Optimal Information Acquisition Under a Geostatistical Model

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Studies examining the value of switching to a variable rate technology (VRT) fertilizer program assume producers possess perfect soil nitrate information. In reality, producers estimate soil nitrate levels with soil sampling. The value of switching to a VRT program depends on the quality of the estimates and on how the estimates are used. Larger sample sizes, increased spatial correlation, and decreased variability improve the estimates and increase returns. Fertilizing strictly to the estimated field map fails to account for estimation risk. Returns increase if the soil sample information is used in a Bayesian fashion to update the soil nitrate beliefs in nonsampled sites.

Key words: estimation risk, geostatistics, nitrogen fertilizer, optimal sample sizes, precision farming, single rate technology, variable rate technology, variogram

Introduction

Use of soil test information to better match fertilizer applications with crop nutrient requirements and nutrient availability has long been proposed as a means for improving soil fertility management and environmental quality (Musser et al.; Fleming, Adams, and Ervin; Babcock and Blackmer). It is widely accepted that uncertainty about soil fertility levels leads to increased applications of nitrogen fertilizer. Advances in mapping and sensing technologies have renewed interest in soil testing as a means of moving to variable rate technologies (VRTs) whereby a farmer varies fertilizer applications across space and/or time. Significant research efforts are underway to develop the knowledge and equipment needed to allow farmers to move to variable rate technologies (National Research Council).

Recent studies examining the potential value of switching to a VRT fertilizer program assume producers possess complete information about soil nitrate levels and have the ability to vary fertilizer applications optimally across the field (Babcock and Pautsch; Lowenberg-DeBoer and Boehlje; Sawyer; Snyder et al.; Solohub, van Kessel, and Pennock; Hertz). In reality, farmers using a VRT strategy will sample only a portion of the field rather than the entire field. The soil samples are then used to estimate the soil nitrate levels at the nonsampled sites. These estimates are summarized and presented

to the producer in the form of a soil nitrate field map where isoclines of equal soil nitrate levels are shown to guide fertilizer rates.

A key factor in such a map is the precision with which the nonsampled points are estimated. Precision can be increased with more soil samples, but at a cost. The purpose of this study is to develop a framework for incorporating estimation risk into a decision model to determine the optimal amount of soil test information when applying nitrogen fertilizer under a variable rate program. The optimal sample size is found by equating the marginal cost of sampling with the marginal benefit of sampling. The marginal benefit of soil sampling is the increased expected returns from an additional soil test. The marginal cost of sampling is the additional cost of obtaining a soil sample.

The analysis also estimates the value of switching to a VRT fertilizer program from the conventional single rate technology (SRT) fertilizer program where a farmer applies fertilizer uniformly across the field. These estimates aid producers, equipment manufacturers, input suppliers, and other agribusiness agents struggling with the adoption and implementation of new precision technologies by providing a benchmark for the initial level of investment that should be made in these new technologies and, once adopted, the optimal level of information acquisition. Finally, the overapplication of nitrogen fertilizer, which is potentially harmful to the environment, is shown to be dependent on adopting these new precision farming technologies and on the cost of information acquisition.

While the specific application of this analysis is to determine the optimal amount of soil test information to obtain, the economic questions addressed here are fundamental to gaining a better understanding of how agricultural technologies in general can be brought into the information age. These questions are: (*a*) How much information should be acquired? (*b*) Is it worth the investment to reduce farming uncertainty and move toward farming under variability? and (*c*) How do the amounts of spatial variability and spatial correlations of soil properties affect the optimal level of information acquisition and the return to investment?

Two methods are used to process soil sample information into soil nitrate estimates for all nonsampled points. Each method provides different estimates for the marginal benefits of soil sampling, and thus each differs on the optimal amount of soil nitrate information to acquire. First, the soil nitrate estimates for the nonsampled points are treated as "truth" and directly inserted into optimal fertilizing rules. This approach is called the plug-in method and is most widely used in agricultural studies (Lence and Hayes 1994, 1995). The plug-in method, however, ignores estimation risk and is not consistent with expected utility maximization (Klein et al.). The second approach accounts for estimation risk by using Bayesian decision rules. While this approach is consistent with expected utility maximization (DeGroot), with the exception of a few studies (Chalfant, Collender, and Subramanian; Lence and Hayes 1994, 1995; and Babcock, Carriquiry, and Stern), it has not been widely used in the farm management literature.

The optimal level of information acquisition and the returns from investing in new precision farming technologies will depend heavily on field characteristics that determine the degree of spatial variability and spatial correlation of soil nitrate levels within the field. The Bayesian decision rule is used to determine how increases in soil nitrate variability and the spatial correlation of soil nitrate across a field affect the optimal number of soil samples. Increased variability and decreased correlation would seem to

increase optimal sample size because more samples are needed to make reliable esti-
mates of nitrate. We show that increased soil nitrate variability increases the optimal
number of soil samples, but increased spatial correlation of soil nitrate levels may
increase or decrease the optimal sample size depending on marginal sampling costs.

The Model

The analysis relies on Monte Carlo simulation rather than the sampling and fertilizing
of an actual field. Figure 1(a) presents an example of a simulated field mapped onto a
6 × 6 unit grid, and thus divided into 36 square grid cells. Each square grid cell is
assumed to possess a single soil nitrate level, and the soil nitrate level varies from cell
to cell. The center of each square cell is assumed to be the soil sampling site for that cell.
     To determine the producer returns from a given sample size, X, the following Monte
Carlo experiment is replicated 1,000 times. First, soil nitrate levels are simulated for
each cell of the field through the use of a geostatistical model. Second, the soil samples
are taken at X evenly spaced sites throughout the field. In figure 1(b), the darkened
circles represent sampled sites, so that in this case the sample size is nine. Third, the
soil sample information is used to create an estimated soil nitrate map of the field.
Fourth, the producer fertilizes according to the estimated soil nitrate map and according
to whether or not estimation risk is taken into account. Finally, the results of using a
VRT fertilizer program are averaged over the 1,000 replications. Another sample size
is then selected and the Monte Carlo experiment is repeated over the same 1,000 draws
of possible soil nitrate levels for the entire field. Soil nitrate levels on a site in a field
vary from year to year because of interactions between soil properties and variable
weather events that occur before soils are tested (Babcock and Blackmer).
     The accuracy of the estimated soil nitrate map depends upon the sample size.
Increasing the sample size increases the amount of soil nitrate information collected,
and thus increases the accuracy of the soil nitrate estimates at the nonsampled points.
The marginal benefit of sampling in a VRT fertilizer program is the change in producer
returns divided by the change in the number of samples. The producer returns from an
SRT fertilizer program are also averaged over the same 1,000 draws of possible soil
nitrate levels for the field and compared with the VRT fertilizer program.

Field Data Simulation

The overall soil nitrate mean and variance for the field are denoted, respectively, as μ
and σ^2. The soil nitrate level at site i (x_i) differs from the soil nitrate level at other sites
within the field. The variance of the difference in soil nitrate levels on two sites i and
j equals

$$E\left[(x_i - x_j)^2\right] = 2\left(\mu^2 + \sigma^2 - E\left[x_i x_j\right]\right).$$

A semi-variogram expresses half of this variance as a function of the distance between
the two sites. If the distance between two sites is beyond some critical level (called the
range), then it is assumed that covariance between the sites equals zero. In this case,

Figure 1(a). An example of a simulated field

Figure 1(b). An example of a sampled simulated field

$E[x_i x_j] = \mu^2$, and the semi-variogram equals the overall soil nitrate variability of the field, σ^2. In other words, when the sites are so far apart that the soil nitrate levels are uncorrelated or spatially independent, half the variance between the two sites equals σ^2. The soil nitrate level at one site provides no additional information about the soil nitrate level at the other site.

