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Modeling the Effect of Uncertainty on Timber Harvest: A Suggested Approach and Empirical Example

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A method is suggested for modeling uncertainty when there is a lack of information concerning the effect of forest management decisions on tree growth. Dynamic programming is used to investigate the optimality of alternative management strategies. The model is illustrated with an empirical example for a boreal forest region of western Canada. Three tentative conclusions follow: (*a*) silvicultural strategies to reduce uncertainty or to increase stand growth may not be worth pursuing, at least in northern forests; (*b*) the discounted cost of ignoring uncertainty may be substantial if taken over the entire forest; and (*c*) given uncertain forest growth, flexible harvest policies are preferred to a fixed harvest age.

Key words: dynamic programming, forest management under uncertainty, silviculture.

Forest management under uncertainty, particularly in the case of catastrophic mortality (*viz.*, fire), has been studied by a number of researchers (Williams; Reed). While Routledge demonstrates that the optimal rotation age can increase under uncertainty, he finds that the tendency is for uncertainty to reduce rotation age. (This result is proven in the more general case of renewable resource markets by Pindyck.) Johansson and Lofgren prove that, under certain restrictive conditions, "the stochastic case does not provide any information about the expected optimal rotation period compared with the optimal rotation in the deterministic case" (p. 264); but, upon relaxing the restrictive assumptions, they come to similar conclusions as Routledge and as Pindyck. Reed and Errico (1986) find that, under certain conditions, the optimal solution to the deterministic model is an approximately optimal solution to the stochastic control problem. However, for forests in the Fort Nelson, British Columbia area, Reed and Errico (1985) find that the optimal rotation age is reduced from 105 years to 93 years as a result of a fire rate of .01. In a study of optimal stocking levels, Kao uses adaptive dynamic programming to update the probabilities of the occurrence of events at each stage, but Kennedy expresses some reservation about this approach, stating that stochastic dynamic programming is preferred (p. 86). Reed reviews other models of uncertainty in forest management, while Williams appraises more general resource models.

The main objective of the current study is to illustrate a method for building growth uncertainty into forest management models when inadequate information concerning the effects of management upon tree growth prevents the use of empirical estimates. An example of this occurs when specific effects of climate change on tree growth and variance of growth are unknown. However, even in the absence of climate change, there are frequently insufficient data about managed stands to permit proper economic analysis; data about managed stands are not only sparse but are narrowly focused (e.g., Yang 1985).

The problem of inadequate knowledge is not only confined to managed stands. Age–volume tables for unmanaged stands do not provide information about variance in growth (Thompson et al.; Alberta Forest Service 1985a, b). For example, the Alberta Forest Service provides only the R^2 values and no standard errors for individual parameter estimates in the nonlinear equations it used to construct yield tables. Since

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individual observations generally are unavailable to the researcher, it is also difficult to construct confidence intervals about the regression line.

In this article, a tree growth model for unmanaged stands is used to derive optimal harvest policies under uncertainty; these are then compared with policies for the deterministic case. Further, the optimality of management policies to promote growth and to reduce variability of growth is investigated. One conclusion is that volume is preferred to age as a criterion for making harvest decisions under uncertainty.

Conceptual Model

Both analytical and numerical solutions to stochastic control models are difficult to obtain. In applied settings, three approaches are employed: (*a*) stochastic models are ignored in favor of deterministic ones, with sensitivity analysis used to investigate alternative scenarios; (*b*) the solution to the deterministic problem is used in a feedback fashion in a stochastic environment; and (*c*) stochastic dynamic programming (DP) is employed. While the first approach generally is regarded as inferior, it is not clear whether (*b*) or (*c*) is more appropriate. In this study, we employ stochastic DP to examine a variety of management strategies to enhance tree growth and reduce uncertainty.

The objective is to maximize the expected net present value of an infinite stream of returns from a uniform stand of trees. The manager can affect the stand's growth by various silvicultural practices (d) such as thinning, reforestation, fertilizing, etc.; these management decisions are identified more clearly below. The fundamental stochastic DP equation for the optimal harvesting problem under uncertainty can be written as:

$$Z_t(V_t) = \max_{d_t}\{R_t(P_t, V_t, d_t) + \frac{1}{1+r}E[Z_{t+1}(V_{t+1})]\}, \tag{1}$$

where $Z_t(V_t)$ is the discounted value of future expected net returns, given that timber volume is V_t at time t and decision d is taken; $R_t(P_t, V_t, d_t)$ is the current period net return as a function of price (P_t) per cubic meter (m^3), stand volume, and the management decision;[1] $Z_{t+1}(V_{t+1})$ is the discounted expected value of stochastic future net returns over the remaining $T - (t + 1)$ years of a T-year horizon, given that timber volume is V_{t+1} at time $t + 1$ and that the optimal path is followed;[2] E is the expectations operator; and r is the real (social) discount rate. In this study, equation (1) is solved by policy iteration (Hastings, pp. 128–36).

