Journal of Agricultural and Resource Economics, 19(2): 409–424

Two-Stage Utility Maximization and Import Demand Systems Revisited: Limitations and an Alternative

George C. Davis and Kim L. Jensen

Two-stage utility maximization theory has been widely used in the literature to estimate import demand for agricultural commodities that are often inputs. This article examines the overlooked conceptual and empirical limitations of applying two-stage utility maximization theory to model the demand for imported commodities that are inputs. A discussion is presented about how the underutilized theory of two-stage profit maximization overcomes these limitations. Also discussed are the conditions under which errors resulting from misapplied utility theory may not be severe. An empirical illustration of the two-stage profit maximization procedure is provided.

Key words: conditional elasticities, import demand, two-stage maximization, unconditional elasticities, weak separability.

Introduction

Estimated elasticities of import demand often have been used to examine policy alternatives and to formulate trade policy (Thompson 1988). The effectiveness of using these estimated elasticities in policy formulation hinges on the appropriate conceptual and empirical specification of the underlying model. The two-stage utility maximization model has been widely used in the estimation of agricultural commodity import demand systems and elasticities (Alston et al.; de Gorter and Meilke; Duffy, Wohlgenant, and Richardson; Goddard; Haniotis; Heien and Pick; Lin and Makus; Sarris). The popularity of this modeling approach likely can be traced to its empirical advantages: with limited degrees of freedom, estimating price parameters across export sources is less problematic (de Janvry and Bieri), and multicollinearity problems are mitigated (Fuss).

However, many imported agricultural commodities are inputs in a production process. Use of utility-based demand systems to estimate import demand for these types of commodities has important conceptual and empirical disadvantages that have not been discussed in the literature. Furthermore, misuse of the results of these models can produce misleading policy implications. The purpose of this article is to point out the disadvantages of misapplying the two-stage utility maximization approach and to show how an underutilized methodology overcomes the approach's disadvantages, but retains its advantages.[1] The parameter bias resulting from misapplying utility theory also is discussed, along with the conditions under which this bias is not severe.

The remainder of the article proceeds as follows. In the next section, we discuss how the conceptual misspecification of many agricultural imports as final goods leads to three

The authors are assistant and associate professors, respectively, in the Department of Agricultural Economics and Rural Sociology at The University of Tennessee, Knoxville.

This research was funded in part by a grant from the U.S. Department of Agriculture Cooperative State Research Service National Research Initiative program.

Appreciation is expressed to Michael Wohlgenant for comments on an earlier draft of this article, to Jim Seale for enlightening discussions, and to Judith Harrison for editorial guidance.

empirical problems. In the third section, we provide a general presentation of the often more appropriate and underutilized theory of two-stage, multiproduct, profit maximization theory which removes the empirical limitations discussed in the second section, while maintaining the empirical advantages of the two-stage utility maximization model. We discuss the significant theoretical and empirical differences between the two-stage utility and two-stage profit approaches. Furthermore, we explicitly show that previously estimated conditional (second-stage) demand systems derived from consumer theory can be justified based on producer theory. The fourth section consists of an empirical illustration designed to demonstrate the econometric issues involved in estimating the model. We specifically concentrate on demonstrating the procedures for testing the sufficient conditions for two-stage maximization which are usually assumed a priori. We also discuss the difficulties associated with estimating the model's first stage and therefore estimating the unconditional elasticities. The article ends with a summary and conclusions section.

Conceptual and Empirical Limitations of Two-Stage Utility Maximization Models

The most prevalently discussed theoretical and empirical limitations regarding agricultural import demand based on two-stage utility maximization have been weak separability and functional form (e.g., Alston et al.). While weak separability and functional form are important, we wish to point out four other disadvantages of basing most agricultural import demands on the theory of two-stage utility maximization. These additional disadvantages seem to have been overlooked. One is conceptual; three are empirical and stem from the conceptual problem.

The obvious conceptual disadvantage of utility-based import demand systems is the overwhelming observational evidence that most imported agricultural commodities are inputs, not final goods. This conceptual misspecification leads to three less obvious empirical disadvantages. First, at the level of aggregation usually considered by agricultural economists, it is difficult to form a consensus when defining the first-stage utility aggregates. For example, in utility-based models, the pragmatic approach has been to choose a commodity such as soybeans (e.g., Heien and Pick; Haniotis), and assume that it is weakly separable from all other goods. Even if the specification of soybeans in the utility function were acceptable, the assumption that soybeans are weakly separable from all other oilseeds and grains, which for logical consistency also must be included in the utility function, is disconcerting. It is further disconcerting that the other commodities also are assumed to in fact help form a single consistent aggregate called "all other goods." Such unique separability conditions are not intuitive and therefore cause the choice of first-stage aggregates to be highly debatable. In addition to the degrees-of-freedom problem, the inability to form a consensus when defining the first-stage utility aggregates may explain why many studies have concentrated on conditional demand systems (i.e., the second stage). Despite these conceptual problems, conditional demand systems have been empirically successful in many applications (Thompson 1981), from wheat (e.g., Grennes, Johnson, and Thursby) to beef (Goddard). This fact raises a puzzling question: If conditional import demand systems based on utility theory are conceptually flawed, why are they empirically successful? We provide a possible answer to this question in the following section.

Because the most prevalent systems estimated have been conditional demand systems, the estimated demand elasticities have been conditional elasticities. This fact leads to the second problem: Because conditional elasticities do not encompass all of the price effects captured by unconditional elasticities, misuse of conditional elasticities can lead to biased inference and erroneous policy prescriptions. For example, Thompson (1988) claimed that import demand elasticities became the single most important policy issue in the 1985 Farm Bill. Yet, almost all of the literature discussed was based on conditional elasticity estimates and not unconditional elasticity estimates.

Even where the unconditional elasticities have been estimated (e.g., de Gorter and

Meilke; Heien and Pick), a third problem arises: Unconditional elasticities derived from misapplied utility theory are not structural parameter (elasticity) estimates, but are instead reduced-form estimates and will differ from those derived from producer theory because the regressors are incorrect. Therefore, many structural hypotheses of consumer theory seemingly may be rejected, when in fact the parameters of interest are not identified and the underlying hypothesis test is inappropriate. More importantly, policy prescriptions again can be erroneous.

