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Recent Duality Contributions in Production Economics

C. Richard Shumway

This article is a limited assessment of the agricultural production economics literature since 1982 that resulted from dual modeling approaches. Contributions have removed several perceived obstacles to dual modeling, such as testing curvature, identifying the technology when prices are collinear, and examining dynamics of production. Some contributions have also removed obstacles to primal modeling. Dual methods have been used in risk applications only recently and still appear less convenient than primal methods. Convenience may become the primary criterion for selecting primal or dual methods.

Key words: collinear prices, duality, dynamics, estimation, production, risk, testing

Introduction

During the 1982 joint AAEA-WAEA meetings, the WAEA sponsored a session on the "Relevance of Duality Theory to the Practicing Agricultural Economist." Although broadly titled, the paper presenters (Pope; Lopez 1982) and discussants (Young; Chambers 1982b; Halter) stressed production applications rather than applications in consumption. Thus, a production focus here has historical precedent.

Prior to 1982, a dozen articles in major agricultural economics journals had used neoclassical duality concepts to specify an economic model of production. Most were published in the *American Journal of Agricultural Economics*. Since 1982, more than 100 such studies have been published. Most have been static analyses using aggregate data. Aggregate data were generally assumed to behave as though they were from a single cost-minimizing or profit-maximizing firm. Some studies used duality in a dynamic context, and a few considered duality and risk. I will assess the problems and promise of using duality in production economics.

Before evaluating recent contributions to dual modeling, the term "duality" deserves comment. Duality has had several meanings in the economics literature. Many of us were first exposed to the term "dual" in an operations research class. For every primal LP or QP program, we learned that there was also a dual program. For example, if the primal problem was to maximize profit subject to an input constraint, the dual problem was to minimize rent subject to shadow price being at least as high as input price. Similarly, if a neoclassical primal problem is to maximize output subject to a cost constraint, its dual could be stated as minimizing cost subject to an output constraint. However, in the neoclassical production literature, the term "primal" has been used most often to refer to an optimization problem

C. Richard Shumway is a professor of agricultural economics, Texas A&M University.
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consisting of a behavioral assumption (e.g., maximize profit) and a set of constraints (e.g., the production function); from a differentiable form of this specification, output supply and/or input demand equations can in principle be derived by solving the first-order conditions. The corresponding dual has referred to the optimized problem from which output supply and/or input demand equations can be derived by invoking the envelope theorem, and their inverted forms can be simultaneously solved to obtain the production function.

The common feature in all the duality approaches is that it changes "the viewpoint of an analytical investigation, that is, a change of variables from [one set of] coordinates to [another]" (Paris, p. 345). The fundamental primal-dual principle is that any problem can be expressed in at least two equivalent ways (e.g., production function and profit function). The view from which I will assess contributions to the dual literature will be the second neoclassical approach mentioned. That is, I will treat the primal as the optimization problem consisting of an explicit form of the behavioral objective function and the dual as the optimized objective function. Primary attention will be given in this article to alternative ways of measuring output supply and input demand functions rather than identifying the production functions.

Duality Assessment, 1982

In the 1982 session, several strengths, weaknesses, and challenges of the dual approach as well as stylized facts from empirical dual analyses were identified.[1] Strengths of duality included conceptual and computational convenience and simplicity (Pope; Chambers 1982b). With duality, no system of first-order equations has to be solved to derive the output supply and/or input demand equations, so a broader range of functional forms can be used for the economic equations (Pope). Less opportunity exists for errors to creep into the computational process of obtaining price elasticities, substitution elasticities, or welfare impacts, or of analyzing market general equilibrium or even noncompetitive behavior (Pope; Lopez 1982b). Data needed for dual models are often more readily available and sometimes more accurate than data needed for primal analyses (Young).

Several weaknesses in dual analyses were also noted. Imposing or testing some of the implications of competitive behavior (e.g., curvature) is difficult. Thus, prior studies generally did not test for curvature (Pope). For those schooled before duality became popular, both technical and economic results of primal models seem easier to interpret for some problems (Pope). Not only are quantities often collinear, so also are many prices, and some are perfectly collinear (e.g., when an input is allocated among multiple outputs or its use is identified by season). When prices were perfectly collinear, it seemed that more information could be recovered from a primal rather than a dual specification of the problem (Pope). Duality works poorly when the objective function and one or more of the constraints are nonlinear (e.g., under risk aversion or some dynamics), and duality with dynamics is difficult

