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Hedging Potential in Grain Storage and Livestock Feeding

Economic Research Service
[redacted]

ABSTRACT

The potential for shifting risk through hedging in commodity futures is analyzed for selected grain storage and livestock feeding situations. Results applying to various locations, grades, and/or classes are reported for wheat, corn, oats, cattle, and hogs. Hedging potential is measured in terms of risk-shifting effectiveness--the proportional reduction in the variance of profits that can be obtained through routine hedging.

The study indicates that hedging provides an effective means of shifting risk in livestock feeding as well as in grain storage. For most of the situations studied, the level of hedging that minimizes overall profit risk ranges between 0.6 and 1.0 unit of futures per unit of cash commodity. About one-third to two-thirds of the price risk can be shifted through hedging at this level.

Hedging effectiveness declines as the distance from the delivery point for the futures contract increases. Hedging effectiveness differs between classes of wheat and among the three wheat futures markets. Grade has little impact on hedging effectiveness in cattle feeding, however. Optimal hedging levels for individual firms are shown to be very sensitive to the firms' price expectations.

Keywords: Futures trading, Hedging, Grain storage, Cattle feeding, Hog feeding, Price analysis.

GLOSSARY

Basis--The difference between the price of a futures contract and a specific cash price.

Hedging--The holding of a temporary position in a commodity futures contract pending an anticipated cash transaction.

Minimum-risk hedge--That level of the futures position relative to the cash position that minimizes the variance of total profits from the two activities.

Optimal hedge--The level of the futures position relative to the cash position that results in the best attainable combination of average profit and risk for a particular individual or firm.

Risk-shifting effectiveness--The degree to which hedging can reduce risk or profit variability for a particular enterprise.

Futures price bias--The tendency for the futures price to lie either below or above the price that traders in the market expect to prevail as the futures contract matures.

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HIGHLIGHTS

Hedging in commodity futures offers typical agricultural producers and marketers opportunities to shift one-third to two-thirds of their price risks on grain storage and livestock feeding operations. In most situations, risk is minimized by holding 0.6 to 1.0 unit of short futures for each unit of cash commodity intended for sale. Price risks can be shifted through hedging almost as effectively in livestock feeding as in grain storage. As a rule, hedging effectiveness tends to decline as distance from the par delivery point for the futures contract increases.

The finding that overall price risk is often minimized by holding less than 1.0 unit of futures for each unit of cash contrasts with frequently used hedging illustrations showing futures and cash positions of the same absolute size. Setting the futures position equal to the cash position minimizes risk if profits from the two positions are equally variable and perfectly correlated. When the correlation is imperfect, risk will normally be minimized by holding a smaller position in futures than in the cash commodity.

Locational differences in hedging potential are most evident for grains. During the 10-year study period, 1960-70, minimum-risk hedging levels and risk-shifting effectiveness were clearly lower in the more remote locations. In cattle feeding, risk-shifting effectiveness tends to decline as distance from the delivery point increases, but for the 1965-71 period the differences between midwestern, southwestern, and California locations were small relative to their standard errors. For hog feeding very little difference in hedging effectiveness was noted between the Corn Belt and the Southeast during the 1966-71 period analyzed.

Quality differences affect hedging potential for grains but appear less important in livestock hedging. Moderate differences in hedging effectiveness were observed among the three wheat futures markets. Each market serves best for a different class of wheat--Hard Spring at Minneapolis, Soft Red Winter at Chicago, and Hard Winter at Kansas City. At Minneapolis, risk-shifting effectiveness appeared greater for high protein wheat than for lower protein wheat, but the differences were not statistically significant. At Kansas City, the situation was apparently reversed so that ordinary protein wheat could be hedged more effectively than 13 percent protein wheat; but again the differences were very small. Risk-shifting effectiveness for Good steers and heifers was essentially the same as for Choice steers and heifers with virtually no difference due to sex.

Historically, prices on live cattle and hog futures markets have tended to rise over the life of the contract, resulting in losses to short hedgers. For the periods analyzed, increases in futures prices averaged about 30 cents per hundredweight per month for both cattle and hogs. Price biases of this magnitude would represent a serious barrier to hedging if they were to persist.

However, these estimates of the bias have large standard errors; they are quite sensitive to the particular period selected for analysis; and competition among traders may be expected to reduce such biases over the long run.

Due to futures trading costs, including commissions and interest foregone on margin deposits, many hedgers will find their optimal futures position to be 2 to 8 percent smaller in absolute size than the position that would minimize risk. Furthermore, relatively small differences in price expectations imply large differences in the optimal hedging level. For example, if a typical cattle feeder expects an increase of 50 cents per hundredweight in the futures price over the feeding period, his optimal short hedging level drops to zero. Thus, the widely differing futures positions held by businessmen in enterprises such as cattle feeding may reflect rational adjustments of individual firms to differing price expectations.

Because of time and data limitations, this study leaves many questions unanswered about desirable hedging policies and hedging effectiveness in grain storage and livestock feeding. Seasonal effects, the effects of diversification among enterprises and over time periods, and the possible gains from varying production and hedging levels in response to futures price levels were not investigated. Such results as the finding that minimum-risk hedging levels were higher for wheat storage and hog feeding than for corn storage and cattle feeding may be largely attributable to sampling error. Nevertheless, they suggest promising areas for further exploration.

HEDGING POTENTIAL IN GRAIN STORAGE AND LIVESTOCK FEEDING

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PROCEDURES FOR MEASURING HEDGING POTENTIAL

Hedging in commodity futures is a well-established business practice which grain merchants in the Midwest have used for nearly a century. Although corn futures trading has set new volume records recently, trading in wheat futures attained its peak volume more than 40 years ago. In contrast, opportunities to hedge livestock feeding operations rose in the last decade. Trading in live cattle futures opened on the Chicago Mercantile Exchange in November 1964, and trading in live hog futures began on the same exchange in February 1966.

Under proper conditions hedging provides an effective means of shifting price risks on commodity inventories. But the effectiveness of hedging varies with the location and quality of the commodity being hedged. Furthermore, operations that involve the transformation of one or more commodities to another commodity--livestock feeding, for example--pose somewhat different hedging problems than the traditional inventory hedge. The hedging of grain inventories is by no means a universal practice and only a small percentage of livestock feeding operations are currently hedged.

Like other traders, agricultural producers may trade in futures to make or increase profits, to reduce risk, or both. Making profits means obtaining returns which exceed costs on the average. Reducing risk involves reducing profit variability; that is, increasing profit stability. Risk avoidance enters into every business operation. For example, businessmen buy insurance and hold cash and other liquid assets in reserve, and bankers require their borrowers to hold prescribed levels of equity. Risk is particularly important to individuals who have substantial assets tied up in commodities exhibiting sharp price fluctuations, such as grain and livestock.

The purpose of this study is to measure the potential for shifting risk through hedging for various types of grain storage and livestock feeding operations. In quantifying risk-shifting potential, the concepts of optimal hedging level, minimum-risk hedging level, and risk-shifting effectiveness were employed. These concepts are discussed in the next section. As an aid to the reader, frequently used terms are defined on p. iii. The appendix provides a more rigorous set of definitions employing mathematical terminology.1/

Successive parts of this report present measures of hedging potential in feed grain storage, wheat storage, cattle feeding, and hog feeding. These four types of activities are combined in one report to make them accessible to agricultural producers and others with multiple interests. By analyzing them together, certain similarities and contrasts in hedging potential between different storage and production situations are revealed. Some of the understanding that has accumulated through long experience in hedging grain inventories can thereby be brought to bear on the problem of hedging livestock feeding activities, where experience is much more limited.

The Nature of Hedging

The principles of hedging livestock feeding are identical to those in commodity storage. Hence, the same analytical approach serves for both situations. Before discussing the analysis, some basic concepts that underlie the idea of hedging will be reviewed.2/

Hedging may be defined as the holding of a temporary position in a commodity futures contract pending an anticipated cash transaction. An example of hedging would be the actions of a cattle feeder who sells a live cattle futures contract as cattle are placed on feed, holds the contract over the feeding period, and buys it back as the finished cattle are sold for slaughter. Similarly, a firm that owns grain in storage may hedge by holding a short position in a grain futures contract over the storage interval. Both are examples of short hedging, which involves balancing a long cash position with a short position in the futures. An example of a long hedger would be a flour miller who, upon making a forward sale of flour, buys and holds a long position in wheat futures pending the purchase of wheat needed for milling into flour to fulfill his flour sales contract. This study is concerned primarily with short hedging, although the methods and some of the conclusions are applicable in long hedging.

Futures trading has evolved primarily to meet the hedging needs of central market merchants and processors, but the contracts are also suitable for use by other firms, such as crop and livestock producers and local merchants. For grains, the contract size is 5,000 bushels. For hogs and cattle, the contract sizes are 30,000 pounds and 40,000 pounds, respectively, While these contract sizes may be too large for some firms, they fit the needs of most commercial operations.

Obviously, either the hedger or the speculator can profit from futures trading if he can forecast futures price movements. However, the hedger may gain over the long run, even without being able to forecast prices, if his futures transactions enable him to stabilize his income. This is the risk-

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1/Most of the material in the appendix and the empirical results reported for cattle feeding are drawn from item (7) in the literature cited, p. 36.

2/The fundamentals of hedging are explained in a number of publications. Examples are (2), (4), and (8).

shifting aspect of hedging which is the focus of this study. The task of appraising the increase in average profits that might arise through various price forecasting and trading schemes is not considered.

Basis and Basis Risk

Hedging can serve as an effective means of shifting risk only if certain relationships prevail between the cash prices at which the hedger buys his inputs and sells his outputs and the futures prices at which he establishes and lifts his hedge. The concepts of basis and basis risk are useful in describing these relationships, particularly in the case of the storage hedge.

The term "basis" is employed with various special meanings among traders and in the literature on futures trading.3/ In this study, basis refers to the difference between the price of a futures contract and a specific cash price. For example, if the corn futures contract that is nearest maturity is selling at $1.20 and the price of corn for immediate delivery at a specified location is $1.18, the basis is 2 cents under the near future.

Few hedgers find it convenient or profitable to close out their futures positions by delivery on the futures contract. The vast majority terminate their futures positions through offsetting futures transactions. This involves buying back the same amount of the futures that they previously sold short or selling the futures that they previously bought. When a hedging position is to be closed by an offsetting futures transaction, the hedger bears a risk that the relationship between the cash price and the futures price may be different than originally expected. For example, the cattle feeder with a short hedge in cattle futures may find at the end of the feeding period that the futures price at which he must buy back his futures contract is unexpectedly high relative to the cash price he obtains for his cattle. This type of uncertainty is called basis risk. The existence of basis risk makes hedging an imperfect method of stabilizing returns.

Basis risk arises not only from uncertainty about price relationships at the end of the storage or production period when the hedge will be lifted, but also from uncertainty about price relationships that will rule at the beginning of the period when the futures position will be established. For example, when the elevator manager buys grain for storage and seeks to hedge by selling futures short, he may find that the price of future grain is unexpectedly low relative to the price of spot grain. Similarly, the cattle feeder may find at the beginning of the feeding period that the slaughter cattle futures price is low relative to the prices he must pay for feeder cattle and feed.