As the two sites become closer, the variance of the difference in soil nitrate levels between the two sites will decrease. The soil nitrate levels at these two closer sites become more correlated or spatially dependent. That is, the sites are close enough so that the soil nitrate level at one site provides additional information about the soil nitrate level at the other site. The soil nitrate variation between any two sites is assumed to follow a spherical semi-variogram. The spherical model is currently the most commonly used semi-variogram in soil science to measure variability in soil properties (Han et al.). The spherical semi-variogram is given by

$$\gamma(h_{ij}) = C_o + C * \left[\tfrac{3}{2}(h_{ij}/a) - \tfrac{1}{2}(h_{ij}/a)^3 \right] \quad \text{for } 0 \leq h_{ij} < a,$$
$$= C_o + C \qquad\qquad\qquad\qquad \text{for } h_{ij} \geq a,$$

where

$\gamma(h_{ij})$ = half the variance in the difference between soil nitrate levels on any site i and site j,
h_{ij} = distance between site i and site j,
a = range,
C = soil nitrate variability that can be explained spatially, and
C_o = soil nitrate variability that cannot be explained spatially.

The overall soil nitrate variance of the field, $\text{var}(x_i) = \sigma^2$, is called the sill and is denoted as $C_o + C$. This overall variation of soil nitrate levels is assumed to consist of a local random component (C_o) called the nugget effect, and a component (C) called the spatial variance. The nugget effect represents measurement error. It is the soil nitrate variability that occurs when two soil samples are taken from the same site, i.e., the variations in soil nitrate levels when distance between the sites is zero. The spatial variance is the variability in the difference of soil nitrate levels on two sites which is attributable to the distance between those two sites. As the distance between any two sites increases, the variability of soil nitrate levels between those sites also increases. In other words, the spatial variance is the variability in soil nitrate levels that can be explained spatially.

The spatial covariance of nitrate levels within the field is represented by

$$\sigma_{ij}(h_{ij}) = C * \left[1 - \tfrac{3}{2}(h_{ij}/a) + \tfrac{1}{2}(h_{ij}/a)^3 \right] \quad \text{for } 0 \leq h_{ij} < a,$$
$$= 0 \qquad\qquad\qquad\qquad\qquad \text{for } h_{ij} \geq a.$$

The covariance (σ_{ij}) of soil nitrate levels between sites i and j depends on the distance between sites i and j. The soil nitrate levels between adjacent sites are more related than nitrate levels from sites further apart. If the distance between sites i and j is greater than or equal to the range, then the corresponding nitrate levels are uncorrelated, $\sigma_{ij} = 0$. Denote the covariance matrix of the soil nitrate levels as $\Phi = [\sigma_{ij}]$.

Cholesky's factorization of the covariance matrix Φ is denoted as $\mathbf{P}$, where $\mathbf{P}$ is a lower triangular matrix and $\mathbf{PP}' = \Phi$. Denote $\mathbf{x}$ as the column vector containing the soil nitrate levels on each of the sites. Let $\mathbf{x}$ equal $\mathbf{P}\mathbf{z} + \mu\mathbf{i}$, where $\mathbf{z}$ is a column vector drawn randomly from a standard normal distribution, $\mathbf{i}$ is the unit column vector, and μ is a scalar constant. In this manner, the soil nitrate levels occurring in the field before fertilizer application are normally distributed with mean μ and covariance structure Φ.

Soil Sampling and Soil Nitrate Maps

To simplify the analysis, it is assumed that the true underlying process (semi-variogram) which generates the spatial distribution of soil nitrate levels is known when making estimates. This assumption represents a first step in combining geostatistical procedures and precision farming concepts to derive optimal sample sizes. If the semi-variogram is not known, then one must be estimated from the sampled values. When using the Monte Carlo simulation technique, such an endeavor is difficult and very time consuming when performed for each replication. Our assumption of a known semi-variogram causes the absolute value of all soil sample information to be higher than if soil nitrate estimates were derived using an estimated semi-variogram. However, the effect of this assumption on the marginal benefit of sampling is indeterminate.

Suppose n different sites are sampled and the soil sample information is represented by the vector $\mathbf{w} = (w_1, \ldots, w_n)'$, where w_j is equal to the soil nitrate reading at the jth sampled site. The sample is then used to estimate the nitrate levels at nonsampled sites. Since the inherent soil nitrate levels are assumed to be normally distributed, the joint distribution of $(x_i, \mathbf{w}')'$, where x_i is the soil nitrate level at a nonsampled site, is multivariate normal with mean vector $(\mu, \mu\mathbf{i}'_n)$ and covariance matrix

$$
\begin{bmatrix}
C_o + C & \mathrm{cov}(x_i, w_1) & \ldots & \mathrm{cov}(x_i, w_n) \\
\mathrm{cov}(w_1, x_i) & \mathrm{cov}(w_1, w_1) & \ldots & \mathrm{cov}(w_1, w_n) \\
\vdots & \vdots & \ddots & \vdots \\
\mathrm{cov}(w_n, x_i) & \mathrm{cov}(w_n, w_1) & \ldots & \mathrm{cov}(w_n, w_n)
\end{bmatrix}
=
\begin{bmatrix}
C_o + C & \varphi'_i \\
\varphi_i & \varphi
\end{bmatrix},
$$

where φ_i is an $\{n \times 1\}$ matrix and φ is an $\{n \times n\}$ matrix. The conditional distribution of x_i, given the sampled information $\mathbf{w}$, is then normal with mean and variance (Graybill):

(1)
$$
\hat{x}_i = E\big[x_i | \mathbf{w}\big] = \mu + \varphi'_i \varphi^{-1}(\mathbf{w} - \mu\mathbf{i}_n),
$$

(2)
$$
\mathrm{var}\big(x_i | \mathbf{w}\big) = (C_o + C) - \varphi'_i \varphi^{-1} \varphi_i.
$$

The covariance of the ith nonsampled point with each of the n sampled points is represented by φ_i. The covariance of the sampled sites with the other sampled sites is represented by φ, and its inverse is denoted as φ^{-1}. Given the sample information $\mathbf{w}$, the soil nitrate estimate at a nonsampled site, $\hat{x}_i$, is then the mean of the conditional distribution of x_i, and the variance of the estimate is the variance of the conditional distribution of x_i. If none of the sampled points are within the range of the ith nonsampled site,

the covariance between it and all the sampled sites is zero. No additional information on the ith nonsampled site is gained, and the soil nitrate estimate and its corresponding variance become the soil nitrate mean and variance for the overall field.

Decision Model

The production decision is the amount of nitrogen fertilizer to apply given the relationship between soil nitrate concentrations and yield, the available technology (SRT versus VRT), and the producer's information concerning inherent soil nitrate levels. The soil nitrate concentration, measured in parts per million (ppm), represents the available nitrate in the top 12-inch layer of soil. A producer can alter the soil nitrate concentration by applying an amount of nitrogen fertilizer (F) measured in pounds per acre. The soil nitrate concentration after applying fertilizer (N^{AF}) is assumed to be a linear function of the nitrogen found naturally in the soil (x) and the amount of nitrogen fertilizer applied (Babcock, Carriquiry, and Stern). The multiplicative constant k indicates the pounds of fertilizer per acre needed to increase the soil nitrate concentration one ppm: $N^{AF} = x + Fk$.

The existence of a corn yield plateau and an approximately linear response to soil nitrates when nitrates are limiting is supported in the literature (Ackello-Ogutu, Paris, and Williams; Cerrato and Blackmer; Paris; Binford, Blackmer, and Cerrato). A review of linear response plateau (LRP) production function research is found in Jomini. The following LRP production relationship is used, assuming that all other input decisions have been made and are at nonbinding levels:

$$Y_i = Y_p - b\left(N^* - N_i^{AF}\right)I_{\{N_i^{AF}<N^*\}}.$$

For each site i, the indicator variable $I_{\{N_i^{AF}<N^*\}}$ equals one when the nitrogen level after fertilizing is less than the critical level of nitrogen (N^*), and equals zero otherwise. The plateau or maximum corn yield (Y_p) is reached when the soil nitrate concentration after fertilizing is greater than or equal to N^*. When the soil nitrate concentration is less than N^*, the corn yield (Y_i) decreases linearly by a constant level (b) for each ppm less than N^*.