[1] As an historical note, one speaker (Halter) also observed that Christensen, Jorgenson, and Lau (CJL) were not the first to introduce the translog functional form to the economics profession. However, he mistakenly credited Halter, Carter, and Hocking (HCH) with that introduction, perhaps because of the common term "transcendental" used in both of their functional form papers. The "transcendental" of HCH is not the same as the "transcendental logarithmic" of CJL. The latter is a second-order Taylor-series expansion in logarithms of all variables while the former is a "double" first-order expansion, i.e., it is a first-order expansion in both the logarithms and the untransformed values of the variables. Although it remained largely unnoticed for more than a decade, the credit for first introducing the translog functional form to the economics profession does actually go to two other agricultural economists, Heady and Dillon (as noted in Berndt and Field, p. 3).

(Pope). The practice of specifying cost functions with output levels as explanatory variables can create simultaneous-equations bias if output levels are not exogeneous (Lopez 1982).

Some stylized (or generally regarded empirical) facts were suggested (Lopez 1982), including moderate price responsiveness of input demands, sizeable substitution possibilities among energy-based inputs and land, nonhomothetic aggregate agricultural production technology, inadequacy of simple production function specifications (such as the Cobb-Douglas, linear, and Leontief), and evidence of cost-minimizing behavior for North American agriculture.

For my assessment of contributions to the duality production literature since 1982, I will focus mainly on the weaknesses cited during the previous session. I will discuss the curvature issue along with tests of the neoclassical theory in the next section. I will then turn in sequence to the issues of collinear prices, risk, dynamics, and estimation procedures for dual models. I will conclude the assessment with a discussion of stylized facts.

Evaluation of Maintained Hypotheses in Dual Models

Maintaining Curvature

The first weakness of dual methods cited above was the difficulty of imposing or testing curvature. While this criticism focused on dual methods, it applies also to primal methods. Curvature (convexity or concavity) is implied when a solution exists to the optimizing problem for a competitive firm, whether the problem is specified in its primal or its dual form.

Since 1982, agricultural economists have developed several ways to impose and test curvature. They include Chalfant and White's Bayesian approach; Talpaz, Alexander, and Shumway's eigenvalue decomposition and Cholesky factorization approaches; and Ball's and Somwaru, Ball, and Vasavada's Cholesky factorization approach. Each of these procedures assures that real symmetric hessian matrices are positive (or negative) semidefinite at the point(s) of approximation, but the Cholesky factorization generally allows finding the solution faster than the eigenvalue decomposition (Talpaz, Alexander, and Shumway). Although computationally intensive, the Bayesian approach is currently easiest to use because it has been included in the popular econometrics package, SHAZAM. Interrelated programs have been written for the Cholesky approach in SAS and MINOS, but they are not yet as convenient to use. Expected developments in the near future in some of the commercial econometric packages should also make that approach more accessible to applied economists for maintaining and/or testing the sign restrictions for curvature. They should also simplify procedures for maintaining and/or testing monotonicity.

Tests of Neoclassical Theory

A recent paper by Fox and Kivanda evaluated the role of agricultural economics literature in testing the neoclassical theory of production. The authors examined 70 empirical applications of static production duality and reported whether the article tested one or more of the implications of neoclassical theory motivating it—linear homogeneity, curvature, and monotonicity of the profit or cost function in the exogenous prices and symmetry of the parameters of the first-derivative equations. Homogeneity, monotonicity, and convexity (concavity) of the profit (cost) function are direct implications of profit-maximizing (cost-minimizing) behavior for a price-taking, risk-neutral firm. Symmetry of the first-derivative equations is also implied if the profit (cost) function is twice continuously differentiable. They observed inadequate testing of the underlying theory and concluded that the "track record of production theory [was] disappointing" when these hypotheses were tested.

In this article, Fox and Kivanda's dataset is expanded with 17 more articles from the same period, two before 1976 (Lau and Yotopoulos; Binswanger), and 24 during 1992 and 1993 published in the same nine journals.[2] These 43 articles are listed in table 1. We also reclassified one-fourth of their 70 papers based on our reading and a broader interpretation of what qualifies as a curvature test.[3] We have 113 articles in our sample that report empirical estimates for 180 datasets and/or models.