Once the initial transactions are consummated, the beginning basis is fixed so that only the closing basis risk remains. Hence, if the hedging decision is postponed until the cash and futures prices for the initial transactions are known, only the closing basis risk needs to be taken into account. Generally, however, the size of the beginning basis and the size of the closing basis are not independent so that the former should be considered in dealing

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3/In (1, pp. 64-70), Arthur gives examples and explains various ways the term "basis" is used by traders.

with the latter. The approach used in this study takes into account the interdependence between the beginning basis and the closing basis. It assumes that the hedger follows a regular hedging program so that his futures trading decisions are made prior to the time the initial basis is known.4/

For the inventory hedge, both the beginning basis and the ending basis may be defined simply as differences between futures prices and cash prices. But for hedging an activity, such as livestock feeding, where commodities are transformed into substantially different commodities, the situation is more complex and the concept of a beginning basis loses its appeal. The beginning basis may be defined as the difference at the beginning of the production period between the futures price for the final product and the cost of production based upon the quantities and prices of the required inputs. This is an awkward concept, however. The conditions for successful hedging can be expressed more conveniently in terms of profit, rather than price, relationships.

Like other problems of decisionmaking under risk, hedging can be described as a problem of setting levels for activities with uncertain rates of profit or return. These activities include cash activities and futures activities. The cash activities may involve holding a commodity in inventory over a prescribed time period or the transformation of one or more commodities into another commodity over time. A futures activity involves holding a long or short position in a specific futures contract over a designated interval.

The conditions for successful hedging can be expressed in terms of relationships among average profits and the variances and covariances of profits on the cash activities and on the futures activities. Profit for a cash activity is the residual after input costs have been subtracted from returns. In storage, the commodity itself is the major input. Profit on a futures activity equals the gain or loss on the futures position minus futures trading costs.

The advantage of expressing the conditions for hedging success in terms of average profits and variances and covariances of profits is that the same conditions apply to the more general production hedge as to the traditional inventory hedge. This avoids using the concept of basis risk which, as shown above, becomes rather awkward in production hedging. It also facilitates comparisons of hedging potential among different types of hedging situations.

The Optimal Hedge

Hedging is frequently illustrated by examples showing the holding of 1.0 unit of futures position for each unit of cash position. This simple view of hedging is misleading when cash profits and futures profits are not perfectly

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4/In principle, a better hedging decision can be made if it is postponed and made conditional to the initial cash and futures prices as they become known at the beginning of the production or storage interval. An analysis of the potential gains from using futures in this way in storing grains is reported by Heifner (6). Such an analysis is outside the scope of this study.

correlated. Such a policy may result in greater risk, with no compensating increase in average return, than would a policy of partial hedging where less than 1.0 unit of futures is held by each unit of cash. The complete hedge--1.0 unit of futures held for each unit of cash--is optimal only under certain conditions.

The optimal hedge is defined here as the level of the futures position relative to the cash position that results in the best attainable combination of average profit and risk for a particular individual. In this analysis, risk is measured in terms of the variance of profit. It is assumed that the hedger is averse to risk. When expected return is held constant, he prefers less risk to more risk; and when risk is held constant, he prefers a higher expected return to a lower expected return.5/

The optimal hedging level may differ from firm to firm due to differences in the degree of risk aversion, and to differences in resources. In general, hedging is most useful to a firm which has fixed resources in the enterprise being hedged--the feedlot operator or the owner of grain storage space, for example.

Differences in profit expectations among individuals and between time periods also result in differences in optimal hedging levels. For example, the short hedger who anticipates a modest increase in the futures price will want to establish a smaller short futures position than the individual who looks for a decline in the futures, other factors being equal. Price expectations are highly subjective, varying from individual to individual according to the information and forecasting tools available to each. It is beyond the scope of this study to show how these price expectations or profit expectations can best be formulated. In a subsequent section, however, the effects of variations in price expectations are explored.

The Minimum-Risk Hedge

Instead of searching for the hedging level that is optimal, in the sense of providing the best available combination of expected profit and risk, the hedging level may be chosen that minimizes risk, given the level of the cash activity. The minimum-risk hedge is easier to calculate than the optimal hedge and under certain conditions the two approximately coincide.

Determination of optimal hedging levels requires estimates of average or expected profits as well as estimates of the variances and covariances of profits for the hedger's cash and futures activities. The expected profits or losses on the futures activities are difficult to estimate because their magnitudes are small relative to their standard errors. The expected profits from

5/Such risk averse behavior may reflect the hedger's desire to avoid a disastrous loss that would put him out of business. The predominance of risk aversion can also be supported by theoretical arguments based on the notion that utility is finite. The assumption of risk aversion does not imply that risk is minimized. It allows the possibility that the decisionmaker may be willing to accept large increases in risk for a small increase in expected returns.

the cash activities must normally be determined as a residual. These expected profits vary from firm to firm depending upon resource availabilities and individual price expectations. In contrast, the level of hedging that minimizes risk can be determined without knowledge of expected profits.

The minimum-risk hedge may be defined as that level of the futures position relative to the cash position that minimizes the variance of total profits from the two activities. It is the optimal hedge when the expected profit on the futures position is zero. No higher level of hedging will be optimal when hedging costs exceed zero. In this sense it represents a limiting value for the optimal hedge.

The condition that expected profit on the futures position is zero requires that futures trading costs be negligible and that no change in the futures price is anticipated. In particular, this calls for the absence of bias in futures prices. Price bias is defined as a tendency for a futures price to fall either below or above the cash price that is expected to prevail when the futures contract matures. For example, if hedgers must pay speculators to assume risks and if there is an excess of short hedging over long hedging, the futures would tend to be priced below the expected cash price. This would be a downward bias. In this case, the futures price would tend to rise as contract maturity approaches. The increase in the futures price would represent a reward for risk bearing or a "risk premium." A risk premium, of course, represents a cost to the hedger and decreases the attractiveness of hedging.

Even when futures prices are unbiased for the market as a whole, individual traders may be able to predict futures price movements. Any trader who can successfully forecast futures price movements will find that his optimal futures position differs from that implied by the minimum-risk hedge. However, price forecasting is no easy task and many hedgers may choose to leave it to others. For such individuals the minimum-risk hedge may closely approximate the optimal hedge.

If the expected profit on the futures position is to be zero, futures trading costs must be zero. Of course, this requirement is met only approximately. Futures trading costs include commissions, interest foregone on margin deposits, and miscellaneous costs. These costs have the effect of making the optimal hedging level smaller in absolute size than the minimum-risk hedging level. However, this effect is frequently small, especially when futures profits and cash profits are highly correlated.

Minimim-risk hedging levels were estimated in this study under the assumption that profits from the specific cash and futures activities analyzed are not correlated with profits from other activities of the firm. This assumption is seldom met exactly. The situation is approached for the specialist who has only one production activity, such as cattle feeding or grain storage, or for the firm whose various activities are not closely related profitwise--the corn producer who feeds cattle, for example.

As shown in the appendix, the minimum-risk hedging level can be estimated conveniently, using a least-squares regression algorithm, with cash profits as the dependent variable and futures profits as the independent variable. Furthermore, the regression algorithm yields estimates of the correlation coefficient or partial correlation coefficient used in assessing risk-shifting effectiveness.

Risk-Shifting Effectiveness

The need to measure and compare risk-shifting potential in different hedging situations leads to the concept of risk-shifting effectiveness. As used here, risk-shifting effectiveness is the degree to which hedging can reduce risk or profit variability for a particular enterprise.

If hedging is to reduce risk, returns from the futures position must be high when returns on the cash position are low and vice versa. In other words, futures profits and cash profits must be negatively related. A convenient measure of this relationship is the correlation between cash profits and futures profits. As this correlation approaches unity in absolute value, the variations in cash profits and futures profits tend to cancel each other, thus enabling the firm to stabilize total profit by hedging at the minimum-risk level. As shown in the appendix, the squared correlation coefficient between cash profits and futures profits provides a direct measure of the proportion of the profit variance that can be shifted by hedging at the minimum-risk level. It, therefore, provides a convenient measure of risk-shifting effectiveness.

When the optimal hedging level differs from the level of hedging that minimizes risk, risk-shifting effectiveness becomes less adequate as a measure of hedging potential. The minimum-risk hedge is the most effective hedge possible from the standpoint of pure risk shifting. But when costs or expected profits from hedging are taken into account a different hedging level is generally to be preferred. Unfortunately, variance of profit and expected profits cannot be combined into a single measure of hedging effectiveness that is meaningful to all firms.

ESTIMATES OF MINIMUM-RISK HEDGING LEVELS AND RISK-SHIFTING EFFECTIVENESS

In this section, estimates of minimum-risk hedging levels and risk-shifting effectiveness are reported for selected hedging situations in grain storage and livestock feeding. The minimum-risk hedge is the level of the futures position relative to the cash position which, when applied consistently, minimizes the variance of profits. It is not necessarily the optimal hedging level for any particular firm. It may, however, be viewed as an upper limit on the optimal hedging level which applies so long as the hedger feels that upward and downward movements in futures prices are equally likely. The reported estimates of risk-shifting effectiveness represent the proportion of the profit variance that can be shifted by hedging at the minimum-risk level.

Estimation of risk-shifting effectiveness and minimum-risk hedging levels requires a sample of paired observations on cash and futures profits. For grains, 120 successive monthly observations for the 10 marketing years 1960-69 were employed. For cattle feeding, successive 4-month feeding periods were taken as observations, giving a total of 18 observations over the 6 years March 1965 to March 1971. For hog feeding, three 3-1/2-month feeding periods per year were used, for a total of 15 observations over 5 years of data, June 1966 to June 1971.

The profit from grain storage was estimated as the value of the commodity at the end of the storage interval minus the value of the commodity at the beginning of the storage interval. Costs for other inputs were disregarded since they are small, relative to the cost of the grain placed in storage, and contribute little to the variance of profits.

In livestock feeding, profit was estimated as the value of the finished animal when sold minus the cost of the feeder animal and the feed priced at the beginning of the feeding period. Grain and roughage were included in feed costs for cattle, and grain and protein supplement in hog-feeding costs. Again, as for grains, other costs, including labor, power, equipment, etc., were disregarded since they vary little from observation to observation and, therefore, do not affect the variance of profits substantially.

To estimate futures profit it was assumed that the futures position was established by selling short when the commodity was placed in storage or when the feeder livestock and feed were purchased and closed by buying the future as the cash commodity was sold. Under this assumption, futures profit equals the change in the futures price over the storage or production interval times the amount of the hedge minus futures trading costs. Futures trading costs including commissions and interest foregone on margin deposits were assumed to be constant.

Price observations were selected from the week nearest the middle of each month. Prices for a single day were used when feasible because they correspond most closely to the actual prices encountered by traders. For grains, Thursday prices were selected since they are readily available in Grain Market News (12). The futures price used was the closing price for the same day. For livestock, weekly average prices were employed since daily prices are not available at all locations for the same days each week. The futures prices for livestock are Wednesday closing prices. For hay and soybean meal, Statistical Reporting Service, USDA, estimates of prices paid by farmers were employed (14).

For both grain and livestock, observations for different seasons were pooled. This was accomplished by using a multiple-regression computer program, with cash profits as the dependent variable and futures profits plus dummy variables for the seasons as independent variables. This procedure permits average profits to vary by season. It is based on the assumption that the variances or standard deviations of profits about their respective monthly or seasonal means are the same throughout the year. Estimates of these standard deviations of cash and futures profits are reported for each hedging situation analyzed. The square of the partial correlation between cash profits and futures profits (r^2) is reported as the estimate of risk-shifting effectiveness. The corresponding regression coefficient is reported as the estimate of the minimum-risk hedging level. The Durbin-Watson statistic (d) is also reported as an indication of the degree of serial interdependence in the sample.

