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The Decision of When to Buy a Frequently Purchased Good: A Multi-Period Probit Model

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Increased availability of scanner-based panel data has enabled researchers to better understand nondurable commodity purchase dynamics. In this study, we focus on one component of the purchase process—when to buy. The relationship between the discrete purchase decision and a set of household and purchase characteristics is quantified using a simulated maximum-likelihood procedure. Given the longitudinal nature of our data, unobserved heterogeneity is addressed by adopting an autocorrelated error structure. Our empirical application is household purchases of cheese. We find evidence of significant persistent unobservable household heterogeneity, which is not eliminated by the inclusion of lagged exogenous variables.

Key words: autocorrelation, discrete decision, panel data, simulated maximum likelihood

Introduction

Increased availability of scanner-based panel data has enabled researchers to better understand nondurable commodity purchase dynamics. A majority of these research efforts have focused on the discrete decision of whether or not a product or brand will be purchased at a particular time (Erdem, Keane, and Sun; Guadagni and Little; Keane 1997; Gupta 1991). For example, Gould (1997, 1998) used event history analysis to examine the determinants of the timing of weekly household purchases of cheese and food fats and oils over a three-year study period. In an early analysis, Gupta (1988) examined the impact of sales promotion on purchase patterns for specific coffee brands. He used separate models to account for brand choice (multinomial logit), interpurchase time (event history), and purchase quantity (ordered regression). These models were estimated independently and, except for the interpurchase time component, did not utilize the panel nature of the data.

Ailawadi and Neslin extended the analysis of Gupta (1988) by using a nested logit formulation to link brand choice and purchase incidence to examine sales promotion impacts on prepackaged foods. A zero-truncated Poisson model links purchase

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incidence and quantity decisions. Although their analysis captures purchase dynamics by incorporating some time-varying exogenous variables such as household commodity inventory and consumption, the authors do not account for household-specific heterogeneity.

An accounting of this heterogeneity is important for several reasons. First, when using micro-level data for demand analysis, heterogeneity may persist over time given that households differ in composition, purchase history, income, and racial/ethnic background. While these characteristics may impact a household's attitudes toward purchase decisions, they are not likely to change over a relatively short study period. Unless this heterogeneity is explicitly incorporated into the econometric model, the possibility of serial correlation increases significantly and may represent a source of misspecification (Hajivassiliou).

In a utility-maximizing framework, this heterogeneity can be accounted for by specifying a utility function whose parameters are impacted by observed consumer attributes (Elrod; Jones and Landwehr; Steckel and Vanhonacker; McCulloch and Rossi). Alternatively, one can define variables that account for the dependence of current utility evaluations on past choices. Examples of such variables include a dummy variable for lagged purchases (Jones and Landwehr), an exponentially smoothed weighted average of past purchases (Guadagni and Little), and various Bayesian updating systems (Fader; Erdem and Keane).

In addition to household-based heterogeneity, a second possible source of serial correlation may arise from learning processes that rely on a history of past purchases as a predictor of future purchase decisions. Habit formation implies state dependence and presence of intertemporal linkages. Keane (1997), using a utility-maximization framework within a multinomial-multiperiod probit model, differentiates between the impacts of state dependence and heterogeneity on the discrete purchase decision. The author found evidence of a significant causal connection between past purchases and current discrete choices for the commodity analyzed.

We approach the problem of investigating the dynamics of the purchase process using an extension to the familiar probit model where the probability of obtaining the entire profile of purchases is incorporated within the likelihood function. Unlike previous binary choice models, we explicitly incorporate serial correlation by specifying an error covariance structure that allows for such dependence (Hajivassiliou).

For this analysis, we follow a panel of households over a 65-week period. An initial 13-week interval is used to initialize a benchmark consumption variable (discussed below). The same 13-week period a year later is used for estimation of model parameters. The empirical application is concerned with the purchases of a fairly broad commodity category—cheese. This is in contrast to previous investigations that have analyzed brand purchase behavior (Keane 1997).

The use of a household panel over such a short time period implies that there will likely be some temporal interdependence of purchases. The possibility of habit formation implies that households with a tendency to purchase cheese as a protein source, versus red meat and other foods, will tend to purchase more cheese throughout the study period. We account for this interdependence by making our estimate of weekly consumption dependent on beginning household inventory. As will be shown below, we specify an autocorrelated error structure which explicitly incorporates serial interdependence.

An Econometric Model of Household Food Purchases

Following Hajivassiliou and Ruud, we assume a random sample of N households observed over T time periods. The latent variable y_{it}^* measures the net benefit to the ith household from undertaking a particular action in period t. Consumers are assumed to compare their welfare at zero levels of purchase (U_0) versus the optimal welfare level if a purchase occurs (U_+) at each time period. We assume the potential consumer decides whether to purchase depending on the net utility obtained from consumption value y_{it}^*:

$$(1) \qquad y_{it} = \begin{cases} 1 & \text{if } y_{it}^* \equiv U_{+it}^* - U_{0it}^* > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \{i = 1, \ldots, N;\ t = 1, \ldots, T\},$$

where y_{it} represents the observed discrete purchase decision. The level of these net benefits is related to an index function of the exogenous variable vector, $\mathbf{X}$, via the following:

$$(2) \qquad y_{it}^* = \mathbf{X}_{it}\beta + \varepsilon_{it} \qquad \{i = 1, \ldots, N;\ t = 1, \ldots, T\},$$

where β is a vector of estimated coefficients, and the error term $(\varepsilon_{it}) \sim N(0, \sigma_\varepsilon^2)$.