The optimal SRT fertilizer rate is the single rate that when applied to the entire field maximizes the producer's expected profit. The spatial correlation and distribution of inherent soil nitrate levels are known, but information on spatial location is not used in SRT. The SRT nitrogen fertilizer optimization procedure is specified as

(3)
$$\underset{F}{\text{Max}}\ E\left[\pi^{SRT}\right] =$$

$$\underset{F}{\text{Max}}\ E\left[\sum_{i=1}^{n} P_c\left(Y_p - b\left(N^* - (x_i + kF)\right)I_{\{x_i+kF<N^*\}}\right) - P_F F\right],$$

where n is the number of grid cells in the field, P_c is the price of corn, and P_F is the price of nitrogen fertilizer. Since each x_i is normally distributed with mean μ and variance $C_o + C$, equation (3) is rewritten as follows (see appendix for details):

(4)
$$\underset{F}{\text{Max}} \sum_{i=1}^{n} \left[P_c \left\{ Y_p - b\left(N^* - (\mu + kF)\right) G\left(\frac{N^* - (\mu + kF)}{\sqrt{C_o + C}} \right) \right. \right.$$
$$\left. \left. - b\sqrt{C_o + C}\ g\left(\frac{N^* - (\mu + kF)}{\sqrt{C_o + C}} \right) \right\} - P_F F \right],$$

where $g(z) = 1/\sqrt{2\pi}\ e^{-z^2/2}$ is the standard normal probability density function, and $G(z) = \int_{-\infty}^{z} g(u)\ du$ is the corresponding cumulative distribution function. The first-order condition for the optimal SRT fertilizer rate is then

(5)
$$P_c G\left(\frac{N^* - (\mu + kF)}{\sqrt{C_o + C}} \right) bk = P_F,$$

from which the optimal SRT fertilizer rate is determined to be

(6)
$$F = \frac{N^* - \mu}{k} - \frac{\sqrt{C_o + C}}{k}\ G^{-1}\left(\frac{P_F}{P_c bk} \right).$$

The producer fertilizes the entire field such that the expected marginal revenue product of nitrogen fertilizer equals the price of fertilizer. When applying the optimal SRT fertilizer rate across the field, the probability of being short of the critical nitrogen level is equal to the ratio of the price of nitrogen fertilizer to its marginal revenue product. The optimal SRT fertilizer rate equals the overall field mean fertilizer rate plus an additional amount of fertilizer, which changes the probability of being short of nitrogen from 50% to the ratio of the price of nitrogen fertilizer to its marginal revenue product conditional on fertilizer being nonbinding ($P_c bk$).

When using variable rate technology to make fertilizer decisions, the producer possesses a field map of estimated soil nitrate levels. The map is based on the soil samples. Let $\mathbf{w}$ represent the vector of sampled nitrate levels at the sampled sites. The producer's posterior belief regarding the ith site's inherent soil nitrate level is denoted by $h(x_i | \mathbf{w})$. The optimal expected VRT profit for the entire field is the sum of the optimal expected profit from each site. The optimal VRT fertilizer rate for the ith site is the rate that maximizes the producer's expected profit on that site:

(7)
$$\underset{F_i}{\text{Max}}\ E\left[\pi_i^{VRT} \right] = \underset{F_i}{\text{Max}} \int_{-\infty}^{+\infty} \left[P_c \left(Y_p - b\left(N^* - (x_i + kF_i) \right) \right) I_{\{x_i + kF_i < N^*\}} \right.$$
$$\left. - P_F F_i \right] h\left(x_i | \mathbf{w} \right) dx_i.$$

The form of the posterior beliefs about the inherent soil nitrate level depends upon whether the site is a sampled or nonsampled site and whether the producer ignores or accounts for estimation risk. Soil sampling errors are assumed to be zero, so that producers have perfect information about the true soil nitrate level at each sampled site.

The posterior beliefs about the soil nitrate level at a sampled site become a point density function at the sampled value. Given perfect soil nitrate information, current prices of corn and nitrogen, and the marginal product of nitrogen fertilizer, the economically optimal fertilizer response is to raise the soil nitrate level to the physically optimum level N^*. If the producer were deciding whether or not to fertilize, then the optimal fertilizer prescription would also include application costs. In our analysis, the producer has already decided to fertilize; thus application costs are ignored and treated as a fixed cost. At each sampled site, a producer fertilizes in the following fashion:

$$(8) \qquad\qquad F_i = (N^* - x_i)/k \qquad \text{if } 0 < x_i < N^*,$$

$$= 0 \qquad\qquad \text{if } x_i \geq N^*.$$

Producers do not possess perfect information about soil nitrogen levels at nonsampled sites. Instead, producers use estimated soil nitrate levels derived from the sampled sites to make their fertilizer decisions. Nitrogen fertilizer decisions are analyzed under two different assumptions. First, producers ignore estimation risk by directly substituting the estimate for the true unknown level of soil nitrate at each nonsampled site. This method is traditionally referred to as the "plug-in" approach. The posterior density, $h(x_i | \mathbf{w})$, in this case is a point density function at the estimated value $\hat{x}_i = E[x_i | \mathbf{w}]$ [equation (1)] for each nonsampled site. The optimal fertilizer rate is found by replacing the true soil nitrate level x_i with its estimate $\hat{x}_i$ in equation (8).

The second procedure accounts for estimation risk by using a Bayesian approach. The posterior distribution of the true soil nitrate level for nonsampled sites is found by updating prior beliefs using Bayes' theorem. The posterior beliefs are then conditional upon the sampled values at the sampled sites. The posterior density, $h(x_i | \mathbf{w})$, is normal with mean $\hat{x}_i = E[x_i | \mathbf{w}]$ and variance $\text{var}(x_i | \mathbf{w})$, given in equations (1) and (2), respectively.

The variable rate fertilizer program maximization problem expressed in equation (7) can be rewritten as follows [this equivalence can be shown in a manner similar to the equivalence of equation (3) and equation (4)]:

$$(9) \qquad \underset{F_i}{\text{Max}} = \left[P_c \left\{ Y_p - b \left(N^* - (x_i + kF_i) \right) G \left(\frac{N^* - (\hat{x}_i + kF_i)}{\sqrt{\text{var}(x_i | \mathbf{w})}} \right) \right. \right.$$

$$\left. \left. - b\sqrt{\text{var}(x_i | \mathbf{w})} \; g \left(\frac{N^* - (\hat{x}_i + kF_i)}{\sqrt{\text{var}(x_i | \mathbf{w})}} \right) \right\} - P_F F \right],$$

where $g(z)$ is the standard normal probability function, and $G(z)$ is the corresponding cumulative distribution function. The first-order condition for the optimal VRT fertilizer rate is

$$(10) \qquad\qquad P_c G \left(\frac{N^* - (\hat{x}_i + kF_i)}{\sqrt{\text{var}(x_i | \mathbf{w})}} \right) bk = P_F,$$

from which the optimal VRT fertilizer rate at site i is determined to be

$$(11) \qquad F_i = \frac{N^* - \hat{x}_i}{k} - \frac{\sqrt{\text{var}(x_i \mid \mathbf{w})}}{k} \, G^{-1}\left(\frac{P_F}{P_c bk} \right).$$

The producer fertilizes each grid cell such that the expected marginal revenue product of nitrogen fertilizer equals its price. The optimal Bayesian VRT fertilizer rate equates the probability of being short of the critical nitrogen level in a grid cell to the ratio of the price of nitrogen fertilizer to its marginal revenue product ($P_c bk$). The optimal Bayesian VRT fertilizer rate equals the plug-in fertilizer rate plus an additional amount of fertilizer that changes the probability of being short of nitrogen from 50% to the ratio $P_F / P_c bk$. Under both the plug-in method and the Bayesian method, if an estimate other than the mean cannot be made on a site due to the lack of locally sampled sites, then no additional information has been gained and the optimal VRT response is to use the SRT fertilizer rate.