An Appealing Alternative

From the above discussion, an alternative conceptual approach will be appealing if it satisfies three criteria: (*a*) it makes estimation of structural (derived demand) parameters possible; (*b*) it retains the empirical advantages of the two-stage utility maximization procedure; and (*c*) it makes defining the first-stage aggregates, and therefore the estimation of unconditional elasticities, easier. These criteria all can be satisfied by modeling import demand in a two-stage, multiproduct, profit maximization framework. Specifically, by using producer theory, the conceptual problem of treating inputs as final goods is immediately overcome and so the estimated parameters will be structural. Also, because this is a two-stage procedure, the empirical advantages of a two-stage optimization procedure are retained. Finally, defining the first-stage aggregates is more intuitive in the profit maximization model, and therefore the estimation of the unconditional elasticities is less debatable. The presentation given here integrates and synthesizes the works of several authors.[2]

First Stage

Assume the multiproduct industry transformation function is well behaved, is intertemporally separable, and is homothetically separable in each time period in the input partition I^n, so that it may be represented as $F(q_1, \ldots, q_m, X_1, \ldots, X_n) = 0$.[3] The variables q and X represent outputs and aggregate inputs, respectively. The aggregate inputs are defined by linearly homogeneous aggregator functions of the form $X_i = X_i(x_{i1}, \ldots, x_{iJi})$, $i = 1, \ldots, n$, where the x_{ij}'s represent disaggregate inputs. Under these conditions, and perfect competition, profit maximization can occur in two stages and will be consistent with a single-stage optimization problem (Bliss, chapter 7, property 3).

The first stage of the profit maximization problem is to solve

$$\Pi(\boldsymbol{p}, \boldsymbol{W}) = \max_{\boldsymbol{q},\boldsymbol{X}}[\boldsymbol{p}\boldsymbol{q}' - \boldsymbol{W}\boldsymbol{X}': F(\boldsymbol{q}, \boldsymbol{X}) \in T]. \tag{1}$$

The $\boldsymbol{p}$ and $\boldsymbol{q}$ are $1 \times m$ vectors of output prices and quantities, respectively; $\boldsymbol{X}$ and $\boldsymbol{W}$ are $1 \times n$ vectors of aggregate input quantity and price indices, respectively. Each W_i is defined by a linearly homogeneous aggregator function of the form $W_i(w_{i1}, \ldots, w_{iJi})$ and is dual to the X_i indices. Each w_{ij} represents the factor price corresponding to the disaggregate input x_{ij}, and T is the technology set.

The left-hand side of (1) is aggregate profit function, and applying Hotelling's lemma gives the output supplies and aggregate input demands:

$$\frac{\partial \Pi}{\partial p_l} = q_l = q_l(\boldsymbol{p}, \boldsymbol{W}), \qquad l = 1, \ldots, m, \tag{2}$$

$$-\frac{\partial \Pi}{\partial W_i} = X_i = X_i(\boldsymbol{p}, \boldsymbol{W}), \qquad i = 1, \ldots, n, \tag{3}$$

which are homogeneous of zero degree in $(\boldsymbol{p}, \boldsymbol{W})$ by Euler's theorem.

Note that the conceptual problem of treating inputs as final goods in the utility-based models is immediately removed by modeling imports as derived demand from a profit maximization problem. The first empirical problem with utility-based models is defining the first-stage aggregates. Defining the first-stage aggregate inputs in the profit model is conceptually more intuitive because the standard input aggregates (i.e., energy, labor, capital, and materials) appeal to Chambers' criterion of "common sense" (Chambers 1988, p. 157) and are therefore less debatable.

To see why, consider how the two-stage profit maximization approach removes the conceptual difficulties associated with the first stage of the utility-based soybean model discussed earlier. First, because soybeans are an input and are no longer placed in a utility function, the question of weak separability between other oilseeds or grains is no longer a concern, unless other oilseeds or grains are used in the production of the output(s). Second, there is no longer the implicit condition that all goods in the utility function must be considered and that all of the goods, other than soybeans, form a single consistent aggregate called "all other goods." Finally, the implications of the homothetic separability assumption are more acceptable in production theory (linear expansion paths) than in utility theory (linear Engel curves).

Second Stage

When the transformation function is homothetically separable in the I^n partition, then Blackorby, Primont, and Russell (BPR) show the sufficient conditions for two-stage optimization are satisfied and conditional demands are obtainable. The conditional demands can be easily obtained from Hotelling's lemma.[4] That is, differentiate the aggregate profit function (1) with respect to the disaggregate input price (w_{ij}) to yield

$$(4) \qquad -\frac{\partial \Pi}{\partial w_{ij}} = x_{ij}, \qquad i = 1, \ldots, n; \;\; j = 1, \ldots, J_i.$$

As in all duality theory, there are two equivalent representations at the optimal point of x_{ij} because there are theoretically two equivalent ways to write the second stage of the two-stage profit maximization problem. These two forms are the same as the two equivalent forms of the second stage of the two-stage utility maximization approach.[5] However, the two solutions to the second-stage problem are empirically different.

One approach is to minimize the cost of obtaining a predetermined level of aggregate input,

$$(5) \qquad C_i(\mathbf{w}_i, X_i) = \min_{x_{ij}} \left[\sum_{j=1}^{J_i} w_{ij}x_{ij} : X_i = X_i(x_{i1}, \ldots, x_{iJ_i}) \right], \qquad i = 1, \ldots, n,$$

which has the solution $x_{ij}^h = x_{ij}(\mathbf{w}_i, X_i)$. The left-hand side of (5) is the cost function for this problem; $\mathbf{w}_i$ is the vector of prices in the ith group, and x_{ij}^h is a conditional Hicksian input demand function, because it is conditional on the aggregate quantity index (X_i) determined in the first stage; x_{ij}^h is homogeneous of zero degree in $\mathbf{w}_i$ by Euler's theorem. By duality theory, the alternative formulation is

$$(6) \qquad X_i(\mathbf{w}_i, C_i) = \max_{x_{ij}} \left[X_i = X_i(x_{i1}, \ldots, x_{iJ_i}) : C_i = \sum_{j=1}^{J_i} w_{ij}x_{ij} \right], \qquad i = 1, \ldots, n,$$

which leads to the solution $x_{ij}^m = x_{ij}(\mathbf{w}_i, C_i)$. The left-hand side of (6) is the indirect production (index) function and is analogous to the indirect utility function; $x_{ij}^m = x_{ij}(\mathbf{w}_i, C_i)$ is homogeneous of degree zero in $(\mathbf{w}_i, C_i)$ by Euler's theorem and is known as the conditional Marshallian (constant cost) input demand function, since it is conditional on the predetermined expenditure, C_i. Chambers (1982) shows that the solutions to equations (5) and (6) are completely analogous to the consumer concepts of Hicksian and Marshallian demand, respectively, and are equivalent at the optimal point. Davis and Kruse have labeled equation (5) and its solution "the conditional Hicksian system," and equation (6) and its solution "the conditional Marshallian system."