With T time periods, there are 2^T alternative combinations of purchase/nonpurchase decisions for a particular commodity. From (1) and (2), the probability of the ith household purchasing a particular commodity in time t $[P(y_{it} > 0)]$ is similar to other cross-section analyses:

$$(3) \qquad \begin{aligned} P(y_{it} > 0) &= P(\mathbf{X}_{it}\beta + \varepsilon_{it} > 0) \\ &= P(\varepsilon_{it} > -\mathbf{X}_{it}\beta) \\ &= P(\varepsilon_{it} < \mathbf{X}_{it}\beta) \qquad \{i = 1, \ldots, N;\ t = 1, \ldots, T\}. \end{aligned}$$

If we assume the distribution of $\mathbf{Y}^*$ (the T-vector of net outcomes for the ith household) is multivariate normal, then the T-dimension probability density function (PDF) of observing a particular purchase history, $\mathbf{Y}_i$ $(= \{y_{i,1}, y_{i,2}, \ldots, y_{i,T}\}')$, is:

$$(4) \qquad f(\mathbf{Y}_i | \beta, \Omega\mathbf{X}) = \varphi\Big((-1)^{(1-y_{i1})}\mathbf{X}_{i1}\beta, \ldots, (-1)^{(1-y_{iT})}\mathbf{X}_{iT}\beta, \Omega\Big) \qquad \{i = 1, \ldots, N\},$$

where φ is the T-dimension multivariate normal PDF, and Ω is the $(T \times T)$ positive semi-definite error term covariance matrix. Equation (4) represents an extension of the traditional multinomial probit model with 2^T alternatives and a covariance matrix that incorporates the assumed serial correlation of the error terms, ε_{it}. The likelihood of observing a particular sequence of choices can be represented as:

$$(5) \qquad \Phi(\Theta | \mathbf{Y}_i, \mathbf{X}_i) = \int_{a_i(\mathbf{Y}_i)}^{b_i(\mathbf{Y}_i)} \varphi(\mathbf{Y}_i^* - \mu_i, \Omega)\, d\mathbf{Y}_i^* \qquad \{i = 1, \ldots, N\},$$

where $\Theta = (\beta, \Omega)$, and the functions a_i and b_i are T-dimension limits of integration:

$$a_{it} = \begin{cases} 0 & \text{if } y_{it} = 1 \\ -\infty & \text{if } y_{it} = 0 \end{cases} \qquad b_{it} = \begin{cases} +\infty & \text{if } y_{it} = 1 \\ 0 & \text{if } y_{it} = 0 \end{cases} \qquad \{t = 1, \ldots, T\}.$$

Given (5), we need to specify a structure for Ω. One approach used in the past is to restrict the variance-covariance matrix to be time and household invariant; that is, $\Omega = \Omega_i = \sigma_\eta^2 \mathbf{I}_T$, where $\mathbf{I}_T$ is a T-dimension identity matrix (Börsch-Supan et al.; Hajivassiliou). This structure yields a pooled cross-sectional model that ignores intertemporal linkages. Under this assumption, the necessity of evaluating a single T-dimension integral in (5) is avoided given that the integral is simplified to T one-dimension integrals. Traditional maximum-likelihood techniques can be used to obtain parameter estimates under this simple assumption.

Hajivassiliou and McFadden; McFadden; Börsch-Supan et al.; and Hajivassiliou assume a number of alternative autocorrelated error structures within the discrete choice model. For the present analysis, we assume a structure that incorporates both random-effect and AR(1) characteristics. That is:

(6) $$\varepsilon_{it} = \eta_i + \zeta_{it},$$

where

$\zeta_{it} = \rho\zeta_{i(t-1)} + \nu_{it};$

$|\rho| < 1;$

ν and η are independent, with $\nu_{it} \sim N(0, \sigma_\nu^2)$ and $\eta_{it} \sim N(0, \sigma_\eta^2)$; and

$$\sigma_\zeta^2 = \frac{\sigma_\nu^2}{(1 - \rho^2)}.$$

The above error structure implies $\text{cov}(\varepsilon_{is}, \varepsilon_{it}) = \rho^{(t-s)}\sigma_\zeta^2 + \sigma_\eta^2$. The full covariance matrix can then be represented as:

(7) $$\Omega = E(\varepsilon'\varepsilon) = \sigma_\zeta^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{T-2} & \rho^{T-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{T-3} & \rho^{T-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{T-4} & \rho^{T-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \rho^{T-4} & \cdots & \rho & 1 \end{bmatrix} + \sigma_\eta^2 \mathbf{J}_T,$$

where $\mathbf{J}_T$ is a $\{T \times T\}$ matrix of ones. As Hajivassiliou and Ruud note, under the multi-period probit model, parameters σ_η^2 and σ_ζ^2 cannot both be identified. The authors impose the arbitrary identification constraint, $\sigma_\zeta^2 + \sigma_\eta^2 = 1$.

Use of Simulated Maximum-Likelihood Techniques

With the adoption of the covariance matrix in (7), the likelihood function based on (5) requires the evaluation of a T-dimension integral. Numerical methods have been developed for evaluating the integral of this function when the dimension is small, i.e., less than or equal to 4 (Johnson and Kotz; Tong). Traditional numerical methods cannot be used to evaluate these integrals with sufficient speed to make the computation of the maximum-likelihood estimator workable with more than four time periods. As an alternative, the likelihood function can be evaluated using simulated maximum-likelihood procedures. There are a number of alternative methods that can be used to evaluate higher order integrals for normal density functions. These simulation methods include: (*a*) the frequency simulator proposed by Leman and Manski; (*b*) the kernel-smooth frequency simulator proposed by McFadden; (*c*) the GHK recursive simulator suggested by Geweke, by Hajivassiliou and McFadden, and by Keane (1993, 1994); and (*d*) the low-variance simulator proposed by Breslaw.

