

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.

Heteroskedasticity in Crop Yield Models

Seung-Ryong Yang, Won W. Koo, and William W. Wilson

This study examines three alternative models of correcting for heteroskedasticity in wheat yield: the time trend variance, the GARCH, and an econometric model that includes the potential sources of heteroskedasticity. Nonnested test results suggest that modeling the sources of heteroskedasticity is the preferred procedure. Including potential sources of heteroskedasticity as explanatory variables removed the heteroskedasticity in the sample wheat yields. The results also suggest that the GARCH specification is a promising model of correcting for heteroskedasticity when the sources cannot be identified. The time trend variance model alone may misspecify the true variance structure.

Key words: GARCH, heteroskedasticity, nonnested orthodox test, yield.

Understanding the behavior of crop yields becomes increasingly important for modeling production functions, forecasting price movements, and understanding farmers' responses to government programs. Variability in crop yields is a principal source of instability in production levels (Hazell 1985), and most studies use the coefficient of variation around the trend to measure the variability in crop production (Hazell 1984; Weber and Sievers; Singh and Byerlee). The maintained assumption of using the coefficients of variation for analysis is that detrended yields are homoskedastic within the sample period.

A phenomenon usually confronted in statistical analysis of crop yields is heteroskedasticity, which seems to be characterized mainly by systematic changes in yield variation over time. Nevertheless, heteroskedasticity has received less attention and frequently has been handled inadequately in empirical analyses.

Hazell (1984) and, similarly, Singh and Byerlee recognized heteroskedasticity in detrended yields and partitioned the residuals into subperiods for analysis. However, the measured coefficient of variation can be misleading if heteroskedasticity occurs more frequently. Gallagher found that detrended yields have an upward trend in variation, and he standardized the data with the predicted standard deviation from a regression against a time trend to correct for heteroskedasticity. This standardization procedure results in heteroskedasticity-adjusted estimates only if the true error structure is known. If the variation is cyclical or changing with some systematic patterns other than trend, this procedure would not correct fully for heteroskedasticity.

Variation in yields may have some patterns due to autocorrelated weather and/or gradual adoption of new technology. If this is the case, the GARCH (Generalized Autoregressive Conditional Heteroskedastic) process developed by Bollerslev can serve as an alternative to the time-trend variance model. The GARCH (p, q) process, which is equivalent to an ARMA process with $m = \max\{p, q\}$ and p in the squared disturbances (Bollerslev), is useful to model systematic changes in yield variation. The GARCH model is similar to Just and Pope's stochastic production function, which allows the relationships of inputs with risk to be independent of the relationships of inputs with production.

Conventional time-trend variance or the GARCH models may explain the variation in crop yields. However, heteroskedasticity may result from model misspecification, most likely from omitted variables (Judge et al.). Offutt, Garcia, and Pinar found that variability of corn yield around a trend increases over time, but including weather variables is likely to remove the heteroskedasticity. Engle (p. 990) claimed:

The existence of ARCH effect would be interpreted as evidence of misspecification.... The ARCH may be a better approximation to reality than making standard assumptions about the disturbance, but trying to find the omitted variables or determine the nature of the structural change would be even better.

Seung-Ryong Yang is a postdoctoral research scientist and Won W. Koo and William W. Wilson are professors in the Department of Agricultural Economics, North Dakota State University, Fargo.

This is Technical Article No. 1964 of the North Dakota Agricultural Experiment Station.

The authors acknowledge B. Wade Brorsen and three reviewers for helpful comments and Jay Larsen for data collection.

An alternative way to model heteroskedastic yields is to incorporate possible sources of heteroskedasticity as a priori information. To remove heteroskedasticity, this approach should be preferred, as Engle suggested, to models that allow heteroskedasticity and approximate the true error structure with time-varying variance models, such as the time-trend variance or the GARCH model. If the analysis fails to identify those sources, then determining how the variance behaves over time becomes important to correctly standardizing the data. If heteroskedasticity results from omitted variables that are dependent, the GARCH process might well approximate the variance. When crop yields follow a GARCH process, ignoring it would bias estimated standard errors and test results (Diebold).

The objective of this study is to specify and evaluate models of heteroskedasticity in crop yields. In this analysis, we conduct a nonnested test of three alternative models of heteroskedasticity: the time-trend variance model, the GARCH specification, and an econometric model that explicitly includes the potential sources of heteroskedasticity.