In order to keep the model simple enough to use in a practical setting, the state transition equation or stochastic constraint consists of the following timber volume growth equation for a stand:

$$V_{t+1} = v(V_t, d_t, e_{1t}) + e_{2t}, \tag{2}$$

where v is the transformation function, e_{1t} is a vector of nonadditive random variables, and e_{2t} is an additive random term. Both e_{1t} and e_{2t} represent sources of growth uncertainty described in greater detail below. The initial condition is: $V_0 = V(0)$. Equation (2) indicates that stand timber volume in one period is a function of timber volume in the previous period, the decision taken in the previous period, and random components.

The rate of change in wood volume is governed by the manager's decisions (d_t) at each point in time. Except when the decision is to clearcut the stand or to do nothing (hold the stand without treatment), management is assumed to increase tree growth above what it otherwise would be. Timber growth is assumed to increase at a decreasing rate so that wood volume approaches an asymptote as the stand ages (fig. 1). Although current management affects tree growth in future years, the simulation model employed in this study requires that only next year's growth be affected by current decisions—the Markov assumption.[3] As discussed below, reality and the simulation model are reconciled through the transformation function and by adjusting management costs.

The principle governing dynamic programming is Bellman's Principle of Optimality: "An optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, p. 83). Numerical methods used to obtain the DP solution require that both the state and the control variables be discrete or that intervals for these variables be constructed. If the states and controls take on too many values (or the intervals are too narrow so that there are too many) or there are too many state and control variables, the so-called "curse of dimensionality" is encountered. Despite this, for many practical problems DP is an excellent tool. DP can be used when the state variables in time $t + 1$ depend upon the state and control variables at time t in a deterministic or stochastic fashion (Williams).

A major problem is that of modeling how management decisions affect growth since information about