Previous analyses have investigated the properties of the GHK simulator (e.g., Börsch-Supan and Hajivassiliou; Breslaw; and Hajivassiliou, McFadden, and Ruud). Using a root mean squared error (RMSE) criterion, Hajivassiliou, McFadden, and Ruud show the GHK algorithm to be the most reliable simulator of the 12 alternatives examined. Geweke, Keane, and Runkle (1994) perform a Monte Carlo evaluation of the use of simulated maximum-likelihood techniques for single-period multinomial probit models and conclude that the GHK simulator significantly outperforms other simulators based on kernel-smoothed probability techniques. Geweke, Keane, and Runkle (1997) undertook a second set of Monte Carlo experiments to compare the sampling distributions of a GHK simulation procedure with that of (*a*) Gibbs sampling and data augmentation, and (*b*) a GHK-based method of simulated moments procedure. The authors conclude these estimators perform reasonably well, but the number of simulations should be increased when the degree of serial correlation increases. For the current application, we adopt the GHK recursive simulator using 500 replicates (as suggested by Hajivassiliou, McFadden, and Ruud). A brief overview of the simulation algorithm is provided in the appendix.

Description of Data Used in the Analysis

We apply the multi-period probit model represented by equations (5)–(7) to a panel of U.S. households. Our empirical application is concerned with weekly purchases of natural and processed cheese for home consumption. Our focus on home consumption limits this analysis to the 35–40% of the U.S. cheese market where cheese is not consumed in food service establishments or used as a food ingredient. We incorporate a number of household as well as purchase-related characteristics as explanatory variables.

Household characteristics included in this analysis are household income (*HHINC*), two variables identifying the race/ethnicity of household heads (*BLACK*, *HISPANIC*), and a series of binary household life cycle variables. These life cycle variables are used to identify how purchase patterns differ across households as the family unit expands or contracts. Examples of the use of similar life cycle variables can be found in Wells and Gubar, and in Murphy and Staples. We follow Huang and Raunikar, and define the following 10 life cycle stages: young single (*YNGSINGLE*), young married without children (*YNGMARNOK*), young married with children (*YNGMARWKD*), young single with children (*YNGSNGWKD*), middle-aged single (*MASINGLE*), middle-aged married without children (*MAMARNOK*), middle-aged married with children (*MAMARWKD*), middle-aged single with children (*MASNGWKD*), senior married (*OLDMARR*), and senior single (*OLDSINGLE*).

The age definitions are based on the age of the female head of household, if present, or the male head. The following age group definitions are used: young (under 35 years old), middle-aged (35–64 years old), and senior (65+ years old). Based on the above division of households into one of the 10 life cycle categories, we used households having middle-aged married heads with children (i.e., *MAMARWKD* = 1) as the base.

Purchase-related characteristics include a binary variable identifying the weeks surrounding the Thanksgiving/Christmas period (*THNKXMAS*), an estimate of benchmark weekly per capita cheese consumption obtained from a 13-week initialization period (Q_d), cheese price (*CH_PRICE*), and a binary variable indicating whether a purchase was made during the previous week (*LAGDUM*).[1]

Cheese is a perishable good which can be consumed any time during the day as a snack or as an ingredient in meal preparation. Using the argument put forth by Ailawadi and Neslin, with most cheeses being refrigerated, there is a tendency for information concerning household stocks to be reinforced each time the refrigerator door is opened. This implies increased consumption rates when inventories are high (Assuncao and Meyer; Folkes, Martin, and Gupta). To capture the impact of such inventories on consumption per capita, beginning total cheese inventory (*BEGCHINV*) is used as an explanatory variable (Ailawadi and Neslin; Bucklin and Lattin; Chintaguanta; Gupta 1991). In addition to the above impact on consumption, inventories are also hypothesized to negatively impact purchase probabilities by reducing the possibility of a stockout (i.e., the household has more of an opportunity to consume the product).

For this analysis, we define beginning inventory as follows:

$$(8) \qquad BEGCHINV_{it} \equiv BEGCHINV_{i,(t-1)} + CHQUANT_{i,(t-1)} - CONSUME_{i,(t-1)},$$

where $CHQUANT_{i,(t-1)}$ is the previous week's quantity purchased by the ith household, and $CONSUME_{i,(t-1)}$ is an estimate of the previous week's quantity consumed.[2]

To obtain an estimate of household inventories via (8), we need an estimate of weekly cheese consumption. Unfortunately, the data set used in this analysis does not explicitly contain this information.[3] Ailawadi and Neslin outline three possible methods that can be used to approximate household consumption: (*a*) status quo, (*b*) spline function, and (*c*) continuous nonlinear function. Under the continuous nonlinear function approach, an estimate of unobserved consumption is obtained from the following:

$$(9) \qquad CONSUME_{it} = BEGCHINV_{it}\left[\frac{Q_{di}}{Q_{di} + BEGCHINV_{it}^{\tau}}\right],$$

where τ is referred to as the flexibility parameter to be estimated, and Q_{di} is a household-specific constant benchmark of average daily cheese consumption. Our analysis encompasses the 13-week period October–December 1992. Similar to the procedures used by Ailawadi and Neslin, we use the same 13-week period in 1991 as an initialization period to obtain an estimate of Q_{di}. Given the shelf life of many cheeses, we assume the initial inventory encompasses two weeks of this benchmark consumption. We adopt (9) for our analysis given that: (*a*) household consumption is allowed to vary continuously in a