Contrary to Fox and Kivanda's result, we found more serious testing of the underlying theory, with 72% of the 113 articles and 82% of the 180 datasets/models reporting tests of one or more of the four hypotheses (see table 2).[4] Curvature was the most tested hypothesis (or most easily examined from reported results), followed in turn by monotonicity, symmetry, and homogeneity. There were big differences between the frequency of testing the first two hypotheses and the latter two hypotheses.

However, like Fox and Kivanda, we found frequent rejection of neoclassical production theory. Monotonicity was the least rejected hypothesis, followed in turn by curvature, symmetry, and homogeneity. Except for monotonicity, each implication of the theory has been rejected more frequently than one would expect if the hypotheses were true and we used the standard alpha level of 0.05. This finding is true for North American agriculture as well as agriculture in several other countries.

What conclusions can be drawn from this examination of the static, applied, dual, agricultural-production economics literature? Does it reverse the 1982 stylized fact that cost-minimizing behavior has not been rejected for North American agriculture? Does it imply that the neoclassical theory is wrong? As usual, the answer is "not necessarily." The reasons for concluding ambiguity are the following:

1. Most of the reported tests do not constitute "critical" tests. The theory is for the firm. The data have generally been aggregated. There is no reason to expect that all (or perhaps any) of the four tested implications would hold for an aggregate of firms even if they held perfectly for each firm. What is really being tested is the hypothesis that the aggregate of firms represented in the data behave collectively as though it were a price-taking, profit-maximizing (cost-minimizing) firm with a twice continuously differentiable profit (cost) function. To conduct a "critical" test of the theory requires micro-level data, data that are even more detailed than that used in most firm-level analyses (Mundlak).[5]

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[2]The help of Wendi Adams in extending this dataset is gratefully acknowledged.

[3]A number of articles made no explicit statement about testing for curvature but reported a complete table of elasticities. Some violations of curvature could be detected by simply checking signs of the diagonal elements and the 2 x 2 principal minors of the elasticity tables.

[4]Also contrary to their results, we found that the percentage of articles reporting tests in the *AJAE* was only a little less than those published in the other eight journals (70% vs. 74%). Between datasets and models, our conclusion was the same as theirs —tests were performed on a larger share of those reported in the other eight journals. With regard to the publication path of these articles, the number of static dual empirical production articles was 4, 22, 37, and 50 in the periods 1972–78, 1979–83, 1984–88, and 1989–93, respectively. The first dual article appeared in the *AJAE* eight years before a dual article was published in one of the other journals. Almost exactly the same number of articles has been published in the *AJAE* as in the others combined, but the trajectory has been different. It has doubled in the other journals in each of the last two five-year periods but has decreased slightly in the *AJAE* in the last period. In addition to an upward trajectory in number of published production duality papers, attention to testing implications of the theory has also increased. Between 1972 and 1982, the number of tests of homogeneity, monotonicity, curvature, and symmetry averaged 1.08 per article. Since 1982, it has averaged 1.45 tests per article.

Table 1. Articles in Nine Agricultural Economics Journals Reporting Applications of Static Dual Production Models, 1972–93

Author(s)	Publication Date
Ali, F., and A. Parikh	1992
Ahrendsen, B. L.	1993
Andrikopoulos, A A., and J. A. Brox	1992
Arnade, C.	1992
Babin, F. G., C. E. Willis, and P. G. Allen	1982
Behrman, J. R., and K. N. Murty	1985
Binswanger, H. P.	1974
Buetre, B. L., and F. Z. Ahmadi-Esfahani	1993
Burrell, A.	1989
Carew, R., P. Chen, and V. Stevens	1992
Chalfant, J. A.	1984
Chambers, R. G.	1982a
Clark, J. S., and C .E. Youngblood	1992
Coxhead, I. A.	1992
Coyle, B. T.	1993a
Coyle, B. T.	1993b
Dixon, B. L., P. Garcia, and J. W. Mjelde	1985
Fernandez-Cornejo, J.	1992
Fulginiti, L. E., and R. K. Perrin	1990
Gallagher, E. W., C. S. Thraen, and G. D. Schnitkey	1993
Huffman, W. E., and R. E. Evenson	1989
Kalirajan, K.	1981
Karagiannis, G., and W. H. Furtan	1993
Lau, L. J., and P. A. Yotopoulos	1972
Lim, H., C. R. Shumway, and T. J. Honeycutt	1993
McIntosh, C. S., and A. A. Williams	1992
Moore, M. R., and D. H. Negri	1992
Newman, D. H., and D. N. Wear	1993
Nghiep, L. T.	1979
Ornelas, F., and C. R. Shumway	1993a
Ornelas, F., and C. R. Shumway	1993b
Polson, R. A., and C. R. Shumway	1992
Rossi, N.	1984
Schroeder, T. C.	1992
Shumway, C. R., and H. Lim	1993
Sidhu, S. S., and C. A. Baanante	1981
Squires, D.	1987
Stefanou, S. E., and S. Saxena	1988
Stranahan, H. A., and J. S. Shonkwiler	1986
Strauss, J.	1984
Surry, Y.	1993
Taylor, T. G., and R. L. Kilmer	1988
Villezca, P. A., and C. R. Shumway	1992b