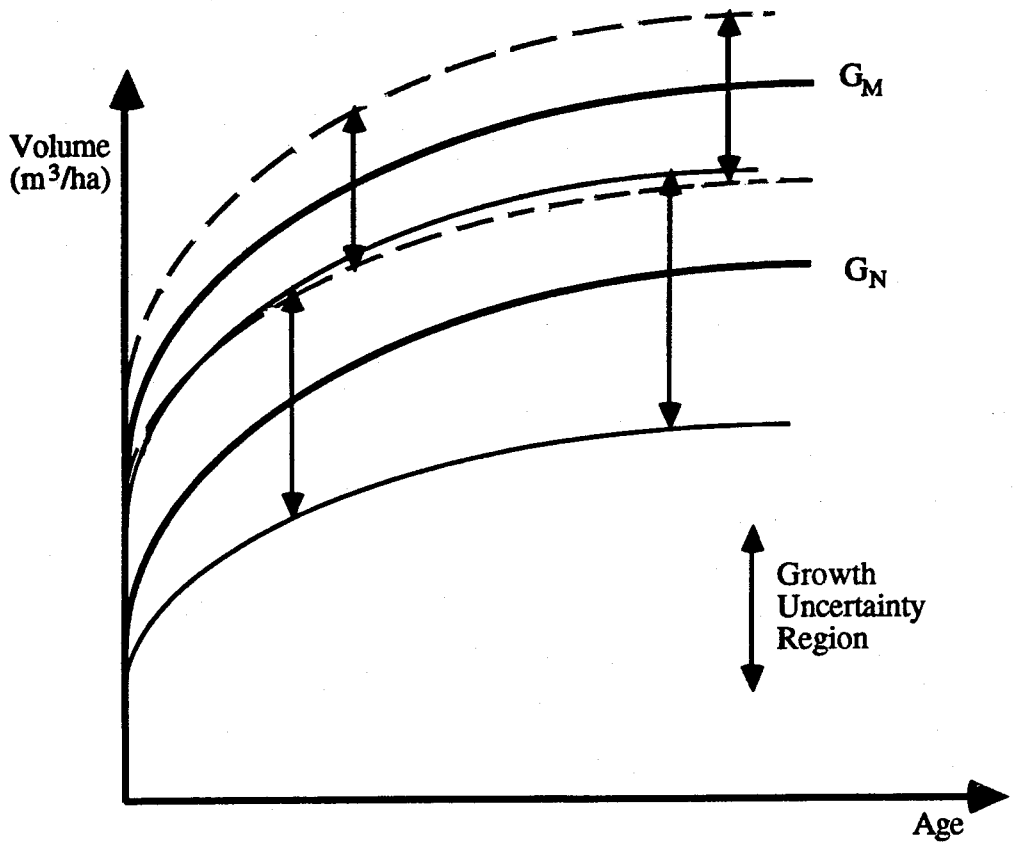

Figure 1. Age-volume relationship: Effect of management

the impact of various silvicultural decisions on both stand growth and its variance is sparse. Suppose that the growth function G_N for an unmanaged stand, along with its probability limits or confidence interval, can be estimated (fig. 1). Given inadequate information about managed stands, it is possible to solicit estimates of the growth function and its probability limits by asking silviculturists the location of the "managed" growth function and the variation about that function, relying on graphical comparisons of various growth functions. (A method for doing so is suggested in the empirical section below.) This is shown in figure 1 by the hypothetical response curve G_M and its probability boundary.[4] An alternative approach is to use secondary data to provide some idea about how growth functions change when a stand is managed. Once the probability distributions about the estimated growth functions are known, it is possible to calculate probability transition matrices for timber volume [which replace equation (2)] by Monte Carlo simulation.

Empirical Timber Growth Relations

The tree growth model employed in this study is a modification of the tree growth relationships estimated by Brown (1989) and described below. The data were obtained from the Alberta Forest Service (AFS). The AFS maintains permanent sample plots (PSP) throughout the province, but only data for the Peace River and Footner Lake Forests of northwestern Alberta were used. PSPs were established by the AFS in the 1960s to acquire data on the dynamics of stand growth, but measurements have been taken at irregular intervals. Although the data available for the PSPs are extensive, only data on diameter at breast height (*dbh*), tree height (*H*), the number of stems per plot, and the dominant species (white spruce) were used. All the sampling plots contained in excess of 100 trees, but measurements for height were available for a limited number of trees, and stems with measurements for two periods were even fewer. Thus, the

number of observations available for estimating a growth relationship were substantially fewer than the number of trees in the PSP plots.

To determine the rate of change of stand volume over time, it was necessary to estimate growth equations for *dbh* and for *H* and then relate these to wood volume (*V*). Logistics growth functions were assumed for both the *dbh* and *H* equations because of their desirable theoretical properties. The functional form for the logistics equation is

$$H = b_0/(1 + \exp[-b_1(A - b_2)], \tag{3}$$

where A is age and b_0, b_1, and b_2 are parameters that measure maximum (asymptotic) tree height, the rate of increase in height, and functional shifts, respectively. Nonlinear seemingly unrelated regression (SUR) was employed since it is reasonable to assume there is a relationship between height and radial growth, and this taken into account by SUR. The regression results for the logarithmic form of the logistics function are as follows (Brown 1990, p. 55):[5]

$$\ln(H) = \underset{(.0169)}{3.4183} - \ln\{1 + \exp[\underset{(.0029)}{-.0317}(A - \underset{(1.3441)}{46.1041})]\}, \text{ and} \tag{4}$$

$$\ln(dbh) = \underset{(.0299)}{3.6196} - \ln\{1 + \exp[\underset{(.0052)}{-.0335}(A - \underset{(2.2911)}{48.9829})]\} \tag{5}$$

Degrees of Freedom = 282; Residual Sum of Squares = 287.9996;
Standard Error of Regression = 1.0213

The asymptotic standard errors of the estimates are provided in parentheses.

The asymptotic values of *H* and *dbh,* found by taking antilogs, are 30.52 meters and 37.32 centimeters (cm), respectively. White spruce located in the study area does not attain such heights, especially with such a small *dbh.* (The incompatibility of the estimated asymptotic *dbh* and *H* could perhaps be accounted for by the small number of observations per tree.) Hosie (p. 64) reports that white spruce generally reaches a height of 80 feet with corresponding *dbh* of two feet. Therefore, the asymptotic values in equations (4) and (5) are set at 24.384 meters and 60.960 cm, respectively. However, the standard errors of the estimates for these parameters still are used in the subsequent analysis.

Finally, the *H* and *dbh* information is converted into volume using the following wood volume relationship for white spruce provided by the AFS (Lakusta).

$$V = .000043 dbh^{1.882751} H^{1.020411}. \tag{6}$$

Since the interest in this study is stand growth, it is necessary to multiply (6) by the number of stems growing on the site at any time. Using data on the number of stems at various age categories and stand volume data from AFS (1985a, pp. 36–40), relation (6) was calibrated by a piecewise linear and nonlinear stem–age relationship.[6]

Economic Data

Prices and costs of harvesting were obtained from a number of sources. The British Columbia Ministry of Forests and Lands (p. 41) indicates that the average stumpage price for spruce in the northern interior of British Columbia, the Prince George Forest, is \$5.94/m^3. This forest is adjacent to the current study region. However, stumpage fees do not reflect the true value of standing timber. Brown (1989, p. 68) estimates that forest companies obtain a net price of \$2.84/m^3 after all costs, including stumpage fees. Adding this to the stumpage fee gives a net price of \$8.78/m^3.

