The Overshooting Hypothesis of Agricultural Prices: The Role of Asset Substitutability

Ching-chong Lai, Shih-wen Hu, and Chih-ping Fan

By allowing for various degrees of asset substitutability between bonds and agricultural products, this paper reexamines the robustness of the overshooting hypothesis of agricultural product prices. It is found, in both a closed economy and an open economy, that the crucial factor determining whether agricultural prices overshoot or undershoot their long-run response following an expansion in the money stock depends upon the extent of asset substitutability between bonds and agricultural goods.

Key words: asset substitutability, commodity prices, overshooting

Introduction

There is a body of empirical literature dealing with macroeconomic impacts on the agricultural sector, much of which has focused on the way agricultural prices adjust to an expansion in the money supply. Relevant studies include Bordo (1980), Bessler (1984), Devadoss and Meyers (1987), Stamoulis and Rausser (1988), Taylor and Spriggs (1989), Lapp (1990), and Choe and Koo (1993). These studies generally conclude that, as the monetary expansion takes place, the speed of adjustment in relation to agricultural products will be greater than that for manufactured products. Bordo (1980) attributes this to the fact that agricultural goods are tradable in auction markets, and manufactured goods are characterized by fixed-price contracts. Devadoss and Meyers (1987) account for this finding based on the elasticities of supply and demand in the agricultural market being relatively low.

In his frequently cited paper, Frankel (1986) develops a theoretical framework embodying this strand of empirical observation to analyze the dynamic adjustment of agricultural prices. Frankel assumes there are two types of goods: agricultural products (or "auction" goods) and manufactured products (or "customer" goods) in the domestic economy. These two commodities have different price adjustment mechanisms, i.e., manufactured product prices adjust sluggishly, while agricultural product prices adjust instantaneously. Given that residents treat both bonds and agricultural goods as perfectly substitutable assets, Frankel concludes the short-run movement of agricultural commodity prices will overshoot their long-run response following an expansion in the money stock.

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Frankel's theoretical finding of agricultural-price overshooting gains support from a number of empirical studies (e.g., Bordo, 1980; Devadoss and Meyers, 1987; Stamoulis and Raussel, 1988; Orden and Fackler, 1989; Saghaian, Reed, and Marchant, 2002). However, other empirical studies reject the prediction of the overshooting hypothesis of agricultural prices. Robertson and Orden (1990) and Belongia (1991) point out that agricultural prices initially respond by a smaller percentage than the level of the money stock, and the economy is characterized by undershooting in relation to agricultural prices.

The conflicting empirical findings have led to attempts by economists to reconcile the controversy concerning whether agricultural prices overshoot their long-run value. In the literature, Lai, Hu, and Wang (1996)—based on the findings of Frankel and Hardouvelis (1985) and Barnhart (1989) that the announced changes in money supply are crucial in determining the evolution of commodity prices—have formulated a theoretical model to challenge the validity of overshooting in agricultural product prices. Lai, Hu, and Wang report that agricultural prices may undershoot their stationary value if the economy experiences an anticipated monetary expansion rather than an unanticipated monetary expansion. Isaac and Rapach (1997) assert that the conflicting results will be resolved when sample periods are updated. In this paper, we propose an alternative approach, namely, the extent of asset substitutability between bonds and agricultural products, to explain the conflicting empirical observations.

Perhaps for operational convenience, Frankel (1986), and Lai, Hu, and Wang (1996) make a special assumption in their analyses: the public treats both bonds and agricultural goods as perfectly substitutable assets. As is evident, the degree of asset substitutability is determined by the agents' subjective preference over the relevant assets, and the assumption of perfect substitutability is somewhat restrictive. Such a specification is inconsistent with the argument of Van Duyne (1979) and Chambers (1984), in which all assets including agricultural products are specified to be imperfect rather than perfect substitutes in the asset holders' portfolios. In addition, In and Mount's (1994) empirical observation reveals that the reward on domestic bonds has a negative impact on the demand for agricultural goods, but the linkage is not significant at the 5% confidence level, implying the degree of asset substitutability between bonds and agricultural goods is very low. In view of these facts, the primary objective of this paper is to extend Frankel's (1986) model to allow for various degrees of asset substitutability between bonds and agricultural goods, and use it to highlight the role of asset substitutability in determining the validity of the overshooting hypothesis in agricultural prices.

International transactions have undoubtedly become increasingly important to every country in the global marketplace. Within the literature, Rausser et al. (1986, p. 399) note that "the rapid expansion of the international market, the emergence of a well-integrated international capital market, and the decreasing barriers between the agricultural economy and other domestic economic sectors have resulted in significant changes in the agricultural sector." As pointed out by Taylor and Spriggs (1989, p. 279), "Just recently Canadian agricultural policymakers have begun to focus on an international source of price instability through the current GATT negotiations." In their empirical studies, Rausser et al. (1986) and Taylor and Spriggs (1989) also conclude that

1 Specific results are reported in In and Mount (1994, table 6.4A).
in an open economy, macro variables are the significant factors influencing agricultural product prices. Based on their empirical observations, an open-economy model is a more plausible and convincing framework when dealing with the dynamic adjustment of agricultural product prices.

It is somewhat intriguing that very few theoretical frameworks have been put forth to analyze how the dynamic patterns of agricultural product prices are related to the openness of the economy. To our knowledge, the study by Saghaian, Reed, and Marchant (2002) is the only exception. In their analysis, an open-economy model is developed under a flexible exchange regime. Their findings indicate the overshooting of agricultural product prices is more likely to be present when the economy experiences a monetary shock. In the Saghaian, Reed, and Marchant model, however, domestic residents do not hold agricultural goods as assets, implying that the degree of asset substitutability between bonds and agricultural products is zero. This specification runs counter to the assumptions presented by Frankel (1986), and Lai, Hu, and Wang (1996). Consequently, in this paper we attempt to set up an open-economy model under flexible exchange rates, and use it to shed light in an open economy on whether the extent of asset substitutability between bonds and agricultural goods will govern the transitional adjustment of agricultural product prices.

The remainder of the paper proceeds as follows. The second section presents an analytical framework in the context of a closed economy. Then, the nature of the steady-state equilibrium is discussed briefly and the dynamic behavior of the economy associated with an expansion in the money stock is analyzed. The next section extends the closed-economy model developed in the second section to an open-economy model, and then calibrates the model. The main findings of the analysis are summarized in the final section.