The optimal Bayesian VRT fertilizer rate for each grid cell [equation (11)] is similar in form to the optimal SRT fertilizer rate for the entire field [equation (6)]. For both programs, the expected proportion of nonsampled sites underfertilized equals the ratio of the price of nitrogen fertilizer to its marginal revenue product. In the absence of soil sampling, as in the SRT fertilizer program, the overall field mean and variance are the best estimates for the soil nitrate level and soil nitrate variability at each nonsampled site. The VRT fertilizer program uses sampling information to improve the quality of these estimates. Subsequently, the overall amount of nitrogen fertilizer overapplied and underapplied will be lower under the VRT Bayesian fertilizer program.

Optimal Sample Size

Figure 2 shows the total benefit of sampling (expected returns over fertilizer costs), *TB*, as a linear and then concave function of the sample size. The linear portion for very small sample sizes reflects the possibility of drawing samples from sites that are at least twice the range in distance from each other. In this case, each sampled site provides information about the same number of nonsampled sites, and the sets of nonsampled sites associated with each sampled site are nonoverlapping. The sampled sites are so spread out that information about each nonsampled site is provided by only one sampled site. Each additional sampled site on average will affect returns the same as the previously sampled sites. However, if more than one sample provides information about a nonsampled site, the latter sample provides less information than the previous samples. As the sample size becomes large, each additional sample provides less and less information about the nonsampled points. Hence, expected returns will eventually become a concave function of the sample size. Expected returns are strictly concave if the range of soil nitrate is high enough such that any two sampled points give information about at least one nonsampled point.

Figure 2 shows the expected marginal benefit of sampling, *MB*, to be constant and then decreasing with the number of samples. The marginal cost of sampling is assumed to be constant. The intersection of the marginal benefit with the marginal cost
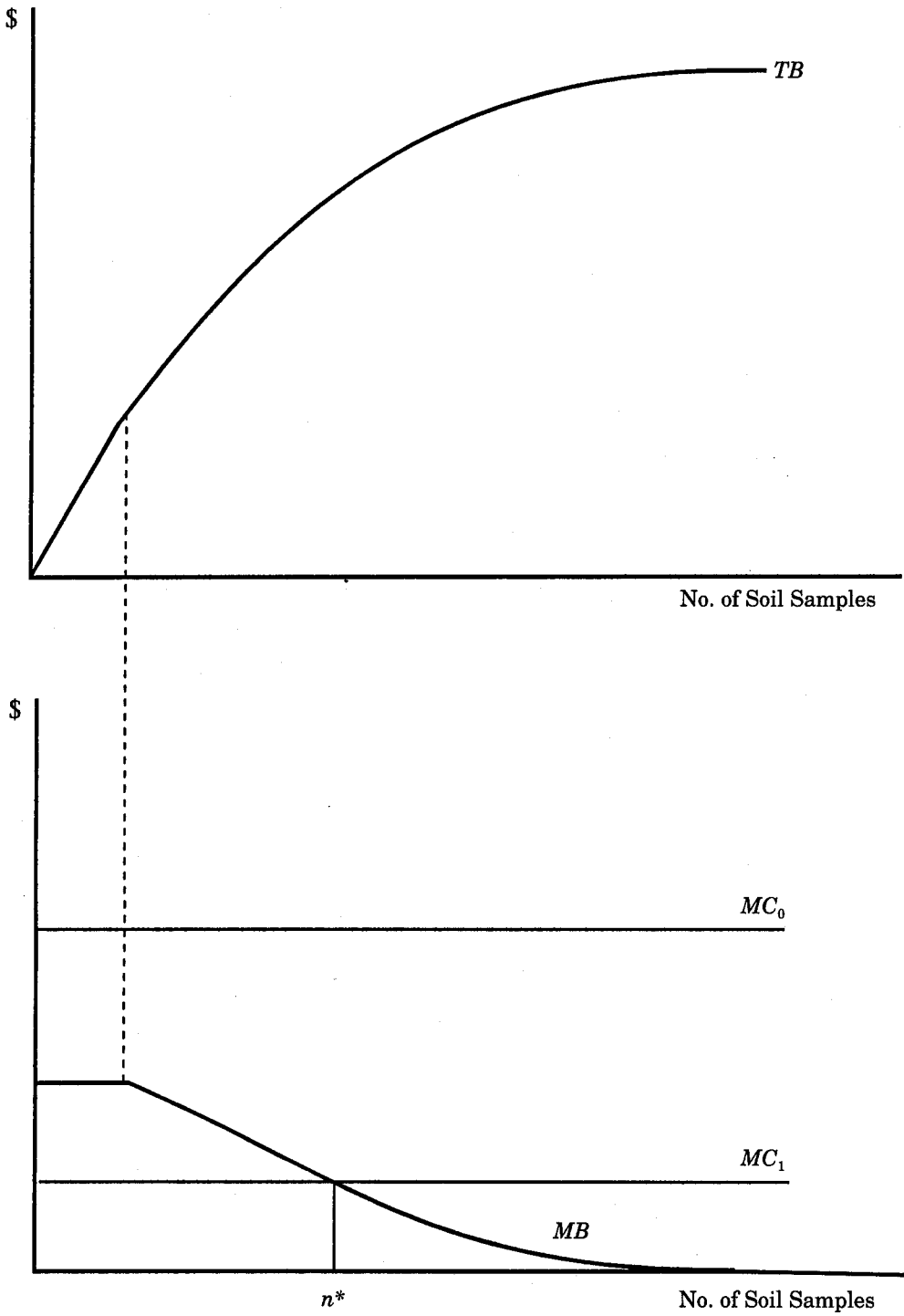
Figure 2. Total and marginal benefits of soil sampling

of sampling determines the optimal sample size. If MC_0 represents the marginal cost of sampling, then the marginal cost of sampling exceeds the marginal benefit at all sample sizes. The optimal producer response is to fertilize the field using a single rate fertilizer program. If the marginal cost of sampling is represented by MC_1, then the optimal producer response is to sample n^* sites and fertilize the field using a variable rate technology program. It is assumed that the cost of investing in the capability of VRT technology has already been made. Otherwise, fixed costs would need to be accounted for in the decision to switch from an SRT fertilizer program to a VRT fertilizer program.

Monte Carlo Experiment

Data and Procedures

In the analysis, a 2,310 × 2,310 foot hypothetical field is mapped onto a 70 × 70 unit grid. The field is then divided into 4,900 square units, each consisting of 0.025 acres. Each square unit, 33 feet long and 33 feet wide, is assumed to possess a homogeneous soil nitrate level. The overall mean and standard deviation for the soil nitrate levels within the field are assumed to be 15 ppm and 5 ppm, respectively. The range of soil nitrate coefficients of variation occurring naturally in Iowa cornfields is estimated to be {0.08, 0.43} (Chin). Our assumed coefficient of variation of 0.33 occurs near the upper end of this interval. Hence, the estimated value of switching to a VRT fertilizer program may be slightly higher than on an average field in Iowa, since greater variability of nitrate levels increases the value of switching to VRT programs (Hennessy and Babcock).

The nugget of the semi-variogram is assumed to be zero. That is, all samples are assumed to be measured without error. The range of the semi-variogram is assumed to be 15 grid cell units (or 495 feet), so that the nitrate level at one point provides some information about the nitrate level at the other points within 15 grid cells. This assumed range is very close to the midpoint of the interval (131 to 900 feet) typically found in precision farming studies of soil nitrate concentrations (Wollenhaupt, Mulla, and Crawford). The range of the semi-variogram provides the spatial covariance structure (Φ) of inherent soil nitrate levels within the field. A Monte Carlo simulation is performed by averaging the results over 1,000 draws on the same field. Each of the 1,000 draws consists of 4,900 correlated soil nitrate values, where each draw is taken from a normal distribution with mean soil nitrate level of 15 ppm, standard deviation of 5 ppm, and covariance structure Φ.

