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The Effects of Including Bankruptcy on Dynamic Investment Decisions

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This article evaluates the effects of including the costs of bankruptcy in a dynamic model of off-farm investment decisions using a stochastic dynamic programming (SDP) model which incorporates the stochastic dynamic nature of investment returns and the interrelationships between financial structure and investment decisions. Our results suggest that in the presence of bankruptcy, optimal investment decisions are affected by financial structure and financial market conditions. Ignoring bankruptcy costs in determining investment decisions results in a high probability of bankruptcy.

Key words: bankruptcy, dynamic investment decisions, stochastic dynamic programming.

Introduction

The agricultural economics literature contains many examples of research that examine the profitability and riskiness of different investment and growth strategies (Young and Barry; Schnitkey, Taylor, and Barry). An important and often contentious issue which arises in these modeling efforts relates to the choice of objective function and treatment of risk. Robison and Barry define risk as arising when bad outcomes can occur. The three most common formalizations of this idea of risk are: (*a*) the probability of disaster, which is commonly referred to as a safety-first criterion; (*b*) the variability of terminal wealth, which is represented by the mean-variance tradeoff; and (*c*) the risk premium required to induce a decision maker to accept a gamble (Collins and Gbur).

Collins and Gbur develop a theoretical model which ties these various concepts of risk together in the presence of bankruptcy and limited liability. They show that when the effects of limited liability are considered, expected utility maximizers should be expected to base their risky decisions partly on the probability of ruin. They suggest that their results help to explain the findings of Masson that sometimes safety-first models explain behavior better than expected utility models that do not incorporate the probability of ruin.

Empirical studies of risk attitudes have identified several potential objective functions. An objective which repeatedly has been identified as being of major importance to decision makers is that of minimizing the probability of default on loans (Fernandez) and avoiding foreclosure or bankruptcy (Patrick and Blake). Kliebenstein et al. found that farmers seemed to perceive some threshold security level above which they preferred to operate. They further suggest that modeling techniques should deal with the issue of increasing returns beyond this threshold level. These findings are consistent with a safety-first objective function (Robison et al.).

A producer survey conducted by Patrick et al. identified substantial safety-first considerations in decision making as well as suggesting the need for research to move towards

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multi-year analysis which reflects important firm-level characteristics, such as financial condition. In a multi-year framework, decisions made in one time period affect decisions in later periods by altering financial structure and the nature of productive assets. The dynamics of asset accumulation are thus a major point of interest as well. The analysis of investment decisions should incorporate these dynamic interrelationships. Studies which have explicitly considered the linkage between financial structure and investment decisions (Schnitkey, Taylor, and Barry) have illustrated the suboptimal nature of more naive decision models.

Our study attempts to develop further the modeling of investment decisions by incorporating important real-world institutions such as bankruptcy into a multi-year analysis. In this study, the idea of a safety-first objective is dealt with by including an objective function in which expected terminal wealth is maximized in the presence of bankruptcy. The occurrence of bankruptcy truncates the expected future accumulations of wealth at zero, while other outcomes allow the process to continue. Thus, bankruptcy results in a zero value in the objective function, while other outcomes are valued at their expected values. The objective function used here explicitly recognizes risk in the form of bankruptcy and limited liability.

Our objective function also follows Antle's notion of risk. Antle states that risk matters if the objective function depends on the parameters of the probability distributions of random variables. The analysis of dynamic models of uncertainty shows that except when certainty equivalence[1] requirements are met, farmers' optimal decisions are affected by risk whether they are risk averse (in the classical expected utility sense) or risk neutral (Antle). Dynamic models with risk-neutral objectives often yield decisions that appear to be risk averse, a result that Taylor (1986) identifies as "pseudo" or "apparent" risk aversion. Further motivation for this approach is provided by the fact that a direct application of von Neumann–Morgenstern utility functions to dynamic choice problems may not yield consistent results (Zacharias).

The purpose of this study is to quantify the impact of including the possibility of bankruptcy on optimal investment decisions. These decisions are investigated using a stochastic dynamic programming (SDP) model which incorporates information on the stochastic dynamic nature of investment returns (both farm and nonfarm) and the interrelationships between financial structure and investment decisions when the costs of bankruptcy are considered. The objective used here is the maximization of expected terminal wealth in the presence of bankruptcy. The implications of this formulation are tested below by comparing results with and without the bankruptcy constraint. Differences in optimal behavior and resulting wealth provide an indication of the costs of ignoring bankruptcy in this type of decision model.