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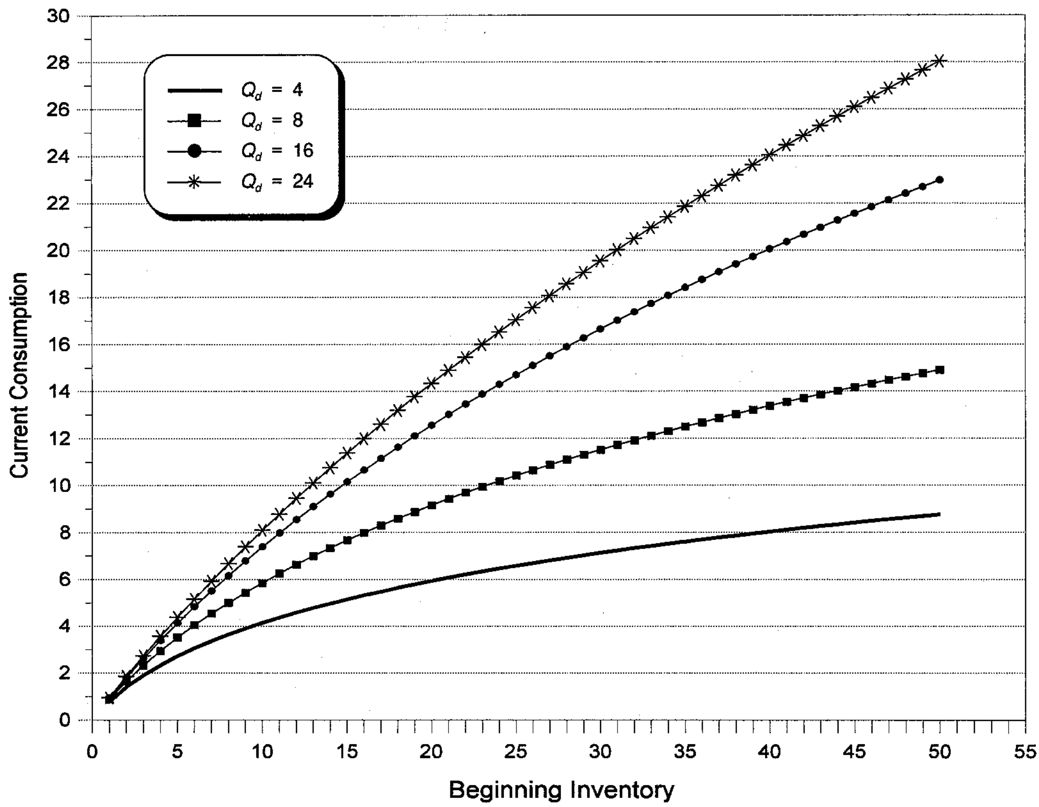
[1] The unit-value variable (*CH_PRICE*) is net of the value of coupons redeemed. Similar to Keane (1997), we did not model the household's coupon redemption decision. Including coupon value in determination of price would increase our estimates of price sensitivity.

[2] All quantities are measured in ounces per capita.

[3] For an overview of the pitfalls of estimating household inventories when such information is not contained within a data set, refer to Jain and Vilcassim.



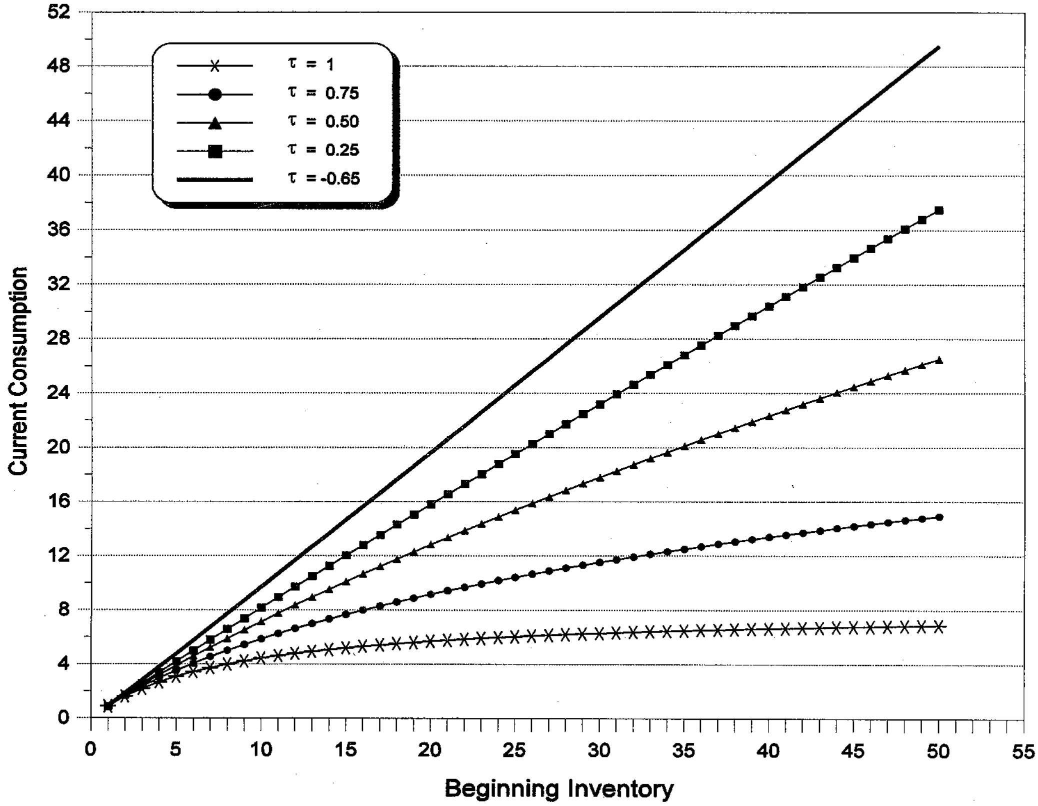
Note: The above assumes a flexibility parameter (τ) value of 0.75.

Figure 1(a). Hypothetical consumption profiles under alternative values of beginning inventory and benchmark consumption (Q_d)

nonlinear manner relative to household inventory, (*b*) only one additional parameter needs to be estimated, and (*c*) consumption will not exceed available inventory, eliminating the need to truncate our estimate of consumption.

Not surprisingly, for a given value of τ, heavy users of cheese (i.e., relatively large values of Q_d) consume more than light users at a particular inventory level. Figure 1(a) shows a hypothetical representation of the relationship between inventories and consumption for alternative levels of a hypothetical benchmark consumption (Q_d), and assumed τ value of 0.75. Figure 1(b) shows the impact of alternative flexibility parameter values, given Q_d. With a negative flexibility parameter (-0.65), households tend to consume almost their entire inventory. With $0 < \tau < 1$, consumption varies in a positive manner with beginning inventory. When $\tau = 1$, households increase their consumption when inventories are low, but they approach their average weekly usage rate (Q_d) for higher inventory levels. By incorporating (9) within the likelihood function, we allow the data to provide us with an estimate of household cheese consumption and how responsive such consumption is to changes in household inventory (i.e., the value of τ).[4]

[4] In the actual estimation, we follow the procedure of Ailawadi and Neslin, and use the household-specific mean-centered version of the *BEGCHINV* variable.