Model Specifications and Test Procedures

Let y_t be the sample yield per planted acre and X_t' be a vector of explanatory variables for the mean process at time t. The three alternative models to be tested are specified as follows. The time-trend variance model is

(1)
$$y_{t} = X_{t}b + \epsilon_{t},$$

$$\epsilon_{t}^{2} = \alpha_{0} + \alpha_{1}T_{t} + \mu_{t},$$

where b and the α s are unknown parameters, T is a trend variable, and T=1 at the first year of the sample period. ϵ_t and μ_t are disturbances, and μ_t is assumed i.i.d. normal. This model indicates that, regardless of the mean process (i.e., X_t), the squared disturbances have a linear trend.

On the other hand, the alternative GARCH process, which is used extensively for heteroskedastic timeseries data (Akgiray; Aradhyula and Holt; Yang), is designed to allow the conditional variance to change over time. Following Bollerslev's proposition, we used the GARCH(1, 1) specification as follows:

(2)
$$y_t = X_t b + \epsilon_t,$$

$$h_t = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 h_{t-1},$$

$$\epsilon_t \sim N(0, h_t),$$

where the β s are parameters, and ϵ_i follows conditionally a normal distribution with zero mean and time-varying variance, h_v , which is determined by lagged squared disturbance and its own lag. The variance equation is equivalent to an ARMA specification in ϵ_i^2 .

Finally, the econometric model specifies that the disturbances ϵ_t in the above equations have a constant variance, i.e., $h_t = \sigma^2$, for all t. In this alternative model, X_t is a vector of explanatory variables, which causes heteroskedasticity in crop yields.

Potential sources of heteroskedasticity considered in this study are changes in technology, government programs, and climatological variables. The time-trend mean process, which is used most often in the analysis of crop yields, models changes in technology with a trend, interpreting the coefficient as the productivity growth rate. However, in addition to yield increases, technology also could change the variability in yields. For example, most crop breeding programs improve disease resistance. Crop yields would be less variable as disease resistance improves.

Government programs have a significant impact on yields in a particular crop year (Houck and Gallagher). Marginal shifts in area planted (due mainly to government programs) are typically on the less productive land. Thus, variation in the area planted on marginal land should affect the variability in yields.

The variation in key climatological variables appears to be nonconstant. For example, weather patterns in the 1980s differed significantly from those in previous decades. Greater variation appeared in both temperature and precipitation in key growing regions in the 1980s. Thus, changes in weather patterns induce apparent changes in variability in yield trends. Weather-related heteroskedasticity seems to influence regional crop yield data more than aggregated national data, i.e., aggregation tends to average out regional weather effects (Offutt, Garcia, and Pinar). We chose two key climatological variables, temperature and moisture.

Finally, we included a lagged dependent variable to remove autocorrelation, which makes inferences unreliable. This lagged variable also models persistence in average productivity, which may be due either

to continuous cropping patterns or to fertilizer and moisture remaining from previous years. This persistence would reduce yield variability.

The mean process for the econometric model is then specified as

(3)
$$y_t = b_0 + b_1 T_t + b_2 T_t^2 + b_3 T_t^3 + b_4 y_{t-1} + b_5 A_t + b_6 W T_t + b_7 W M_t + \epsilon_t$$

where A_t is the planted acres of sample crop, WT_t is the temperature, and WM_t is the moisture. A cubic function of trend is introduced to capture nonlinearity in growth rates.

For the hypothesis test, we used the nonnested orthodox test. The first step for the test is to nest the alternative models in a more general specification. The general model with the same notations as above is

(4)
$$y_{t} = b_{0} + b_{1}T_{t} + b_{2}T_{t}^{2} + b_{3}T_{t}^{3} + b_{4}y_{t-1} + b_{5}A_{t} + b_{6}WT_{t} + b_{7}WM_{t} + e_{t},$$

$$h_{t} = \gamma_{0} + \gamma_{1}e_{t-1}^{2} + \gamma_{2}h_{t-1} + \gamma_{3}T_{t},$$

$$e_{t} \sim N(0, h_{t}).$$

where the γ s are parameters.

