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The Semivariance-Minimizing Hedge Ratio

Calum G. Turvey and Govindaray Nayak

This study presents a new approach to the optimal hedging decision. In some empirical studies, the standard hedge using the mean-variance hedge ratio provides results which are inconsistent with downside risk management. The new approach taken here relates the optimal hedge ratio to semivariance rather than variance. An algorithm to solve for the minimum semivariance hedge is presented, and applied to hedging Kansas City wheat and Texas steers.

Key words: downside risk, optimal hedging ratio, risk management, semivariance hedge ratio

Introduction

The traditional notion of effective hedging is that hedgers manage risk by taking an opposite-to-cash position in the futures market. A hedge is effective if the ex post variance of profits is lower with a hedge in place than without. The minimum-variance hedge ratio represents a specific proportion of the cash position to hedge that maximizes the effectiveness of the hedge. The value of the minimum-variance hedge ratio is determined by the covariance between the respective cash and futures prices (Johnson; Heifner).

However, variance as a risk measure has been judged by many economists as too conservative because it regards all extremes as undesirable. The economics of finance (e.g., Markowitz; Quirk and Saposnik; Samuelson; Tsiang 1972, 1974; Borch; Levy; and Fishburn, among others) warns mean-variance analysis should be used with caution unless the probability distributions used in the analysis satisfy certain restrictions such as quadratic utility functions (Markowitz) or normal distributions with the negative exponential utility function (Freund). There is a contention that even when such restrictions are satisfied, or approximately satisfied, decision makers frequently associate risk with failure to attain a target value or return (Lanzilotti; Mao; Markowitz).

To overcome the limitations of mean-variance analysis, Markowitz proposed using semivariance, a notion which was more fully developed by Mao and by Bawa using the lower partial moments of the assets distribution. The heuristic motivation for using semivariance is that minimization of semivariance concentrates solely on the reduction of losses (Hogan and Warren 1972, 1974), a concept which may be generalized as the failure to achieve a stated standard.

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The objective of this study is to propose an alternative approach to hedging which we call the minimum semivariance hedge. The model builds upon earlier research by Mao; Lanzilotti; Hogan and Warren (1972, 1974); Porter; and others who conclude risk-averse decision makers consistently define or explain risk as the chance of failing to meet a targeted level of returns. This definition suggests a focal point of hedging strategies ought not to be variance, but semivariance—which is more specific to downside risk aversion. DeJong, de Roons, and Veld; Eftekhari; Lien and Tse (1998, 2000); and Chen, Lee, and Shrestha have all discussed and developed various approaches to hedging downside risk, but none have examined semivariance hedging in the context of agricultural commodities.

The semivariance hedging model proposed in this analysis differs from other hedging models in several ways. First, it provides an explicit mathematical representation of the semivariance hedge ratio. Second, an iterative approach, rather than an econometric approach, is outlined for calculating this hedge ratio. Numerical approaches have been used by DeJong, de Roons, and Veld, and by Eftekhari, among others. For example, Lien and Tse (2000) calculate portfolio returns for each possible hedge ratio, construct a probability distribution assuming a normal distribution, and then use a grid search method across lower partial moments to determine the optimal hedge ratio that produces the smallest estimated partial moment. Third, although the semivariance model is entirely consistent with the lower partial moment models of Lien and Tse (1998, 2000), and Chen, Lee, and Shrestha, and the stochastic dominance arguments of Fishburn, the need to estimate risk-aversion parameters is avoided by using the mathematical definition of semivariance. Fourth, our model is distribution free in that it requires no prior assumptions about the underlying probability distribution. Finally, a model is applied to agricultural futures contracts for which the relationship between the cash value of a physical commodity and the futures contract is examined.

Other studies have focused more on financial futures, such as currency futures (DeJong, de Roons, and Veld) or stock market indices (Lien and Tse 1998, 2000). However, use of live cattle and wheat futures allows a qualitative assessment of the effects, if any, on storable versus nonstorable commodities. For example Yang, Bessler, and Leatham found that storability does not affect the cointegrated relationship between futures and cash markets. This article concludes with an empirical application of hedging live cattle and wheat futures, and some final comments.

Background

Semivariance is defined as $E\{[\min(K - T), 0]^2\}$, where E is the expectation operator, K is a random outcome (variable), and T is some reference point. T could be the expected value or a fixed target value. Thus, semivariance is measured as the expected value of squared deviations below a fixed target value.[1] The expected value-semivariance (E-SV) model of portfolio selection (Hogan and Warren 1972) identifies as efficient those portfolios that minimize semivariance for a given expected value or that maximize expected value for a given semivariance.