While these systems are theoretically equivalent at the optimal point, the conditional Hicksian system suffers two empirical problems. First, the conditional Hicksian system suffers the general errors-in-variable problem alluded to in Davis and Kruse, and therefore leads to biased parameter estimates. Second, the conditional Hicksian system is also "separability inflexible" (BPR, chapter 8). Separability inflexible means that once the restrictions for testing weak separability are imposed, the aggregator function also is restricted. Therefore, weak separability may be rejected due to the restrictive form of the aggregator function and not due to a violation of weak separability. Separability inflexible also implies one cannot perform separate tests of the two sufficient conditions for two-stage budgeting (i.e., weak separability and linear homogeneity of the aggregator function). Alternatively, the conditional Marshallian system admits neither the errors-in-variable problem (Davis and Kruse) nor the "separability inflexible" problem (Yuhn), and can therefore be used to test for the two sufficient conditions for two-stage budgeting. So, on empirical grounds, the conditional Marshallian input demand is preferred.

Relationship Between Conditional and Unconditional Elasticities

To obtain unconditional elasticity estimates requires the combination of the first-stage aggregate elasticities and second-stage conditional elasticities. Because the second-stage problem for the two-stage profit and two-stage utility maximization problems is identical, then the estimated conditional elasticities are identical. However, in isolation the conditional elasticities can lead to biased inference, because they do not encompass all of the price effects captured by the unconditional elasticities. Hence, the relationship between unconditional and conditional elasticities is important. The unconditional elasticities are:[6]

$$\frac{\partial \ln(q_l)}{\partial \ln(w_{ij})} = \theta_{li} s_{ij}, \tag{7}$$

$$\frac{\partial \ln(q_l)}{\partial \ln(p_k)} = \epsilon_{lk}, \tag{8}$$

$$\frac{\partial \ln(x_{ij})}{\partial \ln(p_l)} = \Omega_{il}, \tag{9}$$

$$\frac{\partial \ln(x_{ij})}{\partial \ln(w_{km})} = \eta_{ijkm} = \Omega_{ik} s_{km}, \qquad j \in I^i,\ m \in I^k,\ i \neq k, \tag{10}$$

$$\frac{\partial \ln(x_{ij})}{\partial \ln(w_{km})} = \eta_{ijkm} = \eta^{C_i}_{ijkm} + (\Omega_{ii} + 1)s_{im}, \qquad j, m \in I^i,\ i = k, \tag{11}$$

where

$$\Omega_{ik} = \frac{\partial \ln(X_i)}{\partial \ln(W_k)}, \quad \eta^{C_i}_{ijkm} = \frac{\partial \ln(x_{ij})}{\partial \ln(w_{km}) \mid_{C_i}}, \quad \Omega_{il} = \frac{\partial \ln(X_i)}{\partial \ln(p_l)}, \quad s_{km} = \frac{w_{km} x_{km}}{C_k},$$

$$\theta_{li} = \frac{\partial \ln(q_l)}{\partial \ln(W_i)}.$$

Equation (7) shows that the output elasticities with respect to the disaggregate input prices are a product of the output elasticities with respect to the aggregate input price indices and the disaggregate cost share. Equation (8) is the output price elasticity and is determined solely by the first-stage results. Equation (9) shows that all disaggregate inputs in the lth partition must be either normal or inferior factors and have the same elasticity with respect to the output price (p_l) as the aggregate input (X_i). Equation (10) shows that if two disaggregate inputs are not in the same partition, then the unconditional input cross-price elasticities are determined by the aggregate input cross-price elasticities weighted by the cost share. Therefore, all disaggregate inputs between two partitions must be either substitutes or complements, depending on whether the aggregates are substitutes or complements. Equation (10) also shows that each unconditional cross-price elasticity in partition I^i is the same with respect to an element in partition I^k. Equation (11) shows that when two disaggregate inputs are in the same partition, the unconditional input demand elasticities are comprised of two terms. The first term is the conditional elasticity from the second stage. The second term accounts for the change in expenditure allocated to the ith aggregate input due to a change in a disaggregate input price.

The possible biasedness of solely using conditional elasticities is highlighted by equation (11). Equation (11) shows the importance of capturing the first- and second-stage effects of a price change, because the unconditional elasticity may be contrary in sign (substitute or complement) and in elasticity value (inelastic or elastic) to either the isolated aggregate or conditional elasticities. Because total expenditure allocations do change as prices change, this obviously brings into question not only the bias in the magnitude of conditional elasticities, but more importantly, the possibility of the conditional elasticities having the wrong sign. This result could have dire consequences for policy analysis. Therefore, the circumstances under which the conditional elasticities will be approximately equal to the unconditional elasticities are of importance.

From equation (11), there are two cases in which the conditional elasticity is approximately equal to the unconditional elasticity: (*a*) if the own-price aggregate input demand elasticity is approximately unitary, and (*b*) if the cost share is small. If either of these two conditions holds, the error resulting from misapplied conditional utility maximization is small. This is due solely to the fact that the second-stage results from both approaches are equivalent, as discussed. However, at the aggregate level, the aggregate input demand elasticities are expected to be more inelastic because of fewer substitutes. In this case, the unconditional own-price elasticity would be more inelastic than the conditional elasticity. This theoretical result implies that policy implications based on own-price conditional elasticities for any functional form (Alston et al.) can be very misleading, unless the second condition holds. Notice that the unconditional elasticity can be obtained by adding the share to the conditional elasticity if the own-price aggregate input demand elasticity is perfectly inelastic.

Maintaining any of these conditions as assumptions is not advised, and equations (7) through (11) all highlight the need to estimate the first and second stages of the model. It should be noted that if the imported commodity is an input, then the unconditional elasticities given by (7) through (11) will be structural in the conventional sense, because the correct set of regressors is being used in the first stage. The same cannot be said of misapplied utility-based models.

Perhaps the most important theoretical aspect of the two-stage profit maximization model is that it satisfies Quine and Ullian's first virtue of a hypothesis: conservatism. Quine and Ullian state that a hypothesis is preferable to its predecessor if it conserves (validates) all previous hypotheses. The two-stage profit maximization model satisfies this criterion, because it conserves the empirical advantages of the two-stage utility models—efficient use of degrees of freedom and mitigation of multicollinearity. More importantly, it conserves (validates) all previously estimated conditional demand systems which were based on utility theory. For example, the conditional Hicksian system (5) conserves all Armington-based models (e.g., Armington; Davis and Kruse; Duffy, Wohlgenant, and Richardson; Grennes, Johnson, and Thursby; Haniotis; Sarris). The conditional Marshallian system (6) conserves all conditional AIDS-based models (e.g., Alston et al.; de Gorter and Meilke; Heien and Pick).