Three management alternatives are examined: (*a*) to enhance stand growth (*viz.,* thinning, fertilization); (*b*) to reduce variation in stand volume growth (e.g., fire and pest protection and suppression expenditures); and (*c*) a combination of these practices. Based on cost information from a variety of sources (e.g., British Columbia Ministry of Forests and Lands), the one-time costs of managing a stand to enhance growth are assumed to be \$150/hectare (ha), while they are \$85/ha for protection and suppression of fire and pests. A combined strategy for the stand costs somewhat less than the sum of the individual ones due to assumed economies of scale in management; \$200/ha is used for the combined strategy.

Because the discount rate is an important factor determining the economic viability of various silvicultural practices, sensitivity analysis is used. A discount rate of 0% represents a social decision maker charged with ranking activities without regard to the generation to which benefits or costs accrue. A discount rate of 2.5% reflects a low real rate of social discount; the higher real rate of 5% reflects a more realistic social rate of discount and perhaps even a low private rate (Walker and Young).

Modeling Growth

The state transformation equations for an unmanaged stand are given by equations (4), (5), and (6), after changing the intercepts of (4) and (5) as noted above. Using these parameter values, it is possible to "grow" the stand under the strategy of "no management," thereby providing a deterministic relationship between age and wood volume. In practice, it is not possible to use a continuous measure of volume; rather, it is necessary to create volume increments. The size of the increments will determine the dimensions of the problem to be solved, including those of the probability transition matrices. In earlier modeling efforts, an interval width of 6 m^3 was employed. While such an interval width enabled one to solve the DP problem on a microcomputer, it was necessary to reduce the interval width substantially in order to model uncertainty more realistically. Thus, an interval width of .2 m^3 was employed, but this meant that the problem no longer could be solved on a microcomputer, and even a conventional mainframe computer required a substantial amount of time (on the order of 24 hours) to complete the Monte Carlo trials.[7] Of course, this is a form of Bellman's "curse of dimensionality." The problem was solved by accessing the Alberta Research Council's supercomputer which utilizes parallel processing to reduce computational time. Since all aspects of the problem (Monte Carlo simulation and the DP algorithm) could be made parallel, the task could be performed in three hours or less. The first interval is 0 m^3 and represents the case where no timber is growing on the site (due to clearcutting or natural denudation). Subsequent intervals are $0\ m^3 < V < .2\ m^3$, $.2\ m^3 < V < .4\ m^3$, and so on, with the final interval $V > 400\ m^3$.

To gauge the effectiveness of forest management choices, one would like to estimate separate growth equations for each decision. This is not possible given the data limitations and the fact that very little is known about the effect of management on forest stands in the study region. To overcome this problem, we employ the following methodology.

First, we consider only two decision alternatives in each stage of the multistage process, namely, (*a*) do nothing and (*b*) clearcut the stand. Optimal harvest policies are examined for these two options under both deterministic and uncertain growth.

Second, we rely upon results from other studies and crude guesses to illustrate a procedure for estimating stochastic growth under alternative silvicultural choices. In addition to the choices (*a*) or (*b*), we also include the alternatives identified earlier, namely, (*c*) management to enhance growth, (*d*) management to reduce variability of growth, and (*e*) a combination of (*c*) and (*d*). These five management options are chosen for illustrative purposes only. Each alternative is available to the decision maker once every year as this is the time between stages. Given the nature of the management choices, it will be necessary to develop four equations of motion for growth in a stand's wood volume. It is also necessary to determine how uncertainty is generated within the model and how management can reduce uncertainty.

Modeling Uncertainty: The Transition Matrix

To model uncertainty, the volume transformation equation for a particular management choice is replaced by a state probability transition matrix for that decision. The modified equations (4)–(6) serve as a basis for constructing the state transition matrices. By specifying appropriate parameter values and their variances and using Monte Carlo simulation, the probability transition matrices can be constructed directly, as described below. A probability transition matrix gives the probability of attaining a particular wood volume interval i at time $t + 1$, given (*a*) the value of the control variable (management choice) at time t and (*b*) the level of wood volume in the preceding time t. The probability $p(i, j, d)$ that wood volume increases from interval i at time t to interval j at time $t + 1$, given decision d at time t, is used to make the expectation operation in equation (1) explicit.

