related risk. Furthermore, when there is only one "market portfolio" factor—that is, when $f_t = r_{mt} - E(r_{mt})$, with r_{mt} defined as the excess log return on the market portfolio—equation (2) reduces to the familiar CAPM Security Market Line,

$$E(r_t) = \beta_t E(r_{mt}).$$

In developing our empirical procedures, we will therefore view the CAPM as a special (single factor) case of the general K-factor model.

Empirical Analysis

In this article, we follow Chen, Roll, and Ross, and Ferson and Harvey in explicitly specifying the economic/"common" factors which are assumed to generate asset returns per equation (1). The definitions and selection of these factors are discussed later (in the data section).

For a given set of factors, an asset's factor sensitivities over a given interval can be estimated from the time series empirical counterpart to (2),

$$r_{it} = \alpha_i + FB_i + \epsilon^*_{it}, \qquad i = 1, \ldots, N, \qquad t = 1, \ldots, T, \tag{3}$$

where F is a $(T \times K)$ matrix of the factor realizations $(\gamma_{kt} + f_{kt})$; the intercept α_i and sensitivity vector B_i are assumed to be stationary over the estimation interval; and the ϵ^*_{it} are mean-zero disturbances that are assumed to be normal with zero serial correlation (as required by the APT) and stationary contemporaneous correlation, $E(\epsilon^*_t \epsilon^{*\prime}_t) = \Sigma$. Since the regressors in (3) are identical across assets, OLS is equivalent to SUR here; hence, the parameters in (3) can be estimated efficiently by OLS.

Although the APT equations (1) and (2) predict zero intercepts in equation (3), it would not be valid for us to use an equation (3) intercept estimate for an agricultural asset to infer that the agricultural asset has earned more or less than a comparable-APT-risk nonagricultural asset. Rather, the equation (3) regressions provide us with measures of assets' systematic/factor risks, but do not indicate how these risks are priced in capital markets; that is, they do not indicate the expected return premia required for bearing factor risks.

To estimate the capital market pricing of factor risk, we will use the following cross-sectional empirical counterpart to (2), the Security Market Plane (SMP):

$$\bar{r}_i = \gamma_0 + B_i' \gamma_F + u_i, \qquad i = 1, \ldots, N \qquad \Longleftrightarrow \qquad \bar{r} = [i_N : B]\gamma + u, \tag{4}$$

where $\bar{r}_i = \frac{1}{T} \sum_{t=1}^{T} r_{it}$ is the average excess log return on security i, γ_F is the K-vector of factor risk premia (to be estimated), γ_0 is a scalar intercept coefficient (also to be estimated), and the u_i are mean-zero normal disturbances. The right-hand side of (4) expresses the SMP in terms of N-vectors of average returns $(\bar{r})$, ones (i_N), and the residuals (u), as well as the $(N \times K)$ beta matrix (B) and the $(K + 1)$-vector of parameters (γ).

To help explain the need for an estimation of the SMP in (4), it is useful at this point to consider the CAPM version of this model. As pointed out in the introduction, there is an extensive body of research in financial economics that documents the following empirical anomaly in the CAPM: Estimated Security Market Lines, the single "market portfolio" factor versions of (4), indicate that average returns on low-beta assets tend to

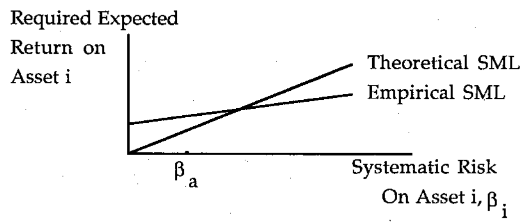

Figure 1. The CAPM Security Market Line

exceed the theoretical required rate of return (i.e., $\hat{\gamma}_0 > 0$), as indicated in figure 1. This anomaly is crucial here since agricultural returns have been found to have betas far below one (Barry; Irwin, Forster, and Sherrick). Therefore, low-beta agricultural assets should be compared to low-beta securities that also tend to exhibit returns in *excess* of the level predicted by the theoretical SML. That is, average agricultural returns should be compared to required agricultural returns that are predicted by an "empirical SML" which allows γ_0 to be non-zero and which is estimated using a broad-based subset of securities, thereby reflecting any empirical anomalies that may exist.[5]

Similarly, multi-explicit-factor SMP estimations have yielded non-zero estimates of γ_0 (e.g., see Chen, Roll, and Ross), thus confounding interpretation of equation (3) intercepts. Given our inability to make inferences from equation (3) agricultural return intercepts, we will make our comparison between agricultural and comparable-risk nonagricultural returns in the following way: Using our equation (3) agricultural asset beta estimates and our equation (4) estimates of the SMP parameters, we can construct a predicted value—and a confidence interval about this value—for the capital-market-required expected return for an asset with the same systematic risks (i.e., the same betas) as the agricultural asset. An average realized agricultural return outside of this interval then will imply a significant difference between agricultural and comparable-risk (same beta) nonagricultural asset returns.

Turning now to the estimation of (4), we first note that this estimation requires the use of an estimated beta matrix, $\hat{B}$, as regressors, rather than the true counterparts, B. The beta estimates can be obtained from equation (3) regressions, but clearly are measured with error, i.e.,

$$\hat{B} = B + v, \tag{5}$$

where v is an $(N \times K)$ mean-zero normal disturbance matrix. The "errors-in-variables" problem implied by (5) leads to bias and inconsistency in standard OLS and GLS estimates of the equation (4) parameters obtained using the regressors, $\hat{B}$. Extant empirical research on explicit-factor APT models has sought to overcome this problem by replacing individual asset betas in (4) with portfolio betas which are estimated with more precision. An apparent problem with this approach is that estimation results have been extremely sensitive to the method chosen for selecting portfolios (Chen, Roll, and Ross, footnote 8), which might be explained by differences between the remaining measurement errors in the different portfolio groupings.

In this article, we overcome the "errors-in-variables" problem directly by using the equation (3) regressions to obtain a consistent estimator of the measurement error covariance matrix and constructing parameter estimates that account for this error. Bjornson and Innes and Litzenberger and Ramaswamy took this direct approach to the errors-in-variables problem in single-factor CAPM analyses; however, to our knowledge, this article is the first to extend this approach to a multi-explicit-factor APT model.

To formally develop our estimation procedure for equation (4), note first that equations (3) and (4) imply the following covariance matrix for u (conditional on the factor realization matrix, F):

$$E(uu') = \Sigma/T, \tag{6}$$

where Σ is the contemporaneous equation (3) residual covariance matrix for the set of market assets. Equation (6) is derived in the appendix. For simplicity in deriving our estimator for γ, we now will proceed on the supposition that Σ is known while noting that asymptotically (as $T \to \infty$) the derived properties of our estimator persist when Σ is replaced by its consistent estimator, $\hat{\Sigma}$, obtained from the equation (3) regression residuals.

Since Σ is not a scalar matrix, the relationship in (6) implies that estimation efficiency can be increased by transforming our data; thus, we premultiply (4) and (5) by $\sqrt{T}\Sigma^{-1/2}$ to obtain the transformed model:

$$\bar{r}^* = X^*\gamma + u^*, \qquad \text{and} \tag{4'}$$

$$\hat{B}^* = B^* + v^*. \tag{5'}$$

The appendix demonstrates that $E(u^{*\prime}v^*) = 0$ and $E(v^{*\prime}v^*) = (F^{*\prime}F^*)^{-1}TN \equiv \sigma_{vv}N$, where expectations are conditional on F, F^* is the $(T \times K)$ mean-differenced factor realization matrix, and σ_{vv} is a $(K \times K)$ measurement error covariance matrix. Drawing on the logic of Fuller, the following estimator now can be shown to be consistent (in N):

$$\hat{\gamma} = G^{-1}\hat{X}^{*\prime}\bar{r}^*, \qquad \text{where} \qquad G \equiv \hat{X}^{*\prime}\hat{X}^* - \begin{bmatrix} 0_1 & 0'_K \\ 0_K & N\sigma_{vv} \end{bmatrix}, \tag{7}$$

$\hat{X}^* \equiv \sqrt{T}\hat{\Sigma}^{-1/2}\cdot[i_N : \hat{B}]$ is the transformed $(N \times (K + 1))$ matrix of ones and estimated betas, $\bar{r}^* \equiv \sqrt{T}\hat{\Sigma}^{-1/2}\cdot\bar{r}$ is the transformed N-vector of average stock returns, $\hat{\Sigma}^{-1/2}$ is the Cholesky decomposition of the inverse estimated $(N \times N)$ contemporaneous residual covariance matrix from equation (3), and 0_k denotes the k-dimensional null vector. $\hat{\gamma}$ in (7) has the following asymptotic covariance matrix (conditional on F):

$$V(\hat{\gamma}) = G^{-1} + G^{-1}\begin{bmatrix} 0_1 & 0'_K \\ 0_K & Q^* \end{bmatrix}G^{-1}, \tag{8}$$

where

$$Q^* \equiv \sigma_{vv}\cdot(\hat{\gamma}'G\hat{\gamma} + N) + N(\sigma_{vv}\hat{\gamma}_F\hat{\gamma}'_F\sigma_{vv} + \sigma_{vv}\hat{\gamma}'_F\sigma_{vv}\hat{\gamma}_F).$$

The appendix contains a proof of consistency and a derivation of equation (8).