A nonstrategic evenly spaced sampling procedure is used in the analysis. Strategic sampling of a field implies gathering additional field information such as topography, soil type, and drainage properties, and examining how soil nitrate levels vary according to these field characteristics (Pocknee et al.). Since all sites in the field are assumed to possess a common mean, soil nitrate levels are assumed to be invariant to other field characteristics. If different portions of the field possessed different mean soil nitrate levels based upon topography, soil type, or drainage, then a producer would use a strategic rather than nonstrategic sampling procedure.

For simplicity, the nonstrategic evenly spaced sampling procedure used for moderate and large sample sizes was to select points at the intersection of every x_1 rows with

Table 1. Intersection Location of Sampled Sites for Sample Sizes of 25 or Greater

Intersection Location	Number of Samples											
	25	36	49	64	81	100	144	196	324	576	1,225	2,450
Row	14th	12th	10th	9th	8th	7th	6th	5th	4th	3rd	2nd	2nd
Column	14th	12th	10th	9th	8th	7th	6th	5th	4th	3rd	2nd	1st

every x_1 columns in the grid. For example, in Figure 1(b), the darkened circled sites represent the sampled sites, where sample points were selected at the intersection of every 2nd row with every 2nd column. Table 1 presents the sampling procedure under the 70 × 70 unit grid when the sampled points are greater than or equal to 25. For smaller sample sizes, the points were selected to maximize the number of nonsampled sites that could be estimated. The results from four different single sites were averaged and represent the first sample point case. Four sample points were chosen such that none of the sites were within 30 grid cells of each other (the range is 15 grid cells in any direction), i.e., no points overlapped with another. A fifth sampled site was added which partially overlapped the previous four sampled sites. A vast majority of the field could be estimated from only five sample points.

The corn yield plateau (Y_p) is 148.21 bushels per acre, the slope coefficient (b) is 3.95 bushels per ppm, and the critical level of inherent soil nitrate concentration (N^*) is 24.45 ppm (Babcock and Blackmer). To raise the soil nitrate concentration 1 ppm, the producer needs to add 7.63 pounds of nitrogen fertilizer $(k = 1/7.63)$ (Babcock and Blackmer). The price of corn is $2.50 per bushel, and the price of nitrogen fertilizer is $0.20 per pound.

Single Rate Fertilizer Program

The SRT fertilizer rate is the field application rate that maximizes the producer's expected profit given knowledge of the distribution of soil nitrate on a field. This rate can be thought of as the single rate of fertilizer an experienced producer applies to the field. In tables 2 and 3, zero sample points represent a single rate fertilizer program. The SRT fertilizer rate is 110.93 pounds of fertilizer per acre and the average per acre returns over fertilizer costs are $344.34. Under the SRT program, producers overapply nitrogen fertilizer (relative to optimal application rates under perfect information) to ensure against possible yield losses (Babcock; Babcock and Blackmer). Given the values of b, k, P_c, and P_F, $G(\cdot)$ equals 0.1545. Therefore, producers overfertilize 85% of the field and overapply the field with 5,059 pounds of nitrogen fertilizer, relative to optimal applications under perfect information. The average per acre yield of 146.61 bushels is 99% of the maximum potential yield. Only 15% of the field is underfertilized, and only 379 pounds of fertilizer are needed for those areas to reach their optimum yield potential.

Table 2. Production and Environmental Summary for Various Sample Sizes Under the Plug-in Approach

Sampled Points	Average Acre Returns ($)	Std. Dev. Acre Returns ($)	Average Acre Yield (bu.)	Average Acre Fertilizer Rate (lbs.)	Percent of Land Under-fertilized (%)	Pounds of Fertilizer Needed (lbs.)	Percent of Land Over-fertilized (%)	Pounds of Fertilizer Wasted (lbs.)	Percent of Land Properly Fertilized (%)
0	344.34	1.56	146.61	110.93	15	379	85	5,059	0
1	343.35	1.99	145.77	105.40	20	577	80	4,580	0
4	340.36	2.91	143.25	88.80	35	1,174	65	3,143	0
5	339.53	3.20	142.55	84.21	39	1,340	61	2,747	0
25	339.42	2.99	141.56	72.42	50	1,573	50	1,536	1
36	340.72	2.44	142.07	72.25	50	1,454	49	1,395	1
49	342.01	2.19	142.59	72.29	50	1,330	49	1,278	1
64	343.27	1.85	143.10	72.36	49	1,210	49	1,165	2
81	344.21	1.79	143.47	72.33	49	1,121	49	1,074	2
100	344.83	1.63	143.72	72.39	49	1,061	48	1,021	3
144	346.11	1.47	144.24	72.47	48	939	48	908	4
196	346.86	1.45	144.54	72.46	48	868	47	836	5
324	348.26	1.33	145.11	72.55	46	734	46	713	8
576	349.60	1.28	145.65	72.59	44	607	43	591	13
1,225	351.34	1.26	146.35	72.63	37	441	37	430	26
2,450	353.06	1.24	147.04	72.67	25	277	24	271	51
4,900	355.98	1.22	148.21	72.72	0	0	0	0	100

Variable Rate Program—Plug-in Method

Table 2 presents the per acre yields, fertilizer rates, and returns over fertilizer costs for various sample sizes under the plug-in approach. If producers ignore estimation risk and use a sample size of less than 100 to generate the soil nitrate map, then they are better off using the SRT fertilizer program than a VRT program. Returns decline because the producer uses a suboptimal decision-making process by treating the soil nitrate esti- mates as completely accurate. This process is equivalent to assuming that the producer no longer overfertilizes to ensure against yield losses. The percentage of acreage over- fertilized and the amount of nitrogen fertilizer overapplied decline. Yields decline by as much as 5.05 bushels per acre as the acreage underfertilized and the amount of nitrogen fertilizer needed to reach maximum yield potential both increase.

Soil nitrate estimates can be generated for every grid cell in the field when the sample size is greater than or equal to 25. In these cases, half of the soil nitrate estimates over- estimate the true soil nitrate level, leaving 50% of the grid cells undersupplied with nitrogen. Similarly, half of the soil nitrate estimates underestimate the true soil nitrate level, leaving 50% of the grid cells oversupplied with nitrogen. The amount of fertilizer overapplied in parts of the field is very close to the amount of fertilizer needed in other parts of the field. Hence, the average fertilizer rate is fairly constant regardless of the amount of information acquired. The misapplication of fertilizer, however, decreases as the sample size increases, since better estimates are being generated from increased soil nitrate information. Reducing the misapplication of fertilizer increases both yields and returns. However, for yields to equal the SRT level, approximately half of the grid cells (2,450) would need to be sampled. The misapplication of fertilizer is completely eliminated and yields reach their maximum potential when the producer has perfect information by sampling all 4,900 grid cells.

The plug-in approach, despite its suboptimal nature, is often prescribed in the agri- cultural economics literature (Swinton and Jones). Producers are typically directed to fertilize so that the average soil nitrate level reaches its critical level. Fertilizer prescriptions are usually equal to the amount of fertilizer needed to raise the average soil nitrate estimate to the critical level of nitrogen.

Variable Rate Program—Bayesian Method

Table 3 presents the per acre yields, fertilizer rates, and returns over fertilizer costs for various sample sizes under the Bayesian approach. The Bayesian approach assumes that producers account for estimation risk. After each sample, producers improve or update their beliefs about the mean and variance of soil nitrate levels. The addi- tional information reduces the amount of misapplication of nitrogen fertilizer—both the amount of fertilizer needed and the amount of fertilizer overapplied. Regardless of the sample size, a variable rate program using the Bayesian approach always produces higher yields, higher returns, and less overfertilization than an SRT fertilizer program.