In earlier studies, Harlow and Rao developed an equilibrium asset pricing model using a "mean-lower partial moment" (mean-semivariance) framework, and Skelton and

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[1] Mean-semivariance analysis about the mean is not consistent with expected utility maximization, while mean-semivariance about a fixed point can be (Selley).

Turvey used semivariance directly in a portfolio selection model applied to agriculture. Porter demonstrated a close relationship between stochastic dominance and the mean-target semivariance model. Except for cases of identical means and semivariances, Porter's results indicate if a cumulative probability distribution $F(K)$ dominates another cumulative probability distribution $G(K)$ by second-degree stochastic dominance, then $F(K)$ dominates $G(K)$ by the mean-target semivariance model.

Despite the intuitive appeal of semivariance and downside risk, semivariance has, until fairly recently, been largely ignored in hedging decisions. Target semivariance has been used to some extent in measuring hedging effectiveness (Hauser and Eales 1986, 1987). In this previous work, hedging effectiveness was not measured by reduced variance, but in terms of the probability of final outcomes falling below a fixed target.

How downside risk affects the optimal hedge has also been explored in relation to government programs in agriculture, capital structure, and liquidity (Turvey and Baker 1989, 1990; Arias, Brorsen, and Harri). Government subsidies and stabilization policies for agricultural commodities reduce business risk by essentially truncating the lower partial moments of the cash price distribution. In the context of financial risk, firms with less debt or excess liquidity have a lower probability of bankruptcy than those with high debt or low liquidity. Again, the concern is how outcomes affect downside risk.

Chen, Lee, and Shrestha; Lien and Tse (1998); and DeJong, de Roons, and Veld have developed various hedging models based on Fishburn's lower partial moments model. In Fishburn's framework, semivariance is equivalent to a lower partial moment of two (2.0). DeJong, de Roons, and Veld, in an application of hedging currency futures, use a direct expected utility approach and provide a semivariance hedge ratio stated as a function of marginal utility and risk aversion. Their model requires an estimate of risk aversion to be operational.

Examining lower partial moment hedge ratios for the Financial Times-London Stock Exchange's FTSE-100 index and futures contract, Eftekhari found that the ratios of lower partial moments would have increased portfolio returns while reducing downside risks. Lien and Tse (1998, 2000) further illustrate how Fishburn's framework can be used to hedge downside risk for Nikkei Stock Average futures contracts. Again, their model requires estimates of the risk-aversion coefficient and, because they do not have an explicit analytical expression for the semivariance hedge ratio, they use GARCH or nonparametric approaches to estimate hedge ratios.

Chen, Lee, and Shrestha propose a mean-generalized semivariance approach to hedging. They defend this argument by noting that the generalized semivariance approach considered by DeJong, de Roons, and Veld, and by Lien and Tse (1998, 2000) is not consistent with stochastic dominance (as in Fishburn). Chen, Lee, and Shrestha still require explicit measurement of risk aversion in their model, and as in previous models, they do not provide an explicit closed-form expression for the hedge ratio. In contrast to the semivariance models proposed by DeJong, de Roons, and Veld; Lien and Tse (1998, 2000); and Chen, Lee, and Shrestha, in this analysis an explicit, step-by-step, expression for the semivariance-minimizing hedge ratio is developed, and an approach to deriving the hedge ratio directly from sample price and futures data is provided.[2]

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[2] Another approach to dealing with downside risk is the mean-Gini coefficient (Kolb and Okuney; Shalit; Lien and Shaffer). The Gini coefficient actually measures the covariance between cash-flow outcomes from a hedged position and high-end probabilities. Unfortunately, the Gini hedge ratio requires an explicit measure of risk aversion and requires solving fairly complex numerical grid search techniques (Lien and Shaffer).

Utility Theory and Expected Value-Semivariance (E-SV) Criteria

Quirk and Saposnik were the first to identify deficiencies in the E-V model and provide theoretical support for the E-SV model. They defined a concept of admissibility which corresponds to the more modern idea of stochastic dominance, and then showed that E-V efficient portfolios may be inadmissible, even when compared with portfolios which are not on the efficient frontier. Based on their observations, Quirk and Saposnik determined the E-SV model is not subject to this potential drawback.