Our equation (7) estimates of the equation (4) SMP parameters, γ, are constructed using a broad-based sample of traded securities, excluding the agricultural assets. Under mild regularity conditions, these $\hat{\gamma}$ estimates are asymptotically independent (as $N \to \infty$) of the equation (3) estimate of the agricultural return beta vector, $\hat{B}_a$.[6] Hence, under the null hypothesis (H_0) that agricultural returns obey the "empirical SMP" (4), the following statistic is a consistent estimator of the required expected rate of return on an agricultural asset:

$$\hat{\bar{r}}_a \equiv \hat{\gamma}_0 + \hat{\gamma}'_F\hat{B}_a. \tag{9}$$

Further, under H_0, the following test statistic is approximately asymptotically (as $N \to \infty$) distributed as a standard normal random variable:[7]

$$z = (\bar{r}_a - \hat{\bar{r}}_a)/\hat{\sigma}_r, \tag{10}$$

$$\hat{\sigma}_r = \left\{\frac{1}{T_a}\hat{\sigma}^a_{\epsilon\epsilon} + \widehat{\mathrm{Var}}(\hat{\gamma}_0) + \hat{B}'_a\widehat{\mathrm{Cov}}(\hat{\gamma}_F)\hat{B}_a + 2\widehat{\mathrm{Cov}}(\hat{\gamma}_0, \hat{\gamma}_F)'\hat{B}_a + \hat{\gamma}'_F\hat{\sigma}^a_{BB}\hat{\gamma}_F + \mathrm{trace}(\widehat{\mathrm{Cov}}(\hat{\gamma}_F)\hat{\sigma}^a_{BB})\right\}^{1/2}, \tag{11}$$

where $\bar{r}_a$ is the average observed agricultural return, $\hat{\sigma}^a_{\epsilon\epsilon}$ is the estimated agricultural resid-ual variance from (3), $\hat{\sigma}^a_{BB}$ is the estimated $(K \times K)$ covariance matrix of $\hat{B}_a$, the $\widehat{\mathrm{Var}}(\hat{\gamma}_0)$

and $\widehat{\text{Cov}}(\hat{\gamma}_F)$ terms are estimated variance and ($K \times K$) covariance relations, and $\widehat{\text{Cov}}(\hat{\gamma}_0, \hat{\gamma}_F)$ is a K-vector of estimated covariances between $\hat{\gamma}_0$ and each of the factor risk premia in the K-vector, $\hat{\gamma}_F$.[8] Thus, at the significance level l, we will reject the null hypothesis that agricultural returns obey the "empirical SMP" (4) if $\bar{r}_a$ is outside the following confidence interval:

$$(12) \qquad [\hat{\bar{r}}_a - \hat{\sigma}_r z_l, \ \hat{\bar{r}}_a + \hat{\sigma}_r z_l],$$

where z_l is the critical value of a standard normal random variable such that the variable takes on a value above z_l with probability (l/2).

Data

Agricultural Returns

Two types of annual agricultural return series (R_a) are used in the analysis. First, annual rates of return to farm assets are obtained from the U.S. Federal Reserve Board's *Agricultural Finance Databook* for the period of 1963 to 1986 (24 years); these rates of return measure capital gains on farmer-owned agricultural assets and farm-level operating income (without interest deductions and with an imputed managerial labor cost deduction), as a proportion of the total farm investment in land, machinery, buildings, and short-term assets. In what follows, this first set of returns will be called "farm asset returns."

Second, annual rates of return to farmland ownership (i.e., capital gains plus rents, as a proportion of land value) are obtained from two sources, Burt's and Alston's land price studies. The Burt series measures returns to Illinois farmland for the period of 1963 to 1986 (24 years). Alston developed farmland return series for each of eight midwestern states based on U.S. Department of Agriculture (USDA) data; the eight states are Minnesota, Ohio, Indiana, Illinois, Iowa, Missouri, North Dakota, and South Dakota. The Alston data extends through 1982 (20 years). In what follows, the nine Burt and Alston return series will be called "farmland returns."

The principal difference between Burt's and Alston's samples is that Burt's rents represent share-crop rents derived from individual farm records (Reiss and Scott) within a homogeneous region of high-value grain land, whereas Alston's (USDA) series represent statewide aggregates with cash rents.

By considering these two types of agricultural return series, farm assets and farmland, we can distinguish between properties of farmer returns and landlord returns, as done in Bjornson and Innes. In contrast, the return series analyzed by Barry and Irwin, Forster, and Sherrick were constructed to represent aggregate returns to all agricultural assets, both those in the hands of farmers and those in the hands of farmland owners/landlords.

Security Returns

Monthly and annual common stock returns (R_i) are calculated from the Center for Research in Security Prices (CRSP) data base for stocks traded continuously on the New York or American Stock Exchanges over each of the following contiguous 60-month and 48-month intervals: (*a*) 1963 to 1967 (60 months), which contains 850 continuously traded stocks; (*b*) 1968 to 1972 (60 months), containing 1,243 stocks; (*c*) 1973 to 1977 (60 months), containing 1,230 stocks; (*d*) 1978 to 1982 (60 months), containing 1,192 stocks; and (*e*) 1983 to 1986 (48 months), containing 1,099 stocks.

The Risk-Free Asset

The U.S. 90-day Treasury Bill is considered to be the risk-free asset, and corresponding monthly and annual log (one plus) returns (R_f) are calculated from data in the U.S. *Federal Reserve Bulletin.*

Explicit Factor Returns, F

Based on the prior work of Chen, Roll, and Ross, and Ferson and Harvey, six economic index variables are considered as possible explicit factors in our study. These six variables, which are defined formally in the appendix, include a stock market portfolio proxy (*Mkt*) and an index of industrial production (*IP*), two variables that have often been cited as indicators of general economic or market conditions. Since inflation is widely regarded as an influence on financial markets and on farm-level returns, we include an unexpected inflation variable (*UI*), as used in the Irwin, Forster, and Sherrick study, and a variable for changes in expected inflation (ΔEI), as used in Burt's and Alston's land price studies. Finally, we employ two variables that characterize the interest rate yield curve, one for the bond default risk premium (*DP*) and another for the bond maturity risk premium (*MP*). Economists variously have identified these variables as important indicators of economic activity and Chen, Roll, and Ross found the last five variables to have significant market risk premia in their explicit-factor APT model.

As noted in the introduction, we also consider two agriculture-related variables as possible explicit factors in our analysis, the first of which represents a percentage index of corn price changes (*Corn*) and the second of which represents an index of meat price changes (*Meat*). The inclusion of these two variables is motivated by the results of Arthur, Carter, and Abizadeh, as is the inclusion of our final possible explicit factor, an index of changes in the Swiss franc/U.S. dollar exchange rate (*Swiss*). These three variables, which are formally defined in the appendix, were found to proxy for significant factors in the Arthur, Carter, and Abizadeh APT analysis. Like the security returns, realizations for all of our nine explicit factors are measured monthly in logged form.

In estimating Security Market Planes [equation (4)], two possible factor structures will be considered, the first including all nine explicit factors described above and the second including only the five factors found by Chen, Roll, and Ross to have significant average risk premia. However, for the sake of brevity, the subsequent analysis will present only results from the Chen, Roll, and Ross five-factor model; qualitatively analogous results were obtained in models with other factor structures.

Analysis and Results

Security Market Plane Estimations

For a given factor structure, a Security Market Plane is estimated for each of the five sequential 60-month and 48-month sample intervals described above. Each of these estimations uses the corresponding sample interval's matrix of factor realizations, F, and monthly common stock returns, r_i, in the following three-step procedure:

(1) Equation (3) OLS regressions are run to obtain the N stocks' estimated beta and residual vectors, $\hat{\beta}_i$ and $\hat{\epsilon}_i$ (where, for example, $i = 1$ to 850 for the 1963–67 interval).

(2) The contemporaneous residual covariance matrix, $\hat{\Sigma}$, is estimated from the $\hat{\epsilon}_i$ obtained in step 1. Since the number of stocks (e.g., $N = 850$ for 1963–67) exceeds the number of time series observations (e.g., $T = 60$ months) in all subintervals, zero-covariance restrictions must be imposed in order to permit estimation. Accordingly, the off-diagonal elements of $\hat{\Sigma}$ are restricted to zero for all pairs of stocks not in the same three-digit SIC code classification (following Connor and Korajczyk).[9] The remaining non-zero elements of the covariance matrix are estimated by $\hat{\sigma}_{ij} = \hat{\epsilon}_i'\hat{\epsilon}_j/(T - K - 1)$.