There is little data available about the variance of stand growth in its natural state beyond that available from estimated equations (4) and (5);[8] there is little information in this regard about managed stands.[9] This makes it difficult to use estimated relations to incorporate uncertainty other than for the unmanaged stand. Therefore, another approach is required to construct the parameter estimates and variances under different management regimes. The method employed here is to introduce uncertainty via a probability density distribution constructed about each parameter in the H and dbh equations, and about the equations as a whole [via the standard error of the estimate (SEE)] (see also Lewis). The first method for making the model stochastic addresses e_1 in equation (2), while the second concerns e_2. The variances of the probability distributions vary with the particular management choice. Finally, independence among the random parameters and across equations is assumed for computational ease.[10].

A triangular distribution for each of the random components is employed because of its desirable properties. In practice, expert opinion can be used (Brown 1990); the modeler can ask tree physiologists and other foresters to describe what happens to certain aspects of growth using three concepts—the most

likely, the maximum, and the minimum outcomes, or the largest, smallest, and most likely values of a parameter.[11] The best approach for accomplishing this is to rely on graphical comparisons of various growth functions. Unfortunately, the scope of the current project did not permit greater interaction among researchers from different disciplines and, therefore, the distributions constructed in this study are less than reliable, based on our opinions obtained from silvicultural studies. As better information becomes available, the model can be modified easily to incorporate it.

Uncertain growth under no management is modeled using the deterministic form of (4), (5), and (6), with the corrected asymptote values, as a starting point. A triangular distribution about each of the parameters and e_2 is then constructed as follows. For an unmanaged stand, the maximum value that a parameter can take is equal to the most likely (deterministic) value plus 2½ times the standard error of the parameter estimate; the minimum value of the parameter is given by the estimated parameter minus 2½ times the standard error of the estimate for the parameter. For e_2, a triangular distribution about zero is used to simulate stochastic growth from one year to the next; minimum and maximum values for the distribution are obtained by respectively subtracting and adding 2½ times the SEE. It also is assumed that a stand can be completely destroyed by fire or pests with probability .015, regardless of how much timber currently is growing on the site. For each management choice, the parameters and their associated triangular distributions, and the probability of natural denudation, are modified.

To incorporate the effect of a strategy to reduce downside variation in stand volume growth, the slope parameters in the H and dbh equations are altered, as is the range of possible parameter values. With the growth rate parameter adjustment, the H and dbh equations can be written as:

$$H = a_0/(1 + \exp[-(1 + Sh)a_1(A - a_2)]$$

and

$$dbh = b_0/(1 + \exp[-(1 + Sd)b_1(A - b_2)],$$

where Sh is the expected change in the growth rate of height and Sd is the expected change in the growth rate of diameter—the change in radial growth due to prevention management. We determine Sh and Sd from differences in basal area between sprayed and unsprayed plots in Minnesota (Balzer). According to Balzer, tree mortality increased 56% in unsprayed stands but only 33% in sprayed stands that were attacked by spruce budworm.[12] Furthermore, basal area growth was reduced by 42% in sprayed versus 67% in unsprayed stands. By factoring out the change in basal area due to tree mortality, the change in basal area due to differences in the radial growth on the plots could be determined. From this, one could determine the percentage change in growth associated with spraying when a stand is infested. Assuming that the probability that insects attack a stand is .05,[13] the increase in growth due to preventative spraying can be determined. It is also important to consider the infestation frequency when considering preventative spraying as it will benefit growth only if there is an insect infestation.

From the information provided above, we know that basal area growth in sprayed stands is 1.76 times (=.58/.33) greater than for unsprayed stands if attacked by spruce budworm. Since the chance of infestation is 5%, we determine the average difference in growth between preventatively managed and unmanaged stands is 2.9%; thus, both Sh and Sd are set to .029. Next, the preceding information indicates that mortality is approximately 50% higher in unsprayed compared to sprayed stands when spruce budworm is present. Given a 5% chance of infestation, it is assumed that the range of possible parameter values is 2.1% lower in sprayed as opposed to unsprayed stands. Thus, the other effect of spraying is to reduce the variance of the appropriate parameter values (plus SEE), but only on the down side of growth, giving an asymmetric triangular distribution about all of the parameters. This is done by raising the "lower" value of the triangular distribution by 2.1%. Finally, it is assumed that the probability of a stand being denuded by fire or insect pests is 60% lower for stands that are managed to reduce variance in growth versus unmanaged stands. The values of the parameters and the maximum and minimum values they can take are provided in table 1.