The hypothesis of the time-trend model is H_0 : $\gamma_1 = 0$ and $\gamma_2 = 0$, while that of the GARCH model is H_0 : $\gamma_3 = 0$. The hypothesis of the econometric model is H_0 : $\gamma_1 = 0$, $\gamma_2 = 0$, and $\gamma_3 = 0$. As is always true of nonnested tests, the test may reject all alternative models or fail to reject any of them. There are eight possible cases of test results, including those mentioned above.²

The data considered in this study are average yields of durum and hard red spring (HRS) wheat produced in North Dakota from 1929 to 1988. We used these state-level data to avoid the possibility that sources of heteroskedasticity would be neutralized or masked in the nationwide data. For temperature we used the average of June's daily maximum temperature in North Dakota, and for moisture we used the sum of growing season precipitation (May to July) and the recharged precipitation (October of the previous year to April) measured in inches in North Dakota,³ The data for yields and planted acres were obtained from various issues of North Dakota Agricultural Statistics (North Dakota State University and U.S. Department of Agriculture). The data for the climatological variables were taken from the U.S. Department of Commerce's Climatological Data for North Dakota.

Time-Trend Variance versus GARCH Models

We first estimated a general model combining the time-trend variance and the GARCH models to confirm the existence of heteroskedasticity in the data since the null hypothesis of the econometric model is no heteroskedasticity. The nonnested test between the time-trend variance and the GARCH models is also important since few studies in the literature have considered dependence in yield variation implied by the GARCH specification. The test is performed with the time-series (Bessler) and time-trend mean processes, which were widely used in past studies.

Time-Series Mean Process

The mean process of each crop yield is identified as an ARMA(1, 1) process through autocorrelation and partial autocorrelation functions (Box and Jenkins). We estimated the ARMA(1, 1) process for the mean equation with the same variance equation as in (4).

Table 1 shows the estimated results of the models for both crops. Both durum and HRS yields are serially correlated. Estimated coefficients of the autoregressive and moving-average terms are significant at the 5% level for each crop, using the t-tests. The Ljung-Box test⁴ does not reject the null hypothesis of no serial correlation in the standardized residuals for each model. The models removed autocorrelation in the sample data.

The results of nonnested tests differ for each crop. The test rejects the time-trend variance model in favor of the GARCH alternative for durum, i.e., the GARCH term (h_{i-1}) is significant while the trend is not statistically different from zero at the conventional levels. On the other hand, the test rejects neither time-trend nor GARCH alternatives for HRS, i.e., all variables are significant.

Time-Trend Mean Process

The sample data were autocorrelated as found in the time-series model, so we included a lagged dependent variable. To avoid heteroskedasticity due to misspecified functional form, trend is introduced into the model using a cubic function as in the general model (4).

The estimated results in table 1 indicate significant changes in productivity due to time-dependent

Table 1. Estimated Results of Time-Series and Time-Trend Models with the Time-Varying Variance Equations

1.68 (1.16) 0.92* (0.07) -0.26* (0.11)	1.69 (1.15) 0.90* (0.07) -0.39* (0.17)	8.96* (2.15)	3.88* (1.82)
(1.16) 0.92* (0.07) -0.26*	(1.15) 0.90* (0.07) -0.39*		
(1.16) 0.92* (0.07) -0.26*	(1.15) 0.90* (0.07) -0.39*		
0.92* (0.07) -0.26*	0.90* (0.07) -0.39*	(2.15)	(1.82)
(0.07) -0.26*	(0.07) -0.39*		
-0.26*	−0.39 *		
(0.11)	(0.17)		
	(0.2.)		
			0.42*
			(0.13)
			0.22*
			(0.07)
			_
			_
		(0.0001)	_
		•	
-0.89	-0.97	13.01	-0.71
(1.62)	(1.00)	(9.94)	(1.09)
-0.11	-0.13*	0.30	-0.11
(0.15)	(0.05)	(0.17)	(0.07)
0.92*	0.99*	-0.73*	0.99*
(0.16)	(0.10)		(0.11)
0.18			0.10*
(0.12)	(0.04)	(0.36)	(0.04)
-173.60	-165.27	-168.91	-164.05
g = 10):			
5.17	7.29	14.65	13.22
		6.10	13.17
	-,		
0.540*	0.331	-0.197	0.189
			0.541
			0.069
	(1.62) -0.11 (0.15) 0.92* (0.16) 0.18 (0.12) -173.60	(1.62) (1.00) -0.11 -0.13* (0.15) (0.05) 0.92* 0.99* (0.16) (0.10) 0.18 0.13* (0.12) (0.04) -173.60 -165.27 g = 10):a 5.17 7.29 3.63 7.15 0.540* 0.331 0.463 0.168	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$

Note: Standard errors are in parentheses. An asterisk indicates statistical significance at the 5% level.

technical changes in each crop. However, the growth rate patterns differ. Yields for HRS increase linearly over time, while those for durum increase, first at an increasing rate and then at a decreasing rate. Thus, the second- and third-order terms are not included in the final models for HRS.