Feed Grain Storage

Corn is the most important feed grain and one of the leading commodities in futures trading. In 1971-72, the total volume of trading in corn futures was about 7.8 billion bushels, compared with 3.5 billion bushels for wheat, about 0.2 billion bushels for oats, and slightly more than 0.1 billion bushels for grain sorghum. Futures trading in grain sorghum did not begin at the Chicago Mercantile Exchange until March 2, 1971.

Corn

Trading in corn futures at Chicago grew into maturity during an era when much of the corn moving in commercial channels passed through Chicago by rail. With the increase in truck and barge hauling of grain, most corn now moves directly from surplus areas to utilization areas or ports, bypassing terminal markets such as Chicago. Consequently, the proportion of commercial corn supplies stored nearby and readily deliverable at Chicago has declined.

The Chicago corn futures contract calls for delivery by warehouse receipt of grain stored in Chicago. The delivery months are March, May, July, September, and December. The corn futures contract calls for delivery of No. 2 Yellow corn, with premiums and discounts for other grades. No. 2 Yellow is the most commonly marketed grade of corn, although lower grades sometimes become important when moisture affects quality.

Table 1 shows estimates of the standard deviation of storage profits, minimum-risk hedging levels, and risk-shifting effectiveness for No. 2 Yellow corn stored at six locations and hedged in the Chicago corn futures.6/ These estimates are based on observations for 120 consecutive 1-month intervals from November 1960 to November 1970. The estimates of risk-shifting effectiveness--the proportion of the profit variance that can be shifted by hedging--range from 0.55 at Chicago down to 0.38 at Omaha.7/ The corresponding estimates of minimum-risk hedging levels measured in terms of bushels of corn futures per bushel of corn in inventory range from 0.80 to 0.57. All the estimates of risk-shifting effectiveness are statistically different from zero at the 1-percent level.8/ The Durbin-Watson values suggest some negative serial correlation in the residuals, but this is not statistically significant at the 5-percent

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6/Corn prices at Denver were obtained from the Denver office of Grain Market News. Prices for other grains and other locations are those reported in weekly issues of Grain Market News (12).

7/The difference in hedging effectiveness between Chicago and Omaha is significant at the 10-percent level, using the Z test as described by Snedecor (10, pp. 173-180). The correlations are 0.741 and 0.613 for Chicago and Omaha, respectively, and the corresponding Z values are 0.952 and 0.713. The standard error of the difference in the Z's is estimated as 2/105 = 0.138. The resulting ratio, t = (.952 - .713)/.138 = 1.73, has a probability only of 0.084 of being exceeded in two samples of this size from the same population.

8/The critical levels for r^2 with 106 degrees of freedom are approximately 0.062 and 0.036, respectively, at the 1-percent and 5-percent levels.

level in four cases and falls in the inconclusive range in the other two cases.

Table 1--Effectiveness of the Chicago corn futures contract for hedging No. 2 Yellow corn stored at selected locations 1/

Location	Standard deviation of profits	Minimum-risk hedge	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Cents/bu.			
Chicago	4.49	-0.80 (.07)	0.55	2.29
Minneapolis	4.02	-.79 (.07)	.54	2.20
Toledo	4.54	-.72 (.08)	.52	2.28
Kansas City	3.97	-.72 (.06)	.47	2.05
St. Louis	4.47	-.65 (.07)	.42	1.92
Omaha	3.88	-.57 (.07)	.38	2.31

1/Based on observations for 120 consecutive 1-month intervals from November 1960 to November 1970. Parentheses contain estimated standard errors. Estimated standard deviation of futures profit is 4.14 cents per bushel.

Oats

Oats have become relatively less important as a feed grain in recent years and trading in oats futures on the Chicago Board of Trade declined from 818 million bushels in 1960-61 to 196 million bushels in 1971-72. For the Chicago Board of Trade contract, No. 3 Extra Heavy White oats, No. 2 Heavy White oats, and No. 1 White oats are deliverable at par; other grades are deliverable at specified premiums and discounts.

Table 2 shows estimates of risk-shifting effectiveness and minimum-risk hedging levels for oats stored at four locations and hedged at Chicago. Risk-shifting effectiveness ranges from 30 to 48 percent at the various locations and indicated minimum-risk hedging levels range from 52 percent to 75 percent. However, the locational differences are not statistically significant at the 10-percent level. The results suggest that the oats contract provides a slightly poorer hedge for oats than the corn contract does for corn at corresponding locations. However, these differences between oats and corn are not statistically significant at the 10-percent level.

Table 2--Effectiveness of the Chicago oats futures contract for hedging oats stored at selected locations 1/

Location and grade	Standard deviation of profits	Minimum-risk hedge	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Cents/bu.			
Chicago, No. 2 Extra Heavy White	3.20	-0.75 (.08)	0.47	2.33
Toledo, No. 2 Heavy White	3.13	-.74 (.07)	.48	1.98
Minneapolis, No. 2 Heavy White	2.88	-.54 (.08)	.30	2.42
Kansas City, No. 2 White	2.55	-.52 (.07)	.36	2.21

1/Based on observations for 120 consecutive 1-month intervals from November 1960 to November 1970. Estimated standard deviation for futures profit is 2.93 cents per bushel.

Grain Sorghum, Barley, and Oats Hedged in Corn Futures

Grain sorghum, barley, and oats are relatively close substitutes for corn in many livestock rations. As a consequence, prices for all four feed grains might be expected to move together. This suggests the possibility of hedging inventories of the three lesser feed grains in corn futures. Since the Commodity Exchange Authority does not recognize cross-commodity hedging, such positions would be classified as speculative and subject to speculative limits. However, many small and moderate-sized firms would not need to approach the 3-million-bushel speculative limit that applies for corn.

Results from analyzing the potential for hedging grain sorghum, oats, and barley stored at selected locations in the Chicago corn futures contract are shown in table 3. Again, the measure of risk-shifting effectiveness is the partial r^2 representing the proportion of the profit variance that can be shifted through hedging at the minimum-risk level. The minimum-risk hedging level is measured in terms of bushels of corn futures per bushel of the specified commodity held in storage. The table shows that risk-shifting effectiveness is significantly different from zero at the 1-percent level for grain sorghum and oats. However, hedging in corn futures would reduce the variance of storage profits only 12 to 20 percent for these grains, and it is doubtful that many firms would consider this worthwhile. For barley, the estimates of risk-shifting effectiveness are not significantly different from zero at the 5-percent level except at one location.

The estimated minimum-risk hedging level for grain sorghum is approximately 0.4 bushel of corn futures per bushel of sorghum in inventory. For oats, the estimated minimum-risk hedging levels are approximately 1 bushel of corn futures for each 4 bushels of oats in inventory. Higher levels of hedging would tend to increase price risks.

Wheat Storage

Wheat is unique among grains in that it is actively traded on three separate futures markets in the United States.9/ The annual volume of trading during the last decade for these three markets is shown in table 4. The Chicago Board of Trade contract is the most heavily traded wheat futures contract, accounting for over two-thirds of the total trading volume. The dominance of the Chicago contract has been attributed to the greater volume of speculation there. Consequently, the Chicago market can absorb hedging better than the markets at Kansas City and Minneapolis. It has been suggested that although hedgers tend to prefer the Minneapolis and Kansas City contracts, which have specifications closer to their needs, they frequently resort to the Chicago contract because of the thinness of the other two markets.10/

Soft Wheat Hedged at Chicago

Although it allows delivery of other classes of wheat, the Chicago contract is generally considered a Soft wheat contract. The primary producing regions for Soft wheat are the Midwest and the Pacific Northwest.

Table 5 presents estimates of risk-shifting effectiveness and minimum-risk hedging levels for Soft wheat stored at selected locations and hedged at Chicago. The estimates of risk-shifting effectiveness for No. 2 Soft Red Winter wheat range from 67 percent at Chicago to 42 percent at Baltimore. Minimum risk-hedging levels are approximately unity at Chicago, Toledo, and St. Louis. At Baltimore, the minimum-risk hedging level drops to 89 percent.

9/In recent years a small volume of grain futures has been traded on a fourth market, the Chicago Open Board of Trade. This market is disregarded here because of its minor role as a hedging market.

10/For more complete discussions of the relationships between these markets, see Ewasiuk (3) and Gray (5).

Table 3--Potential for hedging grain sorghum, oats, and barley at selected locations in the near Chicago corn futures contract[1]

Grain and location	Standard deviation of profits	Minimum-risk hedge [2]	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Cents/bu.			
No. 2 Yellow grain sorghum, Kansas City	6.75	-0.41 (.08)	3/0.20	2.32
No. 2 Yellow grain sorghum, Fort Worth	6.67	-.38 (.08)	3/.18	2.31
No. 2 White oats, Kansas City	2.55	-.23 (.05)	3/.14	2.29
No. 2 Extra Heavy White oats, Chicago	3.20	-.28 (.07)	3/.13	2.19
No. 2 Heavy White oats, Toledo	3.13	-.28 (.07)	3/.14	2.17
No. 2 Extra Heavy White oats, Minneapolis	2.88	-.24 (.06)	3/.12	2.42
No. 3 barley, Kansas City	3.51	-.05 (.08)	.03	2.14
No. 2 Western barley, Los Angeles	2.64	-.14 (.06)	.05	1.86

1/Based on observations for 120 consecutive 1-month intervals from November 1960 to November 1970. Estimated standard deviation of futures profit is 4.14 cents per bushel.
2/Bushels of corn futures per bushel of the commodity specified.
3/Significantly different from zero at the 1-percent level.

Table 4--Volume of trading in wheat futures by exchange, 1961-71

Year beginning July 1	Chicago Board of Trade	Kansas City Board of Trade	Minneapolis Grain Exchange	Total volume1/
	Million bushels			
1961	3,414	459	236	4,141
1962	4,361	535	210	5,152
1963	4,520	582	208	5,355
1964	2,238	434	135	2,826
1965	4,958	761	237	6,000
1966	8,892	1,108	338	10,425
1967	7,566	1,176	365	9,260
1968	5,598	857	328	6,930
1969	2,704	707	209	3,714
1970	2,912	1,007	269	4,235
1971	2,573	650	277	3,535

1/Includes small volume traded on other exchanges.

The Chicago wheat contract does not permit delivery of White wheat; nevertheless, it provides a good hedge for White wheat inventories. At nearby Toledo, the estimated risk-shifting effectiveness and minimum-risk hedging levels at 0.53 and 1.04, respectively, are virtually the same for White wheat as for Red wheat. At Portland, the estimated minimum-risk hedging level drops to 59 percent, resulting in a shift of only 21 percent of the price risk.

The effectiveness of the Chicago wheat contract for hedging Soft wheat at Chicago appears to be slightly higher than the effectiveness of the corn contract for hedging corn. However, the difference is not significant at the 20-percent level.