(3) Having obtained $\hat{\beta}$ and $\hat{\Sigma}$ from step 2, the equation (4) SMP parameters, γ, are estimated according to equation (7).

This SMP estimation procedure has at least two important technical advantages over the implicit-factor (principal component) APT estimation performed by Bjornson and Innes (BI). First, stationarity in the stock market securities' factor sensitivities is imposed only over 60-month intervals at most, a stationarity restriction which is standard in empirical finance work; in contrast, the BI study imposed stationarity in these factor sensitivities over the entire 24-year sample period. Second, sample stocks are restricted to those continuously traded over 60-month intervals; in contrast, BI restricted their sample to stocks traded continuously over their 24-year sample period. As a result, much larger stock market samples are used to estimate the SMPs here than the 288 stocks used to estimate the BI model.

Table 1 reports the SMP risk premia (γ) estimates under the two alternative factor structures. Also reported are the average estimated risk premia over the 1963–82 and 1963–86 intervals. A few implications of these results merit mention:

(1) The intercept estimates in table 1, $\hat{\gamma}_0$, are significantly positive for all subintervals in both factor models. Therefore, capital market assets with low betas tend to earn risk-adjusted returns higher than the APT theory would predict. As noted earlier, this observation is particularly important for the comparison between agricultural and comparable-risk nonagricultural asset returns since agricultural assets have been found in past studies to exhibit little systematic/factor risk (e.g., Arthur, Carter, and Abizadeh).

(2) The corn price factor has a significantly negative average risk premium over both the 1963–82 and 1963–86 periods; in contrast, the meat price factor exhibits an insignificantly small premium in both intervals. We thus find evidence for an agriculture-related (grain price) factor which is priced in general capital markets. Possible explanations for a significant corn price factor come in both supply-side and demand-side forms. On the supply side, it is important to note that, despite the "smallness" of farming in the overall economy, the whole food production and sales sector is not "small" in the economy and is likely to have a pervasive influence on factor markets (including those for land, physical production inputs, and labor) in many sectors. On the demand side, asset-pricing relationships are driven by opportunities for real good consumption (as emphasized, for example, in Breeden's consumption-based CAPM); this observation indicates a possible role in asset pricing for price indexes that measure consumption opportunities in ways distinct from aggregate inflation measures. Regardless of one's interpretation of these results, they suggest that a role for agriculture in capital asset pricing cannot be dismissed.

(3) The Swiss franc exchange rate factor has a significant risk premium in four of the five subintervals and in the 1963–82 period on average; however, the average premium for the Swiss franc factor over 1963–86 is insignificant. These results provide some evidence that a foreign exchange factor may be priced in capital markets, at least in some intervals.

(4) The market portfolio factor exhibits a significant premium in four of the five subintervals and over both the 1963–82 and 1963–86 periods on average. These results contrast with those of Chen, Roll, and Ross (CRR), and Ferson and Harvey, who estimated insignificant premia for their market portfolio proxies when using portfolio grouping methods to deal with the "errors-in-variables" problem.

Excepting the significant market risk premium estimated here, the results in table 1 are broadly consistent with those of CRR. In particular, all of the CRR factors other than that for unexpected inflation (*UI*) exhibit significant average risk premia in both factor models and sample periods. The *UI* factor also exhibits a significant average risk premium in the CRR factor model for 1963–82.

Agricultural Asset Factor Sensitivities

Table 2 reports results from equation (3) regressions of agricultural asset returns (in monthly equivalents) on annual averages of monthly factor realizations. Results are reported for different sample periods in order to shed some light on stationarity in the agricultural asset betas.

Turning to the issue of stationarity, the first panel of table 2 provides informal evidence that the farm asset betas have been stationary over the 1963–82 period, but changed in the 1983–86 period. We have corroborated this evidence in a couple of ways. First, we ran sequences of regressions for the intervals 1963–77, 1963–78, etc., and 1968–77, 1968–

Table 1. Explicit APT Security Market Plane

	1963–67	1968–72	1973–77	1978–82	1983–86	Average 1963–82	Average 1963–86
A. Chen, Roll, and Ross Factors Only							
$\hat{\gamma}_0$	.0051600**	.0044936**	.0027765**	.0019083**	.0076125**	.0035846**	.0042559**
$\hat{\gamma}_{IP}$	−.0010173**	−.0006609**	.0006322*	−.0002779	−.0009861**	−.0003310**	−.0004402**
$\hat{\gamma}_{UI}$	−.0000839*	.0004221**	.0001761**	−.0003068**	.0001986**	.0000519*	.0000101
$\hat{\gamma}_{\Delta EI}$	−.0000179**	.0001147**	.0000259*	.0000441	−.0000678**	.0000417**	.0000234**
$\hat{\gamma}_{DP}$	.0000282**	.0000189	−.0000417**	.0000973**	−.0000238**	.0000257**	.0000174**
$\hat{\gamma}_{MP}$	.0000105	.0001312**	−.0001503**	.0001711**	−.0000686**	.0000406**	.0000224*
B. All Factors							
$\hat{\gamma}_0$	.0025076**	.0065452**	.0042971**	.0011443*	.01044**	.0036235**	.0047596**
$\hat{\gamma}_{IP}$	−.00078877**	−.00050514**	.00067723**	−.00036851	−.00066352**	−.0002463**	−.00031583**
$\hat{\gamma}_{UI}$	−.000022809	.0003691**	.000076243	−.00030833**	−.00020903**	.000028549	−.000011048
$\hat{\gamma}_{\Delta EI}$	−.000016004**	.00010349**	.00001298	.000059791**	−.00002185**	.000040064**	.000029745**
$\hat{\gamma}_{DP}$	.000031199**	.000020073*	−.000025064*	.00010331**	−.000041737**	.00003238**	.000020027**
$\hat{\gamma}_{MP}$	.00003016**	.00013494**	−.000098503**	.00019519**	−.000023705	.000065446**	.000050587**
$\hat{\gamma}_{MKT}$	.005678**	−.0050348**	−.0058154**	.0025892**	.00026094	−.00064574**	−.00049463*
$\hat{\gamma}_{SWISS}$	.00028225	.00075262**	−.0035098**	.00031327	.0031747**	−.00054041*	.000078774
$\hat{\gamma}_{CORN}$	−.0057708**	−.0027329**	.0030154*	−.0021075	−.0075662**	−.001899**	−.0028435**
$\hat{\gamma}_{MEAT}$	−.0013382	−.00023245	.0079685**	−.0047221**	.00076626	.00041893	.00047682

Notes: Single asterisk denotes significant t-statistics, 10% two-tail level; double asterisks denote significant t-statistics, 5% two-tail level.

Table 2. Agricultural Asset Factor Sensitivities from Equation (3) Regressions (Annual Observations; Intercept Coefficients in Monthly Equivalents)

	R^2	$\bar{r}_a$	$\hat{\alpha}$	$\hat{\beta}_{IP}$	$\hat{\beta}_{UI}$	$\hat{\beta}_{\Delta EI}$	$\hat{\beta}_{DP}$	$\hat{\beta}_{MP}$
Farm Assets (Federal Reserve)								
63–77 (15)	.64	.00092	−.0074**	1.05***	4.05	−9.72	2.90*	−.86
68–82 (15)	.92	−.00201	−.0119**	1.29**	6.03***	−12.16*	4.70*	−1.20
63–82 (20)	.90	−.00130	−.0082***	1.22***	5.63***	−13.03**	2.85***	−.87
63–86 (24)	.38	−.00328	−.0059	.93*	3.47	−4.16	.88	−3.39*
Illinois Farmland (Burt)								
63–77 (15)	.35	.00804	.0078	−1.45	−4.98	46.67	1.81	6.34
68–82 (15)	.32	.00532	.0039	.26	5.11	−6.57	1.19	.96
63–82 (20)	.31	.00541	.0023	.35	5.13	−6.03	1.85	.50
63–86 (24)	.20	.00198	.0005	.56	1.69	7.47	1.14	−5.08
Minnesota Farmland (Alston)								
63–77 (15)	.37	.00915	.0137	−1.93*	−9.12	40.05	.68	4.35
68–82 (15)	.20	.00863	.0101	−.61	1.38	6.73	−.05	2.44
63–82 (20)	.19	.00808	.0081	−.50	1.40	7.11	.78	1.94
Ohio Farmland (Alston)								
63–77 (15)	.16	.00868	.0040	−.02	2.51	−6.59	2.33	2.81
68–82 (15)	.43	.00682	−.0010	1.15	6.68	−13.92	3.86	−2.14
63–82 (20)	.40	.00686	−.0009	1.01	6.18*	−10.89	3.80	−1.65
Indiana Farmland (Alston)								
63–77 (15)	.36	.01018	.0130	−1.34	−7.06	40.32	−.48	8.27
68–82 (15)	.30	.00708	.0039	.27	1.50	7.75	1.51	2.73
63–82 (20)	.30	.00773	.0064	.25	1.36	6.61	.35	2.81
Illinois Farmland (Alston)								
63–77 (15)	.40	.00866	.0167*	−2.33**	−12.06	57.25	−.99	6.08
68–82 (15)	.29	.00622	.0212	−.58	2.17	−5.08	−6.62	4.05
63–82 (20)	.25	.00640	.0103	−.33	3.01	−1.89	−1.52	2.59
Iowa Farmland (Alston)								
63–77 (15)	.25	.00996	.0081	−1.06	−6.41	35.18	2.98	2.59
68–82 (15)	.29	.00811	.0049	.17	3.85	−2.23	1.97	.94
63–82 (20)	.27	.00789	.0029	.34	4.25	−3.44	2.88	.06
Missouri Farmland (Alston)								
63–77 (15)	.16	.00843	.0110*	−.60	−4.01	26.15	−.54	.90
68–82 (15)	.55	.00687	.0098	−.05	1.83	5.34	−1.13	.56
63–82 (20)	.45	.00719	.0088*	−.01	1.48	7.06	−.75	.30
North Dakota Farmland (Alston)								
63–77 (15)	.24	.01169	.0052	−.12	1.02	−5.10	4.41	−.12
68–82 (15)	.46	.00967	−.0128	.69	8.51**	−18.08	11.56	−2.25
63–82 (20)	.35	.00952	.0020	.50	7.37**	−23.27	4.54	−.94
South Dakota Farmland (Alston)								
63–77 (15)	.32	.00767	.0128*	−1.49	−6.16	29.80	−.31	2.21
68–82 (15)	.27	.00654	.0096	−.58	2.09	1.83	−.79	1.70
63–82 (20)	.24	.00640	.0093	−.54	1.86	2.32	−.74	1.58