The nonnested test results are consistent with those of time-series mean models. In the case of durum, only the GARCH term is significant, rejecting the time-trend variance model in favor of the GARCH alternative, while neither model is rejected for HRS.

The results of these tests indicate that the conventional use of a time trend in variance misspecified the variance structure. The GARCH effects should be considered when modeling heteroskedastic sample yields. Ljung-Box and Kolmogorov-Smirnov tests of fit (D_{max}) were used as diagnostic checks. These tests indicated that the standardized residuals (\hat{e}_i/\hat{h}_i) satisfy the maintained assumption of i.i.d. normal. The estimated model seems to be a valid specification for the sample data.

Econometric Model and Nonnested Tests

We have shown that the residuals in conventional time-series and time-trend mean processes are heter-oskedastic. We now estimate model (4) and conduct nonnested tests among the three alternatives of the

^a Critical values are 14.45 for the time-series models and 16.01 for the time-trend models.

^b Tests applied to the GARCH residuals.

Table 2. Estimated Results of the General Model for the Nonnested Tests

	Durum		HRS	
	GARCH	OLS	GARCH	OLS
Mean:				
Intercept	33.05*	28.02*	25.31*	32.39*
	(15.38)	(10.39)	(4.73)	(9.13)
y_{t-1}	0.34* (0.11)	0.40* (0.10)	0.43* (0.12)	0.29* (0.10)
T	-1.07 *	-0.83*	0.13*	0.10)
_	(0.44)	(0.34)	(0.05)	(0.06)
T^2	0.05*	`0.04 *		_
	(0.017)	(0.012)	_	_
T^3	-0.0005*	-0.0004*		
Acre	(0.0002) -1.76	(0.0001) -1.14	0.02*	_ 1 12*
Acie	(0.97)	-1.14 (0.78)	-0.93* (0.21)	-1.12* (0.38)
WT	-0.37*	-0.37*	-0.30*	-0.38*
	(0.16)	(0.11)	(0.06)	(0.08)
WM	`0.88*	0.95*	0.65*	0.73*
	(0.27)	(0.18)	(0.12)	(0.15)
Variance:				
Intercept	12.91*	11.59*	3.52	7.98*
	(4.62)	(2.31)	(5.18)	(1.57)
e_{t-1}^{2}	0.09	_	0.44	
1.	(0.08)	_	(0.28)	_
h_{t-1}	-0.28 (1.30)	_	-0.49	
T	(1.30)	<u> </u>	(0.44) 0.23	=
•	_		(0.14)	_
Log-Likelihood	-151.96	-153.55	-144.02	-145.06
LR Test	3.18	-133.33 -	2.08	-145.00
Adjusted R ²	_	.83	_	.87
Ljung-Box Test (L	ag = 10):a			
e_{ι}	16.35	_	5.71	. —
e_t^2	7.98	<u></u>	4.26	
e_i/h_i	15.02	_	5.32	_
$(e_{\iota}/h_{\iota})^2$	5.86	_	3.04	_
Normality Test:b		•		
Skewness	-0.113	_	-0.141	_
Kurtosis	1.109*	_	-0.418	_
D _{max}	0.088		0.063	

Note: Standard errors are in parentheses. Acres are divided by 1,000 to avoid the scaling problem. An asterisk indicates statistical significance at the 5% level.

econometric model and the two time-varying variance models. The trend variable in the variance equation for durum is dropped according to the previous findings, since inclusion of an irrelevant variable reduces efficiency.

The estimated results of the general models for durum and HRS are reported in table 2 along with those from ordinary least squares estimation for comparison. In both cases, planted acres had the expected negative impact on yields but was only significant for HRS. This negative coefficient implies the production exhibits decreasing returns to scale with respect to land. Climatological variables are all significant in explaining yield movements for each crop. Moisture affects yields positively, while temperature affects yields negatively.