The following section contains a conceptual model of an Illinois hog finishing operation which is then incorporated into the SDP model to determine optimal stock investment decisions. Numerical solution of the SDP model and a graphical analysis of the optimal stock investment decision rules are provided in the third section. These decision rules describe how stock investment decisions are affected by different levels of farm returns, stock prices, interest rates, and financial structure. Conditional probability analysis is used to illustrate the effects of optimal stock investment decisions on expected financial structure and risk and to determine the effects of including bankruptcy costs on optimal investment decisions. The article concludes with a discussion of limitations and implications for further research.

Model Development

Farm Model with Stock Investment

In this section, we formulate a model of an Illinois hog finishing operation which is then expressed as a stochastic dynamic programming model to determine optimal stock investment decisions.

Suppose that a manager is considering the investment of funds in nonagricultural opportunities, such as common stocks and money market instruments. Each month, net revenue [gross margin per hog (R_t) times the number of hogs sold] is earned by the marketing of finished hogs. The manager considers two uses for the funds generated by the farm operation. The first use, stock market investment, is achieved through a stock market index mutual fund. The value of stock holdings then is the number of fund units held (S_t) times the value of the index or stock price (P_t). Returns to stock investment are the changes in the price of stock from period to period plus any dividend income.

The second use of funds is investment in financial instruments (F_t) which earn a return equivalent to the short-term rate of interest (I_t). Holdings of financial instruments may represent an asset when held in positive amounts, or a liability when sold short (i.e., operating credit). Purchases or sales of financial instruments can occur each month. Any excess cash flows from farm operations or stock dividends and sales are used to purchase financial assets, and represent a positive investment. Short-term cash requirements for farm operations, family consumption, debt payment, or stock investment are provided by sale of financial instruments (i.e., borrowing), and represent a negative investment.

Beginning wealth is defined as the sum of the values of farm assets (FA) and nonfarm assets. Nonfarm assets are described by the state variables for stocks (P_t, S_t) and financial instruments (F_t). Thus,

$$Wealth_t = FA + P_tS_t + F_t. \tag{1}$$

Farm assets include those assets dedicated to the production of finished hogs, such as buildings, equipment, land, and inventory. It is assumed here that hog finishing is the only farming activity being undertaken and that the land base is minimal.

At the beginning of each period, the investor makes stock purchase/sell decisions.[2] The decision variable (DS_t) is the number of units of stock purchased or sold. The holdings of stocks (S_t) is equal to

$$S_t = S_{t-1} + DS_t. \tag{2}$$

The stock investment decision results in a cash flow equal to the value of stocks bought or sold, DS_tP_t. The investment decision also affects the level of financial holdings (F_t) since stock investment represents a cash outflow and sales produce a cash inflow. Once portfolio decisions are made, the investor realizes income.

$$\begin{aligned} Inc_t = {} & (R_t - FC)NH + (F_{t-1} - DS_tP_t - With) \\ & \cdot(I_t + BLD(F_{t-1}, DS_t, P_t)) + Div\, P_tS_t, \end{aligned} \tag{3}$$

where $(R_t - FC)NH$ is net revenue from farm operations and is defined as gross margin minus fixed costs per hog (FC) times the number of hogs marketed (NH); $(F_{t-1} - DS_tP_t - With)\cdot(I_t + BLD(\bullet))$ is interest revenue (cost) if the first expression in parentheses (which represents the effects of stock purchase/sell decisions and consumption withdrawals on financial holdings) is greater than (less than) zero. *With* represents monthly consumption withdrawals, and $BLD(\bullet)$ is a borrowing/lending differential on interest rates equal to zero for savings (financial holdings greater than zero) and 3% annually on operating credit (financial holdings less than zero). $Div\, P_tS_t$ is dividend income from stock holdings calculated as a percentage of stock value.

End of period financial holdings are

$$F_t = F_{t-1} + Inc_t - DS_tP_t - With. \tag{4}$$

Changes in wealth, or retained earnings, equals

$$RE_t = F_t - F_{t-1} + (P_t - P_{t-1})S_t. \tag{5}$$

The above equations represent the main elements of the dynamic programming model of portfolio choice for the hog feeder operation. Given the firm's wealth level at any point in time, and armed with information on past returns earned by the farm (R_t) and investments (P_t, I_t) and expectations of their future earnings, the manager makes decisions on the levels of investment (DS_t). Following investment decisions, income (Inc_t) is realized, withdrawals for consumption are made, and asset values change, all of which contribute to changes in wealth. The resulting wealth level represents the starting point for investment decisions in the following period. This recursive process is developed formally as a dynamic programming problem in the next section.

