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Efficiency of Weather Derivatives as Primary Crop Insurance Instruments

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This study analyzes efficiency of weather derivatives as primary insurance instruments for six crop reporting districts that are among the largest producers of corn, cotton, and soybeans in the United States. Specific weather derivatives are constructed for each crop/district combination based on analysis of several econometric models. The performance of the designed weather derivatives is then analyzed both in- and out-of-sample. The primary findings suggest that the optimal structure of weather derivatives varies widely across crops and regions, as does the risk-reducing performance of the optimally designed weather derivatives. Further, optimal weather derivatives required rather complicated combinations of weather variables to achieve reasonable fits between weather and yield.

Keywords: agricultural risk management, crop insurance, index insurance, weather derivatives

Introduction

The failure of private insurance markets to provide affordable and comprehensive crop insurance is well documented in the literature [see Glauber and Collins (2002) for a recent survey]. In particular, high systemic risk and agency problems are often cited as major obstacles to viable crop insurance (Ahsan, Ali, and Kurian, 1982; Chambers, 1989; Nelson and Loehman, 1987; Goodwin and Smith, 1995; Skees and Barnett, 1999). Weather events that impact crop yields are often spatially correlated, thus creating problems for traditional insurance, which is designed to pool a large number of small, uncorrelated risks rather than widespread systemic losses (Miranda and Glauber, 1997). Moral hazard and adverse selection problems (Skees and Reed, 1996; Quiggin, Karagiannis, and Stanton, 1994; Smith and Goodwin, 1996; Coble et al., 1997; Just, Calvin, and Quiggin, 1999) result in rather high transaction costs of selling and servicing crop yield insurance policies. Insurers typically pass these expenses on to the buyers by loading premium rates. Consequently, crop yield insurance is heavily subsidized by governments in the United States and other countries. Yet, government premium subsidies are often inefficient and come at high social cost (Skees, Hazell, and Miranda, 1999).

An emerging trend has been the development of new financial instruments (catastrophe options, catastrophe bonds) that allow insurers to securitize correlated risks and circumvent the limitations of traditional insurance markets. With regard to agricultural

insurance, the innovations include area-yield insurance (Miranda, 1991; Skees, Black, and Barnett, 1997) and various exchange-traded area-yield contracts. A characteristic feature of these instruments is that their payoff depends on values of a specially designed measure, or *index*, which is related to the risk being hedged against.

Index contracts potentially may offer many advantages compared to traditional crop yield insurance. Since payoffs are based on a widely available and objectively measured index, there is no need for farm-level loss adjustment. This greatly reduces transaction costs relative to crop yield insurance. The value of the index does not depend on the individual actions of market participants. Thus, there is no adverse selection or moral hazard involved. Since index contracts are designed to provide efficient means of risk transfer rather than risk pooling, systemic risk is not a problem. In fact, index contracts work even better when the risk being transferred is somewhat systemic.¹ Finally, portfolio managers and other investors should be attracted to index-based contracts because the returns on these instruments are largely uncorrelated with returns on traditional financial instruments (stocks, bonds, etc.).

Weather derivatives are a type of index contract whose payoff depends on occurrence or nonoccurrence of specific weather events. While weather risk markets have grown rapidly (Lancaster, 2001; Dischel, 2002), current market participants are largely drawn from the energy sector, although firms in other sectors, such as construction, entertainment, and leisure, have recently purchased weather index instruments. Thus far, weather derivatives in the United States have not been used in agricultural risk management. Applications of other index instruments have been limited to a very small market for area-yield contracts, most likely due to crowding-out by highly subsidized farm-level crop yield insurance.²

Other countries, however, have shown interest in weather indexes as risk management tools for agricultural production. The Canadian provinces of Ontario and Alberta have used weather index instruments to cross-hedge forage production risk. AGROASEMEX, the state agricultural reinsurance company in Mexico, has used weather derivatives to transfer part of its weather-related crop insurance risk. Argentina and Morocco are currently developing rainfall index instruments for use in agriculture (Skees et al., 2001).

Recently, weather derivatives have received considerable attention in the literature as potential risk management instruments for agricultural production (Skees, 2000; Skees et al., 2001; Mahul, 2001; Martin, Barnett, and Coble, 2001; Miranda and Vedenov, 2001; Turvey, 2001a, b; Dischel, 2002). The major focus of these and other papers, however, has been on developing actuarially-fair pricing mechanisms for the contracts (Mahul, 2001; Martin, Barnett, and Coble, 2001; Turvey 2001a, b) and institutional frameworks that would be required to introduce weather-based insurance, especially in developing countries (Miranda and Vedenov, 2001; Skees et al., 2001). While research in this area is an important part of developing weather derivatives for agricultural risk management, it primarily addresses the seller's side of the market.

¹ Indeed, a higher systemic component increases the possibility of correlation between index and losses, thus decreasing basis risk and improving efficiency of weather instruments.

² While there is evidence that area-yield insurance contracts can be quite attractive for corn and soybean producers in the Midwest (iFARM, 2004; Schnitkey, Sherrick, and Irwin, 2003), even the subsidized area-yield products such as Group Risk Plan and Group Risk Income Protection are not as popular as other alternatives offered by the USDA's Risk Management Agency (RMA), with both plans together accounting for only 1.6% of the total book of business in 2002.

Potential purchasers of weather derivatives, on the other hand, are concerned not only with the price but also with how well the contract performs in reducing risk exposure. In other words, the crucial property of a weather derivative as a risk management tool is how well its payoffs are correlated with losses. However, this aspect of weather derivatives has not received serious attention in the literature.

The analysis presented here adds to the existing literature on agricultural applications of weather derivatives by moving beyond pricing issues to consider the efficiency of weather derivatives as risk management instruments for crop production. More specifically, weather derivatives are designed for three different crops (corn, cotton, and soybeans) grown in six crop reporting districts or CRDs (two districts per crop). The efficiency of each instrument is then evaluated for typical crop producers in each district using various risk-reduction measures.