Note: This list of 43 articles supplements the 70 reported for the period 1976–91by Fox and Kivanda.

Table 2. Homogeneity, Monotonicity, Curvature, and Symmetry Test Summary and Comparison

	Fox and Kivanda 1976–91		Augmented Sample, 1972–93			
			Articles		Datasets/Models	
	No.	%	No.	%	No.	%
Sample Size	70		113		180	
Tested one or more hypotheses:						
All nine journals	38	54	81	72	147	82
AJAE	14	42	39	70	61	77
Other eight journals	24	65	42	74	86	85
Frequency of hypotheses tested:						
Homogeneity	6	9	10	9	10	6
Monotonicity	27	39	51	45	106	59
Curvature	32	46	72	64	138	77
Symmetry	12	17	22	20	26	14
Frequency of nonrejection:						
Homogeneity	3	50	5	50	5	50
Monotonicity	26	96	49	96	95	90
Curvature	22	69	46	64	106	77
Symmetry	8	67	14	64	17	65

2. Many of the test rejections are not based on statistical tests but on failure of the unconstrained estimates to satisfy the hypothesis. When the unconstrained estimates are not perfectly consistent with the theory, simulation efforts and comparative static analysis may be hampered. However, the rejection may not be significant in either a statistical or an economic sense. For example, in my own experience modeling multiple-output production with about 60 datasets, I have seldom found curvature to be satisfied by the unconstrained estimates. Yet, this property has been rejected at the 0.05 significance level for only 7% of them. In addition, using nonparametric heuristic tests of price-taking, profit-maximizing behavior with nearly as many aggregate agricultural datasets in the U.S., departures from this hypothesis have been economically trivial in all but two (Lim and Shumway 1992; Shumway 1993; Williams and Shumway).[6] Whatever the reasons for inconsistency with price-taking, profit-maximizing behavior (e.g., different objective function, optimization errors, observation errors, incomplete markets or information, imperfectly competitive market structure, presence of risk, and risk preference structures), their effect has been largely trivial in aggregate American agricultural data.

[5]Mundlak suggests that data on the "available technology, . . . the collection of all the known techniques of production," are needed for such a test of the theory. At best, what is utilized in micro-level tests is the "implemented technology, . . . the collection of all the techniques that are actually used." (pp. 1, 5–7).

[6]I regarded departures as economically trivial if input and output levels consistent with price-taking, profit-maximizing behavior were on average within 3% of observed levels or if maximum profit was no more than 5% greater than observed profit. These criteria are admittedly arbitrary but are much smaller than typical observation errors associated with secondary economic data.

Table 3. Additional Homogeneity and Symmetry Test Results[a]

Reference	Location	Functional Form	Test Conclusion	
			Homogeneity	Symmetry
Ornelas and Shumway (1993a)	Texas	Gen. Leontief	Reject	Reject
		Norm. Quadratic		Reject
Ornelas and Shumway (1993b)	Texas	Translog	Reject	Reject
		Gen. Leontief	Reject	
		Norm. Quadratic		Reject
Shumway and Alexander	Ten U.S. Regions[a]	Translog	Reject	Reject
		Gen. Leontief	Reject	Reject
		Norm. Quadratic		Reject
Shumway and Chesser	District 8N, Texas	Translog		Not Reject
		Norm. Quadratic		Reject
Shumway and Lim	U.S.	Translog	Reject	Reject
		Gen. Leontief	Reject	Reject
		Norm. Quadratic		Reject
Shumway, Saez, and Gottret	U.S.	Translog	Reject	Reject
		Gen. Leontief	Reject	Reject
		Norm. Quadratic		Reject

Note: At 0.05 significance level.
[a]Test conclusion was the same in each of the ten regions.