Notes: Single asterisk denotes significant t-statistics, 10% two-tail level; double asterisks denote significant t-statistics, 5% two-tail level; and triple asterisks denote significant t-statistics, 2% two-tail level.

78, etc., to search for years in which significant changes in beta estimates could be discerned. In both sequences, there was a dramatic difference between the farm asset beta estimates for sequences ending in 1982 or before and those ending in 1983 or later. Second, we constructed two sets of F-statistics for each of the farm asset and Illinois land (Burt) return series; the first set represents test statistics for the null hypothesis that the agricultural asset betas for 1963–73 (1963–74, 1963–75, etc.) are the same as those for 1974–86 (1975–86, 1976–86, etc.); the second set represents analogous test statistics for stationary betas

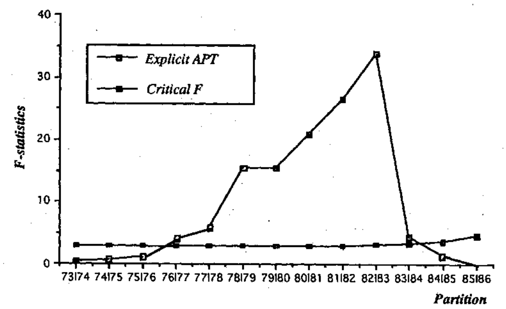

Figure 2. *F*-statistics for stationary farm asset betas in 1963–86

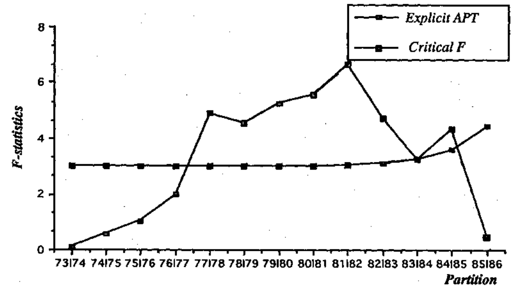

Figure 3. *F*-statistics for stationary Illinois land betas in 1963–86

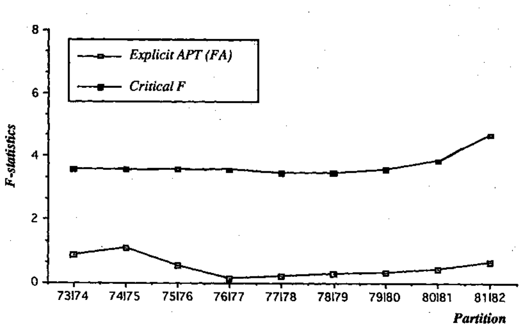

Figure 4. *F*-statistics for stationary farm asset betas in 1963–82

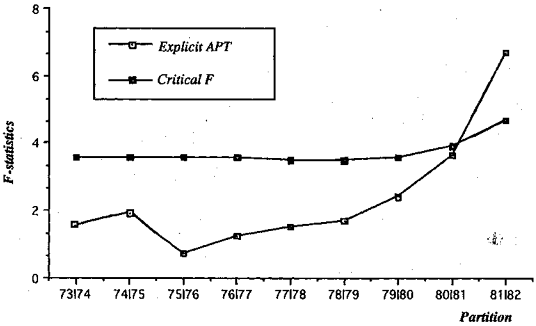

Figure 5. *F*-statistics for stationary Illinois land betas in 1963–82

within the 1963–82 period, rather than the 1963–86 period covered by the first set of F-statistics. Figures 2–5 present these statistics graphically; in these figures, an F-statistic above the critical F indicates that the corresponding null stationarity hypothesis is rejected at the 5% level. Figures 4 and 5 provide evidence of beta stationarity within the 1963–82 period, while figures 2 and 3 indicate that beta stationarity across years that include 1983–86 is generally rejected.

In sum, because agricultural asset returns can only be meaningfully measured annually, our analysis of agricultural return data requires the imposition of a maintained hypothesis that agricultural betas are stationary over our sample intervals. The foregoing results provide evidence in favor of this maintained hypothesis over the 1963–82 period, but against this hypothesis over the 1963–86 period.[10] Therefore, we believe that results from the 1963–86 sample period should be viewed with considerable caution.

Regardless of the sample interval one examines, table 2 indicates that farmland has not been subject to significant systematic factor risk. In contrast, the farm asset returns exhibit significant factor sensitivities over the 1963–82 period; in particular, farm assets have been positively correlated with industrial production (*IP*), unexpected inflation (*UI*), and the default risk premium (*DP*), while negatively associated with changes in expected inflation (ΔEI).

The farmland results in table 2 contrast with those of Irwin, Forster, and Sherrick, who found farmland to be significantly sensitive to unexpected inflation over their 1947–84 sample period in a two-factor version of the APT. To bridge the Irwin, Forster, and Sherrick results, and ours, we have performed equation (3) regressions for our agricultural assets using an Irwin, Forster, and Sherrick two-factor (market portfolio and unexpected inflation) model structure. Like Irwin, Forster, and Sherrick, we find all 10 agricultural return series to exhibit significant positive sensitivity to unexpected inflation in these two-factor regressions.[11] Thus, the differences between the Irwin, Forster, and Sherrick results and ours are attributable to the different factor structures that are assumed to prevail. To the extent that the CRR factors are priced in capital markets, as indicated in CRR and in our SMP analysis, the two-variable Irwin, Forster, and Sherrick model is misspecified; that is, their unexpected inflation factor picks up effects of the missing CRR factors, leading to the spurious conclusion that investors require a lower expected return on farmland because these assets provide a hedge against unanticipated inflation.

The latter conclusion also was tentatively reached by Bjornson and Innes based on correlations between farmland and a complex principal component factor that was correlated with unexpected inflation. The contrasting conclusion implied by table 2 thus illustrates the pitfalls of inferring agricultural assets' systematic/factor risk properties from an implicit factor (principal component) APT analysis.

Relating Agricultural and Comparable-Risk Nonagricultural Asset Returns

In the description of the empirical methodology above, we constructed an estimator—and a confidence interval about this estimator—for the required expected rate of return on an asset with the same systematic risk properties as our subject agricultural asset [recall equations (9) and (12)]. If the realized average agricultural asset return were above (below) the upper (lower) bound of this confidence interval, then we would infer (at the appropriate level of significance) that the agricultural asset has earned higher (lower) mean returns than comparable-APT-risk nonagricultural assets.