The conserving criterion is satisfied because the second-stage problem from the two-stage profit maximization approach is isomorphic (observationally equivalent) with the second-stage problem from the two-stage utility maximization approach. This isomorphism provides a solution to the puzzle that conditional demand systems based on utility theory are conceptually flawed but empirically successful. Because of the isomorphism, it can be argued that previous conditional demand studies actually have estimated the second stage of a two-stage profit maximization procedure, as opposed to the believed second stage of a two-stage utility maximization procedure. The only difference in the second stage between the two-stage profit and utility formulations is in interpretation. While this point has been made other places (e.g., Theil, p. 74), the prevalent use of two-stage utility theory to motivate conditional demands for inputs indicates that it is not widely known or is ignored due to misperceptions about the relative empirical ease of using two-stage utility models. Therefore, we will present an illustration of the empirically appealing two-stage profit maximization model.

An Empirical Illustration

As an empirical illustration, we concentrate on estimating the unconditional input demand elasticities given in equations (10) and (11) for hardwood lumber in Japan. We chose Japanese hardwood lumber demand as an econometric illustration for two reasons. First, Japan uses lumber from temperate and tropical sources as inputs in the furniture and construction industries and we wanted to demonstrate the correct econometric procedures for testing the intuitive notion that temperate and tropical hardwood lumber can be modeled as two conditional demand systems. Second, the data set available for this industry is very rich, yet we were unable to obtain a quantity measure for one of the inputs. Because industry-level data may be difficult to obtain in some cases, this can be a shortcoming of the two-stage profit approach. We wished to explore the theoretical implications of limited data within the model.

The data series span 1970 through 1988. Industry studies (e.g., Timber Research and Development Association) suggest that it is reasonable to consider four aggregate inputs, besides hardwood lumber: capital, energy, labor, and softwood. A listing of the variables, their definitions, units of measure, and sources is displayed in table 1. All price series were in Yen and deflated by the Wholesale Price Index, found in the *Japan Statistical Yearbook* (Japan Management and Coordination Agency). For a more detailed discussion of the data and markets, see Jensen, Davis, and Bevins.

We followed the standard recursive estimation approach of estimating the second stage first and then the first stage (e.g., Barnett; Fuss).[7] Each stage is estimated using Zellner's iterated seemingly unrelated regression (ITSUR) method, which is equivalent to full information maximum likelihood (see Fuss, pp. 99–102). Throughout the estimation we assumed an additive, contemporaneously correlated error structure with finite variances and covariances. We imposed symmetry, adding up, and homogeneity, and only tested the sufficient conditions for two-stage budgeting: weak separability and linear homogeneity in the aggregator functions.

Second-Stage Estimation

In the second stage of the model, we estimated the conditional Marshallian system because it admits no errors-in-variables or separable inflexibility problem as discussed. Yuhn proves the Marshallian system overcomes the separable inflexible problem and suggests the following translog indirect aggregator specification:

$$(12) \qquad \ln(X_i) = \alpha_i + \sum_j \gamma_{ij}\ln(w_{ij}) + \gamma_i\ln(C_i) + \sum_j \gamma_{ijc}\ln(w_{ij})\ln(C_i) + \frac{1}{2}\gamma_{ii}[\ln(C_i)]^2 + \frac{1}{2}\sum_j\sum_k \gamma_{ijk}\ln(w_{ij})\ln(w_{ik}),$$

and by Roy's identity,

Table 1. Variable Names, Definitions, Units of Measure, and Sources

Variable Name	Definition	Units of Measure	Source[a]
First Stage:			
q_1	Quantity of furniture and fixtures	Pieces	JSY
p_1	Price index of furniture and fixtures	1970 = 100	JSY
q_2	Total construction area	1,000 square meters	JSY
p_2	Price index of construction	1970 = 100	JSY
X_1	Hours worked by regular workers, furniture and fixtures manufacturing	1,000 hours	JSY
W_1	Index of hourly wages, furniture and fixtures manufacturing	1970 = 100	JSY
X_2	Quantity of woodworking machinery	Pieces	JSY
W_2	Price index of woodworking machinery	1970 = 100	JSY
W_3	Wholesale price index for petroleum and coal	1970 = 100	JSY
X_4	Quantity index of construction materials and machinery[b]	1970 = 100	JSY
W_4	Price index of construction materials and machinery	1970 = 100	JSY
X_5	Hours worked by regular workers, construction	1,000 hours	JSY
W_5	Index of hourly wages, construction	1970 = 100	JSY
X_6	Quantity index of other wood materials[c]	1970 = 100	YFP
W_6	Price index of other wood materials	1970 = 100	FPP
X_7	Quantity index of temperate nonconiferous lumber	Cubic meters	CTS
W_7	Instrumental Divisia price index of temperate nonconiferous lumber	1970 = 100	CTS
X_8	Quantity index of tropical nonconiferous lumber	Cubic meters	CTS, YFP
W_8	Instrumental Divisia price index of tropical nonconiferous lumber	1970 = 100	CTS, FPP
Second Stage:			
x_{6j}	Quantity of temperate nonconiferous lumber; j = US, Other Temperate[d]	Cubic meters	CTS
w_{6j}	Price of temperate nonconiferous lumber; j = US, Other Temperate	Yen/cubic meter	CTS
x_{7j}	Quantity of tropical nonconiferous lumber; j = Indonesia, Malaysia, Philippines, Japan, Other Tropical[e]	Cubic meters	CTS, YFP
w_{7j}	Price of tropical nonconiferous lumber; j = Indonesia, Malaysia, Philippines, Japan,[f] Other Tropical	Yen/cubic meter	CTS, FPP

[a] JSY = *Japan Statistical Yearbook* (Japan Management and Coordination Agency), YFP = *Yearbook of Forest Products* [United Nations (UN)], FPP = *Forest Products Prices* (UN), and CTS = *Commodity Trade Statistics* (UN).

[b] This is a Divisia quantity index constructed from the prices and quantities of construction machinery including land preparation machinery, cranes/excavators, and concrete machinery, and construction materials including iron and steel, hollow cement blocks, and wooden fiber cement board. The dual Divisia price index is constructed using Fisher's weak factor reversal test.