The Dynamic Programming Model

Empirical formulation of the dynamic programming model requires specification of the relevant state and decision variables, state transition equations, and a recursive objective function. The stock investment model described above contains three stochastic state variables: hog returns (R_t), stock prices (P_t), and return on other financial instruments (I_t). It also contains two deterministic state variables: holdings of other financial instruments (F_t) and stock holdings (S_t). The decision variable is the number of stocks to purchase or sell (DS_t).[3]

The stock investment model is formulated as an expected terminal wealth maximization problem. Denoting the terminal year as T, terminal wealth can be written as a function of the state variables:

(6) $$V_T(R_T, P_T, I_T, F_T, S_T) = (P_T S_T) + F_T + FA_T,$$

where $V_T(\bullet)$ is the recursive objective function for year T, and FA denotes farm assets devoted to the production of finished hogs. This function leads to the following general recursive equation:

(7) $$V_{t-1}(R_{t-1}, P_{t-1}, I_{t-1}, F_{t-1}, S_{t-1}) = \underset{DS_t}{\text{Max}}\ E[V_t(R_t, P_t, I_t, F_t, S_t)],$$

where $E[\cdot]$ is the expectations operator and $V_t(\cdot)$ is the value of wealth assuming that optimal decisions are made.

This maximization is subject to the following state transition equations:

(8a) $$R_t = f_1(R_{t-1})$$

(8b) $$P_t = f_2(P_{t-1}, I_{t-1})$$

(8c) $$I_t = f_3(I_{t-1})$$

(8d) $$S_t = S_{t-1} + DS_t$$

(8e) $$F_t = F_{t-1} + Inc_t - DS_t P_t - With.$$

Rather than discounting returns, as in a present-value maximization, returns in the terminal wealth maximization problem are compounded. Within the stock investment model, compounding is achieved through the retained earnings equation (5) which is affected by beginning financial structure, stock investment decisions, consumption withdrawals, farm returns, stock investment returns, and interest revenue or costs.

The wealth levels which can be achieved in this model are constrained. The lower bound on wealth is a bankruptcy state defined as zero wealth. If the firm becomes bankrupt at any point in time, the assets are presumed liquidated and all business activity ceases. The liability of the decision maker is limited to zero wealth. Bankruptcy also precludes the opportunity to accumulate future wealth. The sensitivity of stock investment decisions to the inclusion of bankruptcy is evaluated in this study by solving for optimal decisions with and without the bankruptcy constraint.

Estimation of Transition Probabilities

Numerical solution of the investment model requires state transition probabilities which, in this study, are derived from estimated state transition equations. The estimated equations describe a continuous Markovian probability density function which gives the future probability distribution for a state variable conditional on the current level of that variable. This section provides a description of the data and estimation procedures for the hog return (R), stock price (P), and interest rate (I) state transition equations.

Monthly hog returns were based on budgets reported in the *Livestock and Meat Situation and Outlook Report* published by the U.S. Department of Agriculture (USDA). Data were adjusted to reflect Illinois costs of production as closely as possible.

Monthly stock prices (S&P 500 index) were collected from the Standard and Poor's Statistical Reporting Service, and dividend data were based on information provided by Ibbotson Associates. Short-term interest rate data were taken from various issues of the *Economic Report of the President* (Congress of the U.S.). The hog return series covered the period from the beginning of 1974 to the third quarter of 1987, while stock price and interest rate series were extended to the end of 1991.

Hog Return Transition Relationship. Examination of autocorrelations and partial autocorrelations suggested that the hog return relationship could be modeled with a second-order autoregressive process, AR(2). A goal of reducing the number of state variables prompted the estimation of a first-order process [AR(1)] as well as the AR(2) model.

Estimation of the AR(1) model produced the following equation (t-statistics are in parentheses):

$$R_t = \underset{(2.58)}{1.895} + \underset{(18.70)}{.811}R_{t-1}$$

$$R^2 = .684, \quad \sigma_e = 8.003,$$

where σ_e is the standard error of the estimate. This formulation resulted in autocorrelated errors (Durbin's h-statistic $\doteq$ 5.76), indicating that an AR(1) model did not adequately capture the series' time-dependent nature.