The considerations behind our choice of analysis units are presented in the next section, followed by a discussion on weather-yield relationships. In the subsequent section, a pricing methodology for weather derivatives is described. Results from an efficiency analysis are then presented, and the implications of our findings are discussed. The final section offers concluding comments.

Experimental Design

Various combinations of weather variables, crops, and weather measuring stations create an enormous number of potential weather derivatives, which obviously cannot be analyzed in a single paper. Therefore, an attempt was made to narrow the list to represent typical crop-producing areas for three major crops: corn, cotton, and soybeans.

A major disadvantage of weather derivatives is the basis risk, reflecting the fact that the underlying weather variables are measured at specific locations and may differ from realizations of the same variables at different locations. Since weather phenomena such as rainfall tend to be fairly localized, even a relatively small distance between the measuring station and the crop field may result in drastic discrepancies between realized losses and weather derivative payoffs.

Ideally, a weather derivative should be written on an index measured at the same location where the derivative is used as a risk management instrument, thereby completely eliminating basis risk. From a practical standpoint, however, this is obviously an unrealistic proposition that would negate the major advantages of the index contracts such as lower transaction cost (relative to traditional crop insurance) and the possibility of risk transfer to capital markets. A contract written on weather measured at a specific farm in Iowa would hardly attract a Wall Street investor or otherwise generate adequate volume of trade.

For practical purposes, a weather derivative contract should be designed for a relatively large geographic area. The level of aggregation one step above a single farm is a county. However, weather derivatives designed at the county level would still face difficulties with both availability of weather data and a limited market for the contracts. Therefore, crop reporting districts (CRDs) have been chosen as the primary analysis units.³ In particular, for each of the three crops included in the analysis, two CRDs were selected

³CRDs are statistical units which typically include eight to ten counties and provide a reasonable intermediate aggregation level between an individual county and a whole state.

Table 1. Crops, Crop Reporting Districts, and Weather Stations Selected for Analysis

Crop	State / District	Harvested Acreage (2001)	Expected Yield per Acre	Price ^a (\$)	Expected Revenue (\$/acre)	Weather Station	
						Name	County
Corn	IA / D50 Central	1,708,000	147.59 bu	\$2.05/bu	\$302.56	Marshalltown	Marshall
Corn	IL / D10 Northwest	1,654,000	143.56 bu	\$2.05/bu	\$294.30	Dixon 1NW	Lee
Cotton	MS / D40 Lower Delta	684,500	869.39 lbs	\$0.34/lb	\$295.59	Belzoni	Humphreys
Cotton	GA / D80 South Central	546,000	712.70 lbs	\$0.34/lb	\$242.32	Tifton Exp. Sta.	Tift
Soybeans	IA / D10 Northwest	1,785,000	46.32 bu	\$4.56/bu	\$211.22	Sanborn	O'Brien
Soybeans	IL / D70 East Southeast	1,666,000	41.15 bu	\$4.56/bu	\$187.64	Windsor	Shelby

^aThe price for corn is the 2001 Chicago Board of Trade (CBOT) November average price on the December contract. The price for soybeans is the 2001 CBOT October average price on the November contract. The price for cotton is the 2001 New York Board of Trade November average price on the December contract. These are the harvest-time prices used to settle 2001 Crop Revenue Coverage federal crop insurance contracts as reported by the RMA.

based on acreage planted and total production in 2001. The selected CRDs, along with 2001 production data, are presented in table 1.

The next step in designing a weather derivative contract is to specify the location where the underlying index is to be measured. The National Weather Service maintains an extensive network of weather stations across the United States, with historical data on various weather variables available in some cases for more than a hundred years. These stations provide a natural choice of reference points that can be used for settlement of weather contracts. In addition, because the stations are supervised by a government agency, the recorded measurements could be trusted to be reliable and objective.

For each CRD, a centrally located weather station was selected as an official source of weather data. An effort was made to ensure that the necessary historical weather data were available for the station for at least 30 years. Names of the specific weather stations and their county locations are also provided in table 1.

Weather and Yield

Temperature and precipitation are arguably among the most important factors contributing to yield variability. However, establishing a link between yields and weather is not a simple task. Part of the problem stems from the tendency of this relationship to be localized and crop dependent. For the crops considered in this investigation, there is a general consensus that temperature and precipitation in summer months (June, July, and August) tend to influence yields. However, the agronomic literature does not provide a single consistent model which can be used to relate climatic conditions and crop yields.⁴ Hence, for purposes of this analysis, an attempt was made to establish the weather-yield relationships for the selected crops and CRDs by employing an econometric approach.

⁴Teigen and Thomas (1995) present a fairly comprehensive attempt to relate weather and yields at the state level. Unfortunately, the high goodness of fit exhibited by their models is mostly due to inclusion of a time trend term rather than the explanatory power of weather variables.

Theoretically, one could create a variety of rather complex models incorporating different weather variables measured over different periods of time, and thus achieve fairly high goodness of fit. However, there are several factors which must be considered in specifying weather-yield models. As the complexity of the model increases, so does the number of variables. While historical weather data can go back as far as a hundred years, the reliable yield series are typically shorter. In addition, a large portion of earlier yield data does not adequately represent variability of yields due to the major technological shifts that occurred in U.S. agriculture, as well as the wave of farm consolidation in the 1950s and 1960s. Even with the appropriate detrending (Ker and Coble, 2003), a data set suitable for estimation would be limited to 30 to 40 observations, creating identification problems for models with a large number of independent variables.