My initial view of the state of scientific inquiry in production economics based on published literature was more positive than Fox and Kivanda's. Nevertheless, it was clear that more attention needed to be given to testing the homogeneity and symmetry hypotheses implied by the theory underlying most dual model specifications. Fox and Kivanda's critique prompted further testing of these hypotheses in several of our aggregate datasets for which curvature and monotonicity had previously been tested.[7] Findings are reported in table 3. Homogeneity was tested in 14 datasets and reejected in each. Symmetry was tested in 15 datasets and rejected in 14. As many as three functional forms were used in the tests. Although only a small number of datasets ere examined and also recognizing that aggregate tests do not constitute "critical" tests of the theory, our nearly complete rejection of both properties raises an important caution. Without specific justification, firm-level implications of the theory of the firm should not be maintained ina ggregate production analysis. Despite the fact that pretest estimators have their own problems, appropriate pretesting may be advisable rather than casually maintaining these properties in such empirical studies.

Collinear Prices and Allocatable Inputs

Pope noted that some prices are highly collinear. For example, prices for a generic input, such as labor or fertilizer, which may be used in the production of any of several outputs, are often perfectly collinear. When the derivative of the multiple-output dual problem is taken with respect to the price of such an "allocatable" input, the envelope theorem gives

[7]I am indebted to Sid Dasgupta for conducting these tests.

total demand for the input used in all outputs, not the demand for its allocation to an individual output. Since 1982, much theoretical and empirical attention has been given by agricultural economists to the allocatable input issue. In fact, the primary theoretical work on this subject has had an agricultural economics emphasis. It has addressed such concerns as (*a*) recovering the allocatable input technology from a dual specification, (*b*) measuring product interdependence (or jointness) from dual models, and (*c*) analyzing commodity-level production decisions with incomplete input allocation data. The first two issues concern dual modeling and will be addressed in turn. For significant contributions on the third issue, see Just, Zilberman, and Hochman; Just, Zilberman, Hochman, and Bar-Shira; and Hornbaker, Dixon, and Sonka.

Recovering Allocatable Input Technology

Besides Pope's expression of concern, others (e.g., Just, Zilberman, and Hochman, pp. 770–71; Shumway, Pope, and Nash 1984, p. 72; and Shumway 1988, p. 729) have asserted that dual approaches can fail to provide some information about the multiple-output technology when prices are collinear, such as would be expected with an allocatable input. Paris is responsible for correcting that misperception. Using a two-stage shadow price approach, he demonstrated that a dual specification can be derived that contains all allocation equations. The system of independent or interdependent production functions can be identified as long as the allocation equations are included in (or appended to) the dual model.

Using the standard concept of a restricted (short-run) profit function, Chambers and Just developed and estimated a dual model that included input allocations. While the envelope theorem does not recover input allocation equations from the multiple-output restricted profit function, it does recover the sum of the allocations. As long as the individual allocation equations sum to the total input demand equation, they can be consistently estimated with other output supply and/or input demand equations. They would each be homogeneous of degree zero in prices, and if the profit function is twice continuously differentiable, symmetry restrictions would apply to the sum (across outputs) of their parameters. Several other studies have empirically estimated input allocation equations (e.g., McGuirk and Mundlak; Moore and Negri; Coyle 1993b; Moore and Dinar). Although none has maintained the symmetry conditions in estimation, these contributions have shown that input allocation and intraseasonal input demand equations can be estimated or recovered from dual specifications even though prices are collinear. Future research can be expected to give more attention to estimation of the allocation equations and to testing for satisfaction of theoretical expectations, including the symmetry conditions.

Jointness

Unless each of the allocatable inputs is variable and each output is produced in a technically independent (or nonjoint) way or their effects are exactly offsetting, the optimization problem for a profit-maximizing or cost-minimizing firm includes the production of multiple outputs. If the allocatable input is fixed, it may give evidence of jointness in production even when the production function for each output is technically independent (Shumway, Pope, and Nash 1984). While product interdependence caused only by a constraining allocatable input ("short-run" or "apparent" jointness) may be different from that caused by technical interdependence ("long-run" or "true" jointness) (Shumway, Pope, and Nash 1988; Chambers and Just), its effect on specification of the supply equations is the same—the exogenous price of each interdependent output appears in the output supply equations. The supply equations are short-run if one or more inputs are fixed and long-run if all inputs are variable.