Estimation of the AR(2) model resulted in the equation:

$$R_t = \underset{(3.60)}{2.430} + \underset{(16.54)}{1.177}R_{t-1} - \underset{(-6.29)}{.439}R_{t-2}$$

$$R^2 = .736, \quad \sigma_e = 7.239,$$

which showed no sign of autocorrelation and yielded normally distributed errors as judged by the Jarque–Bera test statistic.

Based upon these results, and analyses of higher-order models, the AR(2) model was judged to adequately describe the series' Markovian nature. To reduce the dimension of the DP model, only one hog return variable was included.[4] The reduction was accomplished using Burt and Taylor's method of reducing the order of an autoregressive process. This procedure resulted in the following form:

$$R_t = 1.688 + .8177R_{t-1}$$

$$\sigma_e = 8.058.$$

From this equation, transition probabilities were estimated using a hyperbolic tangent method (Taylor 1984).

Interest and Stock Price Transition Relationships. Autocorrelations, partial-correlations, and cross-correlations were examined to identify autoregressive relationships within the first differenced stock price and interest rate series. The plots suggested that these variables were interrelated and that lower-level autoregressive models would adequately capture their Markovian relationships.

Estimation of the AR(1) model for differenced stock price resulted in the following parameters (t-statistics are in parentheses and all variables are in log form):

$$P_t - P_{t-1} = \underset{(3.70)}{.019} + \underset{(3.48)}{.269}(P_{t-1} - P_{t-2})$$

$$R^2 = .073, \quad \sigma_e = .036.$$

This reduced to:

$$P_t = .19 + 1.269P_{t-1} - .269P_{t-2}.$$

Estimation of the AR(1) model for first differenced interest rates resulted in the following parameters:

$$I_t - I_{t-1} = \underset{(.153)}{.000059} + \underset{(6.57)}{.376}(I_{t-1} - I_{t-2})$$

$$R^2 = .406, \quad \sigma_e = .0659.$$

This reduced to:

$$I_t = 1.376I_{t-1} - .376I_{t-2}.$$

Residuals from both of the above equations were normally distributed as judged by the Jarque–Bera statistic.

As was the case with the hog return transitions, a reduction in the number of state variables was preferred to lower the dimension of the DP model. Burt and Taylor's method for reducing the order of autoregressive equations was employed to produce the following equations:

$$P_t = .015 + P_{t-1}; \quad \sigma_e = .037$$

$$I_t = I_{t-1}; \quad \sigma_e = .064$$

$$\rho_{SI} = -.18.$$

These equations describe the stochastic nature of the state transitions for stock price and interest rates. Interest rates depend only on past interest rates (a random walk), and stock price depends on past price (a random walk with drift). The correlation coefficient between stock prices and interest rates is low and negative, which is consistent with the correlations reported in the literature. From these equations, transition probabilities for stock prices and interest rates were estimated using a numerical integration routine (Gerald and Wheatley).

Optimal Stock Investment Decision Rule

In the previous sections, we have described the conceptual model of an Illinois hog finishing operation and the stock investment problem it faces, and we have described the statistical nature of the stochastic state transition equations. The model determines optimal stock investment decisions toward an objective of maximizing expected terminal wealth in the presence of bankruptcy. Five state variables describe the economic characteristics of the world in which these decisions are made. Three stochastic state variables—hog returns (R_t), stock price (P_t), and interest rate (I_t)—whose Markovian structures were given above, describe the earnings possibilities faced by the firm. Two deterministic state variables track the effects of optimal decisions on stock (S_t) and financial (F_t) holdings.

An optimal decision rule identifies the effects of changes in state variables on stock investment decisions. The optimal stock investment decision rule was derived using a value-iteration dynamic programming algorithm. Numerical solution required specification of discrete state and decision variable levels. A hog return range of −\$20 to \$40 was chosen to match historical variation. Five hog return intervals produced state levels of −\$20, −\$5, \$10, \$25, and \$40, respectively.