To address the problems identified above, we selected five model types (see appendix), which incorporate weather variables used previously in the literature (e.g., Teigen and Thomas, 1995; Turvey, 2001a). The models were estimated using historical district-level yield data collected from the USDA's National Agricultural Statistics Service for the period 1972 to 2001 (USDA/NASS, 2002). In order to account for the temporal component, a simple detrending procedure was implemented by fitting a log-linear trend model:

$$(1) \quad \log(Y_t^{tr}) = \alpha_0 + \alpha_1(t - 1971).$$

The detrended yields were then calculated as:

$$(2) \quad Y_t^{det} = Y_t \frac{Y_t^{tr}}{Y_{2001}^{tr}}.$$

The weather data used for analysis were observations⁵ of average monthly temperatures and total precipitation for the three summer months (June, July, and August), as well as cumulative rainfall and cumulative cooling degree days⁶ (CDDs) over the entire period from June 1st to August 31st. Data for each weather station were collected from databases of the National Climatic Data Center (NCDC, 2002).

Insignificant variables were dropped from all models, and models with the highest adjusted R^2 have been used to derive the weather indices for each crop/district combination. The selected models and their corresponding statistics are presented in table 2. The goodness of fit of selected models ranges from 86.6% (Illinois corn) to 35.5% (Iowa soybeans). Note, in all cases, the indices resulted in rather convoluted combinations of individual weather variables with no apparent relation between models for the same crop or within the same state. This finding points to potential difficulties that may arise in designing and marketing weather derivatives. In particular, underlying indices would have to be designed separately for each crop/geographic unit combination. In addition, potential buyers may find it difficult to relate their yield experience to the synthetic model behind the index. However, for purposes of this research, we adopt the indices presented in table 2 in order to analyze performance of weather derivatives in the best-case scenario.

⁵ Both absolute values and deviations from long-term averages have been considered.

⁶ Turvey (2001a) used cumulative CDDs with a basis of 50°F for his analysis of agricultural weather derivatives in Ontario. Since growing conditions in the United States differ from those in Canada, we considered CDDs in two variants, one with a basis of 50°F and the other with a basis of 65°F.

Table 2. Selected Weather-Yield Models by Crop and District

Crop / District	Index / Model	Adjusted R^2
Corn / IA, D50	$Y_{det} = 657.002 + 16.746R_{Jul} - 91.239R_{Aug} - 4.112T_{Jul} - 1.480R_{Jul}^2$ <p>(0.000) (0.001) (0.017) (0.026) (0.000)</p> $- 0.048T_{Aug}^2 + 1.304R_{Aug}T_{Aug}$ <p>(0.009) (0.017)</p>	0.568
Corn / IL, D10	$Y_{det} = -4,053.340 - 42.755R_{Jun} + 10.716R_{Jul} + 119.430T_{Jul} - 0.791R_{Jun}^2$ <p>(0.000) (0.003) (0.000) (0.000) (0.000)</p> $- 0.712R_{Jul}^2 + 0.141R_{Aug}^2 - 0.839T_{Jul}^2 - 0.018T_{Aug}^2 + 0.751R_{Jun}T_{Jun}$ <p>(0.019) (0.004) (0.000) (0.000) (0.000)</p>	0.866
Cotton / MS, D40	$Y_{det} = -1,048.120 + 22.393\Delta R_{Aug} + 28.589\Delta T_{Jun} - 34.676\Delta T_{Jul}$ <p>(0.000) (0.036) (0.029) (0.029)</p> $- 10.017\Delta R_{Aug}^2 - 12.313T_{Jun}^2 - 16.199\Delta T_{Jul}^2 + 4.776\Delta R_{Jul}\Delta T_{Jul}$ <p>(0.009) (0.017) (0.016) (0.083)</p>	0.559
Cotton / GA, D80	$Y_{det} = 820.999 - 48.061\Delta T_{Aug} - 6.396\Delta R_{Jul}^2 - 16.633\Delta T_{Jun}^2$ <p>(0.000) (0.001) (0.067) (0.000)</p> $- 16.442\Delta R_{Jun}\Delta T_{Jun} + 28.405\Delta R_{Aug}\Delta T_{Aug}$ <p>(0.002) (0.001)</p>	0.667
Soybeans / IA, D10	$Y_{det} = 46.642 - 0.600\Delta T_{Jun} - 0.488\Delta T_{Aug} + 0.376\Delta R_{Jun}\Delta T_{Jun}$ <p>(0.000) (0.059) (0.093) (0.030)</p> $+ 0.323\Delta R_{Aug}\Delta T_{Aug}$ <p>(0.007)</p>	0.355
Soybeans / IL, D70	$Y_{det} = 197.669 - 28.884R_{Aug} - 2.136T_{Aug} + 0.399R_{Aug}T_{Aug}$ <p>(0.000) (0.030) (0.000) (0.026)</p>	0.503

Notes: Y_{det} is the detrended district yield. Numbers in parentheses are p -values. Temperature is in degrees Fahrenheit, rainfall is in inches, yield is in bushels/acre for corn and soybeans and pounds/acre for cotton. Independent variables are described in the appendix.

Design and Pricing of Weather Derivatives

In order to formally evaluate the efficiency of weather derivatives in reducing production risk, a particular contract structure must be selected. While an index contract may be structured in many different ways with different coverage layers and provisions, we focus our attention on a specific class of *elementary contracts*. An elementary contract pays an indemnity conditional on realization of an index according to the following schedule (figure 1):

$$(3) \quad f(i | x, i^*, \lambda) = x \times \begin{cases} 0 & \text{if } i > i^*, \\ \frac{i^* - i}{i^* - \lambda i^*} & \text{if } \lambda i^* < i \leq i^*, \\ 1 & \text{if } i \leq \lambda i^*. \end{cases}$$