Mao compared the merits of the E-V and E-SV criteria by examining the specific utility functions from which these criteria are derived. For example, if an investor's utility function is quadratic, then maximizing the expected utility for a random variable of revenues, profits, or returns, R, $E[U(R)]$, implies $\text{Max} E[U(R)] = \text{Max}[a + b\mu - c\sigma^2]$, where μ is the expected value of R, and σ^2 is the variance of R. This investor chooses his or her portfolio solely on the basis of the mean and variance of R. In contrast, a utility function of the following form would imply the E-SV criteria of an investment appraisal: $U(R) = a + b\mu + c[\min(R - T, 0)]^2$. This utility function is a hybrid in that it is quadratic for $R < T$ and is linear for $R \geq T$. The maximization of expected utility implies the maximization of $a + b\mu + c\psi^2$, where ψ^2 is the semivariance of R with respect to T. Thus, E-V and E-SV criteria follow from utility functions of different shapes. Because the quadratic utility function is concave throughout, the decision maker is consistently risk averse. The utility function in the hybrid case indicates a decision maker is averse to risk associated with returns below a target, but is neutral toward risk at higher returns.

Derivation of an Algebraic Expression for Semivariance

Semivariance is defined as $E\{\min(K - T, 0)^2\}$, where K is a random variable and T is the fixed target below which the investor shows aversion. In financial applications, K might represent investment cash flow or portfolio returns. Let

$$K_T = \begin{cases} T & \text{if } K > T, \\ K & \text{if } K \leq T. \end{cases}$$

Semivariance can now be defined as:

$$\begin{aligned} (1) \qquad \psi_K^2 &= E[K_T - T]^2 \\ &= E[\bar{K}_T + \varepsilon_{K_T} - T]^2 \\ &= E[\bar{K}_T^2 + \varepsilon_{K_T}^2 + T^2 + 2\bar{K}_T\varepsilon_{K_T} - 2\bar{K}_T T - 2\varepsilon_{K_T}T], \end{aligned}$$

where $\bar{K}_T$ is the expected value of K_T, and $\varepsilon_{K_T} \sim N(0, \sigma_{K_T}^2)$ is an independent and identically distributed random variable. Taking the expectation of the right-hand side yields:

$$\begin{aligned} (2) \qquad \psi_K^2 &= \bar{K}_T^2 + T^2 - 2\bar{K}_T T + \sigma_{K_T}^2 \\ &= (\bar{K}_T - T)^2 + \sigma_{K_T}^2, \end{aligned}$$

where $\sigma_{K_T}^2$ is the variance of K_T.

In the hedging problem, the variable K represents a return from a portfolio comprised of a cash and a hedge position. The hedge position is a natural long position, so the downside risk is measured relative to price decreases. Consequently, the variable K is actually a function of two random variables: the cash position ($x = \bar{x} + \varepsilon_x$) and the payoff from trading in futures ($n = \bar{n} + \varepsilon_n$). The final outcome is the sum of realizations on these two random variables:

$$x^{net} = x + n, \tag{3}$$

where x^{net} is the total payoff from the hedge. The variable x represents the net proceeds from transactions in the cash market. Thus, x^{net} can represent the cash price of a commodity grown on a farm, or it can represent the difference between the selling price and the buying price of a stock held in a portfolio. According to equation (3), the hedger sells futures contracts in direct (1:1) proportion to the cash position. Because this may not be an optimal semivariance-minimizing hedge, the hedger may elect to hedge a proportion, δ, of the cash position. Now, the firm's net profit equation can be written as:

$$x^{net} = x + \delta n. \tag{4}$$

Substituting x^{net} for K in equation (3), the semivariance for this problem is given by:

$$\begin{aligned}\psi^2_{x^{net}} = {} & \bar{x}^2_T + \bar{n}^2_T + X^2_c + 2\bar{x}_T\bar{n}_T - 2\bar{x}_T X_c - 2\bar{n}_T X_c \\ & + \sigma^2_{x_T} + \sigma^2_{n_T} + 2\sigma_{x_T,n_T},\end{aligned} \tag{5}$$

where $\bar{x}_T$ is the expected value of x_T, $\bar{n}_T$ is the expected value of n_T with variance $\sigma^2_{n_T}$, X_c is the semivariance target, and σ_{x_T,n_T} is the covariance between x_T and n_T. The expected values of x_T and n_T are obtained from:

$$x_T = \begin{cases} (X_c * x)/x + \delta n & \text{if } x + \delta n > X_c, \\ x & \text{if } x + \delta n \le X_c, \end{cases}$$

and

$$n_T = \begin{cases} (X_c * \delta n)/x + \delta n & \text{if } x + \delta n > X_c, \\ \delta n & \text{if } x + \delta n \le X_c. \end{cases}$$