A Closed-Economy Model

The framework we develop here may be regarded as a modified version of the Frankel (1986), and Lai, Hu, and Wang (1996) model. As in these earlier studies, the present analysis is based on a number of simplifying assumptions: (a) two types of goods—agricultural products (or “auction” products) and manufactured products (or “customer” products)—are produced in the economy; (b) domestic residents hold three assets: money, bonds, and agricultural goods; (c) the prices of manufactured products adjust subject to a time lag, and not instantaneously; and (d) market participants form their expectations with perfect foresight.

In accordance with the above description of the economy, the model can be described by the following log-linear relationship:

\[ \dot{p}_m = \pi [\delta (p_e - p_m) - \sigma (i - \dot{p}_m)], \quad \pi > 0, \ \delta > 0, \ \sigma > 0; \]
\[ m - p = -\lambda i + \phi y, \quad \lambda > 0, \ \phi > 0; \]
\[ \beta (p_e - c + \rho - i) - \theta (p_e - p_m) = x (p_e - p_m), \quad \beta > 0, \ \theta > 0, \ x > 0; \]
\[ p = \alpha p_m + (1 - \alpha) p_e, \quad 1 > \alpha > 0. \]
With the exception of the nominal interest rate $i$, storage costs $c$, and convenience yield $\rho$, all variables are expressed in logarithms. The variables are defined as follows: $p_m =$ price of manufactured (customer) products, $p_a =$ price of agricultural (auction) products, $m =$ nominal money supply, $p =$ general price, and $y =$ total output. In addition, $\pi$ denotes the speed of adjustment of manufactured prices, while an overdot denotes the rate of change with respect to time.

Equation (1) shows that the price of manufactured products adjusts sluggishly to excess demand in the manufactured product market. The assumption of sluggish manufactured price adjustment is not only popularly adopted in the already existing theoretical literature (Frankel, 1986; Lai, Hu, and Wang, 1996), but is also consistent with empirical findings (Bordo, 1980; Devadoss and Meyers, 1987; Taylor and Spriggs, 1989). Following Frankel (1986), in equation (1) the demand for manufactured products is specified as an increasing function of the relative price between agricultural and manufactured prices, $p_{ca} - p_m$, and as a decreasing function of the real interest rate, $i - p_m$, while the supply of manufactured products is specified as a decreasing function of the relative price between agricultural and manufactured prices, $p_{ca} - p_m$. Equation (2) is the equilibrium condition for the money market, in which the demand for real money balances is a function of the nominal interest rate and real output.

The equilibrium condition for the agricultural product market is described by equation (3). It specifies that the sum of asset demand and consumption demand for agricultural products is equal to the supply of agricultural products.

The asset demand for agricultural products is specified as an increasing function of the difference between the yield on agricultural products, $p_{ca} - c + \rho$, and that on domestic bonds, $i$. To be more specific, a rise in $p_{ca} - c + \rho$ would make agricultural products more attractive, compared with domestic bonds, causing the public to build up their holding of agricultural commodities. The coefficient $\beta$ shows how the relative yield between domestic bonds and agricultural products affects the asset demand for agricultural products by the public, and hence is designated as the degree of asset substitutability between two assets. Existing studies exhibit a diversity of viewpoints on the extent of asset substitutability between these two types of assets. Frankel (1986), and Lai, Hu, and Wang (1996) assume the public treats both bonds and agricultural goods as perfectly substitutable assets (i.e., $\beta = \infty$). Their special specification is inconsistent with the viewpoint of Van Duyne (1979) and Chambers (1984), in which all assets, including agricultural products, are specified as imperfect rather than perfect substitutes in asset holders’ portfolios (i.e., $0 < \beta < \infty$). Moreover, In and Mount’s (1994) empirical observation supports the view that the degree of asset substitutability between domestic bonds and agricultural products is very low.

The consumption demand for agricultural products negatively depends upon the relative price between agricultural and manufactured prices, and the supply of agricultural

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2 Assuming the production combination of manufactured and agricultural products is determined along the production possibility curve, we then have the following production functions:

$$X_m = X_m(P_m/P_a) \text{ and } X_a = X_a(P_a/P_m); \frac{\partial X_m}{\partial P_m/P_a} < 0 \text{ and } \frac{\partial X_a}{\partial P_m/P_a} > 0,$$

where $X_m$ and $X_a$ are output in the manufactured and agricultural sectors, respectively, $\ln P_m = p_m$ and $\ln P_a = p_a$. By letting $\ln Y = y$ and $\ln P = p$, and defining $Y = (P_mX_m + P_aX_a)/P$, we can easily observe that $Y$ remains constant even if the relative price $P_m/P_a$ changes.

3 For a detailed explanation of the return for holding agricultural (auction) products, see Frankel (1986), Gordon (1987), and Moutos and Vines (1992).
products varies positively with the relative price between agricultural and manufactured prices.

One point concerning the specification in equation (3) should be addressed here. Dividing equation (3) by $\beta$ yields:

$$ (3') \quad (\hat{p}_c - c + \rho - i) - \frac{\theta(p_c - p_m)}{\beta} = \frac{x(p_c - p_m)}{\beta}. $$

As noted above, in their earlier analyses, Frankel (1986), and Lai, Hu, and Wang (1996) made a special assumption that the public treats both bonds and agricultural goods as perfectly substitutable assets ($\beta = \infty$). From equation (3') with $\beta = \infty$, it follows:

$$ (3'') \quad \hat{p}_c - c + \rho = i. $$

Equation (3'') indicates (as specified in Frankel, 1986, and Lai, Hu, and Wang, 1996) a commodity-financial arbitrage condition must hold, as residents treat both bonds and agricultural commodities as perfectly substitutable assets.$^4$

The above inference tells us that, as $\beta = \infty$, the asset demand for agricultural products in the agricultural market [$\beta(\hat{p}_c - c + \rho - i)$] is significantly greater than either the consumption demand for agricultural products [$-\theta(p_c - p_m)$] or the supply of agricultural products [$x(p_c - p_m)$].$^5$ Hence, neither the consumption demand for agricultural products nor the supply of agricultural products plays any role in determining the relevant variables. With this understanding, it is obvious from equation (3) that, as $0 < \beta < \infty$, both the consumption demand for agricultural products and the supply of agricultural products will play appropriate roles in determining the relevant variables. This is the issue to be addressed in what follows.

Finally, equation (4) defines the general price level as a weighted average of manufactured and agricultural commodity prices.