[c] The other wood, temperate nonconiferous, and tropical nonconiferous quantity indices are Divisia indices. The Divisia price index for other wood is the dual Divisia index as in construction. Other wood includes softwood lumber and plywood.

[d] Other temperate sources included Canada, China, the USSR, EC countries, and North and South Korea.

[e] Other tropical sources included Singapore, Thailand, and Brazil. Japan is included as a tropical source because many of the logs that are imported for sawn wood are tropical.

[f] The price of lauan thick boards is used as a proxy for Japanese lumber price. This proxy is plausible because, during the sample period, over 70% of logs consumed were from major tropical exporters.

$$(13) \qquad S_{ij} = -\left[\frac{\gamma_{ij} + \gamma_{ijc}\ln(C_i) + \sum_k \gamma_{ijk}\ln(w_{ik})}{\gamma_i + \gamma_{ii}\ln(C_i) + \sum_k \gamma_{ikc}\ln(w_{ik})}\right], \qquad j, k = 1, \ldots, J_i,$$

where $S_{ij} = w_{ij}x_{ij}C_i^{-1}$ is the ij cost share. We used the entire system, (12) and (13), to

Table 2. Summary of Test Statistics for the Second-Stage Models

Models	Sum of Squared Errors	Calculated χ^2	Degrees of Freedom	Reject/Fail to Reject[a]
Temperate and Tropical System:				
Unconstrained Model	97.17			
Linear Homogeneity	105.63	8.45	8	Fail to reject
Additive Weak Separability	115.00	9.37	8	Fail to reject
Temperate System:[b]				
Unconstrained Model	32.01			
Linear Homogeneity	35.00	3.00	3	Fail to reject
Tropical System:[b]				
Unconstrained Model	74.06			
Linear Homogeneity	80.09	6.03	6	Fail to reject

Notes: To avoid the singularity problem induced by shares summing to one, the Japanese share equation was dropped in the overall system and in the tropical system. The other temperate share equation was dropped in the temperate system.

[a] The appropriate significance level of the test, δ_i, depends on the significance level of the prior test, such that:

$$\delta_i = 1 - \prod_{j=1}^{i} (1 - \delta_j).$$

The significance level selected for δ_1 is .05, so $\delta_2 \cong .10$.

[b] Temperate sources were defined to be the United States and other temperate sources; tropical sources were defined to be Indonesia, Malaysia, the Philippines, Japan, and all other tropical sources (see table 1).

sequentially test for weak separability and then linear homogeneity of the aggregator function. The parametric restrictions for weak separability and linear homogeneity are, respectively,

$$\gamma_{ijm} = \gamma_{ikm} = 0, \qquad j, k \in I^i, m \notin I^i, \tag{14}$$

$$\gamma_i = 1, \quad \gamma_{ii} = \gamma_{ijc} = 0 \; \forall \; j. \tag{15}$$

The results of the tests are shown in table 2. We first assumed that aggregate hardwood lumber (tropical and temperate) is weakly separable from all other inputs. We then tested and failed to reject linear homogeneity in aggregate hardwood lumber expenditures, as assumed by the theory. Next, we tested and failed to reject weak separability between tropical and temperate hardwood lumber in the aggregate hardwood lumber system, so there exist consistent tropical and temperate aggregates. We then tested and failed to reject the hypothesis that the sub-aggregate temperate system and sub-aggregate tropical system were each linearly homogeneous in expenditures, as required by theory. Hence, the sufficient conditions for modeling temperate and tropical hardwood lumber as two separate conditional Marshallian systems are satisfied.

The parameter estimates for the temperate and tropical systems are displayed in table 3, along with their asymptotic t-statistics. Many of the parameters are significantly different from zero. From these estimates we calculated the conditional (second-stage) price elasticities using the formula

$$\eta_{ijm}^{C_i} = -[\delta_{ijm} + s_{ij}^{-1}\gamma_{ijm}], \tag{16}$$

where $\delta_{ijm} = 1$ if $j = m$, and zero otherwise. The conditional demand elasticity matrix is presented in table 4. With the exception of the Philippines, all own-price elasticities are negative. Many of the cross-price elasticities are negative, suggesting that lumber from these sources is considered a complement. But as mentioned, it is important to account for the first-stage results because conditional elasticities in isolation can be misleading. We should note, however, that because the cost shares are small for this market (table 4), the second criterion for unconditional elasticities being approximately equal to conditional

Table 3. Parameter Estimates from the Temperate and Tropical Second-Stage Systems

		Estimated Parameters: γ_{ij}					
Equation	Intercepts	US	OTM	I	P	M	OTR
	γ_i						
US	−.849 (22.39)	.556 (7.05)	−.556 (7.05)				
I	.028 (6.26)			.024 (8.39)	.006 (2.64)	.011 (6.69)	.009 (8.53)
P	−.017 (3.35)				−.020 (4.08)	.003 (1.26)	.008 (7.85)
M	.003 (1.33)					−.001 (.52)	.005 (7.97)
OTR	.009 (6.39)						.0004 (.90)
	α_i						
Temperate	6.663 (574.79)						
Tropical	6.043 (1,468.47)						

Notes: US = United States, OTM = Other Temperate Sources, I = Indonesia, P = Philippines, M = Malaysia, and OTR = Other Tropical Sources. Values in parentheses are the absolute values of the asymptotic *t*-statistics.

elasticities is satisfied, so the first-stage results are likely to have little impact on the unconditional elasticities.

First-Stage Estimation

Because utility-based models make defining the first-stage aggregates difficult, there exists an incentive either to not attempt first-stage estimation or to let available data define the first-stage aggregates. Neither alternative is attractive, because the first yields only conditional elasticities and the second results in aggregates that do not appeal to "common sense." While the first-stage aggregates in the two-stage profit model appeal to "common sense," there is no assurance that all the required industry-level data will be available. An unavailable price series can be circumvented theoretically either by employing a proxy via a perfect competition assumption or by redefining the unrestricted profit function to be the restricted (variable) profit function. Unfortunately, the unavailable quantity series is not handled with such theoretical ease, and implicitly restricts the modeled technology to be joint.