Note: Q_d is assumed to have a value of 8.

Figure 1(b). Effect of beginning inventory on hypothetical consumption under alternative flexibility parameter (τ) values

We include the dichotomous variable *THNKXMAS* to capture the effect of the Thanksgiving/Christmas holiday period on cheese purchases, as this time of year is traditionally one of peak cheese consumption. We hypothesize a positive coefficient for this variable. Following Ailawadi and Neslin, we include the variable *LAGDUM* to account for systematic "swings in purchase and consumption due to eating bouts, binging, special diets, and other situational factors" (Ailawadi and Neslin, p. 393). Again, we would anticipate a positive coefficient for this variable.

For cheese purchase weeks, we obtain an estimate of unit value (*CH_PRICE*) by dividing reported expenditures by quantity purchased. Previous studies (Theil; Houthaker; Cox and Wohlgenant; Deaton 1987, 1988; Nelson; Dong, Shonkwiler, and Capps) have recognized that this method of calculating price reflects not only differences in market prices faced by each household, but also endogenously determined commodity quality.[5] A number of alternative approaches can be used to obtain estimates of the unobserved unit values. Given the complexity of the estimation process for the underlying model,

[5] Observed differences in price paid for cheese across households may be reflecting not only local market conditions but also final product form purchased by this household. For example, households purchasing cheese in block form would be expected to pay a lower price than households purchasing cheese that is pre-sliced or shredded, ceteris paribus, given the increased value-added provided by the manufacturer.

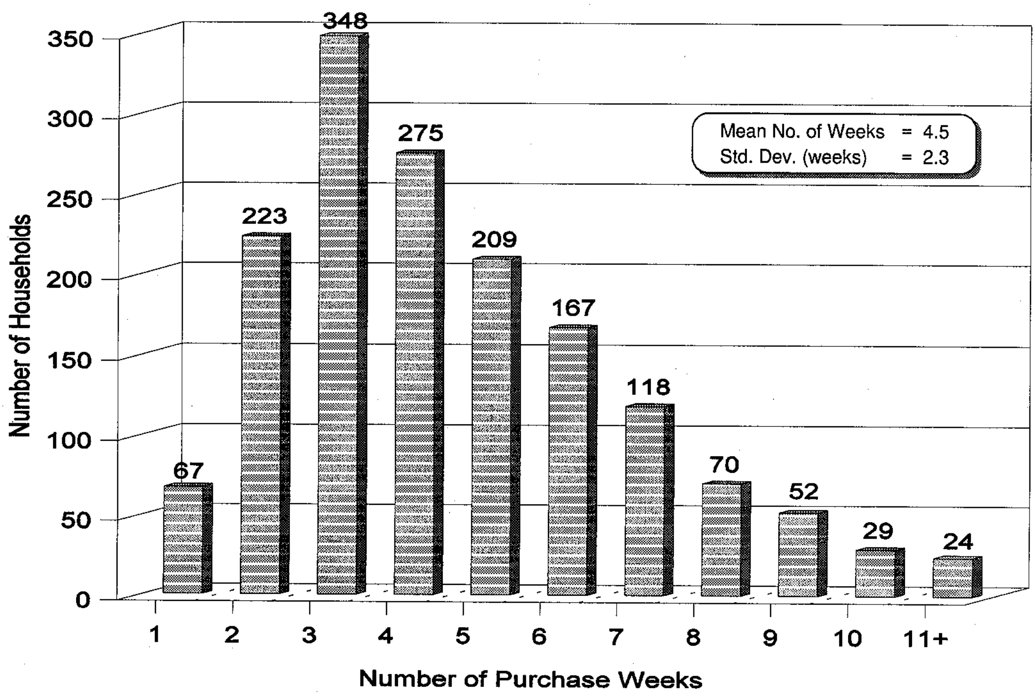


Figure 2. Distribution of household purchase weeks over the 13-week study period

we assume a zero-order correction for missing unit values.[6] That is, for each household the imputed unit value for nonpurchase weeks is set equal to household-specific conditional unit values.

The household panel used here is based on a U.S. consumer panel encompassing the October 1991 through December 1992 period, obtained from Nielsen Marketing Research, Inc. (NMR). Only cheese purchased for at-home consumption is included in this data set. On each purchase occasion, a panel member records the following information: date, UPC code, cheese expenditures, and quantity purchased. We aggregate purchase occasions to a weekly total. Households notify NMR if no purchases occurred during the previous week because of a choice not to purchase, or as a result of being away from home due to vacation, business trip, etc. For this analysis, we include households that reported continuously over the study period. This does not imply that panel households purchased each week, but during weeks where cheese was not purchased for at-home consumption, NMR was given this information.

Given the size of our panel, we randomly selected a set of households for use in this analysis ($N = 1{,}582$). Purchase opportunities and occasions are defined on a weekly basis. Cheese expenditure and quantity are defined as the sum of expenditures and purchases on all types of cheeses except cottage cheese, which is excluded given its unique physical characteristics. Figure 2 shows the distribution of household purchase weeks for our sample over the 13-week study period. The mean number of purchase weeks was found to be 4.5. An overview of household characteristics used in this analysis is provided in table 1.

[6] Alternative methods for estimating these unit values can be found in Cox and Wohlgenant; Dong, Shonkwiler, and Capps; and Dong and Gould.