Because we have estimated Security Market Planes for intervals of four- and five-year lengths over our sample interval, the estimators for our SMP-predicted required expected return, $\hat{\bar{r}}_a$, and the corresponding standard deviation, $\hat{\sigma}_r$, must be obtained by averaging corresponding estimators from subintervals. For example, for the 24-year, 1963–86 sample period, our estimators are:

$$\text{(13)} \qquad \hat{\bar{r}}_a = \sum_{i=1}^{4} \frac{5}{24}\hat{\bar{r}}_{ai} + \frac{4}{24}\hat{\bar{r}}_{a5}, \qquad \hat{\sigma}_r = \sqrt{\sum_{i=1}^{4}\left(\frac{5}{24}\right)^2 \hat{\sigma}_{ri}^2 + \left(\frac{4}{24}\right)^2 \hat{\sigma}_{r5}^2},$$

where i indexes the subintervals, with $i = 1$ (2, 3, etc.) representing the period 1963–67 (1968–72, 1973–77, etc.); $\hat{\bar{r}}_{ai}$ and $\hat{\sigma}_{ri}$ are the corresponding subinterval SMP-predicted expected return and standard deviation as defined in (9) and (11), respectively.[12]

Table 3 reports the estimated required expected agricultural returns, $\hat{\bar{r}}_a$, together with realized average returns, $\bar{r}_a$, indicating whether the latter are significantly higher or lower than the former. The results indicate that farm asset returns have been significantly lower than predicted by the SMP for comparable-risk nonagricultural assets over all four sample intervals considered. In contrast, the Alston farmland returns have been significantly higher than those on APT-comparable-risk nonagricultural assets in all three sample intervals considered. Over the 1963–82 period and subintervals within this period, the Burt Illinois farmland returns have also been higher than those on comparable-APT-risk nonagricultural assets, although insignificantly so in the 1968–82 interval and significantly so in the other intervals at only the 20% level. For the 1963–86 interval, the Burt Illinois farmland returns are significantly less than predicted by the SMP at the 20% level; however, as indicated above, not much stock should be placed in this result due to evidence against the maintained stationary beta hypothesis over the 1963–86 period.

Overall, these results corroborate the conclusions of Bjornson and Innes, indicating that these conclusions are robust to (*a*) different sample intervals within the 1963–82 period, (*b*) an alternative explicit-factor model of capital asset pricing, and (*c*) an allowance for nonstationary stock market factor sensitivities.

CAPM Analysis

For comparison with prior studies, it is instructive to perform our analysis using a single market-portfolio-factor model based on the CAPM. Table 4 presents results from such

Table 3. Explicit APT Required Returns for Agricultural Assets (in Monthly Equivalents)

Asset Period (Years)	Average Return $\bar{r}_a$	Estimated Required Mean Return $\hat{\bar{r}}_a$	Required Return Standard Deviation $\hat{\sigma}_r$	Confidence Interval (12) Level[a] $[\hat{\bar{r}}_a - \hat{\sigma}_r z_l, \hat{\bar{r}}_a + \hat{\sigma}_r z_l]$	Status[b]
Farm Assets					
63–77 (15)	.00092	.00407	.00069	.01	−
68–82 (15)	−.00201	.00282	.00069	.01	−
63–82 (20)	−.00130	.00297	.00051	.01	−
63–86 (24)	−.00328	.00372	.00083	.01	−
Illinois (Burt)					
63–77 (15)	.00804	.00569	.00172	.20	+
68–82 (15)	.00532	.00320	.00214	.20	0
63–82 (20)	.00541	.00355	.00131	.20	+
63–86 (24)	.00198	.00411	.00142	.20	−
Minnesota					
63–77 (15)	.00915	.00488	.00166	.02	+
68–82 (15)	.00863	.00379	.00192	.02	+
63–82 (20)	.00808	.00422	.00119	.01	+
Ohio					
63–77 (15)	.00868	.00431	.00165	.01	+
68–82 (15)	.00682	.00272	.00184	.05	+
63–82 (20)	.00686	.00315	.00118	.01	+
Indiana					
63–77 (15)	.01018	.00503	.00168	.01	+
68–82 (15)	.00708	.00383	.00213	.20	+
63–82 (20)	.00773	.00397	.00131	.01	+
Illinois (Alston)					
63–77 (15)	.00866	.00521	.00167	.05	+
68–82 (15)	.00622	.00306	.00209	.20	+
63–82 (20)	.00640	.00384	.00131	.05	+
Iowa					
63–77 (15)	.00996	.00485	.00164	.01	+
68–82 (15)	.00811	.00338	.00187	.02	+
63–82 (20)	.00789	.00363	.00117	.01	+
Missouri					
63–77 (15)	.00843	.00473	.00115	.01	+
68–82 (15)	.00687	.00357	.00105	.01	+
63–82 (20)	.00719	.00395	.00077	.01	+
North Dakota					
63–77 (15)	.01169	.00416	.00166	.01	+
68–82 (15)	.00967	.00288	.00165	.01	+
63–82 (20)	.00952	.00291	.00110	.01	+
South Dakota					
63–77 (15)	.00767	.00482	.00143	.05	+
68–82 (15)	.00654	.00350	.00157	.10	+
63–82 (20)	.00640	.00400	.00099	.02	+

[a] This column gives the two-tail confidence level (maximum .20) to construct the largest confidence interval such that the observed average return is outside the interval.
[b] This column indicates whether the average return, $\bar{r}_a$, is within (0), below (−), or above (+) the constructed confidence interval.

Table 4. CAPM Results Using S&P 500 Market Proxy (Parameter Estimates Derived Using Monthly Equivalent Returns)

A. Security Market Line

	$\hat{\gamma}_0$	$t_{\gamma 0}$	$\hat{\gamma}_m$	$t_{\gamma m}$
1963–67 (850 stocks)	.00165	2.23	.00525	7.22
1968–72 (1,243 stocks)	.00562	6.95	−.00571	−7.42
1973–77 (1,230 stocks)	.00443	5.28	−.00645	−8.24
1978–82 (1,192 stocks)	.00202	2.56	.00240	2.90
1983–86 (1,099 stocks)	.00952	15.54	−.00076	−1.22
Average 1963–82	.00343	8.63	−.00113	−2.90
Average 1963–86	.00444	12.83	−.00107	−3.14

B. Agricultural Asset Equation (3) Regressions

	R^2	$\hat{\alpha}_a$	t_α	$\hat{\beta}_a$	t_β	$t_{\alpha_{Land} - \alpha_{FA}}$ [a]
1963–86 ($t_{22,5\%} = 1.717$):						
Farm Assets (FA)	.0004	−.00331	−2.12	.0107	.092	—
Land (Burt—Illinois)	.1200	.00267	1.14	−.3023	−1.73	3.21
1963–82 ($t_{18,5\%} = 1.734$):						
Farm Assets (FA)	.0288	−.00137	−.975	.0751	.731	—
Land (Burt—Illinois)	.0514	.00555	2.73	−.1467	−.988	3.37
Land (Alston):						
Minnesota	.0177	.00815	4.73	−.0716	−.569	4.66
Ohio	.0129	.00679	3.45	.0697	.486	4.05
Indiana	.0145	.00765	3.73	.0771	.515	4.20
Illinois	.0469	.00654	3.34	−.1344	−.941	3.81
Iowa	.0199	.00797	4.46	−.0788	−.605	4.86
Missouri	.0332	.00726	5.87	−.0709	−.787	5.48
North Dakota	.0021	.00950	5.34	.0250	.193	5.37
South Dakota	.0583	.00651	4.58	−.1093	−1.06	4.39

C. Agricultural Asset Required Returns

	Mean Return $\bar{r}_a$	Estimated Required Mean Return $\hat{\bar{r}}_a$	Required Return Standard Deviation $\hat{\sigma}_r$	Confidence Interval Level[b] $[\hat{\bar{r}}_a - \hat{\sigma}_r z_l, \hat{\bar{r}}_a + \hat{\sigma}_r z_l]$	Confidence Interval Status[c]
1963–86:					
Farm Assets	−.00328	.00443	.00081	.01	−
Illinois (Burt)	.00198	.00477	.00118	.02	−
1963–82:					
Farm Assets	−.00130	.00334	.00084	.01	−
Illinois (Burt)	.00541	.00359	.00117	.11	+
Land (Alston):					
Minnesota	.00808	.00351	.00101	.01	+
Ohio	.00686	.00335	.00112	.01	+
Indiana	.00773	.00334	.00116	.01	+
Illinois	.00640	.00358	.00113	.02	+
Iowa	.00789	.00352	.00104	.01	+
Missouri	.00719	.00351	.00078	.01	+
North Dakota	.00952	.00340	.00103	.01	+
South Dakota	.00641	.00355	.00087	.01	+

[a] This column reports t-statistics for the differences between the intercepts in each of the farmland return regression equations α_{Land} and the intercept in the farm asset regression intercept α_{FA}. The null hypothesis of a zero difference is rejected in all cases (at the 1% level).
[b] This column gives the two-tail confidence level used to construct the confidence interval.
[c] This column indicates whether the average excess return is within the constructed confidence interval (0), below it (−), or above it (+).

an analysis, which substantially generalizes the analogous Bjornson and Innes CAPM estimation by allowing for nonstationary stock market betas, nonstationary Security Market Line parameters (market risk premia), and a much larger sample of stock market returns. The results indicate that:

(1) SML intercepts are significantly positive in all sample intervals, thus verifying that the "empirical CAPM" relationship documented in earlier studies also holds in our more recent sample.

(2) In the agricultural asset regressions (table 4B), farmland returns have positive intercepts, significantly so in the 1963–82 period. These positive intercepts are consistent with the Irwin, Forster, and Sherrick, and Barry results, but do not indicate anomalous farmland returns for the reasons given earlier in this article.