The other management decision that has an impact on growth is fertilizing. A similar method is used to examine the impact of fertilization on wood volume growth. Radial growth studies conducted by the Canadian Forestry Service on a number of forestry plots throughout Canada have tested for the effects of different fertilizers. Although most of the results of the fertilization studies are for white spruce in eastern Canada, this information is employed in the current study. The magnitude of volume increase due to nitrogen fertilizer was found to be between 6–17%.[14] This is modeled by increasing the H growth parameter by 6% and the dbh growth parameter by 17% as well as the upper and lower bounds of the respective triangular distributions on these parameters (table 1). All the other parameter values are assumed to be the same as for an unmanaged stand.

Finally, the management choice that includes both spraying and fertilizing is modeled. The parameter

Table 1. Parameter Values for the Growth Equations under Various Silvicultural Alternatives

Decision	Variable	Mode Value	Minimum Value	Maximum Value
No Management	*a*0	24.3840	21.8414	26.9266
	*a*1	−.0317	−.0390	−.0245
	*a*2	46.1041	42.7438	49.4644
	*b*0	60.9600	58.3841	63.5359
	*b*1	−.0335	−.0465	−.0205
	*b*2	48.9829	43.2551	54.7107
	*e*2	0	−2.5533	2.5533
Probability of natural denudation = .015				
Reducing Variation in Growth	*a*0	24.3840	22.7684	26.9266
	*a*1	−.0326	−.0401	−.0257
	*a*2	46.1041	42.7438	48.4256
	*b*0	60.9600	59.6102	63.5359
	*b*1	−.0345	−.0479	−.0215
	*b*2	48.9829	43.2551	53.5617
	*e*2	0	−2.4996	2.5533
Probability of natural denudation = .006				
Increasing Wood Volume Growth	*a*0	24.3840	21.8414	26.9266
	*a*1	−.0336	−.0413	−.0260
	*a*2	46.1041	42.7438	49.4644
	*b*0	60.9600	58.3841	63.5359
	*b*1	−.0392	−.0522	−.0240
	*b*2	48.9829	43.2551	54.7107
	*e*2	0	−2.5533	2.5533
Probability of natural denudation = .015				
Reducing Variation and Increasing Growth	*a*0	24.3840	22.7684	26.9266
	*a*1	−.0346	−.0425	−.0273
	*a*2	46.1041	42.7438	48.4256
	*b*0	60.9600	59.6102	63.5359
	*b*1	−.0403	−.0537	−.0252
	*b*2	48.9829	43.2551	54.7107
	*e*2	0	−2.4996	2.5533
Probability of natural denudation = .006				

Note: For the model used in the above table, $H = a0/\{1 + \exp[a1(age - a2)]\}$, $dbh = b0/\{1 + \exp[b1(age - b2)]\}$, and $e2$ is the error term in equation (2).

values for this decision are determined as a composite of the other two choices and also are given in table 1.

The values in table 1 and Monte Carlo simulation are used to generate the required probability transition matrices. Parameters are randomly chosen from each of the triangular distributions at the beginning of each simulation or trial, and these replace the respective parameter values in (4) and (5); e_{2t} in (2) is randomly chosen at each time within trials. For each management choice, tree age is incremented annually from zero to 160 years and, using the values in table 1, wood volume is determined for each of the stages from "seedlings" to mature timber. Thus, a series for $V(t)$ ($t = 1, 160$) is developed for a single set of randomly generated parameters and a particular management choice. Continuing with the same management choice, a second series is developed for volume, but using a new set of randomly generated parameters. This process continues until N series of $V(t)$, or $V(t, n)$ ($n = 1, N$), are simulated, where N is the number of trials, say 100,000. To generate a probability transition matrix for the particular management choice, the $[V(t, n), V(t + 1, n)]$ pairs are arranged in a frequency matrix. Dividing each cell by the total for that row gives the probability transition matrix for that particular management decision. (Each row in the transition matrix must sum to one since it represents the conditional probability density function of moving from one state to the next.) The process is repeated for each management alternative.

Finally, it is necessary to add a row on the top and a column on the left of each matrix to represent denuded forest land (0 m^3). This first row provides the probability of moving from the unstocked state

to one where commercial timber has begun to grow. Since this might take five years on average, elements (1, 1) and (1, 2) in each of the transition matrices are set to .8 and .2, respectively. The first column represents the probability of arriving in the denuded state. For the case of clearcut, all the elements in this first column will be 1 (with the remaining elements of the transition matrix equal to zero), except for the first row. For the other management strategies, this column (except for the first row) represents the probability of denudation due to pests, fire, or some other catastrophic event. These values also are found in table 1. The probabilities in the other cells of the matrix are adjusted to maintain the requirement that rows in the transition matrix sum to one.