The stock price range was chosen to be wide enough to allow for sufficient growth potential given a five-year planning horizon and historical stock index growth values. Stock prices covered 10 intervals ranging from 100 to 325, and stock holdings ranged from zero to 2,000 units in increments of 200. Financial instrument holdings covered the range −\$350,000 to \$350,000 in \$70,000 increments. Interest rates ranged from 6% to

16%, matching the range over the time period for which data were collected, in two percentage point increments. This formulation resulted in 36,300 states. The stock purchase decision was allowed to take on values of −200 (sell), 0, or 200 (buy).

Specifications for the model farm are chosen to match a large commercial finishing operation in Illinois. Farm assets are valued at $350,000, monthly production is 750 hogs, monthly consumption is $2,000, fixed costs are $5 per hog, the borrowing differential is 3% annually, and monthly stock dividends are .083%.[5]

For both models, calculation of the recursive objective function begins in the final year of the planning horizon, denoted here as year T. The final year's recursive objective function $\{V_T(\bullet)\}$ contains the wealth level for each state increment and represents the expected wealth-maximizing objective. The optimal investment decisions and resulting recursive objective functions are calculated recursively. In any year t, the decisions that maximize the expected value of $\{V_{t+1}(\bullet)\}$ are found for all state increments associated with solvency. In the case of the bankruptcy model, optimal decisions were not calculated for state intervals which defined technical bankruptcy.[6]

In order to investigate the effects of the risk formulation chosen above, two alternative models are solved. The first formulation explicity accounts for the possibility of bankruptcy and attendant loss of future earning ability, as described in the conceptual model above. In this case, optimal decisions were found for all state intervals except those combinations which defined technical bankruptcy (i.e., zero wealth). In the case of bankruptcy, the farming operation was presumed to be liquidated. The second formulation (no bankruptcy) does not account for bankruptcy, and has as its objective function the maximization of expected terminal wealth. It is assumed in this model that debt capital is always available and will be provided at prevailing interest rates.

The large size of the optimal decision rule (a matrix with 36,300 elements for each stage) prevents a complete description within the article. Decision rules produced by both models, beginning with the model which incorporates bankruptcy, are summarized below.

Optimal Stock Investment Decisions in the Presence of Bankruptcy

The optimal stock investment decision rule illustrates the effects of changes in levels of state variables on stock purchases. A portion of the optimal decision rule is presented in figure 1. The graphs in figure 1 (panels A–D) will be used to illustrate the effects of changing levels of financial structure, interest rates, and hog return levels on optimal decisions.

For fixed levels of hog returns and interest rates, the graphs illustrate the effects of financial holdings levels on stock purchases. As financial holdings increase, the firm has more funds to purchase stocks. At a 10% interest rate and hog return of −$20 (panel A), for example, financial holdings of less than −$280,000 are associated with stock sales. The range from −$280,000 to −$140,000 is associated with a decision not to purchase or sell stocks, and levels above −$140,000 are associated with stock purchases. This wealth effect is consistent across stock price levels although the breakpoints differ for each stock price.

The financial structure effect described above can be explained by recalling that this formulation incorporates bankruptcy. In this model, the investor is presumed to be striving to maximize expected terminal wealth, recognizing the wealth losses associated with bankruptcy. The optimal investment strategy includes bankruptcy avoidance. Clearly, stock investment must be financed somehow. When the investor does not have sufficient funds in savings, borrowing occurs, which explains the increased sensitivity of decisions to interest rates at low financial holdings levels. At very low wealth levels, borrowing does not occur because the investor is close to technical bankruptcy (i.e., negative wealth resulting from debts exceeding the value of assets); the combination of interest payments on debt, and any drops in stock prices or farm losses would result in bankruptcy. Thus, stock purchases do not occur at low wealth levels.

The graphs in figure 1 also illustrate the dampening effects of higher interest rates on the desirability of stock purchases. For example, at a hog return of −$5 (panel B) and