The introduction of δ as the decision variable capturing the minimum semivariance hedge ratio poses an empirical problem. Specifically, the arguments of semivariance x_T, n_T, $\sigma^2_{x_T}$, $\sigma^2_{n_T}$, and σ_{x_T,n_T} in equation (5) cannot be determined without knowing δ; and δ cannot be solved without determining these arguments. Because these need to be simultaneously determined, there is no explicit closed-form formula for the optimal semivariance-minimizing hedge ratio, δ^*, and the problem must be solved using an iterative approach.

An Iterative Approach to Solve the Minimum Semivariance Hedge Ratio

In this section, an iterative procedure for determining the semivariance hedge ratio is presented (Nayak). Like the approaches used by Eftekhari; DeJong, de Roons, and Veld;

and others, the procedure numerically optimizes the semivariance function. The first step is to initialize the net payoff equation in the first iteration. This is given by:

$$x^{net} = x + \delta_1 n, \tag{6}$$

where δ_1 is an arbitrary proportion to hedge. The semivariance is given by:

$$\psi^2_{x^{net}} = \bar{x}_T^2 + \bar{n}_T^2 + X_c^2 + 2\bar{x}_T\bar{n}_T - 2\bar{x}_T X_c - 2\bar{n}_T X_c + \sigma^2_{x_T} + \sigma^2_{n_T} + 2\sigma_{x_T,n_T}, \tag{7}$$

where x_T and n_T are as defined in (5), but δ_1 is substituted for δ.

The second step is to define a synthetic parameter, Ω, which takes on the value of 1 when semivariance is minimized. For each iteration, Ω can be computed from known variables and will not be equal to 1 if semivariance is not at a minimum. Thus, Ω is introduced into equation (6) by defining:

$$x^{net} = x + \Omega\delta_1 n. \tag{8}$$

As δ_1 is an arbitrary starting point, set $\delta_1 = 1$. Therefore, equation (8) can now be written as:

$$x^{net} = x + \Omega n, \tag{9}$$

and semivariance with respect to a target level, X_c, is given by:

$$\psi^2_{x^{net}} = \bar{x}_T^2 + \Omega^2\bar{n}_T^2 + X_c^2 + \sigma^2_{x_T} + \Omega^2\sigma^2_{n_T} + 2\Omega\bar{x}_T\bar{n}_T - 2\bar{x}_T X_c + 2\Omega\sigma_{x_T,n_T} - 2\Omega\bar{n}_T X_c, \tag{10}$$

where

$$x_T = \begin{cases} (X_c * x)/x + n & \text{if } x + n > X_c, \\ x & \text{if } x + n \leq X_c, \end{cases}$$

$$n_T = \begin{cases} (X_c * n)/x + n & \text{if } x + n > X_c, \\ n & \text{if } x + n \leq X_c, \end{cases}$$

and other variables are as defined previously. Note that expression (10) gives the semivariance if $\Omega = 1$. The process of searching for $\Omega = 1$ in equation (10) involves an iterative procedure where δ is continually adjusted at every iteration until the convergence criterion ($\Omega = 1$) is met. The δ that results in full convergence is the semivariance-minimizing hedge ratio, δ^*.

To simplify the process, a reduced-form expression for Ω is obtained by setting $\partial\psi^2/\partial\Omega = 0$ and solving for Ω. For the first iteration,

$$\Omega_1 = \frac{\bar{n}_T(X_c - \bar{x}_T) - \sigma_{x_T,n_T}}{\bar{n}_T^2 + \sigma^2_{n_T}}. \tag{11}$$

Now use Ω_1 and δ_1 (i.e., $\Omega_1 * \delta_1 = \delta_2$) as the proportion to hedge in the next iteration. The same semivariance expression as in (10) is repeated after substituting for δ_2, and

$$x_T = \begin{cases} (X_c * x)/x + \delta_2 n & \text{if } x + \delta_2 n > X_c, \\ x & \text{if } x + \delta_2 n \leq X_c, \end{cases}$$

$$n_T = \begin{cases} (X_c * \delta_2 n)/x + \delta_2 n & \text{if } x + \delta_2 n > X_c, \\ \delta_2 n & \text{if } x + \delta_2 n \leq X_c. \end{cases}$$