Lopez characterized two types of flexible functional forms for profit functions: flexible functional forms characterized by nonlinear transformations of the dependent variable (NLFFF), and flexible functional forms characterized by linear transformations of the dependent variable (LFFF). When a quantity variable is unavailable, Hotelling's lemma applied to an NLFFF, like the translog, yields a system of product and input share equations which cannot be defined, so the choice of functional forms is limited to the LFFF. But, as Lopez shows, the LFFF implies that the multiproduct transformation function is input-output separable; hence, either production is joint (Hall) or the individual production functions are quasi-homothetic (Lau). Both jointness and quasi-homotheticity are unappealing alternatives. Jointness increases the dimension of the parameter set and decreases degrees of freedom, while quasi-homotheticity implies all production functions for all outputs are identical up to a multiplicative constant. Pragmatism suggests that jointness be tested empirically before adopting a restrictive quasi-homotheticity technology assumption.

Table 4. Conditional Elasticities from the Temperate and Tropical Second-Stage Systems

	Conditional Elasticity with Respect to the Price of Lumber from:						
	US	OTM	I	P	M	OTR	J
US	−1.294	.034					
OTM	.042	−1.042					
I			−3.863	−.764	−1.366	−1.018	6.011
P			−.914	1.917	−.386	−1.121	−.496
M			−1.404	−.331	−.895	−.631	2.261
OTR			−.932	−.857	−.562	−1.045	2.396
J			.052	−.004	.019	.023	−1.091

Notes: Abbreviations are as defined in table 3. The conditional elasticities are calculated at the means. Mean cost shares are: US = .5491, OTM = .4519, I = .0084, P = .0070, M = .0082, OTR = .0092, and J = .9672.

As seen in table 1, the only variable we were unable to obtain was energy consumption by industry, i.e., there is no X_3. We therefore had to choose an LFFF and selected the convenient revenue form of the generalized Leontief (Livernois and Ryan).

$$(17) \qquad r_l = \sum_{l=1}^{m}\sum_{s=1}^{m} \alpha_{ls}\sqrt{p_l p_s} + \sum_{l=1}^{m}\sum_{i=1}^{n} \beta_{li}\sqrt{p_l W_i}, \qquad l = 1, \ldots, m,$$

$$(18) \qquad r_i = \sum_{i=1}^{n}\sum_{l=1}^{m} \beta_{il}\sqrt{W_i p_l} + \sum_{i=1}^{n}\sum_{j=1}^{n} \delta_{ij}\sqrt{W_i W_j}, \qquad i = 1, \ldots, n,$$

where, by symmetry, $\alpha_{ls} = \alpha_{sl}$, $\beta_{li} = \beta_{il}$, and $\delta_{ij} = \delta_{ji}$, and linear homogeneity is implicitly imposed. In this model, $r_l = p_l q_l$; $l, s = 1, 2$; and $r_i = -W_i X_i$ $(i, j = 1, \ldots, 8)$ (see table 1 for variable descriptions). Following Fuss, we formed an instrumental Divisia price index for temperate and tropical lumber from the second-stage results, estimated the system, and tested for nonjointness.

The results of the nonjointness test and the first-stage parameter estimates are given in table 5. Though casual observation suggests that furniture and construction are nonjoint products, we suspected that nonjointness would be rejected based on Chambers and Just's conclusion that limited input data biases the test toward jointness (p. 994). The results from the nonjointness test confirmed our suspicions. Given the restrictive functional form, it should not be surprising that the results of the first stage are mixed. Though some of the parameters are significant and conform in sign to theory, others do not.

To calculate the unconditional hardwood lumber elasticities from equations (10) and (11), we first calculated the four required aggregate price elasticities from table 5, evaluated at the means: temperate own-price ($\Omega_{77} = -.916$), temperate–tropical cross-price ($\Omega_{78} = -.139$), tropical–temperate cross-price ($\Omega_{87} = -.313$), and tropical own-price ($\Omega_{88} = -.01$). In table 6, we present the unconditional hardwood lumber input demand elasticities obtained by using (10) and (11). An obvious pattern in the elasticity matrix in table 6 is that each of the cross-price elasticity rows between elements of the separable group are equal because of (10). Also, because the mean shares of the tropical sources are all approximately equal, all cross-price elasticities of U.S. and other temperate hardwood with respect to Indonesia, the Philippines, Malaysia, Japan, and other tropical sources also are equal. Comparison of the unconditional elasticities in table 6 with the conditional elasticities in table 4 reveals there is very little difference overall in the elasticities within the temperate and tropical blocks, due to the small cost shares (i.e., case *b*).

Summary and Conclusions

The purpose of this article was to present an import modeling framework that removes the previously overlooked conceptual and empirical disadvantages of import demand systems based on two-stage utility maximization, but to retain their empirical advantages.

Table 5. First-Stage Parameter Estimates of Output Revenue and Factor Expenditure Equations

	Estimated Parameters									
Equation	Furniture	Construction	Furniture Labor	Furniture Capital	Energy	Construction Capital	Construction Labor	Other Wood	Temperate Lumber	Tropical Lumber
Furniture	−.272 (1.866)	−.149 (2.804)	.100 (2.031)	−.043 (.887)	−.087 (1.263)	.058 (.679)	.531 (5.622)	.295 (.614)	−.231 (.716)	.132 (.918)
Construction		−.041 (2.804)	.074 (2.195)	.335 (1.371)	−.043 (1.526)	.099 (2.267)	.182 (3.808)	.074 (1.465)	−.077 (.562)	.081 (.780)
Furniture Labor			−.525 (9.985)	−.102 (4.328)	.029 (1.038)	.111 (2.172)	.077 (1.689)	.078 (1.548)	−.310 (2.512)	.074 (.645)
Furniture Capital				.478 (5.361)	−.170 (.136)	−.139 (4.286)	−.057 (1.101)	−.182 (4.608)	−.482 (2.132)	−.154 (2.742)
Construction Capital						.003 (.034)	−.033 (.549)	.002 (.024)	.030 (.130)	−.170 (1.299)
Construction Labor							−1.802 (20.787)	.005 (.082)	−.139 (.595)	.416 (3.66)
Other Wood								.110 (1.144)	−.006 (.026)	−.567 (4.03)
Temperate Lumber									.695 (.690)	.214 (.546)
Tropical Lumber										−.308 (.661)

Wald Test (Nonjointness): 108.73*

Notes: Values in parentheses are the absolute values of the asymptotic t-statistics. The asterisk (*) denotes significance at $\alpha = .00001$.