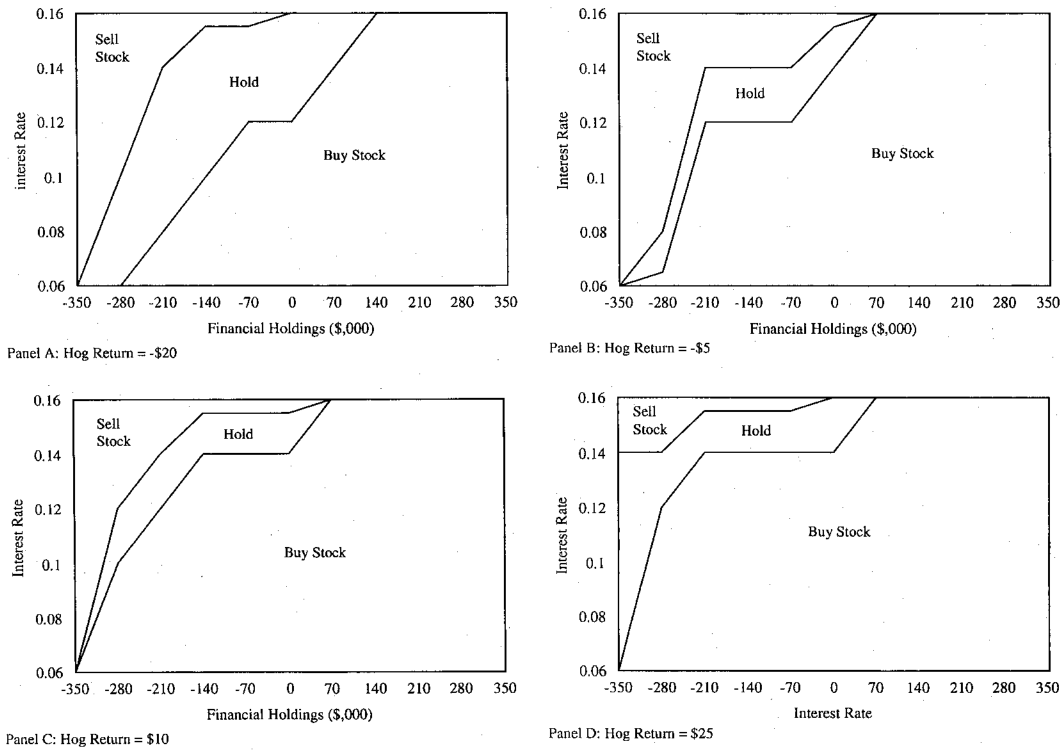

Figure 1. Portions of the optimal decision rule for stock holdings of 400 and stock price of 200

Note: The lines on these graphs identify the regions over which a particular investment action would occur.

financial holdings of −$70,000, stock purchases occur up to an interest rate of 12%. Over a range of 12 to 14%, existing stocks are held; stocks are sold at rates above 14%. This interest rate effect exists for all state variable combinations, although the absolute values of interest rates at which decisions change vary with stock price.[7]

The figure 1 graphs also illustrate the effects of different hog return levels across the range of interest rates and financial holdings. Note that the interest rate and financial holdings effects discussed above exist across the different hog return levels. The effect of changes in hog returns is seen in the positioning of buy and sell breakpoints. For example, at an interest rate of 10%, a hog return of −$20 or −$5 (panels A and B) implies almost no purchases at lower levels of financial holdings. A return level of $10 infers fewer stock sales than does the −$5 return level. A $25 return allows purchases up to an interest rate of 10% at lower levels of financial holdings and infers no stock sales until 12%, even at the lowest level of financial holdings.

The relative increase in stock purchases for each increase in financial holdings becomes greater as hog returns grow. Profits from the farm operation provide cash which may be used for investment or paying off debt. As hog returns improve, there is more cash available, which results in an increase in investment. At the low wealth levels, only very high farming profits generate enough cash for stock purchases to be considered. Higher hog return levels translate into larger stock holdings due to (ceteris paribus) a combination of greater stock purchases and fewer sales.

The optimal stock investment decision rule illustrates the effects of changes in levels of state variables on stock purchases. It is clear that changes in market conditions, hog returns, and wealth all affect stock investment. The decision to purchase stocks as part of the farm portfolio must be made while recognizing the farm's current balance sheet and cash flow situations as well as expectations of future farm and financial market conditions. This suggests that models which do not account for these effects are providing suboptimal results. The effects of ignoring the possibility of bankruptcy in investment decisions are evaluated below.

Optimal Stock Investment Decisions in the Absence of Bankruptcy

The no bankruptcy model produces markedly different decision rules. The stock purchase decisions in this model are fairly simple and may be described by the phrase, "buy stock." The expected returns on stock exceed the expected cost of borrowing, so the model chooses to buy stock. For example, when stock holdings are zero and stock price is 200, the optimal decision rule indicates that stock should be purchased at almost all combinations of financial holdings and hog returns. The only time that stock purchases are not optimal is for some combinations of financial holdings and hog return levels when interest rates exceed 14%.