If $\Omega_2 = 1$, then δ_2 is the semivariance-minimizing hedge ratio. If $\Omega_2 \neq 1$, then iterate once again by defining $\Omega_2 * \delta_2 = \delta_3$, and so on. This iterative process will continue until $\Omega_j = 1$ and is stable at 1 thereafter (i.e., $\Omega_{j+1} = \Omega_{j+2} = \Omega_j = 1$). The δ_j which gives the stable $\Omega_j = 1$ is the semivariance-minimizing hedge ratio, δ^*. Thus the dual problem of defining x_T and n_T without knowing δ, and the inability to solve for δ without defining x_T and n_T, is resolved and the semivariance-minimizing hedge ratio can be calculated.[3]

The Minimum-Variance Hedge Ratio

As discussed previously, the minimum-variance hedge is not necessarily the one that minimizes downside risk. The minimum-variance hedge ratio, δ^*, is derived by taking the variance of equation (4). This variance is equal to $\sigma^2 = \sigma_x^2 + \delta^{*2}\sigma_n^2 + 2\delta^*\sigma_{xn}$. Taking the derivative of $\Delta\sigma^2/\Delta\delta^*$ and setting it at zero gives $\delta^* = \sigma_{xn}/\sigma_n^2$. In this analysis, the minimum-variance hedge ratio was calculated by dividing the sample covariance between cash and futures prices by the variance of the respective futures prices. In the next section, the above concepts are illustrated for a number of agricultural commodity contracts.

Data and Procedures

In the empirical analysis, the minimum semivariance hedge ratio is calculated and compared to the minimum-variance hedge ratio. The investigation covers Kansas cash wheat prices hedged with Chicago Board of Trade (CBOT) wheat futures (¢/bushel, daily from November 1980 through February 2000) and Texas steers hedged with Chicago Mercantile Exchange (CME) live cattle futures ($/cwt, daily from June 1989 through February 2000). The data were purchased on CD-ROM from Technical Tools, Inc. (a Chicago firm) and data were processed using software provided by Technical Tools. All futures data are daily data based on the nearby futures contract with a five-day rollover provision. Once the initial data sets were constructed, the cash and nearby futures prices were processed through a database to ensure cash and futures prices were matched by date. The final data set included 4,711 date-matched observations for the Kansas wheat cash price and wheat futures, and 2,503 date-matched observations for the Texas steer-live cattle hedge.

The calculations of both the mean variance and semivariance hedge ratios used data in levels rather than differenced data. The approach is pragmatic. In practice, it is more useful to set a target relative to an actual price than a price difference. In conventional hedge ratio estimation, differencing data are often used to create a stationary time series

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[3] The iterative procedure is not difficult to program. But like many other iterative techniques, speed and efficiency depend upon the accepted tolerance about the optimal solution. For example, a solution tolerance of $\Omega = \{0.99, 1.01\}$ will converge much quicker than a tolerance of $\Omega = \{0.9999, 1.0001\}$.

Table 1. Sample Statistics of Cash and Futures Positions

Description	Wheat Futures (¢/bushel)	Kansas City Wheat (¢/bushel)	Live Cattle Futures ($/cwt)	Texas Steers ($/cwt)
	<--- Nov. 1980 – Feb. 2000 --->		<--- June 1989 – Feb. 2000 --->	
Minimum	232.50	235.50	54.80	54.50
Maximum	716.50	750.00	83.73	84.75
Mean	352.16	375.75	71.47	71.64
Mode	346.24	376.99	73.17	63.12
Median	348.75	376.50	72.53	72.50
Standard Deviation	63.85	72.98	5.46	6.23
Variance	4,076.24	5,326.42	29.78	38.78
Skewness	1.13	1.09	(0.45)	(0.37)
Kurtosis	4.51	4.80	2.33	2.15
Correlations	0.947		0.940	
No. of Observations	4,711		2,503	

required to satisfy (*a*) a Gaussian assumption of random walks, or (*b*) the asymptotic properties of a maximum-likelihood or other parametric estimators. For example, Lien and Tse (1998) argue that if heteroskedasticity is present in the data, then the minimum-variance and semivariance hedge ratios should also be time varying. Because the semivariance hedge ratios in this study are calculated, rather than estimated, there is no need to correct for heteroskedasticity. For direct comparison of the semivariance and minimum-variance hedge ratios, no heteroskedastic adjustment was made in the calculation of the minimum-variance hedge ratio.