Table 1. Definitions and Mean Values of Household Characteristics

Variable	Definition	Unit	Mean	Std. Dev.	Expected Sign
BLACK	Head of household is Black	0/1	0.047	—	–
HISPANIC	Head of household is Hispanic	0/1	0.055	—	?
HHINC	Household income	$	36,790	22,295	+
THNKXMAS	Thanksgiving/Christmas period	0/1	0.385	—	+
Q_d	Weekly per capita cheese consumption	oz/capita/wk	2.26	1.87	+
CH_PRICE [a]	Conditional cheese price	$/lb.	2.99	0.805	–
Household Composition (10 life cycle variables): [b]					
YNGSINGLE	Young single	0/1	0.009	—	–
YNGSNGWKD	Young single w/children	0/1	0.005	—	?
YNGMARWKD	Young married w/children	0/1	0.040	—	?
YNGMARNOK	Young married w/o children	0/1	0.015	—	–
MASINGLE	Middle-aged single	0/1	0.072	—	–
MASNGWKD	Middle-aged single w/children	0/1	0.031	—	?
MAMARWKD [c]	Middle-aged married w/children	0/1	0.546	—	n/a
MAMARNOK	Middle-aged married w/o children	0/1	0.107	—	–
OLDSINGLE	Senior single	0/1	0.064	—	–
OLDMARR	Senior married	0/1	0.111	—	–

[a] Conditional mean across purchase weeks.

[b] Age definitions are as follows: young = under 35, middle-aged = 35–64, and senior = 65+ (based on age of female head of household, if present, or male head).

[c] Households having middle-aged married heads with children is used as a base of comparison in the econometric model (*MAMARWKD* = 1).

Econometric Results

Parameter estimates are obtained by maximizing the likelihood function in (5) via the use of the MAXLIK routines supplied within the GAUSS software system. Table 2 contains estimated parameter values and standard errors which were obtained from the inverse of the numerically evaluated Hessian matrix. To evaluate the overall fit of the model, we would prefer to use a metric similar to traditional R^2 measures. In binary choice models, the probability of a certain outcome of a choice process is estimated. Ideally we would like to compare the estimated probabilities with the *true* probabilities in order to evaluate the overall performance of the model in explaining the occurrence of the discrete event. But this is not possible given that the true probabilities are not known. Veal and Zimmerman developed a pseudo-R^2 measure of goodness of fit (R^2_{VZ}) based on the ratio of the maximized log-likelihood function [$\log L(\beta^*)$] versus the restricted log-likelihood function [$\log L_0$] where explanatory variable coefficients except the intercept term are set equal to zero:[7]

$$R^2_{VZ} = \frac{2\left[\log L(\beta^*) - \log L_0\right]}{2\left[\log L(\beta^*) - \log L_0\right] + N} \; \frac{2\log L_0 - N}{2\log L_0}. \tag{10}$$

[7] For a review of alternative goodness-of-fit measures applied to binary choice models, refer to Windmeijer. In the restricted model here, in addition to the intercept term being nonzero, coefficients σ_ζ and ρ continue to be estimated.

Table 2. Estimated Multiperiod Probit Parameter Values and Standard Errors

Variable	Estimate	Std. Error	Variable	Estimate	Std. Error
Intercept	0.1342*	0.0571	**Household (cont'd)**		
Purchase/Consumption			*YNGSNGWKD*	-0.2999	0.1795
Characteristics:			*YNGMARWKD*	-0.0761	0.0633
BEGCHINV	-0.0329**	0.0022	*YNGMARNOK*	0.0118	0.0936
ln(*CH_PRICE*)	-0.1856**	0.0346	*MASINGLE*	-0.1815**	0.0520
ln(Q_d)	0.2134**	0.0162	*MASNGWKD*	-0.0086	0.0761
LAGDUM	-0.2842**	0.0509	*MAMARNOK*	-0.1093*	0.0418
THNKXMAS	0.1327**	0.0213	*OLDSINGLE*	-0.3255**	0.0570
τ	0.9000**	0.0343	*OLDMARR*	-0.2034**	0.0425
Household Characteristics:			**Regression Coefficients:**[a]		
BLACK	-0.0627	0.0599	$\beta_{\sigma\zeta}$	1.8788**	0.0560
HISPANIC	-0.0403	0.0549	β_ρ	0.1951**	0.0472
ln(*HHINC*)	0.0546*	0.0209			
YNGSINGLE	-0.1627	0.1349	R^2_{VZ} = 0.3614		

Note: Single and double asterisks (*) denote significance at the .01 and .001 levels, respectively.

[a] Given the constraints that $\sigma_\eta^2 + \sigma_\zeta^2 = 1$, and $|\rho| \leq 1$, the coefficients shown here are used as input into the calculation of a hyperbolic tangent function [tanh] which guarantees that the resulting coefficients fall within the desired range, $\sigma_\zeta = \tanh(\beta_{\sigma\zeta}) = 0.9543$, and $\rho = \tanh(\beta_\rho) = 0.1927$.

As Windmeijer notes, $0 \leq R^2_{VZ} \leq 1$. Using the parameter estimates in table 2, and estimating the restricted model, we obtain an R^2_{VZ} value of 0.361. Given our use of pooled cross-sectional data, we feel this value is reasonable.

As noted in table 2, a nonlinear function is used to restrict our estimate of ρ to be within the 0, 1 range. To evaluate the significance of serial dependence, we use a likelihood-ratio test of the significance of the restriction that $\rho = 0$. That is, we reestimate the likelihood function in (5) with the constraint that $\rho = 0$. The resulting χ^2 statistic of 10.3 is statistically significant at the .001 level, indicating significant serial correlation.

From table 2, we see that household composition has a significant impact on the household purchase propensity. With *MAMARWKD* = 1 used as a base, four of nine composition coefficients are statistically significant and negative. Three household types associated with single-person households differ by the age of the householder. It is not surprising that two of these household types generated negative coefficients relative to the base household. Previous research has shown that single households tend to eat more meals away from home, ceteris paribus (McCracken and Brandt). Combining this trend with the understanding that we are examining only the purchase of cheese for at-home consumption, the negative coefficient implies a lower purchase probability given beginning inventory and other exogenous variables.