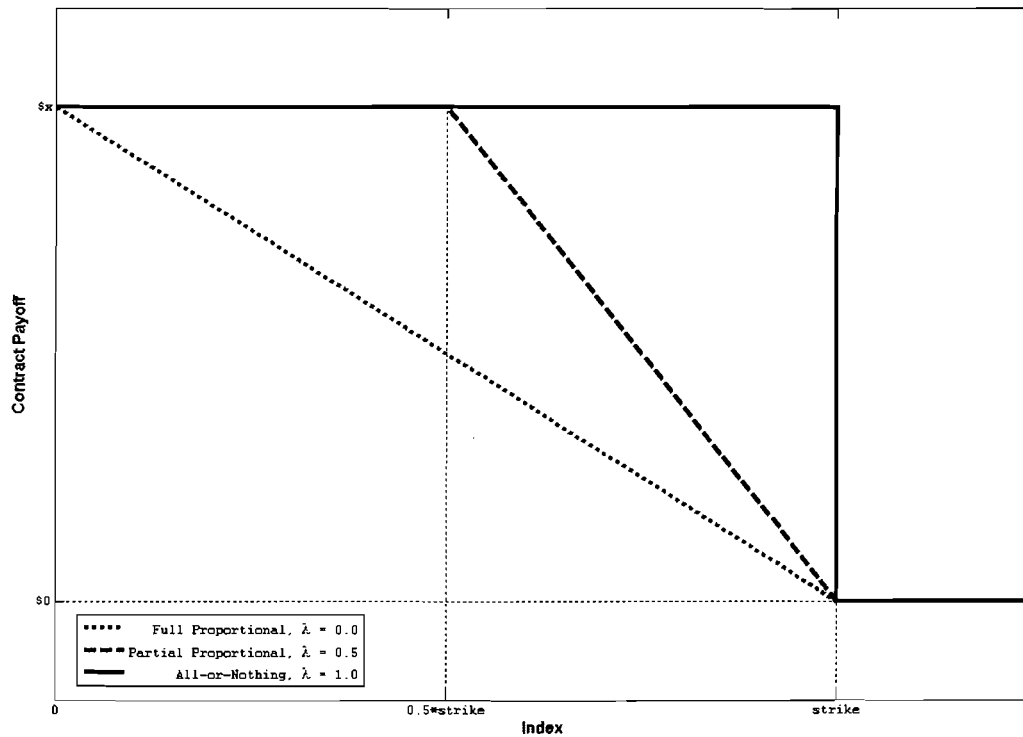


Figure 1. Payoff structure of an elementary contract

In other words, the contract *triggers* (i.e., starts to pay) whenever the index i falls below a specified strike i^* , with indemnity proportional to the difference between the index and the strike. The maximum indemnity (equal to $\$x$) is paid whenever the index falls below the limit λi^* , where $0 \leq \lambda \leq 1$. Thus, an elementary contract can be uniquely identified by fixing three parameters: *strike*, *limit*, and *maximum indemnity*. Note, the use of the terms “strike” and “limit” reflects the fact that an elementary contract may be interpreted as both an insurance contract and a European option on the index.

As an example, consider a hypothetical elementary contract written on the total amount of rainfall measured at a specific weather station during the month of July. Suppose the contract has a strike of 5 inches, the limit of 2 inches ($\lambda = 0.4$), and the maximum indemnity of \$100. The actual indemnity is then determined by the observed level of rainfall. If, for example, the actual rainfall in the month of July were measured at 6 inches (i.e., above the strike), the contract would pay nothing. In contrast, if the total rainfall were only 1.5 inches (i.e., below the limit), the contract would pay the maximum indemnity of \$100. Finally, if the actual amount of rainfall were 4 inches (i.e., between the strike and the limit), the contract would pay $\$100 \times (5 \text{ inches} - 4 \text{ inches}) / (5 \text{ inches} - 2 \text{ inches}) = \33.33 .

Elementary contracts are convenient for analysis, yet offer enough flexibility to construct more complicated instruments. By combining elementary contracts with different triggers i^* , limit parameters λ , and maximum indemnities x , one can recreate or otherwise approximate more complicated, multi-layered indemnification schedules that may enhance risk protection whenever expected losses are not linearly related to the index.

The elementary contract also contains the simple “all-or-nothing” contract as a special case. Specifically, when $\lambda = 1$, the contract pays the maximum indemnity if the index falls below the trigger level i^* , but pays nothing otherwise. Additional treatment of the elementary contract can be found in Martin, Barnett, and Coble (2001).

In order to price an elementary contract, we need to know contract parameters as well as the probability distribution of the underlying index. For weather derivatives, the distribution can be derived based on historical data either by fitting a standard parametric distribution or by using a nonparametric approach (empirical distribution with kernel-smoothing). Regardless of the particular method used, if $h(i)$ is a probability density function of the index, the expected payoff (and hence the actuarially-fair price) of the contract can be determined by:

$$(4) \quad \begin{aligned} \pi(x, i^*, \lambda) &= \int f(i | x, i^*, \lambda) h(i) di \\ &= x \int_0^{i^*} h(i) di + x \int_{i^*}^{i^*} \frac{i^* - i}{i^*(1 - \lambda)} h(i) di. \end{aligned}$$

Note that for any strike i^* and limit parameter λ , if π is the price of a contract with maximum indemnity of \$1, then $x\pi$ is the price of a contract with maximum indemnity of \$ x . Therefore, without loss of generality, we can restrict further analysis to the class of elementary contracts with maximum indemnity of \$1, so that the contract price π is effectively the premium rate. Buyers of the contracts can then purchase the number of \$1 contracts necessary to satisfy their risk management needs.

Pricing equation (4) assumes that the underlying index has a stationary distribution. Alternatively, the index may be treated in the framework of stochastic processes, and stochastic calculus may be used to price the contract (Turvey, 2001b). While one may argue about advantages and disadvantages of each pricing methodology, the issue is somewhat irrelevant from the standpoint of efficiency analysis,⁷ and therefore the simpler method represented by (4) is used here.

Selection of Contract Parameters

Indices determined by models in table 2 were used to calculate payoffs of weather contracts. Strikes were selected as the levels of indices at which the predicted yields were equal to the corresponding long-time averages—i.e., the contracts were designed to pay at least some indemnity whenever predicted yields dropped below the average.⁸ The remaining parameters of the contract, namely the limit parameter and the optimal number of \$1 contracts, were selected for each analysis unit so as to minimize an aggregate measure of downside loss or semi-variance (Markowitz, 1991) over a historic period⁹ (1972–1986). Formally, the parameters x and λ were chosen to solve:

$$(5) \quad \min_{x, \lambda} \sum_{t=1972}^{1986} \left(\max \left\{ \bar{Y} - \left[Y_t^{det} + f(i_t | x, i^*, \lambda) - \pi(x, i^*, \lambda) \right], 0 \right\} \right)^2,$$

⁷ Indeed, any change in the contract price would uniformly shift the ex post revenue of the buyer up or down in all states of nature, but would not affect the payment schedule and thus the correlation between losses and payoffs embedded in the contract structure.