Clearcut versus No Management: Effect of Uncertainty

The effects of both uncertainty and discount rate on the optimal volume at which to cut the stand are provided in table 2. Also provided in this table are the related harvest ages. For the deterministic model, harvest age is equal to the number of years required to attain the optimal wood volume plus the five years required to establish growing trees. The same approach was used to calculate harvest age in the stochastic model, namely, the age in the deterministic model at which the same volume would be realized. However, harvest age is somewhat meaningless in the latter case since the decision to clearcut the stand is based only on standing timber volume and not on the age of the trees in the stand. The true age of the trees could be lower or higher than indicated due to variation in growth.

The results indicate that there is a substantial difference between optimal harvest volumes and ages predicted by the deterministic and stochastic models at all discount rates. Disregarding the effects of other sources of uncertainty (e.g., future prices and stumpage rates), the decision maker who takes into account the effect of uncertainty on tree growth should cut a stand much sooner than otherwise would be the case if uncertainty were ignored. Compared to the deterministic decision rule, optimal critical volumes for cutting the stand are about one-half if uncertainty is explicitly taken into account in deciding when optimal harvest would occur. For a discount rate of 2.5%, for example, the stochastic model indicates that the stand should be clearcut when wood volume exceeds 92 m^3, while the deterministic model indicates that one should wait until stand volume exceeds 177 m^3. The optimal harvest age is reduced from 89 years to 77 years if uncertainty is taken into account. In both models, an increase in the discount rate will result in a shorter harvest age and a lower critical stand volume, as suggested by the Faustmann rule (e.g., see Williams).

If the decision maker wishes to explicitly consider uncertainty, the stochastic model is the correct one and the deterministic model is not. The expected discounted net return from following the stochastic rule should exceed that obtained by following the deterministic rule. The loss associated with adopting the deterministic solution is calculated by forcing the stochastic model to adopt the harvest strategy prescribed by the deterministic solution. (The stochastic model will select a suboptimal path and provide a lower net worth if the deterministic solution is imposed.) The deterministic solution requires postponing harvest. Using simulation and appropriate discounting, we estimate that it costs approximately $30/ha to postpone harvesting and incur the risk that the stand could, for example, be denuded by natural causes.

As a caveat to these results, it should be noted that the deterministic and stochastic models considered here represent two extremes. For the deterministic model, the impacts of catastrophic fires, insect infestations, climate vagaries, and other sources of variability in tree growth are ignored completely. These are taken into account only by the stochastic model. The disparity in the results of table 2 only can be accounted for by the extremes represented by the two models. The decision maker must decide which of the two models, or perhaps some other model, is the closer representation of stand growth in northwestern Alberta.

Forest Decision Making under Alternative Silvicultural Choices

Now consider the five management alternatives that were identified earlier. Given the costs of these alternatives, our model suggests that none of them are profitable and that the results of the previous section hold. The reason is that the silvicultural costs are significantly large so that, even if there is a significant increase in growth or reduction in down-side variability of growth due to management, the stream of future benefits from such silvicultural investments is inadequate to recover such costs. Thompson et al. provide support for this conclusion, particularly at the higher discount rates (5% or greater).

Management decisions in our model only affect next year's growth, while research indicates that their impact is spread over 10 years (Yang 1985, 1989). Therefore, the costs of the management choices were reduced to one-tenth of those noted above. (This can be considered a crude method of annualizing the

Table 2. Optimal Harvest Ages and Critical Timber Volumes: Deterministic versus Stochastic Decision Models

	Deterministic Model		Stochastic Model	
Discount Rate (%)	Harvest Age (years)	Timber Volume (m^3)	Harvest Age (years)	Timber Volume (m^3)
0	131	367	85	152
1.5	104	254	80	112
2.5	89	177	77	92
5.0	85	156	67	61

costs of a particular choice, thereby taking into account silvicultural impacts over a longer period.) Several observations follow from the various simulations with these lower costs.

First, when costs are reduced by one-tenth, but keeping relative costs the same, the model suggests that the forest manager should implement silvicultural techniques that enhance growth, but only in years just before harvest. It is for this reason that Ondro and Constantino find it profitable to fertilize 70-year-old lodgepole pine, while Thompson et al. find investments in early stages of growth to be unprofitable.

Second, if the relative costs of silvicultural investments are changed, the results indicate that a mix of strategies may be desirable, depending on stand volume. However, the combined strategy (management to enhance growth plus reduce variability in growth) never entered into the optimal decision vector.

Third, the effect of increasing the discount rate is to reduce the viability of silvicultural strategies, with no management remaining the only feasible alternative to harvesting at discount rates above 5%.

Fourth, the critical harvest volume for the stand increases when management to enhance growth or reduce its variability is possible in the stochastic model. Optimal harvest volume increases from 152 m^3 to 199 m^3 for a discount rate of 0%, from 112 m^3 to 145 m^3 for 1.5%, from 92 m^3 to 118 m^3 for 2.5%, and from 152 m^3 to 154 m^3 for 5%. The delay in harvest varies from about two years (for low discount rates) to no delay for higher discount rates.