Hard Winter Wheat Hedged at Kansas City and Chicago

Hard Winter wheat is grown primarily in the Southern Plains, where Kansas leads all other States in volume produced. It is the most widely grown class of wheat in the United States, normally accounting for slightly over half of total production. Although the wheat futures contract at Kansas City is for Hard Winter wheat, trading volume there has been small in comparison with Chicago. Only recently has the Kansas City market accounted for as much as one-fifth of total volume in wheat futures trading.

Table 5--Effectiveness of the Chicago wheat futures contract for hedging Soft wheat stored at selected locations[1]

Location and grade	Standard deviation of profits	Minimum-risk hedge	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Cents/bu.			
Chicago, No. 2, Soft Red Winter	8.33	-1.01 (.07)	0.67	1.52
Toledo, No. 2, Soft Red Winter	9.33	-1.02 (.09)	.53	2.01
St. Louis, No. 2, Soft Red Winter	9.77	-1.04 (.10)	.51	2.18
Baltimore, No. 2, Soft Red Winter	9.15	-.89 (.10)	.42	2.26
Toledo, No. 2, Soft White	9.61	-1.04 (.10)	.53	1.97
Portland, No. 1, Soft White	8.82	-.59 (.11)	.21	2.20

1/Based on observations for 120 consecutive 1-month storage intervals from July 1960 to July 1970. Estimated standard deviation of futures profit is 6.72 cents per bushel.

Table 6 presents estimates of risk-shifting effectiveness and minimum-risk hedging levels for Hard Winter wheat hedged at Kansas City. The results suggest that the contract provides a slightly more effective hedge for ordinary protein wheat than for 13 percent protein wheat. Minimum-risk hedging levels are approximately unity in all three cases.

Table 7 presents estimates of risk-shifting effectiveness and minimum-risk hedging levels for Hard Winter wheat stored at various locations and hedged at Chicago. Comparison of these results with the corresponding results for the Kansas City market shown in table 6 provides a basis for assessing the relative performance of the two contracts for hedging Hard Winter wheat. The 63-percent risk-shifting effectiveness for Hard Winter wheat stored at Chicago suggests a slight advantage for the Chicago contract, compared with a corresponding estimate of 55 percent for Kansas City. For Hard Winter wheat stored at Kansas City, the advantage is with the Kansas City contract. There, the estimated risk-shifting effectiveness for Hard Winter wheat of ordinary protein is 50 percent versus 34 percent for the Chicago contract. For 13 percent protein wheat, the Kansas City contract provides 39-percent risk-shifting effectiveness versus 24 percent for the Chicago contract.

The results suggest that each contract is superior to the other for hedging Hard wheat stored locally. Since Kansas City is nearer the major Hard wheat storage areas than is Chicago, the Kansas City market might be expected to be more widely used than it is.11/

Table 6--Effectiveness of the Kansas City wheat futures contract for hedging Hard Winter wheat stored at selected locations1/

Location and grade	Standard deviation of profits	Minimum-risk hedge	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Cents/bu.			
Kansas City, No. 1 Hard Winter, Ordinary Protein	8.20	-1.06 (.10)	0.50	1.95
Kansas City, No. 1 Hard Winter, 13% Protein	8.50	-.97 (.12)	.39	1.91
Chicago, No. 2 Yellow Hard Winter	7.99	-1.08 (.09)	.55	2.08

1/Based on observations for 120 consecutive 1-month intervals from July 1960 to July 1970. Estimated standard deviation of futures profit is 5.46 cents per bushel.

Comparison of table 7 with table 5 shows that the Chicago contract serves almost as well for hedging Hard Winter wheat stored at Chicago as for hedging Soft wheat stored there. Differences in estimated correlations as large or larger than those observed would be found more than 80 percent of the time in different samples from the same population.

Spring Wheat Hedged at Minneapolis and Chicago

Hard Spring wheat is produced in the North Plains primarily in North Dakota, South Dakota, and Montana. This class is differentiated from Hard

11/These conclusions are based on the assumption that individual hedging transactions have an imperceptible effect on price. For large hedging transactions, where the effect on price must be taken into account, the Chicago contract might be more advantageous than this analysis indicates because of the greater number of speculators there to absorb the hedging load.

Winter wheat by quality factors, particularly protein level. In recent years, Hard Spring wheat has accounted for about 13 to 14 percent of U.S. wheat production. The futures contract traded at Minneapolis calls for delivery of Hard Spring wheat. Volume on the Minneapolis contract accounts for only about 6 percent of trading in wheat futures.

Table 7--Effectiveness of the Chicago wheat futures contract for hedging Hard Winter wheat stored at selected locations 1/

Location and grade	Standard deviation of profits	Minimum-risk hedge	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Cents/bu.			
Chicago, No. 2 Yellow Hard Winter	7.99	-0.94 (.07)	0.63	1.51
Kansas City, Hard Winter, Ordinary Protein	8.20	-.71	.34	1.95
Kansas City, Hard Winter, 13% Protein	8.50	-.62 (.11)	.24	1.95
Portland, No. 1 Hard Winter	7.47	-.45 (.10)	.17	1.80

1/Based on observations for 120 consecutive 1-month storage intervals from July 1960 to July 1970. Estimated standard deviation of futures profit is 6.72 cents per bushel.

Estimates of risk-shifting effectiveness and minimum-risk hedging levels for Hard Spring wheat stored and hedged at Minneapolis are shown in table 8. The estimates of risk-shifting effectiveness range from 25 percent for ordinary protein wheat to 41 percent for 15 percent protein wheat. Risk-shifting effectiveness appears to increase as protein level increases, but the differences are not statistically significant even at the 30-percent level.

Estimates of risk-shifting effectiveness for Hard Spring wheat stored at Minneapolis and hedged at Chicago are shown in table 9. The estimates range from 12 to 25 percent and the corresponding minimum-risk hedging levels range from 44 to 57 percent. The estimates of risk-shifting effectiveness are consistently lower than those obtained for Hard Winter wheat stored at Kansas City and hedged at Chicago This suggests that the Chicago wheat futures contract provides a less satisfactory hedge for Hard Spring wheat at Minneapolis than for Hard Winter wheat at Kansas City. Again, the differences are not statistically significant.

Table 8--Effectiveness of the Minneapolis wheat futures contract for hedging Hard Spring wheat stored at Minneapolis[1/]

Location and grade	Standard deviation of profits	Minimum-risk hedge	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Cents/bu.			
Minneapolis, Dark Northern Spring, Ordinary Protein	9.90	-0.94 (.16)	0.25	2.75
Minneapolis, Dark North Spring, 13% Protein	7.81	-.89 (.11)	.36	2.03
Minneapolis, Dark North Spring, 15% Protein	7.05	-.85 (.10)	.41	1.79

1/Based on observations for 120 consecutive 1-month intervals from July 1960 to July 1970. Estimated standard deviation of futures profit is 5.27 cents per bushel.

The performance of the Minneapolis contract for hedging wheat stored locally has been lower than the corresponding performance of the more widely traded contracts at Chicago and Kansas City. For example, the estimated effectiveness of the three contracts, when used to hedge wheat of grade nearest that specified for par delivery, is as follows:

Contract	Grade and class of wheat	Estimated risk-shifting effectiveness
Chicago	No. 2, Soft Red Winter	0.67
Kansas City	Hard Winter, Ordinary Protein	.50
Minneapolis	Dark Northern Spring, 15% Protein	.41

These differences suggest that risk-shifting effectiveness is greater for the more heavily traded contracts.

The above findings tend to confirm that each wheat futures contract plays a distinct role in transferring price risks on wheat inventories. When the quantities involved are so small that their effect on price can be disregarded, the greatest risk-shifting effectiveness is obtained by hedging each type of wheat in the nearby market; that is, Hard Spring wheat at Minneapolis, Hard Winter wheat at Kansas City, and Soft wheat at Chicago. Chicago, which continues to handle the largest trading volume, provides a better hedge for Soft wheat than do the other two markets for their respective wheats.

Table 9--Effectiveness of the Chicago wheat futures contract for hedging Hard Spring wheat stored at Minneapolis1/

Location and grade	Standard deviation of profits	Minimum-risk hedge	Risk-shifting effective-ness (r^2)	Durbin-Watson (d)
	Cents/bu.			
Minneapolis, Dark Northern Spring, Ordinary Protein	9.90	-0.57 (.13)	0.15	2.68
Minneapolis, Dark Northern Spring, 13% Protein	7.81	-.44 (.10)	.15	2.12
Minneapolis, Dark Northern Spring, 15% Protein	7.05	-.52 (.09)	.25	1.94

1/Based on observations for 120 consecutive 1-month storage intervals from July 1960 to July 1970. Estimated standard deviation of futures profit is 6.72 cents per bushel.

Cattle Feeding

Cattle feeders have had a hedging market available in the Chicago Mercantile Exchange live beef cattle futures contract since November 30, 1964. Contracts are traded for delivery every second month beginning in February. The contract calls for delivery of uniform Choice Grade steers weighing 1,050-1,150 pounds and yielding 61 percent, or weighing 1,150-1,250 pounds and yielding 62 percent. Provision is made for substituting a limited number of high Good Grade steers at appropriate discounts. With the August 1969 future, the contract size was changed from 25,000 pounds to 40,000 pounds. Par delivery was at Chicago until the August 1971 contract, with alternative delivery points at Omaha and Kansas City at discounts of 75 cents and $1 per hundredweight, respectively. For the August 1971 contract and subsequent contracts, par delivery is at Omaha with allowances at alternative delivery points as follows: Chicago, + 50 cents; Peoria, + 50 cents; Guymon, Okla.,-$1.

Substantial numbers of feedlot cattle are hedged, but these represent a small fraction of the cattle on feed in the United States. Open contracts averaged 18,993 during 1971-72. All short open positions do not represent hedging, however. In a survey conducted on May 29, 1969, the Commodity Exchange Authority (11, p. 35) found 13,049 short positions classified as hedging or about half of the total. Over half of these short hedging positions were held by livestock farmers, beef producers, and ranchers; and about a third were held by meat packers. However, 13,000 contracts would have covered only about

490,000 cattle--less than 5 percent of the 11 million head of cattle on feed reported for April 1, 1969.

This analysis is based on observations for 18 consecutive 4-month feeding periods beginning March 1965 and ending March 1971. Table 10 lists the cattle-feeding locations analyzed. These were selected to represent the major cattle-feeding regions in the United States and to take advantage of price data collected by Livestock Market News. The markets used as sources of price quotations for slaughter cattle, feeder cattle, grain, and hay are shown for each location.

Table 10--Cattle- and feed-pricing points for the cattle-feeding locations analyzed

Feeding location	Slaughter cattle market	Feeder cattle market	Grain market	Hay price1/
Eastern Corn Belt	Chicago	Kansas City	Corn, Chicago	Illinois
Western Corn Belt	Omaha	Omaha	Corn, Omaha	Iowa
Colorado	Denver, direct	Amarillo, auction	Corn, Denver	Colorado
High Plains	Clovis, N. Mex., direct	Clovis, N. Mex., auction	Sorghum, Ft. Worth	New Mexico
California	Visalia, direct	Visalia, auction	Barley, Stockton Sorghum, Los Angeles	California

1/Hay prices are State averages as reported in Agricultural Prices (14).