Next, we examine the stationary property of the system and address how commodity prices respond as the economy experiences an unanticipated monetary expansion. The system of equations (1)-(4) can be solved for four endogenous variables: $p_m, \hat{p}_c, i$, and $p$. In the long-run equilibrium, $\hat{p}_m = \hat{p}_c = 0$, and $p_m, p_c, i$, and $p$ are at their stationary levels $\hat{p}_m, \hat{p}_c, i$, and $p$. Based on Cramer’s rule, from equations (1)-(4) we have:

$$ (5) \quad \hat{p}_m = m - \phi y + \left[ \frac{\beta \delta \lambda - \beta \sigma (1 - \alpha)}{\beta \delta + \sigma (\zeta + x)} \right] (\rho - c), $$

$$ (6) \quad \hat{p}_c = m - \phi y + \left[ \frac{\beta \delta \lambda + \beta \sigma \alpha}{\beta \delta + \sigma (\zeta + x)} \right] (\rho - c), $$

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$^4$ Specifically, Frankel (1986) uses a simplified version of the arbitrage condition without the convenience yield, i.e., $\hat{p}_c - c = i$. Nevertheless, he recognizes that both bonds and agricultural products might be imperfect substitutes and that there might be a risk premium associated with holding agricultural products. Gordon (1987) criticizes the validity of Frankel’s arbitrage condition, and introduces the convenience yield. However, it is quite easy to show that the inclusion of the convenience yield and a more complicated arbitrage condition in Frankel’s model does not change the overall conclusions reached by Frankel. Departing from Frankel (1986) and Gordon (1987), in which imperfect substitution between bonds and agricultural products is presented using an exogenous parameter, we explicitly specify that the demand for agricultural products consists of two components: asset demand and consumption demand. Under such a model, the extent of asset substitutability between bonds and agricultural goods is found to govern the transitional adjustment of agricultural product prices. Helmerger and Chavas (1996, chap. 6) provide a comprehensive discussion of the arbitrage condition.

$^5$ This implies in equation (3'') that $-\theta(p_c - p_m)/\beta = 0$ and $x(p_c - p_m)/\beta = 0$ as $\beta = \infty$. 
Equations (5)-(8) show that an expansion in the money supply will give rise to long-run proportional increases in all commodity prices, while leaving the long-run interest rate unchanged. Thus, money is neutral in the long run—a result supported by a body of empirical agricultural studies (e.g., Grennes and Lapp, 1986; Orden and Fackler, 1989; Lapp, 1990; Robertson and Orden, 1990).

We now proceed to analyze the dynamic behavior of the economy. Substituting equation (4) into (2) and solving \( i \) from the resulting equation gives:

\[
(2') \quad i = \frac{1}{\lambda} [-m + \alpha p_m + (1 - \alpha) p_e + \phi y].
\]

Substituting equation \( (2') \) into equations (1) and (3) yields:

\[
\begin{align*}
\dot{p}_m &= \pi \left\{ \delta (p_e - p_m) - \sigma \left\{ -m + \alpha p_m + (1 - \alpha) p_e + \phi y \right\} + \alpha \phi p_m \right\}, \\
\beta \dot{p}_e - \beta c + \beta p + \frac{\beta}{\lambda} [-m + \alpha p_m + (1 - \alpha) p_e + \phi y] - \theta p_e + \theta p_m &= x p_e - x p_m.
\end{align*}
\]

From the two expressions above, it is quite easy to derive the following pair of differential equations in \( p_m \) and \( p_e \):

\[
\begin{pmatrix}
\dot{p}_m \\
\dot{p}_e
\end{pmatrix} = \begin{pmatrix}
-\frac{\pi}{1 - \pi \sigma} \left( \delta + \frac{\sigma \alpha}{\lambda} \right) & -\frac{\pi}{1 - \pi \sigma} \left( \delta - \frac{\sigma(1 - \alpha)}{\lambda} \right) \\
\alpha \frac{\theta + x}{\lambda} & 1 - \alpha + \frac{\theta + x}{\beta}
\end{pmatrix} \begin{pmatrix} p_m \\ p_e \end{pmatrix} + \begin{pmatrix}
\frac{\pi \sigma}{(1 - \pi \sigma) \lambda} (m - \phi y) \\
\frac{1}{\lambda} (-m + \phi y) + c - \rho
\end{pmatrix}.
\]

Letting \( S \) be the eigenvalue of the dynamic system, the characteristic function for equation (9) is:

\[
S^2 - \left( \frac{1 - \alpha}{\lambda} + \frac{\theta + x}{\beta} - \frac{\pi}{1 - \pi \sigma} \left( \delta + \frac{\sigma \alpha}{\lambda} \right) \right) S - \frac{\pi}{1 - \pi \sigma} \left[ \frac{\beta \delta + \sigma(\theta + x)}{\lambda \beta} \right] = 0.
\]

Provided \( 1 - \pi \sigma > 0 \), it is clear from equation (10) that two eigenvalues, \( S_1 \) and \( S_2 \), have the following relationship:

\[
S_1 S_2 = -\frac{\pi}{1 - \pi \sigma} \left[ \frac{\beta \delta + \sigma(\theta + x)}{\lambda \beta} \right] < 0.
\]

*Frankel (1984) and Obstfeld (1986) provide a more detailed discussion on the validity of money neutrality.*
Equation (11) reveals the two eigenvalues have opposite signs, implying that the system displays the saddlepoint stability.\(^7\)

The evolution of both agricultural and manufactured prices can be illustrated by means of the phase diagram. It is quite obvious from equation (9) that the slopes of the loci \(\dot{p}_m = 0\) and \(\dot{p}_c = 0\) displayed in the \(p_m\) and \(p_c\) plane are:

\[
\frac{\partial p_c}{\partial p_m} \bigg|_{\dot{p}_n = 0} = \frac{\lambda \delta + \sigma a}{\lambda \delta - \sigma (1 - \alpha)} \geq 0, \quad \text{if } \lambda \delta \geq \sigma (1 - \alpha); \\
\frac{\partial p_c}{\partial p_m} \bigg|_{\dot{p}_c = 0} = \frac{-\alpha \beta - \lambda (\theta + x)}{(1 - \alpha) \beta + \lambda (\theta + x)} \geq 0, \quad \text{if } \alpha \beta \leq \lambda (\theta + x).
\]

Equation (12) indicates the \(\dot{p}_m = 0\) locus can be either upward or downward sloping, depending on whether \(\lambda \delta\) is greater or less than \(\sigma (1 - \alpha)\). More importantly, equation (13) indicates the slope of the \(\dot{p}_c = 0\) locus has an ambiguous sign depending on the relative magnitudes of \(\alpha \beta\) and \(\lambda (\theta + x)\). The \(\dot{p}_c = 0\) schedule is negatively sloped if asset substitutability between bonds and agricultural products is relatively high [i.e., \(\alpha \beta > \lambda (\theta + x)\)], while a positively sloped \(\dot{p}_c = 0\) schedule prevails if asset substitutability is relatively low [i.e., \(\alpha \beta < \lambda (\theta + x)\)].