(3) The agricultural asset betas change substantially between the 1963–82 and 1963–86 periods. For both the farm asset and farmland (Burt) series, stationary betas across the two intervals, 1963–82 and 1963–86, can be rejected at the 5% level.

(4) As in the foregoing APT analysis, farm asset returns have been significantly lower than predicted by the SML capital market standard over all intervals, while farmland returns have been significantly higher than this standard over the 1963–82 period. For the 1963–86 period, the Burt farmland series exhibits significantly lower returns than predicted by the SML, which is also consistent with the APT and cannot be taken too seriously because of the nonstationary beta evidence.

Serial Correlation

Residual autocorrelation is precluded by both APT and CAPM theory (e.g., Chamberlain and Rothschild; Connor). Therefore, a test for serial correlation can provide at least a weak indicator of an agricultural asset's consistency with a CAPM or APT pricing relationship. Table 5 reports results of such autocorrelation tests for both our CAPM and APT analyses.

With the lone exception of Missouri farmland, the null hypothesis of zero autocorrelation can be rejected for all agricultural asset series in the CAPM model. These results are consistent with those of Barry, and Irwin, Forster, and Sherrick and provide strong evidence against the hypothesis that agricultural assets obey a CAPM pricing relationship. Given this evidence (or perhaps in spite of it), the CAPM analysis of agricultural returns was performed with a Cochrane–Orcutt AR(1) transformation; none of the above results were qualitatively altered by the adjustment for autocorrelation.

Turning now to the APT model, the zero-autocorrelation hypothesis can be rejected for both agricultural return series for the 1963–86 sample period. This result, however, is arguably of little import given the doubt that we have already raised concerning the stationary agricultural beta hypothesis for this interval. More important is that, for the 1963–82 sample period, we can*not* reject zero autocorrelation for *any* of the agricultural asset series in the APT model. This observation provides at least some minimal support for the hypothesis that agricultural assets obey an APT pricing relationship.

Summary and Conclusion

This article estimates both a CAPM and an APT asset pricing model in an endeavor to uncover (*a*) the systematic risk properties of returns to agricultural assets, (*b*) the relationship between agricultural returns and returns on comparable-risk nonagricultural assets, and (*c*) the possible relevance of agriculture-related risks in general capital markets. By focusing on an explicit-factor APT model, this analysis permits a clearer identification of the systematic risks to which agricultural assets are exposed than has been possible in implicit (principal component) factor APT analyses performed heretofore. In addition, the article develops an empirical methodology to evaluate agricultural assets' return performance which accounts for empirical anomalies that may prevail in capital markets.

Table 5. Autocorrelation Tests in Agricultural Asset Time Series Market Models

	CAPM	APT
Durbin–Watson Statistics:		
Farm Assets		
1963–82 (20)	.46*	1.31
1963–86 (24)	.39*	.75*
Burt Illinois Farmland		
1963–82 (20)	.45*	1.03
1963–86 (24)	.42*	.76*
Alston Farmland, 1963–82 (20)		
Minnesota	.81*	.90
Ohio	1.16*	1.72
Indiana	.79*	1.18
Illinois	.49*	.84
Iowa	.50*	.86
Missouri	1.68**	2.77**
North Dakota	1.19*	1.49
South Dakota	1.06*	1.17
Durbin–Watson Critical Values (5% significance):		
Lower Values		
1963–82 (20)	1.20	.79
1963–86 (24)	1.27	.93
Upper Values		
1963–82 (20)	1.41	1.99
1963–86 (24)	1.45	1.90

Notes: Single asterisk indicates reject null hypothesis of zero autocorrelation (in favor of positive autocorrelation); double asterisk indicates do not reject null hypothesis of zero autocorrelation. Durbin–Watson statistics not footnoted are in the inconclusive test range.

Specifically, this study uses a large, broad-based subset of traded stock market securities to estimate a capital market standard for an asset's return performance, namely a Security Market Plane (SMP). The SMP gives the relationship between an individual asset's levels of systematic risk and the expected return required for this risk in capital markets. Estimation of the SMP permits both a test for capital market pricing of agriculture-related risks and a comparison between an agricultural asset's observed return performance and the SMP-predicted capital market standard for this performance.

The main conclusions of the analysis are as follows:

(1) Farmer-held assets exhibit significant systematic risk in our sample, with returns that are positively associated with indices of industrial production, unanticipated inflation, and default-risk premia, and negatively associated with changes in expected inflation. In contrast, farmland does not exhibit significant systematic risk in our sample.

(2) Agricultural assets' systematic risks have been stationary over the 1963–82 period, but not stationary between the 1963–82 and the 1963–86 intervals.

(3) A grain-price index has been a "priced factor" in general capital markets; that is, the required expected return on a capital market asset has been negatively related to the asset's "grain-price-factor" sensitivity. In contrast, a meat-price index has not been a priced factor in capital markets.

(4) Over all sample intervals considered in the analysis, mean returns on farmer-held assets have been significantly lower than those on investments in comparable-risk nonagricultural assets, whether "comparable risk" is defined in terms of the CAPM or the APT. In contrast, investments in farm real estate generally have earned significantly higher returns, on average, than investments in comparable-risk nonagricultural assets over our

1963–82 sample interval. The main caveat to this last conclusion is as follows: Over the 1963–82 period, Burt's Illinois farmland returns exhibited insignificantly higher returns than the estimated APT standard for comparable-risk assets.

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Notes

[1] See also Kaplan; Moss, Featherstone, and Baker; Sherrick, Irwin, and Forster; and Young and Barry for analyses of farm assets' presence in mean-variance-efficient portfolios for subsets of traded capital market assets.

[2] For evidence on the "empirical CAPM," see Jacob; Miller and Scholes; Blume and Friend; Friend and Blume; Black, Jensen, and Scholes; and Fama and MacBeth.

[3] In the empirical financial economics literature, it is standard to maintain parameter stationarity over only five-year intervals.

[4] Log returns are used here (and in much empirical finance work) in order to avoid problems of non-normality in unlogged returns and to treat inflation correctly (see Taylor). With regard to the second point, both the CAPM and the APT are real models and, hence, their equilibrium pricing relations are real; differencing log nominal returns implicitly yields differences in log real returns.

[5] If we knew of a single nonagricultural asset with the same systematic/factor risks as the agricultural asset of interest, we could perform a simple difference-of-means test to infer the relationship between the agricultural and "comparable-risk" nonagricultural asset returns. Unfortunately, we do not have a priori knowledge of a "comparable-risk" nonagricultural asset. In addition, one can construct an infinity of nonagricultural asset portfolios with the same estimated factor risks as a given agricultural asset. Therefore, the choice of a particular nonagricultural asset for our "comparable-risk" benchmark would be both ad hoc and subject to the error implicit in the estimation of the asset's factor risks. Both of these problems can be overcome by a statistical comparison between agricultural asset returns and an appropriate Security Market Plane (or SML) benchmark. This comparison is described in detail in what follows.

[6] Let the agricultural return disturbance in equation (3) satisfy the general relation, $\epsilon_a^* = \sum_{i=1}^{N} \theta_i \epsilon_i^* + \epsilon_a^{**}$, where ϵ_a^{**} is independent of all ϵ_i^* and $\sum_{i=1}^{N} \theta_i$ is bounded for all N. Then

$$\mathrm{Cov}(\hat{B}_a, \hat{\gamma}) = \sum_{i=1}^{N} \mathrm{Cov}(\hat{B}_i, \hat{\gamma})\theta_i.$$

By the construction of $\hat{\gamma}$, $\mathrm{Cov}(\hat{B}_i, \hat{\gamma})$ converges to zero as $N \to \infty$. Thus, asymptotic independence of $\hat{B}_a$ and $\hat{\gamma}$ follows from normality.

[7] Since the number of time series observations in our sample is not large ($T = 20$ to 24), our use of the standard normal distribution to evaluate the statistic in (10) biases our test in favor of rejecting the null hypothesis. A use of the t-distribution with $(T - K - 1)$ degrees of freedom would bias our test in favor of accepting the null hypothesis. The results presented below are not qualitatively affected by the bias in our test.

[8] Following Johnston (and others), the standard deviation estimate in (11), $\hat{\sigma}_r$, is derived by (*a*) taking the limit (as $N \to \infty$) of the conditional expectation of $(\bar{r}_a - \hat{\bar{r}}_a)^2$, $E\{[(u_a + \gamma_0 + \gamma_F' B_a) - (\hat{\gamma}_0 + \hat{\gamma}_F' \hat{B}_a)]^2\}$; (*b*) simplifying; and (*c*) substituting consistent estimates of the resulting expectations.

[9] In some cases, the resulting estimated covariance matrix contains two nearly singular blocks of covariance estimates for stocks with similar four-digit SIC codes; in such cases, the stocks associated with each of these blocks are divided evenly and randomly into two groups and the inter-group covariances are restricted to zero in order to permit estimation.

[10] Possible explanations for nonstationary betas between 1963–82 and 1963–86 include the onset of risk management innovations in agriculture (including expansion in commodity option trading) and financial-crisis-induced structural change in agriculture during the 1983–86 period.