Table 6. Unconditional Elasticities from the First- and Second-Stage Systems

	Elasticity with Respect to the Price of Lumber from:						
	US	OTM	I	P	M	OTR	J
US	−1.248	.072	−.001	−.001	−.001	−.001	−.134
OTM	.088	−1.004	−.001	−.001	−.001	−.001	−.134
I	−.172	−.141	−3.85	−.757	−1.358	−1.009	6.97
P	−.172	−.141	−.906	1.924	−.378	−1.111	.462
M	−.172	−.141	−1.396	−.324	−.887	−.622	3.219
OTR	−.172	−.141	−.924	−.850	−.554	−1.036	3.354
J	−.172	−.141	.061	.003	.027	.032	−.134

Note: Abbreviations are as defined in table 3.

The conceptual disadvantage is that most imported commodities are inputs and not final goods. This conceptual misspecification leads to three empirical disadvantages that all imply biased parameter estimates and hence biased inference. We have shown and discussed how a two-stage, multiproduct, profit maximization model overcomes these disadvantages and yet retains the empirical advantages of the two-stage utility models. We also have identified the conditions under which the biased conditional demand elasticities approximately equal the unbiased unconditional demand elasticities. These two points are especially important, because conditional demand systems and elasticities frequently are reported.

We have presented an empirical illustration using Japanese demand for lumber to demonstrate the econometric procedures involved in testing the sufficient conditions for two-stage maximization and in estimating unconditional elasticities. Because estimating unconditional elasticities requires estimating the model's first stage, we have discussed the theoretical implications of limited data on profit functions in the first stage.

The two-stage profit maximization approach is not an empirical panacea, but, as made clear in this study, neither is the two-stage utility maximization approach. As with most models, both approaches suffer weaknesses in the bridge between theory and empirical implementation. Hopefully, the inferential consequences of these weaknesses have been demonstrated. Comparatively, however, the bridge between theory and empirics is stronger for the two-stage profit procedure than for the two-stage utility procedure, when the imported agricultural commodity is an input. Thus, based on an awareness of the limited applicability of the two-stage utility model, inferences and policy implications should be made with considerable caution when the model is applied to commodities that are inputs.

[*Received March 1994; final revision received August 1994.*]

Notes

[1] The producer theory-based alternative presented in this study generally is applicable to all input demand systems; however, the focus of the article on import demand systems is due to the almost exclusive nature in which the theory of two-stage utility maximization has been applied in the literature to model agricultural import demand.

[2] The work in this section is an integration and synthesis of the work of Blackorby, Primont, and Russell (BPR); Bliss; Chambers (1982, 1988); Fuss; Lau; and Yuhn. The work presented here parallels the work of Theil, and more closely, Pinard. However, the work presented differs theoretically and empirically, in order to facilitate a comparison with the agricultural import demand literature cited. None of the works cited comprehensively present all of the points made in this article.

[3] All standard properties of the transformation function, profit function, cost function, and indirect production function are assumed to hold (see Chambers 1982, 1988).

[4] The proof is presented in the appendix.

[5] Compare the second-stage (conditional) problem discussed in this section with that of Deaton and Muellbauer (chapter 5).

[6] The derivations are found in the appendix.

[7] An ongoing debate in the literature concerning this approach is the exogeneity of expenditures in the second stage (e.g., LaFrance). We failed to reject exogeneity of expenditures in all systems with the Hausman test. Exogeneity of expenditures is a moot point, if the consistent conditions for two-stage budgeting are imposed and a double log specification is used, because expenditures drop out of the share equations.

References

Alston, J., C. Carter, R. Green, and D. Pick. "Whither Armington Trade Models?" *Amer. J. Agr. Econ.* 72(1990): 455–67.

Armington, P. "A Theory of Demand for Products Distinguished by Place of Production." *International Monetary Fund Staff Papers* 16(March 1969):159–78.

Barnett, W. *Consumer Demand and Labor Supply.* Amsterdam: North-Holland, 1981.

Blackorby, C., D. Primont, and R. Russell. *Duality, Separability, and Functional Structure: Theory and Economic Applications.* New York: North-Holland, 1978.

Bliss, C. *Capital Theory and the Distribution of Income.* Amsterdam: North-Holland, 1975.

Chambers, R. *Applied Production Analysis: A Dual Approach.* Cambridge: Cambridge University Press, 1988.

———. "Duality, the Output Effect, and Applied Comparative Statics." *Amer. J. Agr. Econ.* 64(1982):152–56.

Chambers, R., and R. Just. "Estimating Multioutput Technologies." *Amer. J. Agr. Econ.* 71(1989):981–95.

Davis, G. C., and N. C. Kruse. "Consistent Estimation of Armington Demand Models." *Amer. J. Agr. Econ.* 75(1993):719–23.

Deaton, A., and J. Muellbauer. *Economics and Consumer Behavior.* Cambridge: Cambridge University Press, 1986.

de Gorter, H., and K. Meilke. "The EC's Wheat Price Policies and International Trade in Differentiated Products." *Amer. J. Agr. Econ.* 69(1987):223–29.

de Janvry, A., and J. Bieri. "On the Problem of Degrees of Freedom in Analysis of Consumer Behavior." *Amer. J. Agr. Econ.* 50(1968):1720–36.

Duffy, P., M. Wohlgenant, and J. Richardson. "The Elasticity of Export Demand for U.S. Cotton." *Amer. J. Agr. Econ.* 72(1990):468–74.

Fuss, M. "The Demand for Energy in Canadian Manufacturing." *J. Econometrics* 5(1977):89–116.

Goddard, E. W. "Export Demand Elasticities in the World Market for Beef." In *Elasticities in International Trade,* eds., C. A. Carter and W. H. Gardiner, pp. 225–54. Boulder CO: Westview Press, 1988.

Grennes, T. J., P. R. Johnson, and M. Thursby. *The Economics of World Grain Trade.* New York: Praeger Publishers, 1978.

Hall, R. "The Specification of Technology with Several Kinds of Output." *J. Polit. Econ.* 81(1973):878–91.

Haniotis, T. "European Community Enlargement: Impact on U.S. Corn and Soybean Exports." *Amer. J. Agr. Econ.* 72(1990):289–97.

Heien, D., and D. Pick. "The Structure of International Demand for Soybean Products." *S. J. Agr. Econ.* 23(1991):134–43.

Japan Management and Coordination Agency, Statistics Bureau. *Japan Statistical Yearbook.* Tokyo. Various issues, 1971–91.

Jensen, K., G. Davis, and S. Bevins. "Japanese Demand for Hardwood Lumber from the United States." Bull. No. 685, Tennessee Agr. Exp. Sta., University of Tennessee, Knoxville, June 1994.

LaFrance, J. "When Is Expenditure 'Exogenous' in Separable Demand Models?" *West. J. Agr. Econ.* 16(1991): 49–62.