The wealth-maximizing decision maker who does not consider the cost of bankruptcy simply chooses the highest yielding investment, which in this case is almost always the stock market. The financial structure effect discussed above is not evident because the decision maker is allowed to continue borrowing money as long as the expected return warrants it. The hog return effect also is not evident because the investor has a ready source of investment capital and need not rely on farm operations as a source of capital.

Comparison of Model Results

Comparison of results from the two models yields a definite conclusion. The explicit recognition of the costs of bankruptcy has a clear impact on the form of the optimal decision rule. When we include bankruptcy and the associated income and wealth losses, financial structure and farm profitability levels have a strong effect on investment behavior. Off-farm investments tend to increase with stronger financial condition and higher farm profits. Financial market conditions also influence investment decisions. Higher interest rates tend to depress stock purchases due to the combined effects of greater costs for debt capital and lower expectations on stock investment.

The inclusion of bankruptcy yields different optimal decisions from a model which ignores this constraint. We should now ask whether or not this difference is important to the investor's financial well-being. The following section contains a conditional probability analysis to determine how stock investment affects expected financial progress of the farm business. A comparison of the profitability and risk levels arising from the two decision rules also is provided.

Conditional Probability Analysis

The optimal decision rules described in the previous section show how stock investment decisions change with different combinations of state variable values. The optimal decisions for the firm will change through time because values of the state variables change. Market conditions will vary over time and the financial structure of the firm will change due to past decisions and realized returns. In order to evaluate the effects of optimal decisions on the financial status of the business, it is important to probabilistically track the changes to the firm as it follows the optimal investment strategy through time.

In this section, discrete conditional probability methods (Howard) are used to determine ex ante probability forecasts of investor wealth, assuming that optimal investment decisions are made each month. This analysis is conducted to investigate the following issues: (*a*) What are the effects of stock investment on the level and riskiness of wealth compared to a farm with no stock investment opportunities, and how do these results change given

Table 1. Expected Wealth and Probability of Bankruptcy in Year 5 for Stock and No Stock Models with Bankruptcy and No Bankruptcy Models

	Expected Wealth ($)	Probability of Bankruptcy (%)
Stock:		
Beginning Financial Holdings		
−$140,000	338,062	.443
$0	583,115	.208
$140,000	830,373	.087
No Stock:		
Beginning Financial Holdings		
−$140,000	316,523	.421
$0	547,331	.179
$140,000	782,212	.059
No Bankruptcy:		
Beginning Financial Holdings		
−$140,000	83,401	.904
$0	314,228	.657
$140,000	613,360	.384

different beginning wealth levels? and (*b*) How do the different assumptions about bankruptcy affect the level and riskiness of wealth?

Financial Effects of Stock Investment

Consider the case of a beginning state with no stock holdings, hog returns of $10, a stock price of 150, and a 10% interest rate. Conditional probabilities were calculated for different initial financial holdings levels ranging from −$140,000 to $140,000.

The relative financial performance of the farm firm, with and without stock purchases, is summarized in table 1. As expected, greater initial financial holdings result in higher levels of terminal wealth and lower probabilities of bankruptcy. The riskiness of hog finishing, especially in the case of low financial reserves, is illustrated by the high bankruptcy probabilities.

The effects of stock purchases are quantified by comparing stock and no stock model results. At $0 beginning financial holdings, following the optimal stock purchase decision rule over a five-year period resulted in an expected increase in wealth of $42,097. In all cases, expected terminal wealth is higher when stock purchases are made compared to the no stock model. The differences between stock and no stock are larger for higher beginning financial instrument holdings. Stock purchases also resulted in slight increases in the probability of bankruptcy, suggesting an increase in risk.

The results also suggest that off-farm investments provide positive returns and should be considered further as potential uses of capital. In this study, only one possible investment was considered—the stock market. It may be possible to find more suitable investments in portions of the stock market which provide wealth-enhancing and/or risk-reducing effects.