To avoid confusing differences due to location with differences due to type of cattle, the same weight, sex, and grade categories were analyzed for each location insofar as possible. Previous studies suggest that short-fed Good and Choice steers and heifers are among the most numerous types of fed cattle produced in each region. Good Grade feeder cattle were assumed to finish out to Good Grade slaughter cattle, and Choice feeders were assumed to finish as Choice slaughter cattle. Feed requirements, costs, and rates of gain are assumed to be the same for Good Grade cattle as for Choice Grade cattle. The assumptions about buying and selling weights and feed consumption are shown in table 11.

Table 11--Assumed buying and selling weights and feed consumption for Good and Choice short-fed steers and heifers

Item	Unit	Steers	Heifers
Initial weight	Pounds	692	667
Days on feed	Number	122	122
Daily gain1/	Pounds	2.87	2.65
Total gain	do.	350	323
Finished weight	do.	1,042	990
Weight after shrink2/	do.	1,000	950
Grain consumed per head:1/			
Corn Belt and Colorado, corn	Bushel	37.9	35.6
High Plains, grain sorghum	do.	43.1	40.5
California:			
Grain sorghum	do.	21.5	20.2
Barley	do.	24.5	23.0
Hay consumed per head1/	Tons	.26	.25

1/Rates of gain are based on National Research Council data (9, p. 22). Feed consumption is based upon TDN requirements reported in the same publication.
2/A 4-percent shrink is assumed.

Buying prices for feeders and selling prices for slaughter cattle are weekly averages reported by USDA's Market News Service for the markets selected. These are calculated by Market News Service as a simple average of the daily prices for each week. The weeks selected include the 15th of the month. The futures quotation used was the closing price on Wednesday.

Grain prices are Thursday prices for the weeks selected as reported in Grain Market News (12). Hay prices are State estimates of monthly prices received by farmers as reported in Agricultural Prices (14).

Hedging was assumed to involve selling the futures short as the cattle were placed on feed and buying back the futures contract 4 months later as the finished cattle were sold. The futures contract selected for hedging was the one maturing the month after the cattle were sold. Since contracts mature only once every 2 months and a contract cannot be held beyond maturity, many cattle must be hedged in contracts maturing a month or more after the cattle are sold. The assumption that the cattle are sold 1 month ahead of contract maturity also avoids the rather sharp upward price movement that has tended to characterize the last month of trading on live cattle futures contracts.

Table 12 summarizes the estimates of minimim-risk hedging levels and risk-shifting effectiveness obtained for four types of cattle at five locations. The estimated minimum-risk hedging levels range from -0.56 to -0.88. These may be interpreted as 0.56 to 0.88 unit of short futures per unit of slaughter cattle produced. The corresponding estimates of risk-shifting effectiveness range from 36 to 57 percent. All but one of the correlation coefficients between cash profits and futures profits differ significantly from zero at the 1-percent level; the exception is significantly different from zero at the 5-percent level. In all of the situations studied, hedging at the minimum-risk level can reduce profit risk.

Location, grade, and sex have little impact on hedging effectiveness, as shown in table 12. The highest correlation was 0.73 for Choice steers in the eastern Corn Belt and the lowest correlation was 0.60 for Good heifers in the western Corn Belt. Sample correlations differing by this amount can be expected to arise more than half the time in samples of this size when the parent populations have identical correlations. Hence, the evidence examined here does not reveal any statistically significant differences in risk-shifting effectiveness among the cattle-feeding situations studied.

Hog Feeding

Trading in live hog futures began February 26, 1966, on the Chicago Mercantile Exchange, but volume remained small until 1969-70, as shown below:

Year beginning July	1,000 transactions
1966	9
1967	11
1968	13
1969	120
1970	191
1971	366

Table 12--Effectiveness of the Chicago Mercantile Exchange live cattle futures contract for hedging feeding operations for four types of short-fed cattle at five locations[1/]

Location and type of cattle	Standard deviation of profit	Minimum-risk hedge	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Dollars/head			
Eastern Corn Belt:				
Steers:				
Choice	21.60	-0.88 (.22)	0.53	2.30
Good	19.56	-.82 (.19)	.57	1.96
Heifers:				
Choice	19.82	-.86 (.21)	.54	2.12
Good	17.79	-.68 (.21)	.43	1.65
Western Corn Belt:				
Steers:				
Choice	20.68	-.80 (.22)	.47	2.63
Good	17.18	-.64 (.19)	.44	2.74
Heifers:				
Choice	19.74	-.75 (.24)	.42	2.84
Good	15.75	-.56 (.20)	.36	2.88
Colorado:				
Steers:				
Choice	19.81	-.84 (.19)	.57	1.85
Good	17.00	-.66 (.18)	.49	2.57
Heifers:				
Choice	18.74	-.83 (.19)	.57	2.73
Good	16.13	-.69 (.17)	.53	2.84
High Plains:				
Steers:				
Choice	19.83	-.74 (.22)	.44	1.85
Good	18.98	-.71 (.21)	.44	2.13
Heifers:				
Choice	18.35	-.70 (.22)	.42	2.40
Good	15.84	-.63 (.18)	.45	2.58
California:				
Steers:				
Choice	19.73	-.76 (.21)	.48	2.50
Good	19.55	-.72 (.22)	.44	2.43
Heifers, Choice 2/	19.42	-.76 (.23)	.45	2.37

1/Based on observations for 18 consecutive 4-month feeding periods starting in March 1965 and ending in March 1971. Estimated standard deviation of futures profit is $17.89 per 1,000 pounds.
2/Prices for 700- to 900-pound heifers were used in the absence of a complete series of prices for heifers in the 900-pound weight range.

During part of this period, contracts maturing each month of the year were listed, but currently only contracts for February, April, June, July, August, October, and December are traded.

The live hog futures contract calls for delivery of U.S. No. 1, No. 2, No. 3, and No. 4 hogs (barrows and gilts) averaging in the 200- to 230-pound weight range. Par delivery was at Chicago prior to the closing of the Chicago stockyards and is now at Peoria. Deliveries are also permited at Omaha, East St. Louis, Sioux City, and St. Paul at an allowance of 25 cents per hundredweight, and at Kansas City at an allowance of 50 cents per hundredweight. Beginning with the April 1972 contract, St. Joseph was added as a delivery point with a 50-cent allowance.

Most of the hogs marketed in the United States are fed out on the same farm where they are farrowed, but separate feeder pig production operations and hog-finishing operations are increasing in importance. To completely hedge a farrowing-finishing operation would require selling live hog futures when the sows are bred and buying back the contracts 9-10 months later as the finished hogs are marketed. However, trading on individual live hog futures contracts has seldom opened early enough to allow producers to hold open positions in one contract for such a long period. As a consequence, this analysis focuses on the hedging of hog-finishing operations where open positions of shorter duration are required. The hedging program thus assumed calls for selling live hog futures as the feeder pigs and feed are purchased and buying back the futures 3-1/2 months later as the finished hogs are sold.

The marketing of feeder pigs is conducted primarily through auctions, and price reporting on auction markets has been limited. Most slaughter hogs are now marketed direct from the producer to the packer and the price-reporting system for direct sales is still evolving. Consequently, this analysis is limited to locations where nearby feeder pig and slaughter hog price quotations are available.

The hog-feeding program analyzed involves the production of 215-pound U.S. No. 1 and 2 barrows and gilts. It is a growing and finishing program starting with U.S. No. 1 and 2 feeder pigs weighing 45 pounds and marketing them 3-1/2 months later at 215 pounds. This is believed to be representative of current practice among many Corn Belt hog producers.

Table 13 lists the four hog-feeding situations analyzed and shows the sources of hog and feed price quotations for each. The feeder pig and slaughter hog prices used are weekly averages reported by Livestock Market News (13); prices for No. 2 Yellow corn are average prices for Thursday reported in Grain Market News (12); and soybean meal prices are midmonth estimates of prices paid by farmers reported in Agricultural Prices (14).

The rate of gain, length of feeding period, and feed consumption assumed in the hog feeding program analyzed are shown below:

Initial weight, lb.	45
Days on feed	106
Daily gain, lb.	1.60
Total gain, lb.	170
Finished weight, lb.	215
Pounds of feed, per lb. of gain	3.3
Total feed per pig, lb.	561
Corn per pig, bu.	8.61
Soybean meal per pig, lb.	67.4

These figures are based on estimates derived from Van Arsdall (15). It should be noted that the grading system for slaughter hogs was changed in July 1968 and that for feeder pigs in April 1969. Also, quotations for southern Missouri feeder pigs were not available prior to November 1967. Mexico, Mo., feeder pig quotations were used for the period prior to that data. In the initial runs, dummy variables were introduced to permit shifts in the relationships on these dates. In no case did the coefficients on the dummy variables exceed their respective standard errors in absolute value. The other coefficients differed little from those obtained without use of the dummy variables for grade change and quotation change.

As shown in table 14, the estimated risk-shifting effectiveness of the Chicago live hog futures contract ranged from 70 percent for Indiana to 78 percent for Iowa. Estimated minimum-risk hedging levels were approximately 100 percent. All locational differences observed were quite small relative to their standard errors. However, the Durbin-Watson statistics fell in the area of indeterminancy, indicating that the usual statistical tests may not be valid. In general, the results suggest that the live hog futures contract provides a high level of risk protection.

DETERMINING OPTIMAL HEDGING LEVELS

When expected returns on the futures position differ from zero, the optimal hedging level differs from the level that minimizes risk. For the individual hedger, expected returns on the futures position may be nonzero for several reasons. First, there are costs associated with futures trading such as commissions and interest foregone on margin deposits. Second, the futures price may be biased. Third, individual traders may be able to forecast futures price movements.

This section considers the problem of determining optimal hedging levels when expected returns on the futures position differ from zero. In contrast to the minimum-risk hedging levels reported in the preceding section, optimal hedging levels can only be determined if the expected profits from both the cash and futures activities are specified. Expected profits are a function of costs and price expectations, both of which differ among firms. Therefore, estimates of optimal hedging levels tend to be much more narrowly applicable than the corresponding estimates of minimum-risk hedging levels. Consequently, this section focuses on procedures for determining optimal hedging levels using only a limited number of examples.

Table 13--Hog- and feed-pricing points for the hog-feeding locations analyzed

Feeding location	Slaughter hog market	Feeder pig market	Corn market	Soybean meal price[1]
Iowa	Interior Iowa and southern Minnesota	Southern Missouri[2]	Omaha	Iowa
Illinois	Illinois, direct	Illinois, auctions	Chicago	Illinois
Indiana	Indianapolis	Kentucky, auctions	Toledo	Indiana
Georgia	Georgia, Florida, Alabama, direct	Kentucky, auctions	Chicago	Georgia

1/Soybean meal prices are State averages as reported in Agricultural Prices (14).
2/Mexico, Mo., auction prices used prior to November 1967.

Table 14--Effectiveness of the Chicago live hog futures contract for hedging hog-feeding operations, selected locations[1]

Location	Standard deviation of profits	Minimum-risk hedging level	Risk-shifting effectiveness (r^2)	Durbin-Watson (d)
	Dol./head			
Iowa	6.74	-1.06 (.17)	0.78	1.06
Illinois	6.91	1.04 (.20)	.71	1.22
Indiana	6.65	-.99 (.20)	.70	.89
Georgia	6.55	-.99 (.19)	.71	1.52

1/Based on observations for 15 3-1/2-month feeding periods starting every 4 months from June 1966 to June 1971. Estimated standard deviation of futures profit is $5.59 per head.