^a Critical values are 19.02 for the residuals and 16.01 for the standardized residuals.

^b Tests applied for the residuals (not standardized).

The nonnested tests support the econometric model against the other two time-varying variance models. Neither the ARCH nor GARCH terms are significant at conventional levels for either crop. Also, the trend is not significant in the HRS model. The null hypothesis of no heteroskedasticity is also supported by the likelihood ratio (LR) test (H_0 : $\gamma_1 = 0$, $\gamma_2 = 0$, and $\gamma_3 = 0$) for the two crops. The diagnostic tests (Ljung-Box and Kolmogorov-Smirnov tests) show that the residuals (not standardized) are approximately normal with no linear or quadratic dependence. These test results indicate that the econometric model specified in this study corrects for heteroskedasticity found in the time-series and time-trend mean processes.

The standard errors of GARCH estimates are compared to those of OLS. Since the econometric specification removed autocorrelation and heteroskedasticity, OLS estimates should be more efficient. The results support this proposition for the durum model in which all standard errors from OLS are smaller than those from the GARCH model. However, results for HRS are mixed, i.e., some are smaller and some larger.

Finally, to evaluate a farmer's revised perception of yield risk from the econometric model, we calculated standard deviations of raw yields and residuals of the econometric models. The measured unconditional and conditional standard deviations are 8.63 and 3.48 for durum and 8.26 and 2.94 for HRS, respectively. The conditional standard deviations in this study represent farmer risk near harvest time because information about the weather variables in the model is not available until the end of July.

Summary and Conclusions

Most studies on crop yields tend to ignore heteroskedasticity or handle it improperly. Simple linear timetrend as well as time-series models usually encounter variances that change over time. The conventional correction for heteroskedasticity is to standardize the data with their predicted standard deviations. However, we suggest that including factors that cause systematic changes in yield variation should be preferred to models that allow heteroskedasticity and approximate the variance structure. The underlying hypothesis is that heteroskedasticity in crop yields results from omitted variables, explaining the systematic changes in yield variability. To confirm this proposition, we conducted nonnested tests among three alternatives: the time-trend variance, the GARCH, and an econometric model that includes potential sources of heteroskedasticity for durum and HRS produced in North Dakota from 1929 to 1988.

The empirical results showed that the variances of sample yields were serially correlated and/or trending when yields were modeled conditional to time trend only or modeled as ARMA specifications. This heteroskedasticity disappeared when mean yields were conditioned on trend, planted acres, and climatological variables. The results imply that these variables caused heteroskedasticity when they were omitted from the model. Among the variables included, the climatological variables seemed most important for explaining heteroskedasticity. The sample yields conditional only on the weather variables were not heteroskedastic. This was not true for models that only included the time trend (as shown in the previous section) or planted acres. The conventional use of the OLS estimator is acceptable for these sample data, which is convenient for empirical studies.

The results of this study support our proposition that heteroskedasticity may result from model misspecification. Efforts to identify sources of heteroskedasticity may not be too difficult. Climatological variables used in this study were easily incorporated. The conventional time-trend variance model alone could not correctly approximate the systematic changes in the sample yield variation. Past analyses that used monotonically increasing risk in crop yields might provide misleading results.

Another important implication is that the GARCH specification shows promise in modeling heteroskedastic yields. The GARCH effects were significant for the models that did not include those econometric variables. This seems to result from omission of climatological variables that are dependent. If these variables are neither identified nor available, the GARCH specification or the GARCH with a trend in variance would be an important model of heteroskedasticity. In addition, the GARCH model predicts the conditional variance for each observation. Thus, this model should be preferred to a procedure that arbitrarily partitions and standardizes the sample data.

Since this study was restricted to a small set of crop data in North Dakota, drawing general conclusions is difficult. However, the implications of this study should generate a broader discussion of issues on heteroskedasticity in crop yields. A natural extension would be to include other important crops, such as corn, barley, and soybeans, for various levels of aggregation.

[Received June 1990; final revision received November 1991.]

Notes

- ¹ Hereafter we call this model a time-trend variance model.
- ² With n alternatives, there are 2^n possible cases of nonnested test results.

³ The recharged precipitation is for the period preceding wheat seeding in May. Therefore, the moisture represents the total precipitation from October of the previous year to July of the current year. Spring wheat matures in August and harvesting is usually completed by early September; thus, precipitation during the omitted period rarely affects crop yields. Also, in North Dakota, these two months are usually dry and almost all precipitation is lost to evaporation. Precipitation during the period does not add moisture for the next crop year.