Evaluation of the Effects of Bankruptcy

The previous section described the results of the conditional probability analysis of the effects of stock investment decisions on firm wealth in the presence of bankruptcy costs. The optimal decisions for the no bankruptcy case did not show the sensitivity to financial structure and farm returns that was evident in the bankruptcy model results. The effects of this difference in decisions can be described in terms of expected wealth and probability of bankruptcy. A comparison of these results clearly shows the costs associated with ignoring bankruptcy costs in making stock investment decisions. At the end of year 1, the probability of bankruptcy for the bankruptcy-included model is only .003, while the no bankruptcy model has a bankruptcy probability of .254. By the end of year 5, these probabilities are .218 and .657, respectively. Optimal decisions in the no bankruptcy model were not sensitive to financial structure. These results are useful in illustrating the importance of recognizing the effects of investment decisions on financial structure and the costs which may result.

Summary and Conclusions

This study attempts to develop further the modeling of investment decisions by incorporating important real-world institutions such as bankruptcy in a multi-year analysis. The idea of a safety-first type objective is dealt with by including an objective function in which expected terminal wealth is maximized in the presence of bankruptcy. The objective function used here explicitly recognizes risk in the form of bankruptcy and limited liability. The implications of this formulation are tested by comparing results with and without the bankruptcy constraint. Differences in optimal behavior and resulting wealth provide an indication of the costs of ignoring bankruptcy in this type of decision model.

The optimal stock investment decision rule illustrates the effects of changes in levels of state variables on stock purchases. Comparison of optimal decisions from the two models illustrates the point that the explicit recognition of the costs of bankruptcy has a clear impact on the form of the optimal decision rule. When we include bankruptcy and the associated income and wealth losses, financial structure and farm profitability levels have a strong effect on investment behavior. Off-farm investments tend to increase with stronger financial condition and higher farm profits. Financial market conditions also influence investment decisions. Higher interest rates tend to depress stock purchases due to the combined effects of greater costs for debt capital and lower expectations on stock investment.

The costs associated with ignoring bankruptcy in making stock investment decisions also are identified. Failure to account for bankruptcy in determining the optimal decision rule results in very high probabilities of bankruptcy. By not considering bankruptcy costs, investment decisions result in such high debt levels that bankruptcy is almost assured. These results are useful in illustrating the importance of recognizing the effects of investment decisions on financial structure and the costs which may result.

The model presented here lends itself to problems such as portfolio investment and firm growth where optimal combinations of farm and off-farm investments could be selected. The sensitivity of dynamic investment decisions to different objective functions, different farm types, and various stages of the farm's life cycle are other questions which may be addressed using this approach.

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Notes

[1] Certainty equivalence requirements are met if the different moments of random variables enter the objective function linearly (Simon; Theil). Certainty equivalence allows for random variables to be replaced by their expected values.

[2] It is assumed here that there are no transactions costs for purchases and sales of stocks or financial instruments. There are currently a wide range of mutual fund investment groups or families where investors can move capital between stock market and money market accounts without transactions costs. Transactions costs are no longer a significant factor in these types of investments.

[3] Recall that financial holdings are affected by stock investment decisions. Therefore, the decision to purchase or sell stock is effectively a decision to trade off between stock and financial holdings. Thus, financial holdings are, in effect, an implicit decision variable.

[4] Recall that the size of the SDP model increases exponentially with the number of state variables. Therefore, from a modeling perspective, it is desirable to limit the number of state variables while still capturing the dynamic nature of the problem. The state variable reduction technique employed here allows reduction of the number of state variables while retaining the essential characteristics of the estimated Markovian relationship.

[5] Asset values, production levels, and fixed costs were based on budgets prepared for hog finishing operations in the Midwest. Changes in the value of farm assets (i.e., depreciation of buildings and equipment) were handled by amortizing their values over appropriate lifetimes and adding this cost to fixed costs. Thus, the model assumes that the individual puts money away to keep the facilities in operation. Stock dividends are based on historical averages and the borrowing/lending differential is based on banking industry averages.

[6] Optimal decision rules were generated until the optimal decision rules converged, which occurred by month 6 of year 3. Thus, the converged decision rule was applicable to all years up to the 30 months before the end of the planning horizon. For example, if the planning horizon is 10 years long, the converged decision rule would be applicable from year 1 through month 6 of year 7.

[7] The interest rate effect on stock purchases is not intended as a market timing rule for stock trading. This result is due in part to the influences of several interrelated factors, particularly wealth and the costs of bankruptcy. For instance, at the higher wealth levels in figure 1, there is little suggested switching of stock and financial instruments. The stock sales at lower wealth levels are spurred on in part by the desire to pay off debt and avoid the possibility of bankruptcy.

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