Table 3 shows for many of the sample sizes that the land underfertilized is approxi- mately 15%. With a VRT program, the first-order condition for the optimal fertilizer rate given the updated beliefs is given in equation (9), where $G(\cdot)$ represents the probability

Table 3. Production and Environmental Summary for Various Sample Sizes Under the Bayesian Approach

Sampled Points	Average Acre Returns ($)	Std. Dev. Acre Returns ($)	Average Acre Yield (bu.)	Average Acre Fertilizer Rate (lbs.)	Percent of Land Under-fertilized (%)	Pounds of Fertilizer Needed (lbs.)	Percent of Land Over-fertilized (%)	Pounds of Fertilizer Wasted (lbs.)	Percent of Land Properly Fertilized (%)
0	344.34	1.56	146.61	110.93	15	379	85	5,059	0
1	344.42	1.56	146.62	110.63	15	376	85	5,020	0
4	344.68	1.54	146.65	109.74	15	369	84	4,904	0
5	344.76	1.53	146.66	109.51	15	366	84	4,872	0
25	346.13	1.41	146.87	105.18	15	318	84	4,294	1
36	346.93	1.37	146.97	102.48	15	294	84	3,939	1
49	347.69	1.31	147.08	100.00	15	268	84	3,610	1
64	348.44	1.28	147.18	97.59	15	244	83	3,290	1
81	349.01	1.27	147.25	95.63	15	227	83	3,033	2
100	349.37	1.26	147.31	94.49	15	215	83	2,882	2
144	350.12	1.26	147.41	92.04	15	191	82	2,557	3
196	350.57	1.25	147.47	90.49	15	178	81	2,355	4
324	351.40	1.25	147.58	87.79	15	153	78	1,998	7
576	352.19	1.24	147.69	85.18	14	129	74	1,654	12
1,225	353.23	1.24	147.83	81.74	12	96	63	1,201	25
2,450	354.25	1.23	147.97	78.39	8	61	42	756	50
4,900	355.98	1.22	148.21	72.72	0	0	0	0	100

that the soil nitrate level after fertilizing is less than the critical level of nitrogen (N^*), or equivalently that yield is less than the maximum potential yield. As mentioned, given the values of b, k, P_c, and P_F, $G(\cdot)$ equals 0.1545. Therefore, each nonsampled grid in a field has a probability of 15.45% of being underfertilized and a probability of 84.55% of being overfertilized. Hence, approximately 15% of the land that is not properly ferti- lized will be underfertilized.

Comparing tables 2 and 3 reveals that VRT per acre returns over fertilizer costs are always higher with the Bayesian approach than with the plug-in approach. The Bayesian approach deals with estimation risk in a manner that is consistent with expected profit maximization; the plug-in approach is easier to implement, but it is not consistent with expected profit maximization (Lence and Hayes 1994, 1995). Producers using a VRT fertilizer program that strictly fertilizes according to an estimated map (plug-in approach) are using a suboptimal decision-making process.

Variability of SRT and VRT Returns

Tables 2 and 3 also present the standard deviation of per acre returns due to variations in soil nitrate. This measure reflects the variability of producer returns when using the SRT and VRT fertilizer programs. Under an SRT fertilizer program, the variability of producer returns is very low at $1.56 per acre. The overfertilization incurred under the SRT program has a stabilizing effect on returns by reducing the risks of yield losses. Under a Bayesian VRT fertilizer program, the variability of producer returns declines even further as the sample size increases. The increased soil nitrate information leads to better mapping accuracy and better decision making, reducing the variability of returns and overfertilization. Under a plug-in VRT fertilizer program, the increased information is used suboptimally, leading to suboptimal decision making and increasing the variability of returns. Eventually, enough information is acquired (and used sub- optimally) to reduce the variability of returns below the SRT level.

Marginal Benefit and Cost of Sampling

Table 4 presents the VRT marginal production benefits and an indication of the environ- mental benefits from sampling. Under the plug-in approach, the marginal returns are first negative, then increase to $14.48, and subsequently decline. The indicator of marginal environmental benefit is very large at first, 479.02 pounds of fertilizer for the field, and then declines to 0.11 pounds of fertilizer. The large environmental benefit and large reduction in returns with very small sample sizes result as producers are no longer overfertilizing to ensure against yield losses. Instead, producers are treating imperfect soil nitrate maps as truth and, as a result, are suffering from yield losses. If the marginal cost of sampling and other additional VRT costs exceed $4.02 per sample, producers are better off with an SRT fertilizer program than a VRT program that fertilizes strictly to an estimated map.

Under the Bayesian approach, marginal returns over fertilizer costs and marginal environmental benefits decline as the sample size increases. If the marginal cost of sampling and other additional VRT costs exceed $10.30 per sample, the profit from an

Table 4. Marginal Production Benefits and an Indication of the Environmental Benefits from Sampling

Sampled Points	Plug-in Approach		Bayesian Approach	
	Marginal Returns ($)	Marginal Environmental Benefit (lbs. of fertilizer)	Marginal Returns ($)	Marginal Environmental Benefit (lbs. of fertilizer)
0	—	—	—	—
1	-121.74	479.02	10.30	38.66
4	-121.74	479.02	10.30	38.66
5	-102.31	395.87	10.27	32.26
25	-24.11	60.56	8.61	28.93
36	14.48	12.76	8.49	32.21
49	12.18	9.05	7.21	25.29
64	10.27	7.49	6.09	21.35
81	6.79	5.38	4.12	15.10
100	4.02	2.80	2.33	7.98
144	3.56	2.56	2.08	7.37
196	1.76	1.38	1.07	3.89
324	1.34	0.97	0.79	2.79
576	0.65	0.48	0.38	1.36
1,225	0.33	0.25	0.20	0.70
2,450	0.17	0.13	0.10	0.36
4,900	0.15	0.11	0.09	0.31

Table 5. Spatial Correlation Coefficients for Various Values of the Range

Range (grids)	Distances Between Sites					
	FEET: 0	31.75	63.50	158.75	317.50	476.25
	GRIDS: 0	1	2	5	10	15
1	1.00	0.00	0.00	0.00	0.00	0.00
15	1.00	0.90	0.80	0.52	0.15	0.00
99	1.00	0.98	0.97	0.92	0.85	0.77

SRT fertilizer program exceeds that of a VRT fertilizer program. The marginal environmental benefit is also quite low. The first four sample points each reduce overfertilization in the field by 38.66 pounds (or 154.64 pounds total).

The marginal cost of obtaining a soil nitrate sample is approximately $9 per sample (Lowenberg-DeBoer and Swinton). Hence, a variable rate fertilizer program using the Bayesian approach appears to be feasible for only very small sample sizes, i.e., five or fewer sample points, or sampling approximately 0.1% of the possible points in the field. However, other costs of moving to variable rate technology should be included, such as new fertilizer spreaders, computer hardware and software, global positioning systems, and any additional labor costs. This suggests that soil sampling to guide nitrogen fertilizer rates in a variable rate technology program is not likely to be financially feasible unless soil sampling costs are greatly reduced.

Effect of Variability and Correlation on Optimal Sample Size

In this section, we examine how changing the spatial correlation and variability of soil nitrate levels within a field affects the marginal benefits from sampling and the optimal sample size. Marginal costs are assumed to remain constant. The Bayesian method, not the plug-in method, of using estimated soil nitrate mappings is highlighted, since it is consistent with expected profit maximization.