[11] These regressions were performed over the 1963–82 sample period using our Standard & Poors 500 market proxy, as well as over the 1963–84 sample period using Irwin, Forster, and Sherrick's U.S.-market-portfolio proxy.

[12] The variance in (13), $\hat{\sigma}_r^2$, is a simple average of corresponding subinterval variances due to the maintained hypothesis of no serial correlation in equation (3) residuals and, hence, in the prediction error, $\bar{r}_{ai} - \hat{\bar{r}}_{ai}$.

References

Alston, J. "An Analysis of Growth of U.S. Farmland Prices, 1963–1982." *Amer. J. Agr. Econ.* 68(1986):1–9.

Arthur, L., C. Carter, and F. Abizadeh. "Arbitrage Pricing, Capital Asset Pricing, and Agricultural Assets." *Amer. J. Agr. Econ.* 70(1988):359–65.

Barry, P. "Capital Asset Pricing and Farm Real Estate." *Amer. J. Agr. Econ.* 62(1980):548–53.
Bjornson, B., and R. Innes. "Another Look at Returns to Agricultural and Nonagricultural Assets." *Amer. J. Agr. Econ.* 74(1992):109–19.
Black, F., M. Jensen, and M. Scholes. "The Capital Asset Pricing Model: Some Empirical Tests." In *Studies in the Theory of Capital Markets,* ed., M. Jensen, pp. 79–121. New York: Praeger, 1972.
Blume, M., and I. Friend. "A New Look at the Capital Asset Pricing Model." *J. Finance* 28(1973):19–33.
Breeden, D. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." *J. Financ. Econ.* 7(1979):265–96.
Brewster, J. "Society Values and Goals in Respect to Agriculture." In *Goals and Values in Agricultural Policy,* pp. 114–37. Ames IA: Iowa State University Press, 1961.
Burt, O. "Econometric Modeling of the Capitalization Formula for Farmland Prices." *Amer. J. Agr. Econ.* 68(1986):10–26.
Chamberlain, G., and M. Rothschild. "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets." *Econometrica* 51(1983):1281–1304.
Chen, N., R. Roll, and S. Ross. "Economic Forces and the Stock Market." *J. Bus.* 59(1986):383–403.
Connor, G. "A Unified Beta Pricing Theory." *J. Econ. Theory* 34(1984):13–31.
Connor, G., and R. Korajczyk. "Risk and Return in Equilibrium APT: Application of a New Test Methodology." *J. Financ. Econ.* 21(1988):255–89.
Fama, E., and M. Gibbons. "A Comparison of Inflation Forecasts." *J. Monetary Econ.* 13(1984):327–48.
Fama, E., and J. MacBeth. "Risk, Return, and Equilibrium: Empirical Tests." *J. Polit. Econ.* 38(1973):607–36.
Ferson, W., and C. Harvey. "The Variation of Economic Risk Premiums." *J. Polit. Econ.* 99(1991):385–415.
Friend, I., and M. Blume. "Risk and the Long Run Rate of Return on NYSE Common Stocks." Work. Pap. No. 18-72, Rodney L. White Center for Financial Research, Wharton School of Commerce and Finance, 1972.
Fuller, W. *Measurement Error Models.* New York: John Wiley and Sons, 1987.
Irwin, S., D. Forster, and B. Sherrick. "Returns to Farm Real Estate, Revisited." *Amer. J. Agr. Econ.* 70(1988): 580–87.
Jacob, N. "The Measurement of Systematic Risk for Securities and Portfolios: Some Empirical Results." *J. Financ. and Quant. Anal.* 6(1971):815–34.
Johnston, J. *Econometric Methods.* New York: McGraw-Hill Book Co., 1984.
Kaplan, H. "Farmland as a Portfolio Investment." *J. Portfolio Manage.* 11(1985):73–78.
Litzenberger, R., and K. Ramaswamy. "The Effect of Personal Taxes and Dividends and Capital Asset Prices: Theory and Empirical Evidence." *J. Financ. Econ.* 7(1979):163–95.
Miller, M., and M. Scholes. "Rates of Returns in Relation to Risk: A Reexamination of Recent Findings." In *Studies in the Theory of Capital Markets,* ed., M. Jensen, pp. 47–78. New York: Praeger, 1972.
Moss, C., A. Featherstone, and T. Baker. "Stocks, Bonds, Bills, and Farm Assets: A Portfolio Analysis." Paper presented at the annual meetings of the American Agricultural Economics Association, Reno NV, 1986.
Reiss, F., and J. Scott. "Landlords and Tenant Shares." Dept. Agr. Econ. AEER Series No. 1959–81, University of Illinois, 1982.
Ross, S. "The Arbitrage Theory of Capital Asset Pricing." *J. Econ. Theory* 13(1976):341–60.
Sherrick, B., S. Irwin, and D. Forster. "Returns to Capital in Agriculture: A Historic View Using Portfolio Theory." Paper presented at the annual meetings of the American Agricultural Economics Association, Reno NV, 1986.
Standard & Poors Corporation. *Standard & Poors Security Price Index Record.* New York: S&P, 1988.
Taylor, S. *Modelling Financial Time Series.* Chichester, England: John Wiley and Sons, 1986.
U.S. Department of Commerce, Bureau of Economic Analysis. *Survey of Current Business.* Washington DC, selected issues, 1962–87.
U.S. Federal Reserve Board of Governors. *Agricultural Finance Databook.* Washington DC, 1987.
———. *Federal Reserve Bulletin.* Washington DC, selected issues, 1962–87.
Young, R., and P. Barry. "Holding Financial Assets as a Risk Response: A Portfolio Analysis of Illinois Grain Farms." *N. Cent. J. Agr. Econ.* 9(1987):77–84.

Appendix

Derivation of Equation (6)

First note that the true equation (3) disturbances, stacked into the NT-vector $\underline{\epsilon}$, have the covariance matrix

(A1) $$E\{\underline{\epsilon\epsilon'}\} = \Sigma \otimes I_T,$$

where I_T is the T-dimensional identity matrix. Note further that the true equation (4) and true equation (3), now stacked, are both conditioned on F and related as follows:

(4′) $$\bar{r} = X\gamma + u = Zr = Z\{[I_N \otimes [i_T{:}F]][\alpha_1, B_1', \alpha_2, B_2', \ldots, \alpha_N, B_N']' + \underline{\epsilon}\},$$

where r is the stacked NT-vector of returns, $Z \equiv [I_N \otimes i_T']/T$ is an $(N \times NT)$ temporal averaging transformation (with kronecker product operator $\otimes$, $I_N \equiv N$-dimensional identity matrix, and $i_T \equiv T$-vector of ones. Conditioned on F, $E(u) = 0$, $E(\underline{\epsilon}) = 0$, and $E(u) = ZE(\underline{\epsilon})$, which implies

$$X\gamma = [I_N \otimes [i_T{:}F]]\,[\alpha_1, B_1', \alpha_2, B_2', \ldots, \alpha_N, B_N']',$$

and hence, from (4′), $u = Z\underline{\epsilon}$. Thus,

$$E\{uu'\} = ZE\{\underline{\epsilon\epsilon'}\}Z' = Z[\Sigma \otimes I_T]Z' = \frac{1}{T}\Sigma.$$

Derivation of E{u*′v*} *and* E{v*′v*} *[see equations (4′) and (5′)]*

By construction,

$$v^* = \Sigma^{-1/2}v\cdot\sqrt{T} \qquad \text{and} \qquad u^* = \Sigma^{-1/2}u\cdot\sqrt{T},$$

where $v_i = \epsilon_i' F^*(F^{*\prime}F^*)^{-1}$, $\epsilon_i \equiv T$-vector of true equation (3) residuals for asset i, $\epsilon \equiv [\epsilon_1{:}\epsilon_2{:}\ldots{:}\epsilon_N]$, $v = \epsilon' F^*(F^{*\prime}F^*)^{-1}$, $u = \epsilon' z$, $z \equiv (1/T)i_T$, $i_T \equiv T$-vector of ones, and $F^* \equiv (T \times K)$ mean-differenced factor state variable matrix $\equiv (I_T - zi_T')F$. Thus,

$$\begin{aligned} E\{v^{*\prime}v^*\} &= E\{(F^{*\prime}F^*)^{-1}F^{*\prime}\epsilon\Sigma^{-1}\epsilon' F^*(F^{*\prime}F^*)^{-1}\}\cdot T \\ &= (F^{*\prime}F^*)^{-1}F^{*\prime}(N\cdot I_T)F^*(F^{*\prime}F^*)^{-1}\cdot T = (F^{*\prime}F^*)^{-1}\cdot NT = N\sigma_{vv}, \end{aligned}$$

where $\sigma_{vv} \equiv (F^{*\prime}F^*)^{-1}\cdot T$ and the second equality follows from using standard trace manipulations to evaluate $E\{\epsilon\Sigma^{-1}\epsilon'\}$ term by term. Similarly,

$$\begin{aligned} E\{v^{*\prime}u^*\} &= E\{(F^{*\prime}F^*)^{-1}F^{*\prime}\epsilon\Sigma^{-1}\epsilon' z\}\cdot T = (F^{*\prime}F^*)^{-1}F^{*\prime}(N\cdot I_T)z\cdot T \\ &= (F^{*\prime}F^*)^{-1}F^{*\prime}z\cdot NT = 0. \end{aligned}$$