⁸ In terminology of traditional insurance, the strike was set so as to provide 100% coverage.

⁹ The second half of the data (1987–2001) was used for out-of-sample performance analysis presented later in the paper.

where Y_t^{det} denotes detrended historical yields; \bar{Y} is the long-term average (target) yield; i_t represents historical realizations of the index; instrument payoff $f(\cdot)$ and price $\pi(\cdot)$ are given by (3) and (4), respectively; and summation is over the historic period 1972–1986.

Note, the decision process in (5) imitates that of a farmer who attempts to select an optimal instrument so as to minimize the expected net loss with the contract based on his or her knowledge of historical relationship between yields and weather. This approach is in line with an argument often used in the literature on weather derivatives (Turvey, 2001a): individual producers have the best information regarding the relationship between their yields and specific weather variables, and thus should be able to choose the parameters of the contract offering the best protection against their specific losses.

Minimization of the semi-variance instead of full variance is chosen because only downside losses are of major concern to crop producers (Miranda, 1991; Miranda and Glauber, 1997). This approach has been developed in the literature as an alternative to the traditional mean-variance analysis for situations where reduction of losses or failure to achieve a certain target is of importance (Hogan and Warren, 1972). It has also been shown to be consistent with the expected utility maximization (Selley, 1984).

In order to compute contract prices used in (5), the following approach was implemented. For each of the weather models presented in table 2, realizations of the index were calculated based on historical weather observations at the corresponding weather stations.¹⁰ Each realization was assigned an equal probability weight to construct empirical distributions. A kernel-smoothing methodology was then used to arrive at continuous probability density functions $h(i)$. Formally, for index realizations i_t , $t = 1, \dots, T$, the index density function was calculated as:

$$(6) \quad h(i) = \frac{1}{T\Delta} \sum_{t=1}^T K\left(\frac{i - i_t}{\Delta}\right),$$

where $K(\cdot)$ is a kernel function, and Δ is a degree of smoothness or bandwidth¹¹ (Härdle, 1991). The prices of weather contracts for a given set of parameters $\{x, i^*, \lambda\}$ in (5) were then calculated according to (4) and (6). In order to convert yields into monetary units, the harvest-time prices reported by the RMA in 2001 were used.¹² The commodity prices, expected yields, and corresponding expected revenues have been summarized earlier in table 1. The strikes, limits, and maximum liabilities for optimal contracts are reported in table 3.

The variety of indices and contract types presented in tables 2 and 3 indicate that weather derivatives cannot be designed in a one-size-fits-all manner, even for the same crop (e.g., cotton in Mississippi vs. cotton in Georgia) or within the same state (e.g., Iowa corn vs. Iowa soybeans). Differences in growing conditions, location, and agricultural practices translate into the need for different weather derivatives. Table 2 also suggests that even fairly sophisticated models fail to completely capture the relationship between

¹⁰ The historical series used to derive index distributions do not have to be of the same length as those used to establish weather-yield relationship. Therefore, all available weather data have been used, with some of the series going as far back as 1903.

¹¹ Normal kernel and optimal bandwidth have been used for practical computations.

¹² The harvest-time price for corn is the 2001 Chicago Board of Trade (CBOT) November average price on the December contract. The harvest-time price for soybeans is the 2001 CBOT October average price on the November contract. The harvest-time price for cotton is the 2001 New York Board of Trade November average price on the December contract. These are the harvest-time prices used to settle 2001 Crop Revenue Coverage (CRC) federal crop insurance contracts as reported by the RMA. Note that the prices act simply as scale multipliers similar to price election parameters of traditional APH contracts.

Table 3. Parameters of Optimal Weather Instruments

Crop / District	Strike	Limit		Maximum Liability (\$/acre)	Premium (\$/acre)	Premium Rate ^a
		Absolute Value	% of Strike			
Corn / IA, D50	147.59 bu/ac	110.69 bu/ac	75.0%	\$124.05	\$26.88	21.7%
Corn / IL, D10	143.56 bu/ac	108.10 bu/ac	75.3%	\$91.24	\$20.71	22.7%
Cotton / MS, D40	869.39 lbs/ac	866.78 lbs/ac	99.7%	\$141.88	\$64.18	45.2%
Cotton / GA, D80	712.70 lbs/ac	463.97 lbs/ac	65.1%	\$145.39	\$33.88	23.3%
Soybeans / IA, D10	46.32 bu/ac	46.00 bu/ac	99.3%	\$33.80	\$12.51	37.0%
Soybeans / IL, D70	41.15 bu/ac	31.65 bu/ac	76.9%	\$60.05	\$10.37	17.3%

^a The premium rate is the ratio of premium to maximum liability.

weather and yield for some district/crop combinations (e.g., Iowa soybeans). As seen from table 3, the premium rates associated with the weather contracts turned out to be surprisingly high, ranging from 17.3% for Illinois soybeans to 45.2% for Mississippi cotton.

Efficiency Analysis

The efficiency analysis was performed under the assumption that the contracts were bought by a representative farmer who produced only one crop, used weather derivatives to decrease his or her exposure to yield risk on that crop, and did not purchase any other risk management instruments such as subsidized crop insurance, futures, or options. Further, the representative farmer's yields were assumed to exactly match the crop reporting district (CRD) yields. Obviously, the actual farm-level yields may differ significantly from the district yields both in magnitude and especially in variability, and the effect of weather derivatives on risk exposure at an individual farm may be different from the effect experienced at the district level. However, the additional variability of actual farm yields would only decrease the risk-reducing efficiency of weather derivatives in practical applications compared to their performance for the representative farmer.