A Graphical Method of Determining Optimal Hedging Levels

At least two elements are required to determine the optimal hedging level: (1) the correlation between cash profits and futures profits and (2) the ratio between expected profits on the futures position and expected profits on the cash position. For a more precise estimate of the optimal hedging level, the ratio of the standard deviations of cash profits to futures profits also must be taken into account. However, this ratio frequently approaches unity and the errors introduced by assuming that it is unity may be inconsequential.

Figure 1, which is derived from the equations in the appendix, provides for determining the optimal hedging level when the correlation between cash profits and futures profits and the ratio of expected profits for the cash and futures activities are specified. The figure applies directly when cash profits and futures profits have equal variances. It also may be used where the variances differ by following the procedure described below.

The horizontal axis on the figure measures the ratio of expected futures profit to expected cash profit after each is standardized by dividing by its standard deviation. The figure has been limited to the range of profit ratios from -0.5 to +1.0, thus covering most of the situations of interest. It must be noted that expected futures profit is measured with respect to a long futures position. For use in this figure, a positive cost (or negative expected profit) on a short futures position is equivalent to a positive expected profit on a long futures position. Thus, for example, if the expected profit on the cash activity is $10 per unit and the expected loss on the short futures position is $2 per unit, the profit ratio to be located on the horizontal axis is 0.2.

The vertical axis measures the correlation between cash profits and futures profits. Each curved line connects points where the same optimal hedging ratio applies when the standard deviations are equal. For example, if the correlation between cash profits and futures profits is 0.8 and the standardized profit ratio is 0.2, the optimal hedging ratio is determined by locating the point directly to the right of 0.8 on the vertical axis and directly above 0.2 on the horizontal axis. This point lies just above the -0.7 hedging ratio line, indicating an optimal hedging ratio of approximately -0.71.

To determine the optimal hedging ratio when the variance of cash profits and the variance of futures profits differ, the following procedure may be employed:

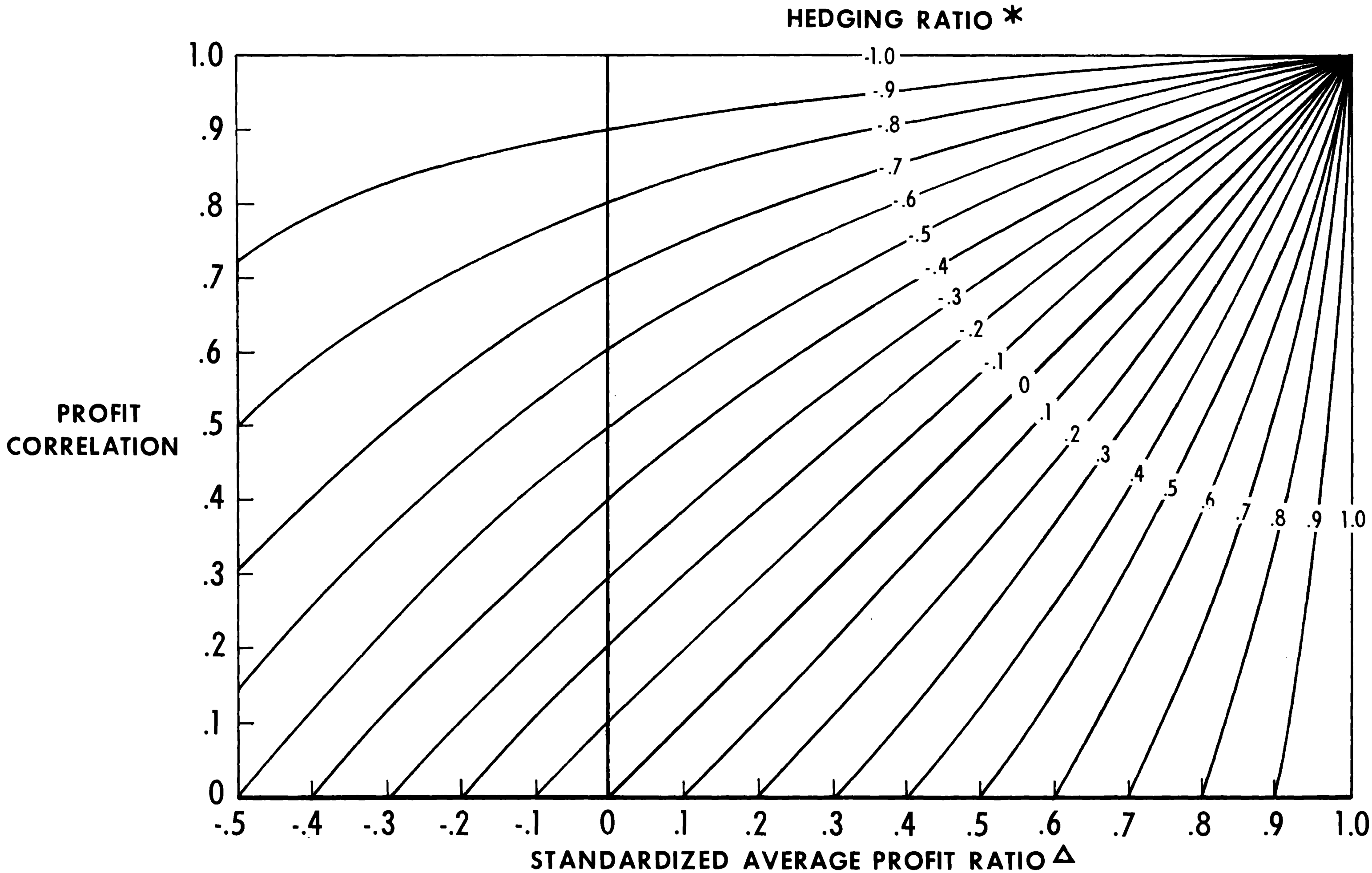
1. Calculated the ratio Θ of the standard deviation of futures profits to the standard deviation of cash profits, $\Theta = \sqrt{\sigma_{11}} / \sqrt{\sigma_{22}}$.

2. Determine the standardized profit ratio, $(\mu_1 / \mu_2) \div \Theta$.

3. Locate the point on the figure corresponding to the appropriate correlation and standardized profit ratio and determine the indicated hedging ratio.

4. Divide the hedging ratio obtained from the figure by Θ.

OPTIMAL HEDGE ESTIMATOR



* UNITS OF FUTURES PER UNIT OF CASH. DIVIDE BY THE CORRESPONDING RATIO OF STANDARD DEVIATIONS, $\sqrt{\sigma_{11}} / \sqrt{\sigma_{22}}$, TO OBTAIN THE OPTIMAL HEDGING RATIO.

△ RATIO OF AVERAGE FUTURES PROFIT TO AVERAGE CASH PROFIT AFTER EACH IS DIVIDED BY ITS STANDARD DEVIATION, $(\mu_1 / \sqrt{\sigma_{11}}) / (\mu_2 / \sqrt{\sigma_{22}})$.

Figure 1

Consider the feeding of Choice steers in the western Corn Belt, for example. For this enterprise, the squared correlation between cash profits and futures profits shown in table 12 is 0.47. The corresponding correlation is 0.69. Assume that expected cash profit is $10 per head and that expected loss on the short futures position equals futures trading costs, $1.21 per head. Noting again that a loss on a short position is treated the same as an equivalent gain on a long position, the ratio of expected futures profits to expected cash profits is 1.21/10.00 = 0.121. By finding the point on the figure that lies directly above 0.121 on the horizontal axis and directly to the right of 0.69 on the vertical axis, the optimal hedging ratio is determined to be -0.61.

Somewhat greater precision may be obtained by taking into consideration the differences in variances between cash profits and futures profits. Using the data for Choice steers in the western Corn Belt, $\Theta = 17.89 \div 20.68 = 0.865$. The standardized profit ratio is $(1.21/10.00) \div \Theta = 0.140$ and the correlation is 0.69. The hedging ratio obtained from the figure is -0.61. When divided by Θ, the corrected hedging ratio is -0.71. In this case, the high variance of cash profits relative to the variance of futures profits has a considerable effect on the optimal hedging level.

Certain basic relationships between correlations, profit expectations, and optimal hedging ratios are illustrated in the figure. Of primary interest is the area above and to the left of the 45-degree line that passes through the origin. This is the area where a normal hedge involving a futures position opposite in sign to the cash position is implied. Three observations are noteworthy. First, as the correlation between cash profits and futures profits declines, the optimal hedging level becomes smaller. Second, so long as the correlation is less than 1, an increase in expected futures profits relative to expected cash profits (or equivalently an increase in the expected loss from a short futures position) makes the optimal hedging level smaller. Third, the changing slopes of the lines show that the two effects are interrelated. A given increase in futures trading costs has a larger impact on the optimal hedging level when the correlation is low than when it is high.

The limiting cases represented in the figure are of special interest. First, if the correlation between cash profits and futures profits is +1, the optimal hedging level is -1 as long as the profit ratio is less than 1. Thus, if basis risk is completely absent the optimal hedging level is unity so long as futures trading costs do not exceed expected cash profits and futures profits and cash profits have the same variance.

Second, the points on the vertical line that passes through zero on the horizontal axis represent minimum-risk hedging ratios. The figure shows that the minimum-risk hedging ratio equals the correlation between cash profits and futures profits when the profit variances are equal. When these variances are not equal, the correlation must be divided by the ratio of standard deviations, Θ. This is exactly equivalent to the procedure used to calculate the minimum-risk hedging levels reported previously.

Below the 45-degree line on the figure, the optimal futures position has the same sign as the cash position. For normal hedging to be implied, the correlation between futures profits and cash profits must exceed the ratio of expected futures profits to expected cash profits.

Futures Trading Costs

Having demonstrated how futures trading costs tend to depress optimal hedging levels below the levels that minimize risk, the next concern is to obtain the required estimates of costs and expected profits on the futures and cash positions. The costs of futures trading will be considered first.

Commissions represent a major cost of futures trading. Standard commission rates for nonmembers are set by the exchanges. Other costs of trading commodity futures include interest foregone on margin deposits and miscellaneous costs. Although the exchange sets minimum margin requirements, actual margin requirements vary among brokers and even among customers for the same broker. Commission rates and margin requirements change over time. Table 15 shows commissions and typical costs of interest on margin deposits for the commodities in this study. In estimating futures trading costs, monthly interest cost on the margin deposit is multiplied times the number of months the position is held and added to commission costs.

Compared with the value of the commodities traded, futures trading costs are small. But when futures trading costs are compared with profits on the cash position, they are significant.

Table 15--Round-term commissions and typical cost of interest foregone on margin deposits for futures trading in five commodities

Commodity	Contract size	Commission		Margin deposit		Monthly interest cost1/
		Per contract	Per unit	Per contract	Per unit	
	Bushels	Dollars	Cents/bu.	Dollars	Cents/bu.	Cents/bu.
Corn	5,000	30	0.6	700	0.14	0.0875
Oats	5,000	25	.5	500	.10	.0625
Wheat	5,000	30	.6	800	.16	.1000
	Pounds		Cents/cwt.		Dollars/cwt.	Cents/cwt.
Live cattle	40,000	36	9.0	500	1.25	0.0781
Live hogs	30,000	30	10.0	400	1.33	.0831

1/Based on an interest rate of 7-1/2 percent per annum.