Proof of Consistency of $\hat\gamma$ in (7) and Derivation of Cov($\hat\gamma$) in (8)

First stack v^* into an NK-vector, $v^{**} = (v_1^*, \ldots, v_N^*)'$, and note that $(v^{**\prime}, u^{*\prime})'$ has the $N(K + 1)$-dimensional block-diagonal covariance matrix,

(A2) $$\begin{bmatrix} I_N \otimes \sigma_{vv} & 0 \\ 0 & I_N \end{bmatrix}.$$

Now expand $\hat\gamma$ in (7) around γ by a Taylor series,

$$\begin{aligned} \hat\gamma &= [(X^{*\prime}X^*)^{-1} - (X^{*\prime}X^*)^{-1}H(X^{*\prime}X^*)^{-1}]\left(X^{*\prime} + \begin{pmatrix} 0_N' \\ v^{*\prime} \end{pmatrix}\right)(X^*\gamma + u^*) + O_p(N^{-1}) \\ &= [I_{K+1} - (X^{*\prime}X^*)^{-1}H]\left\{\gamma + (X^{*\prime}X^*)^{-1}\begin{pmatrix} 0_1 \\ v^{*\prime}X^*\gamma + v^{*\prime}u^* \end{pmatrix} + (X^{*\prime}X^*)^{-1}X^{*\prime}u^*\right\} + O_p(N^{-1}), \end{aligned}$$

where $H \equiv \begin{bmatrix} 0_1 & 0_K' \\ 0_K & v^{*\prime}v^* - N\sigma_{vv} \end{bmatrix}$, a $(K + 1 \times K + 1)$ matrix, and I_{K+1} is the $(K + 1)$-dimensional identity matrix. Multiplying the two bracketed matrices and rearranging now yields:

$$\begin{aligned} \hat\gamma = \gamma &+ (X^{*\prime}X^*)^{-1}\begin{pmatrix} 0_1 \\ v^{*\prime}X^*\gamma + v^{*\prime}u^* \end{pmatrix} + (X^{*\prime}X^*)^{-1}X^{*\prime}u^* - (X^{*\prime}X^*)^{-1}H\gamma \\ &- (X^{*\prime}X^*)^{-1}H\left((X^{*\prime}X^*)^{-1}\begin{pmatrix} 0_1 \\ v^{*\prime}X^*\gamma + v^{*\prime}u^* \end{pmatrix} + (X^{*\prime}X^*)^{-1}X^{*\prime}u^*\right) + O_p(N^{-1}). \end{aligned}$$

Since the last two terms in the last equation converge to zero as $N \to \infty$, $\sqrt{N}(\hat\gamma - \gamma)$ converges in distribution to

(A3) $$\begin{aligned} \lim_{N\to\infty} \sqrt{N}(\hat\gamma - \gamma) = \sqrt{N}\Bigg\{&(X^{*\prime}X^*)^{-1}\begin{pmatrix} 0_1 \\ v^{*\prime}X^*\gamma + v^{*\prime}u^* \end{pmatrix} \\ &+ (X^{*\prime}X^*)^{-1}X^{*\prime}u^* - (X^{*\prime}X^*)^{-1}\begin{bmatrix} 0_1 \\ (v^{*\prime}v^* - N\sigma_{vv})\gamma_F \end{bmatrix}\Bigg\}. \end{aligned}$$

The right-hand side of (A3) has zero expectation (since v^*, u^*, $v^{*\prime}u^*$, and $v^{*\prime}v^* - N\sigma_{vv}$ are mean zero from above), thus establishing consistency of $\hat\gamma$ in (7).

The asymptotic covariance matrix for $\hat\gamma$ is the expected cross-product, $E(ZZ')$, where Z is the bracketed sum of matrices in (A3). This expectation is simplified by noting that (*a*) normally distributed random variables such as v^* and u^* have vanishing third moments; (*b*) (A2) implies zero correlation between v^* and u^*, as well as v_i^* and v_j^* for assets i not equal to j; and (*c*) by expanding $E(v^{*\prime}u^*\gamma_F'v^{*\prime}v^*)$ term by term and then appealing to (A2) and properties of the normal distribution (Fuller, p. 89), this expectation is found to be zero. Thus, we have

(A4) $$\operatorname{Cov}(\hat\gamma) \approx (X^{*\prime}X^*)^{-1} + (X^{*\prime}X^*)^{-1}\begin{bmatrix} 0_1 & 0_K' \\ 0_K & Q \end{bmatrix}(X^{*\prime}X^*)^{-1},$$

where

(A5) $$Q \equiv E(v^{*\prime}X^*\gamma\gamma'X^{*\prime}v^*) + E(v^{*\prime}u^*u^{*\prime}v^*) + E[(v^{*\prime}v^* - N\sigma_{vv})\gamma_F\gamma_F'(v^{*\prime}v^* - N\sigma_{vv})].$$

Now, expanding the first right-hand-side matrix in Q term by term, it is seen to equal $\sigma_{vv}\cdot\gamma'X^{*\prime}X^*\gamma$ from (A2). Further, since u^* and v^* are uncorrelated (and hence independent), the second term equals $E(v^{*\prime}E(u^*u^{*\prime})v^*) = N\sigma_{vv}$. Finally, expanding the third covariance matrix term by term and invoking properties of normal distribution moments (again Fuller, p. 89), this matrix is seen to equal $N(\sigma_{vv}\gamma_F\gamma_F'\sigma_{vv} + \sigma_{vv}\gamma_F'\sigma_{vv}\gamma_F)$. Thus,

(A5′) $$Q = \sigma_{vv}\cdot(\gamma'X^{*\prime}X^*\gamma + \tilde{N}) + N(\sigma_{vv}\hat{\gamma}_F\hat{\gamma}_F'\sigma_{vv} + \sigma_{vv}\hat{\gamma}_F'\sigma_{vv}\hat{\gamma}_F).$$

Substituting consistent estimators for the unobservables in (A4) and (A5′) yields (8).

Definitions of Explicit Factors

(1) $IP_t \equiv$ growth rate in U.S. industrial production $\equiv \log[P_t] - \log[P_{t-1}]$.
(2) $UI_t \equiv$ unexpected inflation $\equiv \log[CPI_t] - \log[CPI_{t-1}] - \log[1 + E(I_t|t-1)]$.
(3) $\Delta EI_t \equiv$ change in expected inflation $\equiv \log[1 + E(I_{t+1}|t)] - \log[1 + E(I_t|t-1)]$.
(4) $DP_t \equiv$ default risk premium $\equiv \log[1 + R^*_{Baat}] - \log[1 + R^*_{LTGt}]$.
(5) $MP_t \equiv$ maturity risk premium $\equiv \log[1 + R^*_{LTGt}] - \log[1 + R^*_{ft}]$.
(6) $Mkt_t \equiv$ market excess return $\equiv \log[1 + R^*_{mt}] - \log[1 + R^*_{ft}]$.
(7) $Swiss_t \equiv$ percentage change in Swiss franc/U.S. dollar exchange rate $\equiv \log[SF_t] - \log[SF_{t-1}]$.
(8) $Corn_t \equiv$ percentage change in corn price $\equiv \log[C_t] - \log[C_{t-1}]$.
(9) $Meat_t \equiv$ percentage change in meat price $\equiv \log[MT_t] - \log[MT_{t-1}]$,

where for month t:

P_t ≡ index of U.S. industrial production from the *Survey of Current Business* (U.S. Department of Commerce);
CPI_t ≡ Consumer Price Index for month t;
$E(I_t|t-1)$ ≡ expected inflation rate for month t from information at month $t - 1$ as estimated in Fama and Gibbons;
R^*_{LTGt} ≡ return (unlogged) on long-term government bonds from the *Federal Reserve Bulletin*;
R^*_{ft} ≡ risk-free rate (unlogged) as defined in Chapter 2 for U.S. T-bill returns;
R^*_{Baat} ≡ return (unlogged) on low grade corporate bonds from the *Federal Reserve Bulletin*;
R^*_{mt} ≡ (unlogged) return on Standard & Poors' index of 500 stocks (capital gain plus divided yield) from *Standard & Poors Security Price Index Record*;
SF_t ≡ Swiss franc/U.S. dollar exchange rate from the *Federal Reserve Bulletin*;
C_t ≡ average spot price of No. 2 yellow corn at Chicago, ERS, USDA; and
MT_t ≡ average wholesale price of steer beef carcass, Choice, at midwest markets, USDA.