Methodology

The risk-reducing performance of weather derivatives was analyzed both in- and out-of-sample by comparing producers' revenues with and without the contract. Three different criteria were used to measure the change in risk exposure experienced by producers who bought the designed contracts—the mean root square loss (MRSL), value-at-risk (VaR), and certainty-equivalent revenues (CERs).

The yield/index series from 1972 to 2001 was split into two equal subsets with 15 observations each. The first subset was used to derive the parameters of optimal contracts (as described in the previous section) and to measure the in-sample performance. The second subset was used to evaluate out-of-sample efficiency of the designed weather instruments. For each of the subsets, the revenues without the contract were calculated as

$$(7a) \quad R_t = pY_t^{det}, \quad t = 1, \dots, T,$$

while revenues with the contract were calculated as

$$(7b) \quad R'_t = pY_t^{det} + f(i_t | x, i^*, \lambda) - \pi(x, i^*, \lambda), \quad t = 1, \dots, T,$$

where p is the corresponding commodity price (table 3).

The mean root square loss, which is a simple function of the semi-variance used in (5), was calculated for revenues without and with the contract as follows:

$$MRS L_{without} = \sqrt{\frac{1}{T} \sum_{t=1}^T [\max(p\bar{Y} - R_t, 0)]^2},$$

$$MRS L_{with} = \sqrt{\frac{1}{T} \sum_{t=1}^T [\max(p\bar{Y} - R'_t, 0)]^2}.$$

For a given distribution of revenues R , the value-at-risk at an $\alpha\%$ level is defined as a level of revenue Var_α (Manfredo and Leuthold, 1999), such that

$$\Pr\{R < Var_\alpha\} = \alpha,$$

and can be found by using the cumulative density function of R . For purposes of analysis, the series of revenues without and with contract in (7a) and (7b), respectively, were used to generate the corresponding empirical distributions by assigning each realization an equal weight $1/T$ and applying a kernel-smoothing procedure (6). Value-at-risk was then calculated for both distributions at 5%, 10%, and 20% levels both in- and out-of-sample.¹³

Finally, the certainty-equivalent revenues were calculated by using a negative exponential utility function:

$$U = 1 - \exp(-\gamma R),$$

where the risk parameter γ was calibrated separately for each revenue distribution to correspond to a prespecified risk premium θ (Babcock, Choi, and Feinerman, 1993; Schnitkey, Sherrick, and Irwin, 2003). First, the expected revenue ER without a contract has been calculated using the kernel-smoothed empirical distribution in (7a). Then, for a given level of risk premium θ , the parameter γ has been computed numerically so that the expected utility of the revenue without the contract is equal¹⁴ to the utility of certain revenue $(1 - \theta) \times ER$, i.e.,

$$EU(R) = E_R(1 - \exp(-\gamma R)) = 1 - \exp(-\gamma \times (1 - \theta) \times ER) = U((1 - \theta) \times ER).$$

Once the risk parameter of the utility function has been determined, the certainty-equivalent revenues (CERs) without and with weather contract were calculated from the conditions

¹³ If the change in value-at-risk resulting from the purchase of the weather derivative is positive (negative), the weather derivative is risk-reducing (risk-enhancing). This is in contrast to the mean root square loss measure where a positive (negative) change implies the weather derivative is risk-enhancing (risk-reducing).

¹⁴ The interpretation of this procedure is that a risk-averse producer would be willing to sacrifice $\theta\%$ of his or her expected revenue to eliminate the uncertainty.

Table 4. Efficiency of Weather Derivatives as Measured by Mean Root Square Loss (MRSL)

Crop / District	In-Sample (1972–1986)			Out-of-Sample (1987–2001)		
	Without Contract (\$/acre)	With Contract (\$/acre)	Percent Change	Without Contract (\$/acre)	With Contract (\$/acre)	Percent Change
Corn / IA, D50	\$9.20	\$5.16	-43.91%	\$10.92	\$4.46	-59.16%
Corn / IL, D10	\$7.22	\$1.65	-77.10%	\$10.88	\$6.80	-37.45%
Cotton / MS, D40	\$11.45	\$3.79	-66.88%	\$7.83	\$11.30	44.41%
Cotton / GA, D80	\$13.01	\$6.59	-49.32%	\$7.36	\$9.07	23.35%
Soybeans / IA, D10	\$4.03	\$2.53	-37.20%	\$5.68	\$4.74	-16.56%
Soybeans / IL, D70	\$5.80	\$4.10	-29.35%	\$3.86	\$2.51	-34.98%

Note: Decrease (increase) in MRSL corresponds to lower (higher) risk exposure.

$$U(CER_{without}) = E_R U(R) \quad \text{and} \quad U(CER_{with}) = E_R U(R'),$$

respectively, where the distributions of R and R' again were derived from the series of revenues in (7a) and (7b). The above procedure was performed for the risk premium levels of 0% (risk-neutrality), 5%, and 10%. Tables 4–6 report the results of the analysis.

Results

Risk-reducing efficiency of weather derivatives as primary insurance instruments varies significantly both across crops and districts (table 4). There also appears to be no consistent relationship between contract performance in- and out-of-sample. The contract for Iowa corn performed better out-of-sample than in-sample, while the contract for Illinois corn was not as effective out-of-sample as in-sample. With soybean contracts, the situation reversed itself and performance of the Illinois contract was better out-of-sample than in-sample. Contracts for both Georgia and Mississippi cotton, on the other hand, showed excellent in-sample performance (second only to Illinois corn) but actually increased the producer's risk exposure in out-of-sample tests.

Note also that the performance of weather/yield models does not necessarily translate into performance of weather derivatives based on these models. For example, models for Iowa corn and Mississippi cotton exhibited close explanatory power (56.8% and 55.9%, respectively, table 2), while the corresponding weather derivatives exhibited drastically different performance, especially out-of-sample (59.2% decrease in MRSL for Iowa corn vs. 44.4% increase in MRSL for Mississippi cotton, table 4).