⁴ Under the null hypothesis of no serial correlation, the Ljung-Box test statistic with sample size T is

$$Q(K) = T(T+2) \sum_{\tau} p(\tau)^2/(T-\tau), \qquad \tau \leq K,$$

where $p(\tau)$ is the τ th autocorrelation coefficient. When applied to residuals, Q has asymptotically a chi-square distribution with K-s degrees of freedom to adjust for the estimated parameters, and s is the number of lagged dependent variables.

⁵ A reviewer correctly pointed out that the trend term may reflect technical changes, but it may also reflect other, unspecified changes. Most studies use the trend as a proxy for technology. This is mainly because technology as a whole is hardly quantifiable. In this study, the econometric model includes other variables such as weather, acres, and persistence in production. There may not be many other omitted factors that can significantly influence the trend term. To this extent, the trend variable can serve as a proxy for technology change.

⁶ Since yield is the average product of land $(y = O/A = AP_I)$, the negative coefficient implies that $(MP_I - AP_I)/A$

< 0 and $MP_L < AP_L$.

When assessing farmer risks in yields, conditional standard deviations may be more accurate measures than the raw standard deviations. However, the conditional standard deviation is subject to information set at a specific time. For example, near harvest, all information about the variables in the econometric model is available, while at seeding, the data for weather variables are not known.

⁸ The mean equations only with the planted acres showed the GARCH effects. We did not report these results because of space limitations.

References

- Akgiray, V. "Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts." J. Business 62(1989):55-80.
- Aradhyula, S., and M. Holt. "GARCH Time Series Models: An Application to Retail Livestock Prices." West. J. Agr. Econ. 13(1988):365-74.
- Bessler, D. "Aggregated Personalistic Beliefs on Yields of Selected Crops Estimated Using ARIMA Processes." Amer. J. Agr. Econ. (November 1980):666-74.
- Bollerslev, T. "Generalized Autoregressive Conditional Heteroskedasticity." J. Econometrics 31(June 1986):307-27. Box, G., and J. Jenkins. Time Series Analysis: Forecasting and Control. San Francisco: Holden-Day, 1976.
- Diebold, F. "Empirical Modeling of Exchange Rate Dynamics." Lecture Notes in Economics and Mathematical Systems, No. 303. New York: Springer-Verlag, 1988.
- Engle, R. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation." Econometrics 50(July 1982):987-1007.
- Gallagher, P. "U.S. Soybean Yields: Estimation and Forecasting with Nonsymmetric Disturbances." Amer. J. Agr. Econ. 69(November 1987):796-803.
- Hazell, P. "Sources of Increased Instability in Indian and U.S. Cereal Production." Amer. J. Agr. Econ. 66(1984): 302-11.
- -. "Sources of Increased Variability in World Cereal Production Since the 1960's." J. Agr. Econ. 36(1985):145-58. Houck, J., and P. Gallagher. "The Price Responsiveness of U.S. Corn Yields." Amer. J Agr. Econ. 58(November 1976): 731-34.
- Judge, G., W. Griffiths, R. Hill, H. Lutkepohl, and T. Lee. The Theory and Practice of Econometrics, 2nd ed. New York: John Wiley and Sons, 1984.
- Just, R., and R. Pope. "Stochastic Specification of Production Functions and Economic Implications." J. Econometrics 7(1978):67–86.
- North Dakota State University and U.S. Department of Agriculture Statistical Reporting Service. North Dakota Agricultural Statistics, various issues.
- Offutt, S., P. Garcia, and M. Pinar. "Technological Advance, Weather, and Crop Yield Behavior." N. Cent. J. Agr. Econ. 9(January 1987):49-63.
- Singh, A., and D. Byerlee. "Relative Variability in Wheat Yields Across Countries and Over Time." J. Agr. Econ. 41(January 1990):21-33. U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Data Center. Clima-
- tological Data for North Dakota, various issues. Weber, A., and M. Sievers. "Observations on the Geography of Wheat Production Instability." Quart. J. Internat.
- Agr. 24(1985):201-11.
- Yang, S. "The Distribution of Speculative Price Changes." Unpublished Ph.D. thesis, Purdue University, 1989.