Because different degrees of asset substitutability will create different adjustment patterns of agricultural product prices when the economy experiences a monetary shock, in what follows, we consider two situations: high substitutability of assets and low substitutability of assets.

**High Substitutability Between Bonds and Agricultural Products**

When bonds and agricultural products are highly substitutable [i.e., \(\alpha \beta > \lambda (\theta + x)\)], we observe from equation (13) that the \(\dot{p}_c = 0\) locus is downward sloping. Figures 1a and 1b depict the phase diagrams associated with \(\lambda \delta > \sigma (1 - \alpha)\) and \(\lambda \delta < \sigma (1 - \alpha)\), respectively. As indicated by the directions of the arrows in both figures, we can sketch all possible trajectories. In the phase space plane, the lines SS and UU represent the stable and unstable branches, respectively. The convergent saddle path SS is always downward sloping, while the divergent branch UU may be either upward or downward sloping.\(^6\)

We are now ready to address the dynamic adjustment of \(p_m\) and \(p_c\) in response to an expansion in the money supply. Figure 2a illustrates the situation where \(\lambda \delta > \sigma (1 - \alpha)\), while figure 2b portrays the situation where \(\lambda \delta < \sigma (1 - \alpha)\). In both figures, suppose the initial equilibrium, where \(\dot{p}_m = 0(m_o)\) intersects \(\dot{p}_c = 0(m_o)\) is at \(Q^0\); the initial manufactured and agricultural prices are \(p^0_m\) and \(p^0_c\), respectively. The unique convergent path

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\(^7\) As manufactured product prices adjust with a time lag, \(p_m\) is treated as a predetermined variable. If we assume that \(1 - \tau_o < 0\) rather than \(1 - \tau_o > 0\), from equation (10) we have:

\[
S_1 + S_2 = -\frac{1}{\lambda} \cdot \frac{\theta + x}{\theta} \cdot \frac{\pi}{\theta - \sigma \alpha} \left( \frac{\delta - \sigma \alpha}{\lambda} \right) > 0 \quad \text{and} \quad S_3 + S_4 = \frac{-\pi}{\theta - \sigma \alpha} \left( \frac{(\delta - \sigma \alpha)}{\lambda} \right) > 0.
\]

The system is then characterized by two positive roots, implying the number of negative roots is smaller than the number of predetermined variables, and the perfect foresight equilibrium does not exist. See, for example, Burmeister (1980) and Buiter (1984) for a detailed discussion of this issue.

\(^6\) The detailed derivations of the slopes of both lines SS and UU are provided in appendix A.
Figure 1a. Phase diagram under $\alpha\beta > \lambda(\theta + x)$ and $\lambda\delta > \sigma(1 - \alpha)$

Figure 1b. Phase diagram under $\alpha\beta > \lambda(\theta + x)$ and $\lambda\delta < \sigma(1 - \alpha)$
Figure 2a. Commodity price dynamics of monetary expansion under $\alpha\beta > \lambda(\theta + x)$ and $\lambda\delta > \sigma(1 - \alpha)$

Figure 2b. Commodity price dynamics of monetary expansion under $\alpha\beta > \lambda(\theta + x)$ and $\lambda\delta < \sigma(1 - \alpha)$
SS(m₀) will pass through point $Q^0$. On encountering a permanent monetary shock, both $p_m = 0(m₀)$ and $p_c = 0(m₀)$ shift rightward to $p'_m = 0(m₁)$ and $p'_c = 0(m₁)$; $p_m = 0(m₀)$ intersects $p'_c = 0(m₁)$ at point $Q^*$, with $p_m$ and $p'_c$ being $p_m^*$ and $p'_c^*$, respectively. Meanwhile, the $SS(m₁)$ will also pass through point $Q^*$. Both $Q^0$ and $Q^*$ should be located on a 45° line going through the origin, as money is neutral in the long run. Since the public becomes aware that the money stock exhibits a once-and-for-all expansion, the stability (transversality) condition requires that the economy move to a point exactly on the saddle path $SS(m₁)$ at the moment of monetary expansion. Because manufactured product prices adjust sluggishly, agricultural product prices must rise from $p_m^0$ to $p_m^*$, and the economy will therefore jump from $Q^0$ to $Q^*$ on the $SS(m₁)$ locus at the instant of policy implementation. Thereafter, agricultural prices will continue to fall and manufactured prices will continue to rise as the economy moves along the stable branch $SS(m₁)$ toward its new stationary equilibrium $Q^*$. It is clear from both figures 2a and 2b that the short-run movement of agricultural prices $p_m^0p'_c$ will overshoot its long-run equilibrium response $p_m^0p'_c$.

In their previous studies, Frankel (1986), and Lai, Hu, and Wang (1996) assume that bonds and agricultural products are perfectly substitutable assets, and find that agricultural product prices will overshoot their long-run value following an expansion in the money stock. Clearly, the relation $\alpha \beta > \lambda (\theta + x)$ holds when both assets are perfect substitutes (i.e., $\beta$ is constrained to be $=\omega$). Our results exhibited in figures 2a and 2b are thus consistent with Frankel’s (1986) assertion.

**Low Substitutability Between Bonds and Agricultural Products**

We now consider the case where the substitutability between bonds and agricultural products is relatively low [i.e., $\alpha \beta < \lambda (\theta + x)$], so that the slope of the $p'_c = 0$ locus stated in equation (13) turns positive. The phase diagrams associated with $\lambda \delta > \sigma (1 - \alpha)$ and $\lambda \delta < \sigma (1 - \alpha)$ are presented in figures 3a and 3b, respectively. With arrows of motion as shown in both figures, we can sketch all possible trajectories. In the phase space plane, the lines $SS$ and $UU$ represent the stable and unstable branches, respectively. The stable arm $SS$ is always upward sloping, while the unstable arm $UU$ may be either upward sloping or downward sloping (refer to appendix A).