Efficiency analysis based on the value-at-risk criterion (table 5) and certainty-equivalent revenues (table 6) generally produced results similar to those obtained with MRSL. In-sample, all contracts reduced value-at-risk for all district/crop combinations (with the exception of Iowa corn at 20%) and increased certainty-equivalent revenues for risk-averse producers (with the exception of Georgia cotton). Out-of-sample, contracts for both Georgia and Mississippi cotton increased producers' risk exposure regardless of the VaR levels or risk-aversion levels assumed. The degree of risk protection provided by the contracts generally tends to increase with the degree of risk aversion and decrease with the value-at-risk level.

Table 5. Efficiency of Weather Derivatives as Measured by Value-at-Risk (VaR)

Crop / District	In-Sample (1972–1986)			Out-of-Sample (1987–2001)		
	Without Contract (\$/acre)	With Contract (\$/acre)	Change (\$/acre)	Without Contract (\$/acre)	With Contract (\$/acre)	Change (\$/acre)
<----- VaR _{0.05} ----->						
Corn / IA, D50	\$190.10	\$239.85	\$49.75	\$182.99	\$255.84	\$72.84
Corn / IL, D10	\$213.78	\$272.69	\$58.92	\$156.55	\$222.19	\$65.65
Cotton / MS, D40	\$178.20	\$250.47	\$72.27	\$215.82	\$190.08	(\$25.74)
Cotton / GA, D80	\$103.81	\$176.73	\$72.92	\$159.57	\$145.85	(\$13.73)
Soybeans / IA, D10	\$168.73	\$185.12	\$16.39	\$132.32	\$152.95	\$20.64
Soybeans / IL, D70	\$128.18	\$135.56	\$7.38	\$144.00	\$160.35	\$16.35
<----- VaR _{0.10} ----->						
Corn / IA, D50	\$246.95	\$252.28	\$5.33	\$198.98	\$262.94	\$63.96
Corn / IL, D10	\$226.40	\$277.74	\$51.34	\$238.19	\$249.13	\$10.94
Cotton / MS, D40	\$200.97	\$262.35	\$61.38	\$231.66	\$208.89	(\$22.77)
Cotton / GA, D80	\$132.98	\$188.74	\$55.77	\$175.02	\$164.72	(\$10.30)
Soybeans / IA, D10	\$176.02	\$189.37	\$13.35	\$191.80	\$180.87	(\$10.93)
Soybeans / IL, D70	\$138.20	\$164.57	\$26.37	\$160.88	\$165.10	\$4.22
<----- VaR _{0.20} ----->						
Corn / IA, D50	\$272.71	\$267.38	(\$5.33)	\$277.16	\$273.60	(\$3.55)
Corn / IL, D10	\$280.27	\$283.64	\$3.37	\$259.23	\$262.59	\$3.37
Cotton / MS, D40	\$229.68	\$282.15	\$52.47	\$253.44	\$231.66	(\$21.78)
Cotton / GA, D80	\$174.16	\$203.33	\$29.17	\$193.89	\$189.60	(\$4.29)
Soybeans / IA, D10	\$188.15	\$195.44	\$7.28	\$202.11	\$196.65	(\$5.46)
Soybeans / IL, D70	\$160.35	\$175.12	\$14.77	\$170.37	\$170.90	\$0.53

Note: Higher (lower) VaR corresponds to lower (higher) risk exposure.

Implications

While most of the literature to date has focused on the pricing of agricultural weather derivatives, this analysis emphasizes the importance of considering the efficiency of these instruments as risk reduction tools. The results of this study indicate that for the district/crop combinations considered, weather derivatives may indeed provide fairly substantial decreases in risk exposure regardless of the particular criterion used. However, rather complicated combinations of weather variables must be used in order to achieve reasonable fits of the relationship between weather and yield. This makes contracts less transparent and may complicate marketing the products to potential buyers. In addition, high predictive power of weather/yield models does not necessarily translate into high performance of corresponding weather derivatives.

The in-sample performance did not necessarily translate into out-of-sample performance, as contracts for both Georgia and Mississippi cotton resulted in increased risk exposure out-of-sample. Inconsistency between in- and out-of-sample performance also creates a potential problem in designing and marketing the contracts. At any given point in time, efficiency analysis can only be implemented based on historical data that may not always adequately reflect correlation between the yields and weather indices.

Table 6. Efficiency of Weather Derivatives as Measured by Certainty Equivalent Revenues (CERs)

Crop / District	In-Sample (1972–1986)			Out-of-Sample (1987–2001)		
	Without Contract (\$/acre)	With Contract (\$/acre)	Change (\$/acre)	Without Contract (\$/acre)	With Contract (\$/acre)	Change (\$/acre)
Risk Premium 0%						
Corn / IA, D50	\$307.69	\$300.99	(\$6.70)	\$296.80	\$301.41	\$4.61
Corn / IL, D10	\$302.43	\$297.71	(\$4.71)	\$285.72	\$280.21	(\$5.51)
Cotton / MS, D40	\$290.78	\$317.32	\$26.54	\$299.83	\$287.35	(\$12.48)
Cotton / GA, D80	\$239.45	\$236.80	(\$2.65)	\$242.99	\$236.11	(\$6.87)
Soybeans / IA, D10	\$210.24	\$207.88	(\$2.36)	\$212.02	\$208.54	(\$3.47)
Soybeans / IL, D70	\$190.14	\$188.00	(\$2.14)	\$184.92	\$185.58	\$0.65
Risk Premium 5%						
Corn / IA, D50	\$292.74	\$294.69	\$1.96	\$282.35	\$296.97	\$14.63
Corn / IL, D10	\$287.55	\$295.90	\$8.35	\$271.84	\$276.75	\$4.91
Cotton / MS, D40	\$290.75	\$317.31	\$26.56	\$299.82	\$287.33	(\$12.49)
Cotton / GA, D80	\$239.41	\$236.79	(\$2.62)	\$242.97	\$236.10	(\$6.87)
Soybeans / IA, D10	\$199.82	\$204.51	\$4.69	\$201.54	\$202.33	\$0.79
Soybeans / IL, D70	\$180.83	\$183.20	\$2.37	\$175.76	\$180.94	\$5.18
Risk Premium 10%						
Corn / IA, D50	\$277.26	\$289.76	\$12.50	\$267.42	\$293.51	\$26.08
Corn / IL, D10	\$272.38	\$294.39	\$22.01	\$257.46	\$273.95	\$16.48
Cotton / MS, D40	\$290.72	\$317.31	\$26.58	\$299.81	\$287.30	(\$12.50)
Cotton / GA, D80	\$239.37	\$236.78	(\$2.59)	\$242.95	\$236.08	(\$6.87)
Soybeans / IA, D10	\$189.30	\$201.00	\$11.70	\$190.93	\$197.46	\$6.53
Soybeans / IL, D70	\$171.30	\$177.40	\$6.10	\$166.50	\$177.53	\$11.03