Only when outputs are technically interrelated and are economic complements (i.e., they exhibit economies of scope) does a firm maximize long-run profit by producing multiple outputs (Sakai). When outputs are apparently joint because of allocatable fixed inputs, short-run profit may be maximized by producing multiple outputs that are economically competitive (Moschini). In such a case, some output supplies can increase with an increase in input price. However, fixed allocatable inputs create an incentive for a firm to produce outputs that are economically competitive in the short run only if at least one commodity exhibits short-run diseconomies of size (Leathers). Consequently, greater attention needs to be given to examining the effects of alternative output prices on supplies and to measuring economies of scope and size when short-run jointness is tested in multiple-output production.

Risk

Of all the weaknesses of the dual approach cited in the 1982 session, progress has been slowest in dealing with the problem of nonlinear objective functions caused by risk. Only within the last few years has significant progress been made. Coyle (1992) reported the first tractable dual model that considered risk aversion and price uncertainty. He later generalized his mean-variance approach to consider production risk and nonlinear mean-variance utility (1994b).

Saha also generalized the duality model with risk to the nonlinear mean-standard deviation case (1994a) and to the expected-utility-maximization case (1994b). Both Coyle and Saha found that important refutable hypotheses apply to dual models with risk as well as with certainty.

Specification of cost functions with yield uncertainty and risk aversion has been addressed by Pope and Chavas, but the empirical literature using dual approaches to model risk-nonneutral decisions remains embryonic. Consequently, to determine the usefulness of these newly developed dual procedures, a critical immediate research need is to empirically apply them to risky production problems.

Dynamics

The agricultural economics literature applying dual methods within the calculus-of-variations framework to dynamic problems began to appear at the same 1982 joint AAEA-WAEA meetings as the previous duality session was held. About a dozen articles have appeared since that rely on the theoretical work of McLaren and Cooper and of Epstein, in which the envelope theorem relates derivatives of the dynamic dual problem only to current period (realized) levels of endogenous variables. See, for example, Vasavada and Chambers (1982, 1986); Lopez (1985); Taylor and Monson; LeBlanc and Hrubovcak; Howard and Shumway; Weersink and Tauer; Vasavada and Ball; Somwaru, Ball, and Vasavada; Halvorson; Luh and Stefanou; and Fernandez-Cornejo et al.

With the goal of maximizing net present value subject to a set of assumptions (concerning static disequilibrium cause, adjustment mechanism, and production function regularity conditions), the value function is specified as a dynamic optimization problem.[8] This function satisfies the Hamilton-Jacobi equation that solves the optimization problem by maximizing current profit plus the marginal value of the optimal change in net investment.

Economic equations can be obtained from the Hamilton-Jacobi equation either by a primal approach using Euler equations or by a dual approach using the envelope theorem. With suitable flexible functional forms, either approach to dynamic optimization is tractable. Although nonlinear estimation methods are required, econometric specification and estimation are only slightly more involved than for the static representation. They permit formal and exhaustive testing for instantaneous adjustment (i.e., variable inputs), independent adjustment of inputs, and implications of the theory motivating model specification. They also allow derivation of short-, intermediate-, and long-run elasticities of input demands and output supplies, scope and scale economies, and rates of productivity growth.

Dynamic optimization adds realism previously lacking in dual economic analyses. Yet, the dynamic structures remain highly restrictive and beg for tractable relaxation, a subject addressed by Vasavada and Cook and by Vasavada and Thijssen. Stefanou, Coyle (1994a), and Arnade and Coyle have examined the effect of uncertainty on dynamic investment decisions. Caputo, LaFrance and Barney, and Kamien and Schwartz have broadened the dynamic dual foundations by approaching the problem from the more general optimal control theory.

Estimation Procedures

A strength cited for the dual approach in 1982 was that simpler estimation procedures could be used than those used for primal problems. One issue is that when nonexperimental data are used, instrumental variable procedures may be required to avoid simultaneity bias from primal model estimates because endogenous variables appear on both sides of the production function. Simultaneity bias is avoided by ordinary least squares or feasible generalized least squares estimation of a dual model for a competitive firm if no endogenous variables appear on the right-hand side of the equations. However, the applied duality literature has given little attention to assessing exogeneity of the right-hand-side variables.[9] When cost functions are estimated, output levels are generally treated as exogenous. Yet in the absence of regulation or a policy-induced constraint, they are seldom predetermined either for the firm or for the market. When profit functions are estimated, expected prices are treated as independent of output quantities produced and input quantities used. Yet they are independent only for a price-taking firm or an industry small enough that it does not impact prices by its output or input levels. For aggregates, both prices and quantities are frequently endogenous.