We now examine the kind of pattern the transitional adjustment of $p_m$ and $p_c$ will exhibit in response to an expansion in the money supply. Figure 4a illustrates the situation where $\lambda \delta > \sigma (1 - \alpha)$, while figure 4b portrays the situation where $\lambda \delta < \sigma (1 - \alpha)$. In conjunction with the initial money supply $m₀$, the initial equilibrium of the economy is established at point $Q^0$, where $p_m = 0(m₀)$ intersects $p_c = 0(m₀)$. The initial manufactured and agricultural prices are $p_m^0$ and $p_c^0$, respectively, and the stable branch $SS(m₀)$ will pass through point $Q^0$. In response to a sudden, permanent rise in the money supply from $m₀$ to $m₁$, both $p_m = 0(m₀)$ and $p_c = 0(m₀)$ will shift to $p'_m = 0(m₁)$ and $p'_c = 0(m₁)$. The new equilibrium occurs at $Q^*$, where $p_m = 0(m₁)$ intersects $p'_c = 0(m₁)$, with $p_m$ and $p'_c$ being $p_m^*$ and $p'_c^*$, respectively. Meanwhile, the $SS(m₁)$ will also pass through point $Q^*$. At the instant of monetary expansion, the prices of agricultural products must immediately jump from $p'_m$ to $p'_m^*$, while the prices of manufactured products remain intact at their initial level $p_m^0$. As a consequence, the economy will jump vertically from $Q^0$ to $Q^*$ on the $SS(m₁)$ schedule. Subsequently, both agricultural and manufactured product prices will
Figure 3a. Phase diagram under $\alpha \beta < \lambda (\theta + x)$ and $\lambda \delta > \sigma (1 - \alpha)$

Figure 3b. Phase diagram under $\alpha \beta < \lambda (\theta + x)$ and $\lambda \delta < \sigma (1 - \alpha)$
Figure 4a. Commodity price dynamics of monetary expansion under $\alpha \beta < \lambda(\theta + x)$ and $\lambda \delta > \sigma(1 - \alpha)$

Figure 4b. Commodity price dynamics of monetary expansion under $\alpha \beta < \lambda(\theta + x)$ and $\lambda \delta < \sigma(1 - \alpha)$
continue to rise as the economy moves along the stable branch $SS(m_t)$ toward its new stationary equilibrium at $Q'$. An important feature emerging in both figures 4a and 4b is that the short-run adjustment of agricultural prices $p'_0p'_t$ undershoots its ultimate response $p'_0p'_*$.\textsuperscript{9}

Based on findings of the graphical analysis in this section, the tendency for agricultural prices to overshoot or undershoot their long-run response will depend on the relative size between $a\beta$ and $\lambda(\theta + x)$. As indicated from earlier studies (e.g., Klein, 1990; In and Mount, 1994; Swinton and Thomas, 2001), $\lambda = 5$, $\theta = 0.13$, $x = 0.29$, $\alpha = 0.6$, and $\beta = 0.0405$. This implies that $a\beta < \lambda(\theta + x)$, and hence the undershooting of agricultural prices is more likely to be present.

**An Open-Economy Model**

This section extends the closed-economy model developed in the previous section to an open-economy model under flexible exchange rates, and uses it to shed light on whether the extent of asset substitutability between bonds and agricultural goods will govern the transitional adjustment of agricultural product prices.\textsuperscript{10}

The open-economy model can be described by the following equations:

\begin{equation}
  p_m = \pi \left\{ \delta(p_c - p_m) - \sigma(p' - p_m) + \left[ \psi(p' + e - p_m) - \eta y \right] \right\},
  \pi > 0, \delta > 0, \sigma > 0, \psi > 0, \eta > 0;
\end{equation}

\begin{equation}
  m - p = -\lambda i + \phi y, \quad \lambda > 0, \phi > 0;
\end{equation}

\begin{equation}
  \beta(p_c - c + p - i) - \theta(p_c - p_m) = x(p_c - p_m), \quad \beta > 0, \theta > 0, x > 0;
\end{equation}

\begin{equation}
  i = i^* + \epsilon;
\end{equation}

\begin{equation}
  p = \alpha_1p_m + \alpha_2p_c + (1 - \alpha_1 - \alpha_2)(e + p'_m), \quad 1 > \alpha_1 > 0, 1 > \alpha_2 > 0,
\end{equation}

where $e$ denotes the exchange rate (defined as the price of foreign currency in terms of domestic currency), $p'_m$ denotes the price of non-agricultural products in terms of foreign currency, and $i^*$ denotes the foreign interest rate.

Equation (14) states that manufactured prices adjust sluggishly in response to excess demand in the manufactured products market. When compared with equation (1), the demand for manufactured products in the open economy includes an additional component: net exports, which is expressed in the square brackets on the right-hand side of equation (14). As is common in the international finance literature, net exports are specified as an increasing function of the relative price between foreign and domestic manufactured prices, $p'_m + e - p_m$, and as a decreasing function of real output $y$.

\textsuperscript{9} It should be noted that, although our analysis focuses on the substitutability between bonds and agricultural products, the demand and supply parameters in the agricultural product market (i.e., $\theta$ and $x$) and other parameters (e.g., $\lambda$ and $\alpha$) also play an important role in determining adjustment patterns of agricultural product prices.

\textsuperscript{10} An anonymous referee, to whom we are grateful, brought this issue to our attention.
The equilibrium conditions for the money market and the agricultural product market are respectively expressed by equations (15) and (16). The specifications in equations (15) and (16) are the same as those in (2) and (3), and hence we do not restate the economic rationale of the behavioral functions here. With the assumption of perfect capital mobility, equation (17) describes the interest rate parity. Finally, equation (18) defines the general price level which is comprised of a weighted average of domestic manufactured product prices $p_m$, domestic agricultural product prices $p_c$, and foreign manufactured product prices $p^* + e$.

The system of equations (14)–(18) determines five endogenous variables: $p_m$, $i$, $p_c$, $e$, and $p$. At the long-run equilibrium in the context of an open economy, $\dot{p}_m = \dot{p}_c = \dot{e} = 0$, and $p_m, i, p_c, e$, and $p$ are at their stationary levels $\bar{p}_m, \bar{i}, \bar{p}_c, \bar{e}$, and $\bar{p}$. According to Cramer's rule, from equations (14)–(18) the following long-run relationships are derived:

\[
\bar{p}_m = m - vy - \frac{\beta(\psi \alpha_2 - \delta \alpha_3)}{(x + \theta)\psi}(p - c) + (\lambda + \omega - \mu)i^*,
\]
\[
\bar{i} = i^*,
\]
\[
\bar{p}_c = m - vy + k(p - c) + \left(\frac{\lambda \psi - \sigma \alpha_3}{\psi} - k\right)i^*,
\]
\[
\bar{e} = m - \left(\frac{1}{\psi} - \frac{\eta}{\psi}\right)y - p^* - \omega(p - c) + \left[\omega + \left(\frac{1}{\psi}\right)(\alpha_1 + \alpha_2)\right]i^*,
\]
\[
\bar{p} = m - \phi y + \lambda i^*,
\]

where

\[v = [\eta \alpha_3 + \phi \psi]/\psi > 0,\]
\[\alpha_3 = 1 - \alpha_2 - \alpha_3 > 0,\]
\[\mu = [\sigma \alpha_3(x + \theta) + \beta \delta]/(x + \theta)\psi > 0,\]
\[k = \beta[\psi \alpha_1 + (\delta + \psi)\alpha_3]/(x + \theta)\psi > 0,\]

and
\[\omega = \beta[\delta \alpha_1 + (\delta + \psi)\alpha_2]/(x + \theta)\psi > 0.\]

Equations (19a)–(19e) reveal that money neutrality is valid in the long run under the regime of flexible exchange rates. Specifically, an expansion in the money supply will cause long-run proportional increases in all commodity prices and the nominal exchange rate.