Previous research has investigated the role of household member age structure on cheese purchases. Yen and Jones, using Bureau of Labor Statistics consumer expenditure data, found the marginal impact of having an additional household member over the age of 65 on conditional household cheese purchases was the lowest of the five age groups analyzed (except for young children). Gould and Lin, using annual purchase data over the March 1991/92 period, estimated a single-equation model of total at-home cheese demand that contained an endogenously determined cheese "adult equivalence scale." The authors found both male and female equivalence scales possess similar profiles when plotted against household member age. Maximum scale values were obtained before the age of 30, and continuously decreased after the age of 65. Gould, Cornick, and Cox examined the demand structure for reduced fat cheeses using a censored expenditure system framework. For the major cheese types, they found the marginal impact of additional household members was the least for household members over the age of 65, and greatest for household members between the ages of 18 and 40 (p. 375).

In terms of the results obtained here, both household types corresponding to *older* households generated statistically significant negative coefficients. The implied lower purchase probabilities may be due to several factors—including decreased dietary requirements, a cohort effect of special diets that have relatively lower amounts of cheese as a component, or decreased cheese intakes due to health concerns.

Cheese demand typically increases during the Thanksgiving/Christmas period. The positive and statistically significant coefficient associated with the *THNKXMAS* variable indicates increased purchase probabilities for the weeks before/during the holiday period. The positive income effect is consistent with previous analyses of cheese purchase probability, purchase timing, and conditional demand. In contrast to the results reported in Gould (1992) and those of Yen and Jones, we found that race/ethnicity had no impact on purchase frequency.

Ailawadi and Neslin, in their analysis of yogurt and ketchup purchases, obtained a positive estimated *LAGDUM* coefficient. In contrast, we obtained a significant negative coefficient. If a household purchased cheese in the previous period, there is diminished purchase probability in the current period. Combining the significance of the estimated *LAGDUM* with the statistically significant correlation coefficient provides evidence of persistent unobservable household-based heterogeneity.

As noted above, we used a zero-order method to estimate cheese price during non-purchase weeks. Based on this approach, the negative and significant price coefficients imply that a price increase will decrease purchase probability.

Two of the more statistically significant coefficients were associated with benchmark cheese consumption (Q_d) and beginning cheese inventory (*BEGCHINV*). Consistent with our initial null hypothesis, the greater the benchmark of estimated consumption, the greater the likelihood of purchasing. Again, this may be reflecting increased rates of stockouts, ceteris paribus. The negative inventory coefficient also supports our initial hypothesis that with larger per capita household inventories, a household is likely to purchase cheese.

The estimated flexibility parameter was found to be close to 1. Such a value implies that households would initially increase consumption as cheese inventory increases from relatively low levels. For higher inventory levels, consumption approaches a maximum. Figure 3 provides a representation of the estimated consumption profile, given the endogenously determined consumption under alternative values of Q_d and inventory levels. At the mean value of Q_d, estimated consumption approaches 2.5 ounces/capita/week, which is similar to estimates obtained from commercial disappearance data.[8]

[8] Putnam and Allshouse estimate that total U.S. per capita cheese disappearance was 26 pounds in 1992. With approximately 35% of this disappearance being cheese consumed directly at home, this implies a per capita consumption of 0.175 pounds/capita/week, or 2.8 ounces/capita/week.

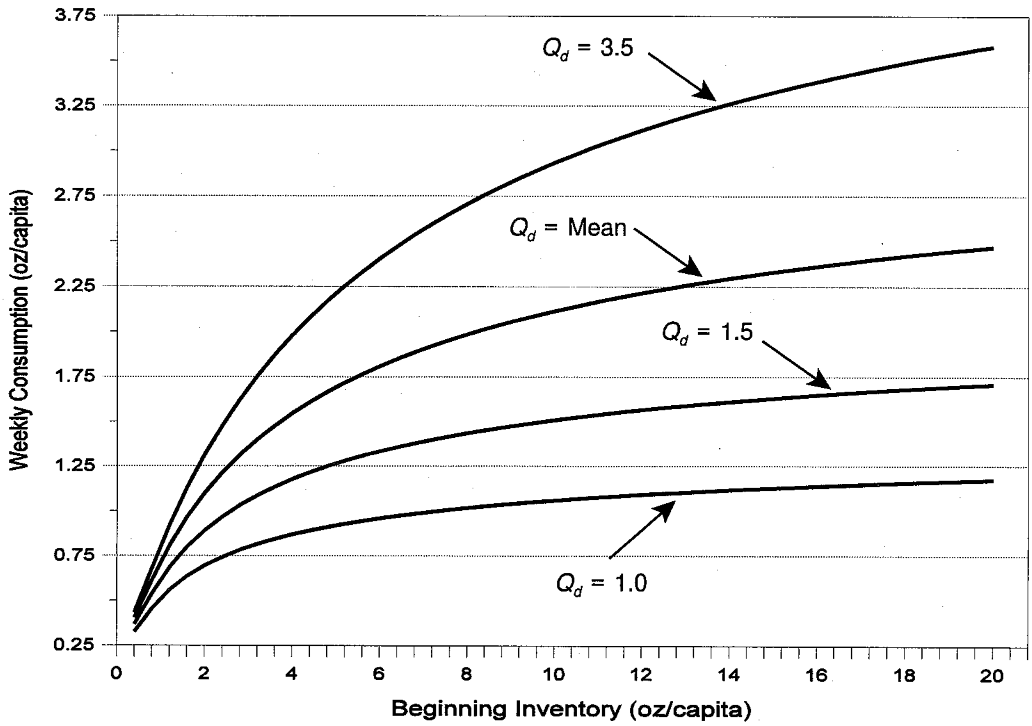


Figure 3. Estimated relationship between beginning inventory, benchmark consumption (Q_d), and weekly consumption (ounces per capita)

Based on estimated parameter values, table 3 contains estimated elasticities of changes in several explanatory variables on conditional and joint purchase probabilities for weeks 8 and 9 of the survey period, assuming no purchase in week 8. As hypothesized, the elasticity of the conditional probability of purchasing in week 9 was negatively related to estimated inventory. The elasticity, although small, was statistically significant. A similar negative relationship was found in the joint probability.