Bias in Grain and Livestock Futures Prices

Futures price bias may be defined as the tendency for the futures price to lie either below or above the price that traders in the market expect to prevail as the future matures. The theory of normal backwardation suggests that the bias would be downward; that is, the futures price would tend to lie below the expected price. When bias exists it tends to be small relative to the magnitude of the price fluctuations and therefore difficult to isolate and quantify. Consequently, past efforts to measure bias or risk premiums in futures have met with mixed success and somewhat contradictory conclusions. Similarly, in this study the data do not firmly resolve the question of bias, but only add to the accumulated evidence and indicate some apparent differences among commodities.

In principle, the measurement of futures price bias is simply finding the difference between the futures price and the traders' expected price. The task is difficult because the expected price is not observable. One approach is to assume that traders' price expectations, in contrast to the futures prices themselves, are unbiased. In other words, errors in expectations are distributed symetrically about zero so that they average zero over the long run. This assumption allows one to estimate the bias in futures prices by averaging futures price changes over a series of observations. In this study, estimates of the bias were constructed by averaging successive month-to-month changes in the price of the near future.

If traders' price expectations are to be completely unbiased, prices must be free of trend or traders must be able to anticipate trends. Consequently, the assumption of unbiased price expectations may not be justified when prices have exhibited a pronounced trend over the period of observations available. An alternative approach is to assume that traders completely lack ability to anticipate trends and instead base their price expectations on current price levels plus the normal seasonal price movement. This results in an estimate of the futures price bias that differs from the estimate obtained under the assumption that expectations are unbiased. The difference equals the average trend over the period of observations. It seems likely that traders only partly anticipate trends and that the true bias lies somewhere between the estimates obtained by these two alternative approaches.

Estimates of price bias for seven futures contracts using both approaches are reported in table 16. The estimates of futures price bias under the second assumption were derived from the first set of estimates by adding the monthly trend in the cash price which most nearly corresponds to the futures price in terms of location and grade.

The results reported in table 16 show that the available data do not permit sufficiently precise estimates to determine whether futures price bias is likely to be of economic consequence. For grains, the magnitude of the estimated bias tends to be small relative to its standard error and not statistically significantly different from zero. However, in several cases, and particularly for Minneapolis wheat, the estimated bias is about 0.5 cent per bushel per month, a magnitude which has important economic implications when hedging. Hence, with the exception of corn, the possibility of economically significant bias cannot be ruled out.

Table 16--Estimates of bias in futures prices based upon two assumptions about traders' price expectations 1/

Contract	Period of observation	Assumptions: Unbiased expectations 2/	Assumptions: Expectations based upon current prices
		---Cents/bushel---	
Chicago corn	Nov. 1960 to Nov. 1970	-0.18 (.37)	+0.17
Chicago oats	July 1960 to July 1970	-.18 (.28)	-.52
Chicago wheat	July 1960 to July 1970	+.51 (.61)	-.09
Minneapolis wheat	July 1960 to July 1970	-.58 (.48)	-.61
Kansas City wheat	July 1960 to July 1970	+.03 (.48)	-.40
		--- Cents/pound---	
Chicago live cattle	May 1965 to May 1971	-.30 (.12)	-.21
Chicago live hogs	May 1966 to May 1971	-.31 (.15)	-.41

1/Based on monthly observations of price changes in the near future.
2/Figures in parentheses are standard errors.

For livestock, evidence of a negative bias in futures prices is stronger. From May 1965 to May 1971, holders of long positions in cattle futures, on the average, gained at the expense of the holders of short positions. The average gain during the 6-year period was 30 cents per hundredweight per month, with an estimated standard error of 12 cents per hundredweight. However, about one-fifth of this estimated bias is attributable to a single observation, the price movement from January to February 1971. Also, the upward movement in the futures price was sharper during the last 2 months of trading on each contract than during earlier periods in the lives of the contracts. The cash price of slaughter cattle increased about $6.50 per hundredweight over the period, or about 9 cents per month. The alternative estimate of the bias is 30 - 9 = 21 cents per hundredweight per month. An average bias of 21 cents per hundredweight per month amounts to a cost of over $8 per head for the cattle feeder fully hedging 1,000-pound cattle over a 4-month feeding period. This would be a prohibitively high hedging cost if it were to persist. However, a price bias of this magnitude provides very handsome returns for long speculators. Consequently, competition among traders may be expected to reduce or elimiate such bias over time.

The average return for holding a long position in the near live hog future over successive 1-month intervals starting in May 1966 and ending in May 1971 was 31 cents per hundredweight. The standard error of this estimate of the mean is 15 cents per hundredweight. The average decrease in the cash price for No. 1 and 2 200- to 220-pound slaughter hogs in Iowa and southern Minnesota was 10 cents per month over the same period. The alternative estimate of the bias in the futures price is, therefore, 31 + 10 = 41 cents per hundredweight over each 1-month period. The estimated cost of price bias per hog for hedging 215-pound hogs over a 3-1/2-month feeding period is 2.15 x 3.5 x .31 = $2.33, when the first estimate of the bias is used. When the alternative estimate of the bias is used, the estimated cost of the bias per hog is $3.09. Costs of this magnitude would represent a considerable barrier to hedging of hog-feeding operations. They also represent a substantial profit for persons holding long positions in hog futures and for this reason seem unlikely to persist.

Based upon the evidence examined here, it seems reasonable to assume that corn futures prices are unbiased. For the other grain futures contracts, the question of bias must remain open. For the livestock contracts, there is a strong indication of a negative price bias. But experience with these contracts is limited and the bias may diminish as trading expertise grows.

The Impact of Price Expectations

Even if futures prices are unbiased, individual traders may be able on occasion to predict price changes. Indeed, the belief that they can forecast price movements is the primary reason that speculators enter the market. When the hedger thinks that he can forecast futures price movements, his futures position may be adjusted accordingly.

Common sense dictates that when prices are expected to rise, the trader should shift his net position toward the long side. Indeed, if the expected price increase is large enough, he may find it advantageous to hold long positions in both cash and futures instead of holding opposite positions. Similarly, if the futures price is expected to decline, a larger short position is desirable than when no change in the futures price can be foreseen. The figure on p. 28 provides for entering such varied expectations in determining optimal hedging levels. All that is required is that expected prices be used in calculating the expected profit ratio. The impact of different price expectations can thus be explored. If, for example, futures prices are expected to increase, the ratio of futures profits to cash profits will be large. Therefore, to determine the optimal hedging level a shift to the right on the figure is necessary. This results in a smaller hedge. If the profit ratio is larger than the correlation, one must pass to the right of the 45-degree line and enter the area where the cash position and futures position have the same sign.

Alternatively, if futures prices are expected to decline by an amount which exceeds futures trading costs, a shift to the left side of the figure is required. In this case, the optimal hedge calls for a larger short position than does the minimum-risk hedge which is determined along the vertical line that passes through zero on the horizontal axis.

Consider again the individual feeding Choice steers in the western Corn Belt. Recall that the correlation between futures profits and cash profits was 0.69 and Θ, the ratio of standard deviations in profits, was 1.16. Again assume that the feeder expects $10 per head cash profit on the cattle. This time, however, instead of a neutral view on the futures price, he expects a 50-cent-per-hundredweight increase in the futures price. When this is added to futures trading costs, his expected loss on the futures is $5 + $1.21 = $6.21 per head for 1,000-pound steers. The raw profit ratio is $6.21/$10.00 = 0.62 and the standardized profit ratio is 0.62 x 1.16 = 0.72. When this ratio and the correlation of 0.69 are applied to the figure, the optimal hedging ratio obtained is between 0 and + 0.1. This compares with an optimal hedging ratio of -0.71 for the case of neutral futures price expectations. Obviously, rather small shifts in price expectations can imply large shifts in the optimal hedging ratio.

Estimating Expected Profits on the Cash Position

Determining the appropriate level of expected cash profits to use in calculating the optimal hedging level requires considerable care. The cash profit figure needed equals total expected receipts minus out-of-pocket costs less costs charged for fixed resources. The last category of costs poses the greatest problem.

The fixed resources of the firm may include physical capital such as feedlot or grain storage facilities. They may also take the form of human capital such as special skills in managing livestock feeding and grain storage operations. The basic problem is to determine what costs should be charged to the enterprise for these fixed inputs. The appropriate procedure is to price each input at the price the firm would be willing to pay for an additional unit of the input in the short run. But since the value imputed to each limiting resource cannot be determined without conducting a complete analysis of the firms' activities, the costs for fixed inputs can only be approximated.12/ First, the costs for any resource not used to full capacity may be set at zero. For those remaining resources that are fully utilized, some portion of their costs--depending upon their best alternative use--may be subtracted from profit.

In some cases, such as an individual having cattle fed in a custom feedlot or a firm hiring grain storage services, virtually all costs may be variable. This leaves a very small residual expected profit. The ratio of expected futures profit to expected cash profit might then be quite large. In viewing the figure, it is apparent that as the expected profit ratio approaches 1, the optimal hedging level becomes very sensitive to small changes in the correlation. Empirical estimates of means, correlations, and variances generally lack the precision needed for drawing conclusions about optimal hedging levels in this area on the right side of the figure. Consequently, little can

12/Such an analysis could be accomplished through use of quadratic programing. To obtain a unique solution, the firm's level of risk aversion would have to be known.

be said about optimal futures positions in situations such as custom feeding where virtually all inputs are variable.

Conclusions

This analysis of hedging potential shows that futures trading costs tend to depress optimal hedging levels below the levels that would be optimal without such costs. Figure 1 illustrates that this effect depends upon the correlation between cash profits and futures profits and becomes more marked as this correlation declines. Since both cash profit expectations and futures profit or loss expectations vary from firm to firm, estimates of optimal hedging levels are applicable only to specific firms. Such estimates are subject to error because of difficulties in specifying expected profits and sampling errors in measuring correlations and variances. Since profit expectations vary widely among firms, few quantitative conclusions about optimal hedging levels can be drawn. However, some conditional conclusions about the impact of futures trading costs seem justified. Most of the correlations between cash profits and futures profits found in this study range between 0.5 and 0.9. For many firms, the ratio of futures trading costs to cash profits is about 0.1 or 0.2. The figure shows that under these conditions optimal hedging ratios differ from minimum-risk hedging levels by 0.02 to 0.16. This amounts to a 2- to 8-percent reduction in the absolute size of the futures position below the level that minimizes risk. For firms with lower cash profit expectations, the effect would obviously be larger.

A second important implication is that optimal hedging levels are very sensitive to futures profit expectations and cash profit expectations. Even a relatively small expected increase in the futures price may force the optimal hedging level to zero or make it advantageous to be long in both cash and futures. Thus, two individuals with similar operations may find markedly different hedging levels desirable because of differences in their individual price expectations.

LITERATURE CITED

(1) Arthur, Henry B.
1971. Commodity Futures as a Business Management Tool. Harvard Univ.

(2) Chicago Board of Trade
1971. Commodity Trading Manual. Pp. 29-40.

(3) Ewasiuk, W. J.
1962. Hedging Practices and Futures Contracts in the Marketing of High Protein Spring Wheat. Mont. Agr. Expt. Sta. Bul. 564.

(4) Gold, Gerald
1966. Modern Commodity Futures Trading. Commod. Res. Bur., Inc., New York.

(5) Gray, Roger W.
1961. The Relationship Among Three Futures Markets: An Example of the Importance of Speculation. Food Res. Inst. Stud. 2:21-32.