Substituting equation (18) into (15), and solving $i$ from the resulting equation, gives the following expression:

\[(15') \quad i = \frac{1}{\lambda} \left[-m + \alpha_1 p_m + \alpha_2 p_c + (1 - \alpha_1 - \alpha_2)(e + p^*_m) + \phi y\right].\]

Substituting equation (15') into (14), (16), and (17) and manipulating the resulting equations, we obtain the following differential equations associated with $p_m, p_c,$ and $e$:
where $A = \pi \sigma / (1 - \pi \sigma)$.

Let $\Theta$ be the characteristic root of the dynamic system. From equation (20) we derive the following characteristic equation:

$$
\left( \begin{array}{c}
\frac{\Delta m}{\beta} \\
\Delta p_c \\
\Delta e
\end{array} \right) =
\left( \begin{array}{ccc}
\frac{A}{\sigma} \left( \delta + \frac{\alpha_1}{\lambda} + \psi \right) & \frac{A}{\sigma} \left( \delta - \frac{\alpha_2}{\lambda} \right) & \frac{A}{\sigma} \left( \psi - \frac{\alpha_3}{\lambda} \right) \\
\frac{\alpha_1}{\lambda} & \frac{\alpha_2}{\lambda} & \frac{\alpha_3}{\lambda} \\
0 & 0 & 0
\end{array} \right)
\left( \begin{array}{c}
\Delta m \\
\Delta p_c \\
\Delta e
\end{array} \right)
$$

Equations (21) indicate that three characteristic roots, $\Theta_1$, $\Theta_2$, and $\Theta_3$, have the following relationship:

$$
\Theta_1 + \Theta_2 + \Theta_3 = \frac{x + \Theta}{\beta} + \frac{A[1 - \alpha_1 - \pi(\sigma + \lambda(\delta + \psi))]}{\lambda \pi \sigma} \geq 0,
$$

$$
\Theta_1 \Theta_2 \Theta_3 = -\frac{A \psi (x + \Theta)}{\beta \lambda \sigma} < 0.
$$

Equations (22a) and (22b) indicate that, of the three characteristic roots of the system, two are positive and one is negative. In what follows, it is assumed that $\Theta_1 < 0$, $\Theta_2 > 0$, and $\Theta_3 > 0$. Since the dynamic system reported in equation (20) has one predetermined variable ($p_m$) equal to the number of negative roots, the system thus displays saddle-point stability. It generates a unique stable path leading the economy to the stationary equilibrium.

The stable path describing the relation between $p_m$ and $p_c$ is given by.**
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\[ p_c = \bar{p}_c + \frac{(x + \theta)(\alpha_3 - \Theta_1 \lambda) + \Theta_1 \alpha_3 \beta}{(x + \theta)(\alpha_3 - \Theta_1 \lambda) - \Theta_1 \beta(1 - \Theta_1 \lambda - \alpha_1)} (p_m - \bar{p}_m). \]

The \( \bar{SS} \) locus is defined as representing the pairs of \( p_m \) and \( p_c \) that satisfy equation (23). It is clear from equation (23) that the slope of the \( \bar{SS} \) line is:

\[ \frac{\partial p_c}{\partial p_m}_{\bar{SS}} = \frac{\Omega}{(x + \theta)(\alpha_3 - \Theta_1 \lambda) - \Theta_1 \beta(1 - \Theta_1 \lambda - \alpha_1)} \gtrless 0, \text{ if } \Omega \gtrless 0, \]

where \( \Omega = (x + \theta)(\alpha_3 - \Theta_1 \lambda) + \Theta_1 \alpha_3 \beta \). Figures 5a and 5b depict the \( \bar{SS} \) line associated with \( \Omega < 0 \) and \( \Omega > 0 \), respectively. Agricultural-price overshooting is illustrated in figure 5a. As shown in equation (24), the slope of the \( \bar{SS} \) locus is negative if \( \Omega < 0 \). The initial equilibrium is at \( Q_0 \) on the \( \bar{SS}(m_0) \) line; the initial manufactured and agricultural prices are \( p_m^0 \) and \( p_c^0 \), respectively. To simplify our graphical analysis, we assume \( p_m^0 = p_c^0 \) initially, and hence \( Q_0 \) is located on a 45° line going through the origin. Upon an unanticipated permanent rise in the money supply, the \( \bar{SS}(m_0) \) line shifts rightward to \( \bar{SS}(m_1) \). Given that money neutrality is valid in the long run, point \( Q^* \), where the \( \bar{SS}(m_1) \) line intersects the 45° line, is the new equilibrium point.

Because manufactured product prices adjust with a time lag, agricultural product prices consequently must immediately rise from \( p_c^0 \) to \( p_c' \), and the economy will jump vertically from \( Q^0 \) to \( Q' \) on the \( \bar{SS}(m_1) \) locus at the instant of monetary expansion. Thereafter, agricultural prices will continue to fall and manufactured prices continue to rise as the economy moves along the stable branch \( \bar{SS}(m_1) \) toward its new stationary equilibrium \( Q^* \).