Spatial Correlation

Changing the range in the spherical semi-variogram alters the spatial correlation of soil nitrate levels. The spatial correlation coefficient of soil nitrate levels for a spherical semi-variogram is

$$\rho(h_{ij}) = \frac{C * \left[1 - \frac{3}{2}(h_{ij}/a) + \frac{1}{2}(h_{ij}/a)^3\right]}{C_o + C} \qquad \text{for } 0 \le h_{ij} < a,$$

$$\qquad\qquad = 0 \qquad\qquad\qquad\qquad\qquad \text{for } h_{ij} \ge a.$$

Table 5 shows that increasing the range increases the spatial correlation of soil nitrate readings. If the range is one grid unit, then all the soil nitrate levels in the field are uncorrelated. Sampling at a site provides information only about that site. On the other hand, if the range is 99 grid units, then sampling at one site provides some information about the nitrate levels at all the other sites in the field. The previous analysis assumed the range was 15 grids. For example, the spatial correlation coefficient for sites five grids (or 158.75 feet) apart is 0.52 when the range is 15 grids, and 0.92 when the range is 99 grids.

To see how an increase in spatial correlation affects the marginal value of soil sampling, note first that the range does not affect either the optimal SRT fertilizer rate or the value of fertilizing according to the SRT rule because a does not appear in either equations (4) or (6). Next note that the value of fertilizing according to VRT under
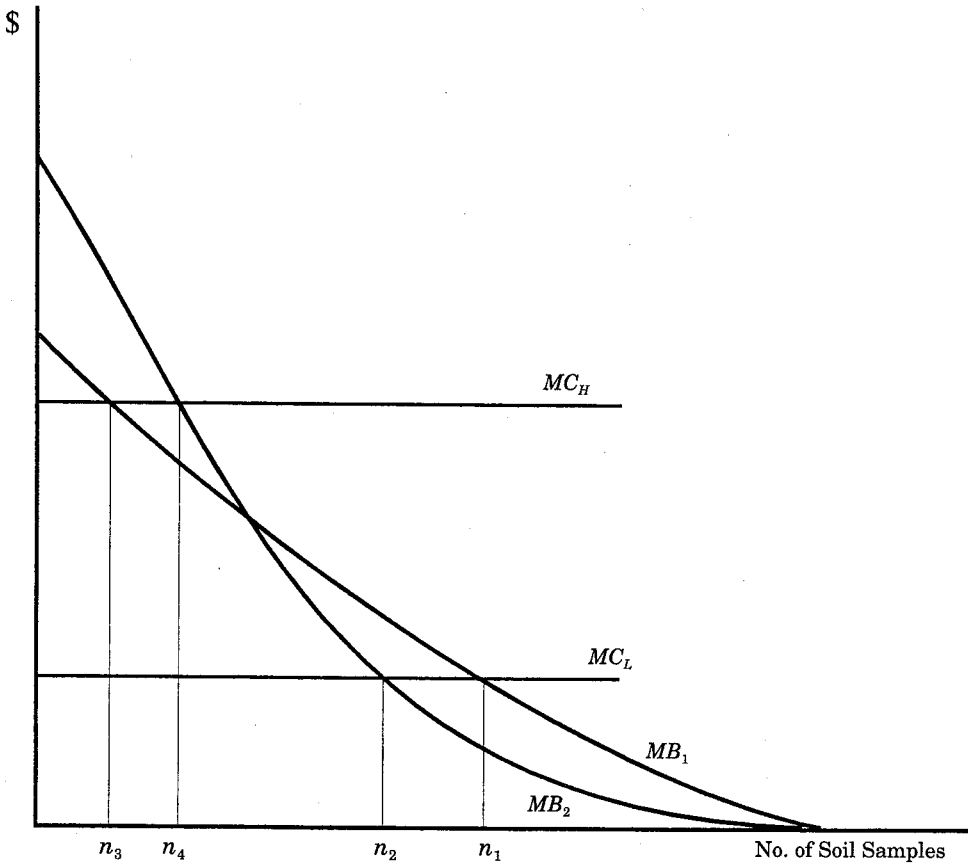
Figure 3. Effect of increased correlation on optimal sample size for different marginal costs of soil sampling

perfect information is not affected by spatial correlation. Under perfect information, a does not appear in equations (9) or (11) because $\text{var}(x_i | \mathbf{w}) = 0$. Thus the value of moving to VRT under perfect information is unaffected by an increase in spatial correlation. That is, the maximum value that can be obtained from soil sampling in a field is the same regardless of the degree of spatial correlation.

This result does not imply that the marginal benefit curves of VRT are unaffected by spatial correlation. But, because the area under a marginal benefit curve equals the value of perfect information, we know that the area under two marginal benefit curves which differ only with respect to spatial correlation must be equal.

Figure 3 shows the implication of this result. An increase in spatial correlation rotates marginal benefits from MB_1 to MB_2. An increase in correlation increases the marginal benefit when sample size is low because each sample point reveals more information about adjoining nonsampled points. The two curves must cross at least once, however, because the areas under MB_1 and MB_2 are equal. That is, there is a finite amount of value that can be obtained from soil sampling.

When marginal costs are relatively low (such as MC_L in figure 3), an increase in spatial correlation reduces the optimal sample size from n_1 to n_2 because marginal benefits at this high optimal sample rate decrease. This decrease in marginal benefit is a result of the increase in prediction capability of all previous sample points. That is, there is a finite amount of information to be obtained, and with a higher degree of spatial correlation, a greater proportion of this information is revealed by previously sampled points. However, when marginal cost is high (such as MC_H), then an increase in correlation increases the optimal sample size from n_3 to n_4. Thus, whether an increase in correlation increases or decreases optimal sample size depends on the level of marginal cost.

Spatial Variability

Increased variability in a field increases the potential gain from moving to variable fertilizer applications. To see this, note first from equation (6) that increased variability (C) increases the optimal single rate of fertilizer application. Thus the potential amount of fertilizer saved as one moves to variable applications increases with C. This implies that the total area under the marginal benefit curve of soil samples increases with increased spatial variability. If increased variability results in an upward shift in marginal benefits for all sample sizes, then increased variability increases the optimal number of soil samples. However, if increased variability results in a crossing of marginal benefit curves, then the optimal sample size may increase or decrease depending on the level of marginal cost, as was the result under increased spatial correlation.

Figure 4 presents expected marginal benefits for three levels of soil nitrate variability (0.16, 0.33, and 0.5) at nine soil sample levels using the Bayesian decision rule. In this range of variability and for these sample sizes, it is apparent that marginal benefits increase with increased variability. The marginal benefit of each sample increases, since each sample provides more information. The size of the increase in marginal benefits is initially quite large and then decreases rapidly as the sample size increases. Thus, given an interior solution, the optimal sample size increases with increased variability.

Figure 4 also shows that increased variability increases the likelihood of an interior solution, which will also result in increased optimal sample size. Suppose the cost of a soil sample is $6. When the coefficient of variation of nitrate is 0.16, the optimal sample size is 0. That is, marginal benefits are never greater than marginal costs. Increasing variability to 0.33 creates an interior solution and the optimal sample size increases to between 25 and 36 samples. This simply reflects that there is a critical amount of variability that must exist before moving to a variable-rate application method becomes economically feasible.