A second issue is that tobit and probit procedures may need to be used when analyzing economic problems with firm-level data. Quantity data are often censored because some firms do not produce all outputs and/or do not use all inputs. Thus, such procedures are clearly needed when production functions are estimated as the primal model (or a part of it). They are also frequently needed in econometric analysis with dual models. For estimation efficiency, the dual specification and its first-derivative equations are often estimated as a system. The derivative equations are generally input and/or output quantities or shares,

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[8]The assumptions include the following: (*a*) smooth, symmetric, and convex costs of adjusting stocks of quasi-fixed inputs are the only cause of static disequilibrium; (*b*) the partial adjustment (or flexible accelerator) mechanism can approximate the equilibrium solution of a dynamic optimization problem; and (c) regularity conditions on the production function include twice continuous differentiability, concavity, and positive marginal products of all inputs, and negative marginal products of net investments in quasi-fixed inputs.

[9]Hausman's specification test is one procedure that can be used to test for variable exogeneity.

which may also be censored. Unfortunately, the popular econometric packages do not yet include systems estimation options using these procedures. In addition, some other econometric issues (e.g., aggregation, joint production, and pooled observations) are not all jointly resolvable with the censored data problem using current econometric techniques (Moore and Negri). Further, it was previously noted that assuring consistency of primal or dual econometric estimates with optimizing behavior may require the ability to maintain and/or test sign restrictions implied by curvature and monotonicity.

Although output and input prices are often less collinear than are their quantities, this is not always the case. When cross-sectional data collected for farms or households are confined to relatively small regions, there may be very little variation in prices. Since several important econometric and data issues require attention in both primal and dual models, it is not apparent that procedures needed to estimate primal models are much more demanding than those needed to estimate dual models.

The production literature of the last 12 years using models specified for consistency with optimizing behavior has had a heavy dual orientation. Most tests for empirical consistency with a behavioral objective have been carried out using dual models. Although fewer in number, several primal models have also been estimated that included both the production function and first-order conditions (e.g., Bailey et al.; Jegasothy, Shumway, and Lim; Adelaja).

Stylized Facts[10]

Five stylized facts were presented in the 1982 session. Of those five, two have received increased empirical support in the intervening years: (*a*) nonhomothetic aggregate agricultural production technology (Karagiannis and Furtan; Shumway 1993), and (*b*) inadequacy of simple forms for production functions (e.g., Cobb-Douglas, linear, or Leontief). However, additional analysis has not brought increased agreement on the other three: (*a*) North American agriculture consistent with cost-minimizing behavior, (*b*) moderate price responsiveness of input demands, or (*c*) sizeable substitution possibilities among energy-based inputs and land.

For example, of the 113 articles cited previously that report static dual analyses, 50 assume cost-minimizing behavior. Not all are for North American agriculture, but many are. Of the 34 studies of North American agriculture, three tested for homogeneity, 19 for monotonicity, and 23 for concavity. Of those that tested for these properties, 33%, 100%, and 83%, respectively, fail to reject these hypotheses. These percentages compare to a larger percentage nonrejection of homogeneity (50%) and a lower percentage nonrejection of monotonicity (90%) and curvature (77%) among all articles. Whether agricultural producers individually or collectively behave as though they are cost minimizers, profit maximizers, utility maximizers, or nonoptimizers remains an empirical question.

It is hard to quantify "moderate" responsiveness. The dual estimates of short-run own-price input demand elasticities likely to have been available to Lopez for North America in 1982 (Binswanger; Lopez 1980; Ray; Lopez and Tung) were in the range of -0.1 to -1.1. Most were between -0.3 and -0.9, and both the largest and smallest ones were from restricted profit function (as opposed to cost function) estimates. More than a dozen articles

[10]The interested reader will find additional insights in several other recent review articles: Chambers (1989); Hallam; Lee; Gempesaw; Guyomard, LeMouel, and Vasavada; Just; Shumway (1993).

have added estimates for some part of North America since 1982. Own-price input demand elasticity estimates now cover at least the range from 0 to –3.2. Even using the same data sources and model specification, estimates vary widely among inputs and among geographic areas. Seemingly modest changes in model specification, for example, functional form, error assumption, or theoretical restrictions, often cause large changes in elasticity estimates, even on those that are significant under each specification. For example, in our estimates for seven states using the same data sources and model specification, elasticity estimates ranged from – 0.04 to – 0.8 for one of the least elastic inputs and from – 0.8 to –2.3 for the most elastic input (Villezca and Shumway 1992b; Lim, Shumway, and Honeycutt). Using U.S. data and varying only the functional form among common locally flexible forms, elasticities ranged from – 0.01 to –1.0 for the input with the narrowest range and from – 0.9 to –3.2 for the input with the widest range of estimates (Shumway and Lim). Others have obtained similarly sensitive results.