We now illustrate another situation where agricultural-price undershooting is present. It is clear from equation (24) that the \( \bar{SS} \) locus is upward sloping, i.e., \( \Omega > 0 \). In response to an increase in the money supply, agricultural prices, on impact, will rise from \( p_c^0 \) to \( p_c' \) and then continue to increase until their long-run value, \( p^*_c \), is reached. As can be seen from figures 5a and 5b, whether agricultural prices overshoot or undershoot their long-run response depends on \( \Omega < 0 \) or \( \Omega > 0 \). The value of the negative root \( \Theta_1 \) is determined by the characteristic equation in equation (21), and hence is crucially related to the extent of asset substitutability between bonds and agricultural goods \( \beta \). As a consequence, the value of \( \Omega \), which is equal to \( (x + \theta)(\alpha_3 - \Theta_1 \lambda) + \Theta_1 \alpha_3 \beta \), is a nonlinear function of \( \beta \). Given the function’s complexity, the issue of how \( \beta \) is related to the value of \( \Omega \) is addressed here via numerical simulations. In order to illustrate how the value of \( \beta \) governs the adjustment patterns of agricultural prices, it is convenient to establish a benchmark case. The benchmark case involves the following parameter configuration:

- The parameters in the manufactured market: \( \sigma = 2, \psi = 0.4, \delta = 1.2, \eta = 0.1, \pi = 0.1, \) and \( y = 5 \);
- The parameters in the money market: \( \lambda = 5, \phi = 0.2, \) and \( m = 5 \);
- The parameters in the agricultural market: \( \theta = 0.13, x = 0.29, c = 0.05, \rho = 0.1, \) and \( \beta = 1 \) and 100;
- The parameter in the foreign exchange market: \( i^* = 0.05 \);
- The parameters in the definition of the general price level: \( \alpha_1 = 0.6, \alpha_2 = 0.2, \) and \( \alpha_3 = 0.2, \) and \( p^*_m = 3.75 \).
**Figure 5a.** Commodity price dynamics of monetary expansion in an open economy under $\Omega < 0$

**Figure 5b.** Commodity price dynamics of monetary expansion in an open economy under $\Omega > 0$
Some of the parameters we utilize are adopted from Bhandari (1987), Klein (1990), and Swinton and Thomas (2001), while certain other parameters are chosen to reflect the model's plausibility. In addition, given that the initial values of output $y$, the money supply $m$, the storage cost $c$, the convenience yield $p$, the foreign interest rate $i^*$, and non-agricultural product prices in terms of foreign currency $p_n^*$ do not affect the adjustment patterns of relevant variables, these values are chosen in an ad hoc manner.

The critical parameter on which we focus is the extent of asset substitutability $\beta$, and we discuss only two benchmark economies that depend on the degree of asset substitutability between bonds and agricultural products. These benchmark economies are:

(a) high substitutability between bonds and agricultural products, which is reflected by $\beta = 100$; and

(b) low substitutability between bonds and agricultural products, as reflected by $\beta = 1$.

We first consider the case where bonds and agricultural products are highly substitutable, i.e., $\beta = 100$. Given the structural parameters identified above, the following calibrated reduced values are derived: $\Theta_1 = -0.29$, $\Omega = -16.51$, $p^0_m = p^0_c = 3.95$, $p^0 = 4.25$, $i^0 = 0.05$, $e^0 = 1.7$ (where $z^0$ denotes the initial value of $z$, $z = p_m, p_c, p, i, e$). The dynamic transition path of agricultural product prices following an increase in the money supply from 5 to 6 is illustrated in figure 6a. As indicated in the figure, agricultural product prices increase from 3.95 to 5.20 at the instant of monetary expansion. Thereafter, agricultural prices continue to fall toward their new stationary level of 4.95. It is clear that the short-run adjustment of agricultural prices overshoots its long-run response.

Next, we examine the case where the substitutability between bonds and agricultural products is relatively low, i.e., $\beta = 1$. Based on the structural parameters reported above, the following calibrated reduced values are generated: $\Theta_1 = -0.18$, $\Omega = 0.36$, $p^0_m = p^0_c = 3.95$, $p^0 = 4.25$, $i^0 = 0.05$, $e^0 = 1.7$. Figure 6b illustrates the dynamic adjustment path of agricultural product prices in response to an expansion in the money supply from 5 to 6. In this benchmark case, agricultural product prices increase from 3.95 to 4.57 at the instant of monetary expansion. Thereafter, agricultural prices increase steadily toward their new stationary level of 4.95. Hence, the short-run adjustment of agricultural prices undershoots its long-run response.

In view of the calibrated results for the two benchmark economies, in the context of an open economy, we conclude the extent of asset substitutability also plays a critical role in determining the transitional adjustment of agricultural product prices. Moreover, the calibrated dynamic patterns of agricultural product prices in both figures 6a and 6b are conformable to theoretical results, which are also illustrated in both figures 5a and 5b.

Before ending this section, one point should be mentioned here. In order to highlight the role of the substitutability between bonds and agricultural products, two benchmark economies associated with $\beta = 100$ and $\beta = 1$ are calibrated. However, if we adopt the empirical evidence reported by In and Mount (1994) (i.e., $\beta = 0.0405$), the dynamic pattern of agricultural product prices in conjunction with $\beta = 0.0405$ is the same as the pattern in conjunction with $\beta = 1$. Hence, in the context of an open economy, the undershooting of agricultural prices is more likely to occur.
Figure 6a. Calibrated dynamic path of agricultural product prices in an open economy under $\beta = 100$

Figure 6b. Calibrated dynamic path of agricultural product prices in an open economy under $\beta = 1$
Concluding Remarks

In their innovative paper, Rausser et al. (1986, p. 411) remind us that the “analysis of agricultural market dynamics must take into account not only real demand and supply forces directly related to the sector but also the effects of monetary and fiscal policies.” Using a macroeconomic model, Frankel (1986) concludes that agricultural product prices exhibit a tendency to overshoot following an unanticipated expansion in the money supply. However, the empirical studies reveal conflicting observations: some support the overshooting hypothesis of agricultural prices, while others reject it.

This analysis has extended Frankel’s (1986) closed-economy model and offered a reconciliation of conflicting empirical results. The key feature of the model we have developed is that it allows for various degrees of asset substitutability between bonds and agricultural goods. Our findings suggest that the crucial factor determining whether agricultural prices will overshoot or undershoot their long-run response depends on the extent of asset substitutability between bonds and agricultural goods. An expansion in the money stock will result in an overshooting of agricultural prices if asset substitutability is relatively high, while undershooting will prevail if asset substitutability is relatively low.

This paper has addressed the issue of whether or not the extent of asset substitutability between bonds and agricultural goods still governs the transitional adjustment of agricultural product prices in the context of an open economy. Using numerical simulations, we have shown that the results for a closed economy may be applied to an open economy. Specifically, in an open economy the calibrated dynamic patterns of agricultural product prices are closely related to the extent of asset substitutability between bonds and agricultural goods.