Finally, and as expected, one would implement silvicultural strategies only if they yield a higher net return, and this is precisely what the simulations indicate. In each case where a management strategy was optimal for the state variable, the discounted value of net returns at harvest time was also somewhat higher. For example, assuming the lower costs, the managed stand yields almost $250/ha more than the unmanaged stand at harvest time assuming a discount rate of 2.5%, but less than $10/ha at a 5% discount rate.

The main reason for each of these outcomes is that, at higher discount rates, both costs and returns are worth less today than would be the case at lower rates of discount. Since the eventual return from harvesting the forest occurs at a distant date, the current value of returns is worth less than the current value of costs.

Discussion

This study illustrated an approach to forest management under uncertainty when information is lacking. However, the problems with this inquiry are that it was based on too few observations, the empirical method of incorporating uncertainty was ad hoc, and the economic data probably were not reflective of true costs and prices (which were not available). Further, uncertainty regarding prices and costs was ignored, although costs and prices are likely important factors in making management decisions (including harvest).

Nonetheless, the results are indicative of the effects that uncertainty and alternative choices will have on actual management strategies. These effects are twofold. First, if uncertainty in forest growth is taken into account, the forest should be cut earlier than otherwise would be the case. Uncertain growth simply reduces the optimal time between harvests. Second, given the high costs of forest management (e.g., forest fire prevention, fertilization, chemical spraying, etc.), it is unlikely that management is worth pursuing in the boreal forest region of northwestern Alberta. Even if the costs are small, it appears that, as the discount rate rises, management choices are reduced to ones that cost little or no money. This has a policy implication when interest rates are high, since private forest operators will need special encouragement to pursue management options that require an outlay with no hope of repayment for a long period. While private forest companies will wish to shift the burden of silvicultural costs onto the government since these are

unprofitable from a private perspective, the government still may wish to undertake such expenditures for reasons having to do with recreation, watershed, etc. other than timber.

The results indicate that the savings to society, and to private forest companies, from explicitly taking into account uncertainty may not be inconsequential. In our model these turn out to be approximately $30/ha for the timber-producing land in the Peace River and Footner Lake Forests of northwestern Alberta. They may be even higher in regions that experience a less severe climate.

Finally, given uncertain forest growth, decisions to manage the forest, including when to harvest the forest stand, should not be based solely on the age of the trees in the stand, but on levels of commercial timber volume, insect infestation, and so on. Management needs to be flexible with regard to harvest. In this model, the state variable was timber volume, but it also could be some combination of timber volume and other state variables (e.g., level of insect infestation). Whatever state variables are used to represent the system, with uncertain growth the actual decisions should not be based solely on the time since last harvest, but on the state of the stand as well as the larger forest.

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Notes

[1] Price is assumed to be known; uncertainty shows up in the transformation function.

[2] By letting T approach infinity, there is no concern about the soil expectation or land value after a clearcut.

[3] The Markov assumption frequently is employed in forest management. For example, Lembersky and Johnson employ a Markov decision process to investigate optimal policies for managed forests stands, although they do not compare their stochastic results with those obtained from a deterministic model.

[4] This approach was suggested by Oscar Burt, but in the context of climate change.

[5] Site indexes were estimated and used to narrow the data to observations on 72 stems of white spruce from five plots, with observations on each tree at two distinct points in time.

[6] The exact specification can be provided to the reader upon request.

[7] Given that the transition matrices have dimensions about 2,000 × 2,000, a large number of simulations (about 100,000 trials) is required to "fill" the matrix.

[8] The standard errors of the parameter estimates in equations (4) and (5), as well as the standard errors of the regressions, provide some information about variance, at least for the PSP data.

[9] Studies by Yang (1985, 1989) and others provide some information about the effect of management on growth, but there tends to be no data about how variance differs from the unmanaged case.

[10] This exaggerates the variance from sampling errors on the parameter estimates since these are typically highly correlated.

[11] The triangular distribution is useful since, when there is more than one respondent, fuzzy methods can be used to combine responses (see Kickert). Further, the triangular distribution is easy to understand and construct and can be made asymmetric.

[12] Monenco Consultants Ltd. (p. 21) indicate that spruce budworm can reduce radial growth by 50%.

[13] As a result of spruce budworm, two to three defoliations can be expected in a 60-year period (Monenco Consultants Ltd., p. 20).

[14] The rate of application of nitrogen was 112 kilograms (kg)/ha and was aerially applied. Yang (1985) reports results of a similar magnitude for lodgepole pine in Alberta.

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