Conclusions

Studies examining the value of switching to a VRT fertilizer program assume the producer possesses perfect soil nitrate information (Babcock and Pautsch; Lowenberg-DeBoer and Boehlje; Sawyer; Snyder et al.; Solohub, van Kessel, and Pennock; Hertz).
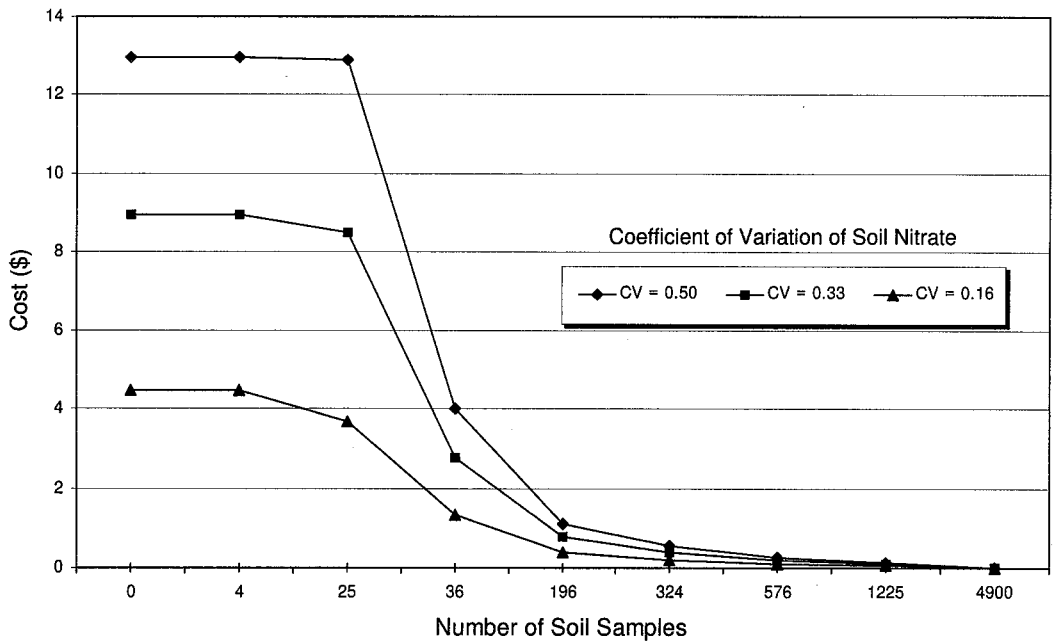
Figure 4. Effect of increasing variability on marginal benefits of soil sampling

In reality, producers estimate soil nitrate levels with soil sampling. The value of switching to a VRT program from an SRT program depends greatly on how the producer uses the estimates and on the precision of the estimates at nonsampled points.

Producers failing to account for estimation risk by strictly fertilizing to the estimated soil nitrate map are not following a VRT strategy consistent with expected profit maximization. Despite the inconsistencies, this strategy has been used in the precision farming literature (Swinton and Jones). To be consistent with expected profit maximization, producers should acknowledge that the soil nitrate mapping is a collection of estimates and does not provide perfect information at nonsampled sites. The soil sample information should be used in a Bayesian fashion to fine-tune or update the producer's beliefs about the soil nitrate levels in nonsampled sites.

The accuracy of the soil nitrate estimates depends on the sample size as well as the degree of spatial correlation and variability among nitrate levels within the field. Larger sample sizes, increased spatial correlation, and decreased variability improve the accuracy of the estimates and increase producer returns.

The marginal benefit of sampling increases for smaller sample sizes when there is a high degree of spatial correlation among nitrate levels. A few sampled sites are able to provide better information for a larger proportion of the field when the degree of correlation is high. Since the marginal cost of soil sampling is substantial, switching to a VRT fertilizer program appears to be more plausible for fields with a high degree of spatial correlation.

The marginal benefit of sampling increases for all sample sizes when there is greater variability in soil nitrate levels. The optimal sample size increases under a VRT

fertilizer program. However, expected per acre returns decline under both SRT and VRT fertilizer programs due to the increased uncertainty surrounding soil nitrate levels. Switching to a VRT fertilizer program from an SRT fertilizer program appears to be more plausible for fields with greater soil nitrate variability.

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Appendix

The equivalence of text equation (3) and equation (4) is outlined below. Equation (3) is restated as

$$E[\pi^{SRT}] = E\left[\sum_{i=1}^{n} P_c\left(Y_p - b\left(N^* - (x_i + kF)\right)I_{\{x_i+kF\leq N^*\}}\right) - P_F F\right].$$

Rearranging terms yields

$$E[\pi^{SRT}] = \sum_{i=1}^{n}\left[P_c Y_p - P_F F\right] - P_c b(N^* - kF)\sum_{i=1}^{n} E\left[I_{\{x_i \leq N^* - kF\}}\right]$$
$$+ P_c b \sum_{i=1}^{n} E\left[x_i I_{\{x_i \leq N^* - kF\}}\right].$$

Since the soil nitrate level at site i (x_i) is normally distributed with mean μ and variance $C_o + C$, the expected value of the indicator variable at the ith site will be equal to the standard cumulative normal distribution at the critical nitrogen value $N^* - kF$:

(A1)
$$E[\pi^{SRT}] = \sum_{i=1}^{n}\left[P_c Y_p - P_F F\right] - P_c b(N^* - kF)\sum_{i=1}^{n} G\left(\frac{N^* - Fk - \mu}{\sqrt{C_o + C}}\right)$$
$$+ P_c b \sum_{i=1}^{n} E\left[x_i I_{\{x_i \leq N^* - kF\}}\right].$$

The expected value of the soil nitrate level at the ith site multiplied by its indicator variable is rewritten as

$$E\left[x_i I_{\{x_i \le N^* - kF\}}\right] = E\left[(x_i - \mu)I_{x \le Z}\right] + \mu E\left[I_{x_i \le Z}\right], \quad \text{where } Z = N^* - Fk$$

$$= \sqrt{C_o + C} \int_{-\infty}^{Z} \left(\frac{x_i - \mu}{\sqrt{C_o + C}}\right)\left(\frac{1}{\sqrt{2\pi}}\right) \exp\left[\frac{-(x - \mu)^2}{2(C_o + C)}\right] \frac{dx_i}{\sqrt{C_o + C}}$$

$$+ \mu G\left(\frac{Z - \mu}{\sqrt{C_o + C}}\right).$$

Substituting

$$v = \frac{x_i - \mu}{\sqrt{C_o + C}} \quad \text{and} \quad dv = \frac{dx_i}{\sqrt{C_o + C}}$$

into the integral and integrating yields

(A2) $$E\left[x_i I_{\{x_i \le N^* - kF\}}\right] = \frac{-\sqrt{C_o + C}}{\sqrt{2\pi}} \exp\left[\frac{-(N^* - Fk - \mu)^2}{2(C_o + C)}\right] + \mu G\left(\frac{N^* - Fk - \mu}{\sqrt{C_o + C}}\right).$$

Substituting equation (A2) into the expression for the expected profit under an SRT fertilizer program, equation (A1), and rearranging terms yields

$$E[\pi^{SRT}] = \sum_{i=1}^{n}\left[P_c Y_p - P_F F\right] - P_c b(N^* - kF - \mu)\sum_{i=1}^{n} G\left(\frac{N^* - Fk - \mu}{\sqrt{C_o + C}}\right)$$

$$+ P_c b \sum_{i=1}^{n} \frac{-\sqrt{C_o + C}}{\sqrt{2\pi}} \exp\left[\frac{-(N^* - Fk - \mu)^2}{2(C_o + C)}\right].$$

Since the soil nitrate level at site i (x_i) is normally distributed with mean μ and variance $C_o + C$, the expected profit under an SRT fertilizer program can be rewritten using the standard normal density:

$$E[\pi^{SRT}] = \sum_{i=1}^{n}\left[P_c Y_p - P_F F\right] - P_c b(N^* - kF - \mu)\sum_{i=1}^{n} G\left(\frac{N^* - Fk - \mu}{\sqrt{C_o + C}}\right)$$

$$- P_c b\sqrt{C_o + C} \sum_{i=1}^{n} g\left(\frac{-(N^* - Fk - \mu)^2}{2(C_o + C)}\right).$$

Finally, rearranging terms yields equation (4) of the text:

$$E[\pi^{SRT}] = \sum_{i=1}^{n}\left[P_c\left\{Y_p - b\left(N^* - (\mu + kF)\right)G\left(\frac{N^* - (\mu + kF)}{\sqrt{C_o + C}}\right)\right.\right.$$

$$\left.\left. - b\sqrt{C_o + C}\, g\left(\frac{N^* - (\mu + kF)}{\sqrt{C_o + C}}\right)\right\} - P_F F\right].$$