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References


Appendix A: Detailed Derivation of the SS and UU Schedules

According to Gandolfo (1980, pp. 263–265) and Turnovsky (1995, pp. 138–139), it follows from text equation (9) that the general solution for \( p_m \) and \( p_c \) can be expressed as:

\[
\begin{align*}
  p_m &= \hat{p}_m + B_1 \exp(S_1 t) + B_2 \exp(S_2 t), \\
p_c &= \hat{p}_c + \frac{(1 - \pi \sigma) \lambda S_1 + \pi (\lambda \delta + \sigma a)}{\pi [\lambda \delta - \sigma (1 - a)]} B_1 \exp(S_1 t) \\
  & \quad + \frac{(1 - \pi \sigma) \lambda S_2 + \pi (\lambda \delta + \sigma a)}{\pi [\lambda \delta - \sigma (1 - a)]} B_2 \exp(S_2 t),
\end{align*}
\]

where \( B_1 \) and \( B_2 \) are undetermined coefficients. Equipped with the relation \( S_1 S_2 < 0 \) reported in text equation (11), we assume, for expository convenience, \( S_1 < 0 \) and \( S_2 > 0 \).

The stable branch SS is associated with the value \( B_2 = 0 \) in equations (A1) and (A2), i.e., the positive (unstable) root is excluded in the trajectory. Hence, the stable branch SS satisfies the following relation:

\[
\begin{align*}
  p_c - \hat{p}_c &= \frac{(1 - \pi \sigma) \lambda S_1 + \pi (\lambda \delta + \sigma a)}{\pi [\lambda \delta - \sigma (1 - a)]} (p_m - \hat{p}_m).
\end{align*}
\]

From equation (A3), with \( S_1 < 0 \), the slope of the saddle path SS is:

\[
\frac{\partial p_c}{\partial p_m}_{|_{SS}} = \frac{(1 - \pi \sigma) \lambda S_1 + \pi (\lambda \delta + \sigma a)}{\pi [\lambda \delta - \sigma (1 - a)]}.
\]

It follows from equation (10) in the main text that two characteristic roots have the following relationship:

\[
S_1 + S_2 = \frac{1}{\lambda} + \frac{\theta + x}{\beta} - \frac{\pi}{1 - \pi \sigma} \left( \frac{\delta + \sigma a}{\lambda} \right) \geq 0,
\]

\[
S_1 S_2 = -\frac{\pi}{1 - \pi \sigma} \left[ \frac{\beta \delta + \sigma (\theta + x)}{\lambda \beta} \right] < 0.
\]

By substituting these two relationships into equation (A4) and engaging in some complicated computations, we obtain:

\[
\frac{\partial p_c}{\partial p_m}_{|_{SS}} = \frac{\alpha \beta - \lambda (\theta + x)}{\lambda \beta S_1 - \beta (1 - \alpha) + \lambda (\theta + x)} \geq 0, \quad \text{if } \alpha \beta \leq \lambda (\theta + x).
\]

On the other hand, the unstable branch UU is associated with the value \( B_1 = 0 \) in equations (A1) and (A2), i.e., the negative (stable) root is excluded in the trajectory. Accordingly, the unstable branch UU satisfies:

\[
\begin{align*}
  p_c - \hat{p}_c &= \frac{(1 - \pi \sigma) \lambda S_2 + \pi (\lambda \delta + \sigma a)}{\pi [\lambda \delta - \sigma (1 - a)]} (p_m - \hat{p}_m),
\end{align*}
\]

The slope of the unstable branch UU is thus:

\[
\frac{\partial p_c}{\partial p_m}_{|_{UU}} = \frac{(1 - \pi \sigma) \lambda S_2 + \pi (\lambda \delta + \sigma a)}{\pi [\lambda \delta - \sigma (1 - a)]} \geq 0, \quad \text{if } \lambda \delta \geq \sigma (1 - a).
\]

Apparently, the slopes of both SS and UU exhibited in figures 1a, 1b, 3a, and 3b are consistent with the results stated in equations (A5) and (A7).
Appendix B: General Solutions for $p_m$, $p_e$, and $e$

The general solutions for $p_m$, $p_e$, and $e$ can be described by:

(B1) \[ p_m = \bar{p}_m + H_1 \exp(\Theta_1 t) + H_2 \exp(\Theta_2 t) + H_3 \exp(\Theta_3 t), \]

(B2) \[ p_e = \bar{p}_e + \frac{(x + \Theta)(a_3 - \Theta_1 \lambda) + \Theta_1 a_3 \beta}{(x + \Theta)(a_3 - \Theta_1 \lambda) - \Theta_1 \beta(1 - \Theta_1 \lambda - \alpha)} H_1 \exp(\Theta_1 t) \]

\[ + \frac{(x + \Theta)(a_3 - \Theta_2 \lambda) + \Theta_2 a_3 \beta}{(x + \Theta)(a_3 - \Theta_2 \lambda) - \Theta_2 \beta(1 - \Theta_2 \lambda - \alpha)} H_2 \exp(\Theta_2 t) \]

\[ + \frac{(x + \Theta)(a_3 - \Theta_3 \lambda) + \Theta_3 a_3 \beta}{(x + \Theta)(a_3 - \Theta_3 \lambda) - \Theta_3 \beta(1 - \Theta_3 \lambda - \alpha)} H_3 \exp(\Theta_3 t), \]

and

(B3) \[ e = \bar{e} - \frac{(x + \Theta)(a_3 - \Theta_1 \lambda) - \Theta_1 \beta a_3}{(x + \Theta)(a_3 - \Theta_1 \lambda) - \Theta_1 \beta(1 - \Theta_1 \lambda - \alpha)} H_1 \exp(\Theta_1 t) \]

\[ - \frac{(x + \Theta)(a_3 - \Theta_2 \lambda) - \Theta_2 \beta a_3}{(x + \Theta)(a_3 - \Theta_2 \lambda) - \Theta_2 \beta(1 - \Theta_2 \lambda - \alpha)} H_2 \exp(\Theta_2 t) \]

\[ - \frac{(x + \Theta)(a_3 - \Theta_3 \lambda) - \Theta_3 \beta a_3}{(x + \Theta)(a_3 - \Theta_3 \lambda) - \Theta_3 \beta(1 - \Theta_3 \lambda - \alpha)} H_3 \exp(\Theta_3 t), \]

where $H_1$, $H_2$, and $H_3$ are undetermined coefficients.

The stable branch $\tilde{SS}$ is associated with the restriction $H_2 = H_3 = 0$ in equations (B1) and (B2), i.e., two positive (unstable) roots are excluded in the trajectory. Hence, the stable branch $\tilde{SS}$ satisfies the following relation:

(B4) \[ p_e = \bar{p}_e + \frac{(x + \Theta)(a_3 - \Theta_1 \lambda) + \Theta_1 a_3 \beta}{(x + \Theta)(a_3 - \Theta_1 \lambda) - \Theta_1 \beta(1 - \Theta_1 \lambda - \alpha)} (p_m - \bar{p}_m), \]