Lau, L. "Profit Functions of Technologies with Multiple Inputs and Outputs." *Rev. Econ. and Statis.* 54(1972): 281–89.

Lin, B., and L. Makus. "Japanese Barley Imports: Implications of the 1988 Japanese Beef Market Access Agreement." *Can. J. Agr. Econ.* 39(1991):423–37.

Livernois, J., and D. Ryan. "Testing for Non-Jointness in Oil and Gas Exploration: A Variable Profit Function Approach." *Internat. Econ. Rev.* 30(1989):479–504.

Lopez, R. "Structural Implications of a Class of Flexible Functional Forms for Profit Functions." *Internat. Econ. Rev.* 26(1985):593–601.

Pinard, J. "A Disaggregated International Linkage Model: The Profit Function Approach." Unpub. Ph.D. diss., Econ. Dept., University of California–San Diego, 1975.

Quine, W. V., and J. S. Ullian. "Hypothesis." In *Introductory Readings in the Philosophy of Science,* eds., E. D. Klemke, R. Hollinger, and A. D. Kline, pp. 239–45. New York: Prometheus Books, 1988.

Sarris, A. "European Community Enlargement and World Trade in Fruits and Vegetables." *Amer. J. Agr. Econ.* 65(1983):235–46.

Theil, H. *System-Wide Explorations in International Economics, Input-Output Analysis, and Market Research.* Amsterdam: North-Holland, 1980.

Thompson, R. L. "Discussion and Concluding Comments." In *Elasticities in International Trade,* eds., C. A. Carter and W. H. Gardiner, pp. 300–301. Boulder CO: Westview Press, 1988.

———. *A Survey of Recent U.S. Developments in International Agricultural Trade Models.* Bibliographies and Literature of Agriculture Pub. No. 21, U.S. Department of Agriculture/Economic Research Service, Washington DC, September 1981.

Timber Research and Development Association (TRADA). *Timbers of the World,* Vols. I and II. Bath, Ontario, Canada: Pitman Press, 1979.
United Nations, Food and Agriculture Organization. *Commodity Trade Statistics.* UN/FAO, New York. Various issues, 1971–89.
———. *Forest Products Prices.* UN/FAO, Rome, Italy. Various issues, 1971–89.
———. *Yearbook of Forest Products.* UN/FAO, Rome, Italy. Various issues, 1971–89.
Yuhn, K. "Functional Separability and the Existence of Consistent Aggregates in U.S. Manufacturing." *Internat. Econ. Rev.* 32(1991):229–50.

Appendix

Hotelling's Lemma in Disaggregate Inputs

Proof: From the aggregate profit function (1) and the chain rule, we have

$$\text{(A1)} \qquad \frac{\partial \Pi}{\partial w_{ij}} = \frac{\partial \Pi}{\partial W_i}\frac{\partial W_i}{\partial w_{ij}} = -X_i\frac{\partial W_i}{\partial w_{ij}}.$$

To prove (A1) equals x_{ij}, note that because of linear homogeneity of the aggregator functions,

$$\text{(A2)} \qquad W_i(w_{i1}, \ldots, w_{iJ_i})X_i(x_{i1}, \ldots, x_{iJ_i}) = \sum_{m=1}^{J_i} w_{im}x_{im}.$$

Therefore, differentiating (A2) with respect to x_{ij} yields

$$\text{(A3)} \qquad W_i\frac{\partial X_i}{\partial x_{ij}} = w_{ij}, \qquad j, m = 1, \ldots, J_i.$$

Differentiating (A2) with respect to w_{ij}, and some algebra, yields

$$\text{(A4)} \qquad \frac{\partial W_i}{\partial w_{ij}}X_i \equiv x_{ij} + \sum_{m=1}^{J_i}\left[w_{im} - W_i\frac{\partial X_i}{\partial x_{im}}\right]\frac{\partial x_{im}}{\partial w_{ij}}.$$

Substituting (A3) into (A4) yields

$$\text{(A5)} \qquad \frac{\partial W_i}{\partial w_{ij}}X_i = x_{ij}.$$

Substituting (A5) into (A1) yields equation (4) in the text.

Unconditional Elasticity Derivations

From the first stage of the model, we have

(A6) $\quad q_l = q_l(p, W) \quad$: lth output supply,

(A7) $\quad X_i = X_i(p, W) \quad$: ith aggregate input demand,

and

(A8) $\quad C_i = W_i(w_{i1}, \ldots, w_{iJ_i})X_i(p, W) \quad$: ith expenditure.

From the second stage and by linear homogeneity of the aggregator functions, we have

(A9) $\quad x_{ij}^m = C_i f(w_i) \quad$: jth constant cost input demand,

(A10) $\quad X_i = C_i h(w_i) \quad$: ith indirect production,

and

(A11) $\quad C_i = X_i g(w_i) \quad$: ith indirect cost.

From (A11), cost minimization and linear homogeneity imply

$$\text{(A12)} \qquad \ln(W_i) = \ln(C_i) - \ln(X_i),$$

so, by Shephard's lemma, and X_i being predetermined,

$$\text{(A13)} \qquad \frac{\partial \ln(W_i)}{\partial \ln(w_{ij})} = s_{ij}.$$

Substituting (A8) into (A9), placing in log form, and differentiating now yields

$$\text{(A14)}\quad \frac{\partial \ln(x_{ij})}{\partial \ln(w_{km})} = \frac{\partial \ln(W_i)}{\partial \ln(w_{km})} + \frac{\partial \ln(X_i)}{\partial \ln(W_k)} \frac{\partial \ln(W_k)}{\partial \ln(w_{km})} + \frac{\partial \ln[f(w_i)]}{\partial \ln(w_{km})}$$

$$= \Omega_{ik} s_{km}, \qquad j \in I^i, \quad m \in I^k, \quad i \neq k,$$

which is equation (10) in the text. Note the first and last terms are zero because $i \neq k$. When $i = k$, equation (11) in the text is

$$\text{(A15)}\quad \frac{\partial \ln(x_{ij})}{\partial \ln(w_{km})} = \frac{\partial \ln(W_i)}{\partial \ln(w_{km})} + \frac{\partial \ln(X_i)}{\partial \ln(W_k)} \frac{\partial \ln(W_k)}{\partial \ln(w_{km})} + \frac{\partial \ln[f(w_i)]}{\partial \ln(w_{km})}$$

$$= s_{im}(1 + \Omega_{ii}) + \eta_{ijkm}^{C_i}, \qquad j, m \in I^i, \quad i = k.$$

Equations (7)–(9) within the text are derived similarly.