Substitution possibilities among energy-based inputs and land obtained from North American duality studies by 1982 reflected wide elasticity estimates (0.1 to 1.8), but they each suggested that these inputs were substitutes (Binswanger; Lopez 1980; Lopez and Tung). Recent estimates also reflect a wide range of elasticities, but they do not consistently conclude that these inputs are substitutes. Several (Shumway and Alexander; Shumway, Saez, and Gottret; Fernandez-Cornejo) suggest that some energy-based inputs are complements to land (or real estate). Thus, the effect of additional duality research has not been supportive of earlier stylized facts about input responsiveness. In fact, recent research has challenged our ability to extract a stylized fact concerning input demand elasticities.

Several other stylized facts have been claimed in recent literature that were not mentioned in the previous duality session: (*a*) technical change is not Hicks neutral; (*b*) production of some outputs satisfies short-run (apparent) nonjointness in inputs; (*c*) some (possibly incomplete) partitions of outputs and inputs in all datasets are both separable and homothetic; and (*d*) partitions that are nonjoint, separable, or homothetic vary widely among observation units and model structures (Karagiannis and Furtan; Villezca and Shumway 1992a; Shumway 1993).

Conclusions and Prognosis

Since 1982, several times as many papers have been published in the agricultural economics journals using dual methods than before 1982. This extensive use of dual methods is consistent with the prognosis of participants in the 1982 WAEA session on duality. All participants perceived important strengths of the dual method that have made it a popular analytical tool.

Although cited weaknesses of the dual method were also valid, their limitations have been reduced considerably. Unrestricted estimates often fail to satisfy the curvature sign constraints, but procedures have been developed that permit the constraints to be maintained and/or tested (at least locally) without undue burden. Software currently is not very user-friendly, but expected developments in commercial econometric packages should overcome that problem. Two important challenges will remain in using such software—avoiding arbitrariness in choosing the point(s) of enforcement and ensuring that impacts of binding constraints on parameter estimates and empirical inferences are examined.

Considerable dynamic analysis has had a dual focus, and that will likely increase. Recent contributions have demonstrated that input allocation and intraseasonal input demand equations can be estimated or recovered from dual models even though prices are collinear. More dual models will be estimated that include input allocation equations. It is likely that tests for short-run jointness in multiple-output production will increasingly be accompanied by measurement of scope and size economies.

The most substantive recent contributions relate to nonlinear objective functions and nonlinear constraints. Although still in working-paper stage, tractable procedures have been developed for estimating nonlinear risk problems within a dual approach. However, it is too early to forecast the eventual role of duality in empirical risk applications. It is clear that dual analysis of risk is possible, but primal approaches still appear to be simpler for many purposes.

Although of relatively recent vintage (two decades), we now have more experience estimating dual systems than we do estimating primal systems that maintain theory. Theoretical expectations can be maintained with either system, and data and econometric challenges associated with estimating primal systems of equations are not much greater than those associated with estimating dual systems. Thus, it is likely that the choice of a primal or dual model specification will be determined largely as a matter of convenience for achieving particular research objectives. The same could be said for type and form of primal or dual specification. If implications of primary concern can be derived more easily from a dual model, the dual will generally be estimated; if they can be derived more easily from a primal model, the primal will be estimated. We can also expect to see increased attention to pretesting for a variety of specification issues, such as functional form, data aggregation, nonjointness, and satisfaction of firm-level theoretical implications (particularly in aggregate studies).[11]

Stylized facts in 1994 include two from 1982 (agricultural production functions are not homothetic nor describable by simple first-order functional forms) and several newer ones relating to nonneutral technical change, separable and nonjoint partitions. Probably the most important prognosis with regard to stylized facts is the trivial observation that they are not static. As additional evidence is gathered by examining problems from more perspectives, the stylized facts will continue to change.

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[11] Although not treated in this article, increased pretesting is likely to focus also on cointegration, heterogeneity, and error term assumptions.

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