The optimal contracts varied across districts and crops considered. Such lack of consistency in indices across the crops and geographical units should be of concern in practical applications of weather derivatives. Based on these results, the contracts have to be highly localized, which comes at additional cost and raises questions about potential markets for the contracts. Stated differently, any practical application of weather derivatives in agriculture will require an in-depth analysis, similar to that reported here, for each crop/region combination under consideration. The results presented here clearly caution against blanket assessments of the feasibility of weather derivatives in agricultural applications. Rather, the analysis must be conducted on a case-by-case basis.

Conclusion

The efficiency of weather derivatives was analyzed for three major crops in six crop reporting districts. The analysis was conditioned on a number of unrealistically favorable assumptions. For each crop/district combination, the relationship between yield and selected weather variables was estimated for five alternative functional forms, and a weather derivative was constructed based on the function which best fit the data. The

strike on the weather derivative was set at an unrealistically high level whereby it began to pay an indemnity whenever weather events were such that the predicted yield was less than the average yield. Since the weather derivatives were assumed to have actuarially-fair prices, an unrealistically high strike implies an unrealistically favorable assessment of the risk reduction provided by the weather derivative. The limit parameter and the number of \$1 contracts purchased were optimized so as to minimize the aggregate losses in-sample. This extremely unrealistic assumption implies that the purchaser knows the exact value of these parameters for maximizing the risk-reducing capability of the weather derivative. Finally, this analysis was conducted using yields measured at the crop reporting district (CRD) level. It seems reasonable to assume that weather variables measured at a point central to the CRD should be more highly correlated with CRD-level yields than with yields on most individual farms located within the CRD. The constructed weather derivatives provided risk protection against yield shortfalls. However, the contracts did not perform consistently out-of-sample, and in some cases actually increased overall risk exposure for the crop/district combinations considered.

Further research should investigate the potential of weather derivatives as agricultural risk management tools for other crops and regions. In addition, future research could focus on the potential for using weather derivatives to reinsure traditional farm-level yield or revenue insurance products.

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Appendix: Weather-Yield Models

Table A1. Families of Weather-Yield Models Used in Estimations

Model Type	Functional Form
Monthly Observations:	
Quadratic in absolute values	$Y_{det} = \alpha_0 + \alpha_1 \mathbf{R} + \alpha_2 \mathbf{T} + \alpha_3 \mathbf{R}^2 + \alpha_4 \mathbf{T}^2 + \alpha_5 \mathbf{RT} + \varepsilon$
Log-log in absolute values	$\log(Y_{det}) = \alpha_0 + \alpha_1 \log(\mathbf{R}) + \alpha_2 \log(\mathbf{T}) + \varepsilon$
Quadratic in deviations	$Y_{det} = \alpha_0 + \alpha_1 \Delta \mathbf{R} + \alpha_2 \Delta \mathbf{T} + \alpha_3 \Delta \mathbf{R}^2 + \alpha_4 \Delta \mathbf{T}^2 + \alpha_5 \Delta \mathbf{R} \Delta \mathbf{T} + \varepsilon$
Cumulative Observations:	
Quadratic in absolute values	$Y_{det} = \alpha_0 + \alpha_1 R_{Cum} + \alpha_2 CDD_{65/50} + \alpha_3 R_{Cum}^2 + \alpha_4 CDD_{65/50}^2 + \alpha_5 R_{Cum} CDD_{65/50} + \varepsilon$
Log-log in absolute values	$\log(Y_{det}) = \alpha_0 + \alpha_1 \log(R_{Cum}) + \alpha_2 \log(CDD_{65/50}) + \varepsilon$

Variables are defined as follows:

- Y_{det} is the detrended district yield;
- R_{Cum} is cumulative rainfall between June 1st and August 31st;
- $\mathbf{R} = \{R_{Jun}, R_{Jul}, R_{Aug}\}$ is a vector of monthly rainfall observations for June, July, and August;
- $\Delta \mathbf{R} = \{\Delta R_{Jun}, \Delta R_{Jul}, \Delta R_{Aug}\}$ is a vector of the same observations expressed as deviations from the corresponding long-time averages;
- $CDD_{65/50}$ is a number of cumulative cooling degree days with either 65°F or 50°F base measured between June 1st and August 31st;
- $\mathbf{T} = \{T_{Jun}, T_{Jul}, T_{Aug}\}$ is a vector of average monthly temperatures for June, July, and August;
- $\Delta \mathbf{T} = \{\Delta T_{Jun}, \Delta T_{Jul}, \Delta T_{Aug}\}$ is a vector of the same observations expressed as deviations from the corresponding long-time averages.

Note: All products in models 1 and 3 are element-by-element; e.g., the last term in model 1 is a vector, $\mathbf{RT} = \{R_{Jun}T_{Jun}, R_{Jul}T_{Jul}, R_{Aug}T_{Aug}\}$.