(6) Heifner, Richard G.
1966. The Gains from Basing Grain Storage Decisions on Cash-Future Spreads. Jour. Farm Econ. 48:1490-1495.

(7) ________
1972. Optimal Hedging Levels and Hedging Effectiveness in Cattle Feeding. Agr. Econ. Res. 24:25-36.

(8) Hieronymus, Thomas A.
1971. Economics of Futures Trading for Commercial and Personal Profit. Commod. Res. Bur., Inc., New York, pp. 173 ff.

(9) National Academy of Sciences
1970. Nutrient Requirements for Beef Cattle. No. 4, Washington, D.C.

(10) Snedecor, George W.
1956. Statistical Methods. 5th ed., Iowa State Univ. Press, Ames.

(11) U.S. Department of Agriculture
1970. Trading in Live Beef Futures. Commod. Exch. Authority.

(12) ________
1965-70. Grain Market News, Weekly Summary and Statistics. Consum. and Mktg. Serv.

(13) ________
1965-71. Livestock, Meat and Wool Market News, Weekly Summary and Statistics. Consum. and Mktg. Serv.

(14) ________
1965-71. Agricultural Prices. Statis. Rptg. Serv.

(15) Van Arsdall, Roy N.
1965. Resource Requirements, Investments, Costs and Expected Returns from Hog Production System in Illinois. Ill. Agr. Expt. Sta. Bul. AE-4074.

APPENDIX

Like other problems of decisionmaking under risk, hedging can be described as a problem of setting levels for activities with uncertain rates of return. In hedging, both cash activities and futures activities are included. Cash activities may involve holding a commodity in inventory over a prescribed time period or the transformation of one or more commodities into another commodity over time. The futures activities involve holding long or short positions in specific futures contracts over designated intervals.

Let

$$(1) \quad R = \sum_k x_k r_k$$

be the total profit obtained by the firm in a particular time period where

x_k = level of activity k, a constant set by the decisionmaker, and

r_k = profit per unit of activity k, a random variable with mean μ_k, variance σ_{kk}, and covariances with profits from other activities σ_{kh} for $h \neq k$.

The profit rate, r_k, is defined to equal revenue minus variable costs minus rents imputed to fixed resources--all on a per unit basis. Rents are imputed only to those fixed resources which limit the output of the firm. When no resources are fully used, profit equals revenue minus variable costs.

For a particular activity, the mean profit rate, μ_k, may vary from firm to firm, depending upon resource availabilities. In longrun competitive equilibrium, the mean profit rate would approach the market rate of return for bearing risk. In the short run, the profit rate may include not only the market rate of return for risk bearing but also an additional return earned by the fixed resources of the firm. In this shortrun situation, when the firm has fixed resources committed to production, hedging becomes important.

The Optimal Hedge

Assume that the hedger seeks profit and is averse to risk. Risk is measured in terms of variance of total profit. Mathematically, the hedger's objective is to maximize,

$$(2) \qquad \psi = \sum_k x_k \mu_k - \lambda \sum_{kh}\sum x_k x_h \sigma_{kh}$$

where λ is an unknown weight assigned to the variance of total profit relative to mean profit. In general, λ may differ from individual to individual depending upon differences in the degree of risk aversion.

Without knowledge of λ, direct maximization of ψ is impossible. However, in the hedging problem the primary concern is the optimal level of the futures position relative to the cash position. Knowledge of λ is not required to determine the relative level of the various activities that will prevail when ψ is maximized.

When ψ is at a maximum the partial derivatives of ψ with respect to the levels of the activities will be zero:

$$(3) \qquad \frac{\partial \psi}{\partial x_k} = \mu_k - 2\lambda \sum_h x_h \sigma_{kh} = 0 \qquad k = 1,2,\ldots,n$$

Let x_1 represent the level of the futures position and let x_2 represent the level of the cash position. Combining the first two equations in (3) and eliminating λ results in,

$$(4) \qquad x_1 = \frac{(\mu_1\sigma_{22} - \mu_2\sigma_{12})x_2 + \sum_{h>2} (\mu_1\sigma_{2h} - \mu_2\sigma_{1h})x_h}{(\mu_2\sigma_{11} - \mu_1\sigma_{12})}$$

Equation (4) provides a general condition for specifying the optimal level of the futures position, given the levels of the other activities of the firm.

The absence of λ in equation (4) shows that the optimal level of the futures position is independent of the degree of risk aversion, so long as the levels of the other activities and the means, variances, and covariances of profits remain constant. Thus, the optimal hedging level is the same for all risk-averse firms with the same mix of production activities and the same set of profit expectations and profit variances and covariances. Consequently, a single estimate of the optimal hedging level may apply to a group of similar firms.

The second term in the numerator of equation (4) introduces the effects of other activities of the firm on the optimal level of hedging. Because the mix of production activities differs from firm to firm, the exact solution to equation (4) is specific to each individual firm. However, the second term vanishes if the other activities are uncorrelated with profits on activities 1 and 2. In this case, equation (4) reduces to the following expression for the optimal ratio of the futures position to the cash position:

$$(5) \qquad \frac{x_1}{x_2} = \frac{\mu_1\sigma_{22} - \mu_2\sigma_{12}}{\mu_2\sigma_{11} - \mu_1\sigma_{12}}$$

The optimal hedging ratio is closely related to the correlation between cash profits and futures profits. However, it also depends upon the means and variances of returns for the two activities. These relationships can be demonstrated by expressing the optimal hedging ratio as a function of three arguments: the correlation between futures profits and cash profits; the ratio of their standardized expected profits; and the ratio of their standard deviations. First, introduce standardized expected profits (defined as $\mu_1' = \mu_1/\sqrt{\sigma_{11}}$ and $\mu_2' = \mu_2/\sqrt{\sigma_{22}}$). Substituting for μ_1 and μ_2 in equation (5) results in the following:

$$(6)\quad \frac{x_1}{x_2} = \frac{\mu_1'\sqrt{\sigma_{11}}\sigma_{22} - \mu_2'\sqrt{\sigma_{22}}\sigma_{12}}{\mu_2'\sqrt{\sigma_{22}}\sigma_{11} - \mu_1'\sqrt{\sigma_{11}}\sigma_{12}}$$

This can be rewritten as follows by factoring out $\sqrt{\sigma_{22}}/\sqrt{\sigma_{11}}$ and dividing both numerator and denominator by $\mu_2'\sqrt{\sigma_{11}\sigma_{22}}$.

$$(7)\quad \frac{x_1}{x_2} = \frac{\sqrt{\sigma_{22}}}{\sqrt{\sigma_{11}}} \cdot \frac{\mu_1'/\mu_2' - \sigma_{12}/\sqrt{\sigma_{11}\sigma_{22}}}{1 - (\mu_1'/\mu_2')(\sigma_{12}/\sqrt{\sigma_{11}\sigma_{22}})}$$

Noting that the correlation is defined as $\rho = \sigma_{12}/\sqrt{\sigma_{11}\sigma_{22}}$, the standardized profit ratio is $\pi' = \mu_1'/\mu_2'$, and the ratio of standard deviations is $\Theta = \sqrt{\sigma_{11}/\sigma_{22}}$, the expression can be rewritten

$$(8)\quad \frac{x_1}{x_2} = \Theta^{-1}\frac{\pi' - \rho}{1 - \pi'\rho} .$$

In many cases, particularly for inventory hedging, the ratio of standard deviations, Θ, will be about 1 so that π' will approximately equal the raw expected profit ratio, μ_1/μ_2. In these cases, the optimal hedging ratio can be derived solely from the correlation and the ratio of expected profits.

The Minimum-Risk Hedge

The condition for the optimal hedge can be simplified if the expected profit on the futures position is zero. This situation is approached when the market rate of return for risk bearing approaches zero and futures trading costs are negligible. Under these conditions, the profit rate on the futures activity, μ_1, is zero and equation (5) reduces to

$$(9)\quad x_1/x_2 = -(\sigma_{12}/\sigma_{11}) .$$

Equation (9) also defines the hedging ratio that minimizes risk given the level of the cash activity. This can be shown as follows:

The variance of total profit for activities 1 and 2 is

$$(10) \quad V = x_1^2\sigma_{11} + 2x_1x_2\sigma_{12} + x_2^2\sigma_{22}.$$

Differentiating with respect to x_1 produces,

$$(11) \quad \frac{\partial V}{\partial x_1} = 2x_1\sigma_{11} + 2x_2\sigma_{12}.$$

Noting that the second derivative is positive indicating a minimum, we set (11) equal to zero and find that it reduces to equation (9). The hedging ratio specified by equation (9) will be referred to as the minimum-risk hedge, while keeping in mind that it is also the optimal hedge when the expected costs or returns from hedging are zero.

To estimate the minimum-risk hedge, the sample ratio of the covariance and variance, s_{12}/s_{11}, may be employed where these are calculated individually by the standard formulas. This is exactly equivalent to the standard procedure that would be used to calculate the regression of unit cash profits on unit futures profits. Therefore, the standard least-squares regression algorithm provides a convenient means of approximating the minimum-risk hedge.

Unfortunately, as is the case for many ratio estimates, the properties of s_{12}/s_{11} as an estimator of σ_{12}/σ_{11} are not easily specified. The estimate is consistent but apparently biased in small samples. Examination of the first few terms of the Taylor expansion of s_{12}/s_{11} suggests that when profits are from a bivariate normal distribution the bias is positive and small for the size of sample used here (7, p. 35).

Risk-Shifting Effectiveness

Risk-shifting effectiveness can be measured as the proportional reduction in profit variance obtained through hedging. Let $H = x_1/x_2$ represent the size of the futures position relative to the cash position. Assuming once again that the cash and futures profits are uncorrelated with profits from other activities, risk-shifting effectiveness is represented as,

$$(12) \quad Z = 1 - (\sigma_{22} + 2H\sigma_{12} + H^2\sigma_{11})/\sigma_{22}.$$

This simplifies to

$$(13) \quad Z = -(2H\sigma_{12} + H^2\sigma_{11})/\sigma_{22}.$$

With complete hedging, $H = -1$, which reduces to

$$(14) \quad Z_c = (2\sigma_{12} - \sigma_{11})/\sigma_{22}.$$

In this case, risk-shifting effectiveness exceeds zero, if and only if the numerator is positive. This requires that

$$(15) \quad \sigma_{12}/\sigma_{11} > 0.5.$$

The term on the left is identical to the negative of the minimum-risk hedge as shown in equation (9).

The risk-shifting effectiveness of the minimum-risk hedge is

$$(16) \quad Z_m = -[2(-\sigma_{12}/\sigma_{11})\sigma_{12} + (-\sigma_{12}/\sigma_{11})^2\sigma_{11}]/\sigma_{22}.$$

This reduces to

$$(17) \quad Z_m = (\sigma_{12})^2/(\sigma_{11}\sigma_{22}),$$

which is the square of the correlation between cash profits and futures profits. Thus, the r^2 between cash profits and futures profits measures the risk-shifting effectiveness of the minimum-risk hedge.

When the expected return on the futures position differs from zero, the optimal hedge differs from the minimum-risk hedge. Where they differ, the optimal hedge is less effective in shifting risk than the minimum-risk hedge. Unfortunately, there is no single measure of hedging effectiveness that takes into account both risk and expected returns and is equally meaningful to individuals with differing levels of risk aversion.