A positive and significant relationship was found between beginning inventory level and the probability of not purchasing in week 9. The household income elasticity was estimated to be positive with respect to the conditional and joint probability of purchasing, and the opposite impact on the probability of not purchasing. The benchmark average consumption level (Q_d) was found to have a significant and positive impact on purchase probability. Significant negative price elasticities were obtained.

Summary

The availability of scanner data has enabled researchers to examine food purchase dynamics. In this study we focus on one component of the purchase process—when to buy, and in this application we are interested in the purchases of a particular type of food—cheese. We quantify the relationship between the discrete purchase decision and a set of household and purchase characteristics using a simulated maximum-likelihood

Table 3. Estimated Probabilities and Impacts of Exogenous Variable Changes for Weeks 8 and 9 of the Survey Period

	PROBABILITY TYPE							
	$P(y_t > 0 \mid y_{t-1} = 0)$ Est'd Prob = 0.362		$P(y_t = 0 \mid y_{t-1} = 0)$ Est'd Prob = 0.638		$P(y_t > 0,\ y_{t-1} = 0)$ Est'd Prob = 0.234		$P(y_t = 0,\ y_{t-1} = 0)$ Est'd Prob = 0.375	
Elasticity	Value	Std. Err.	Value	Std. Err.	Value	Std. Err.	Value	Std. Err.
CH_PRICE	-0.180**	0.0339	0.102**	0.0194	-0.137**	0.0294	0.231**	0.0433
BEGCHINV	-0.052**	0.0033	0.030**	0.0015	-0.040**	0.0052	0.067**	0.0038
HHINC	0.053*	0.0201	-0.030*	0.0117	0.040*	0.0156	-0.068*	0.0264
Q_d	0.207**	0.0151	-0.118**	0.0103	0.157**	0.0188	-0.266**	0.0218

Notes: Single and double asterisks (*) denote significance at the .01 and .001 levels, respectively. Estimates assume no purchase in week 8.

procedure. For the most part, we find results consistent with our initial hypothesis concerning the direction of the impacts of these explanatory variables.

From our analysis, we find evidence of significant persistent unobservable household heterogeneity, which is not eliminated by the inclusion of a variable identifying the purchase in the previous time period. This result is important for cheese sellers as it indicates that some households more consistently purchase cheese than others. The question remains as to whether this purchase heterogeneity exists with respect to purchase quantity. Given the censored nature of these purchases, such an analysis would require a reformulation of the traditional Tobit model into a dynamic setting. Hajivassiliou has developed such a model in the analysis of debt repayment by less developed countries. In the application of this model to consumer expenditures, we could adopt a similar autocorrelated error term structure. The dynamic extension of the traditional Tobit model would allow for an analysis of both purchase timing and quantity. Given the structure of the Tobit model, which results in the sign of the marginal impacts of exogenous variable changes on purchase probability and conditional purchases being the same, we may want to adopt a different model structure that separates purchase timing from the quantity decision. For example, we could adopt a double-hurdle structure such as that given in Cragg, but extend it to our panel application.

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Appendix: A Brief Overview of the Simulation Algorithm

Suppose we need to evaluate the following multivariate distribution function:

$$PR[\mathbf{A} \le \boldsymbol{\mu} \le \mathbf{B}] = \int_{a_M}^{b_M} \int_{a_{M-1}}^{b_{M-1}} \cdots \int_{a_2}^{b_2} \int_{a_1}^{b_1} \varphi(x)\, dx,$$

where $\boldsymbol{\mu} \sim N(\mathbf{0}, \Sigma)$. This probability can be approximated by:

$$\frac{1}{R} \sum_{r=1}^{R} \prod_{i=1}^{M} Q_{ir},$$

where R is the number of replications, and Q_{ir} is the probability of the ith recursive truncated normal for replication r, where:

$$Q_1 = PR[a_1/c_{11} \le \kappa_1 \le b_1/c_{11}],$$
$$Q_2 = PR[(a_2 - c_{21}\kappa_1)/c_{22} \le \kappa_2 \le (b_2 - c_{21}\kappa_1)/c_{22}],$$
$$Q_3 = PR[(a_3 - c_{31}\kappa_1 - c_{32}\kappa_2)/c_{33} \le \kappa_3 \le (b_3 - c_{31}\kappa_1 - c_{32}\kappa_2)/c_{33}],$$
$$\ldots$$
$$Q_M = PR\left[\left(a_M - \sum_{j=1}^{M-1} c_{mj}\kappa_j\right) / c_{MM} \le \kappa_M \le \left(b_M - \sum_{j=1}^{M-1} c_{mj}\kappa_j\right) / c_{MM}\right].$$

From the above, c_{ij} is the ijth element of the Cholesky factorization of Σ, and κ is a univariate truncated normal variate which can be obtained from the following:

$$\kappa = \Phi^{-1}[\Phi(b)\Gamma + (1 - \Gamma)\Phi(a)],$$

where the distribution of Γ is uniform (0, 1), and $\kappa \sim N(0, 1)$ such that $(a \le \kappa \le b)$. This simulator produces smooth unbiased multivariate probability estimates (Breslaw).

The results shown in text table 2 are based on 500 replicates (i.e., $R = 500$). This number of replications was suggested by Hajivassiliou, McFadden, and Ruud. We compared our parameter estimates using R set at 100, 200, and 500. Parameter estimates varied little as we increased the number of replicates. Geweke, Keane, and Runkle (1994); Hajivassiliou, McFadden, and Ruud; and Breslaw have undertaken extensive evaluation of the performance of the probability simulators under a variety of conditions. For this analysis, we obtained the computer code used by Hajivassiliou, McFadden, and Ruud in their 1996 review article.