Some sample statistics are presented in table 1. The empirical distributions of wheat and cattle cash and futures prices are shown, with their normal distribution approximations, in figure 1 (panels A–D). As observed from the summary statistics reported in table 1, on average, cash prices exceed future prices for wheat, but are about the same for cattle. The variance of the cash price is greater than the futures price, and with similar skewness coefficients, the cash to futures track closely. Risk is varied. The coefficient of variation (σ/E) for wheat is more than twice that of the livestock; however, based on the annualized (250-day)[4] volatility in the percentage daily change in cash prices, the cash market risks for Kansas wheat and Texas steers are almost identical at about 0.22. In contrast, the volatility in wheat futures is about 30% higher than the volatility in cattle futures.

Under the assumption of bivariate normality in the cash-futures price distributions and an unbiased futures market, Lien and Tse (1998) show the minimum-variance hedge ratio will be equal to the minimum semivariance hedge ratio. In figures 1A and 1B (and in table 1), wheat prices (¢/bushel) are shown to be positively skewed and quite leptokurtic, indicating there is a large density of downside risk probabilities. In figures 1C and 1D, Texas steers ($/cwt) and live cattle futures ($/cwt) prices are shown to be slightly negatively skewed and the tails are not as fat as with the wheat prices.

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[4] The use of 250 days represents the approximate number of trading days in one year.

Chi squared, Anderson-Darling, and Kolmogorov-Smirnov tests all reject the null hypothesis of normality at the 1% level. Chen, Lee, and Shrestha also reject the assumption of bivariate normality in their evaluation of S&P500 futures contracts. Formally, this means the minimum semivariance hedge ratio and the minimum-variance hedge ratio will differ. However, the correlation coefficients for both commodity pairs are about 0.94 (table 1), so it is expected the hedge ratios will be in the neighborhood of 1.

The minimum-variance and semivariance hedge ratios were calculated using the following methods.[5]

- The cash position was assumed to be equal to the mean of the sample distribution, and so the expected gain from the hedge position is zero. For both commodities, it was assumed the hedger was initially long in the cash asset.
- The minimum-variance hedge ratio was calculated by dividing the covariance between the cash and futures positions by the variance of the futures position.
- Semivariance targets were generated relative to the mean cash position. Targets were set according to $X_c = \bar{x} + \omega\sigma$, where $\bar{x}$ is the mean cash price with a standard deviation σ. The variable ω is a weight, ranging from -1.0 to 1.0 in 0.10 step increments, to obtain a total of 21 different targets.
- For each of the weights in the preceding step, δ was initialized to 1.00, and the program iterated about equation (8) until Ω [equation (9)] fell within a tolerance of 1.0 ± 0.00001. The resulting δ was recorded as the minimum semivariance hedge ratio.

Results

The results are presented in tables 2 and 3. The first column indicates the number of standard deviations used to set the targets reported in the second column. Column 3 reports the semivariance hedge ratio for each target, and column 4 presents the square root of the calculated semivariance associated with a particular hedge ratio. Column 5 shows the standard deviation of the payoff, and columns 6 and 7 provide the respective minimum and maximum payoff values. The minimum-variance hedge ratio is found in the second to last row, and the last row in the tables shows the risk associated with the unhedged cash position.

Results for the wheat hedge are reported in table 2. The minimum-variance hedge ratio was estimated at 1.08. The minimum payoff for the minimum-variance hedge ratio was 286.66¢/bushel and the maximum was 475.99¢/bushel. At the highest target level of 447.19¢/bushel, the semivariance hedge ratio for wheat was 1.078 and the range of payoff values was 286.99¢/bushel to 476.51¢/bushel. The lowest hedge ratio of 0.894 occurred for a target of 314.18¢/bushel (at -0.8 standard deviations) with a payoff range of 313.79¢/bushel to 518.24¢/bushel. In contrast, the range of the unhedged cash position was a low of 235.50¢/bushel and a maximum of 750.00¢/bushel.

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[5] The semivariance-minimizing hedge ratio was calculated using a visual basic program written in a Microsoft Excel spreadsheet. This model and data are available from the senior author upon request.

Table 2. Minimum Semivariance Hedge Ratios for Kansas Wheat (¢/bushel)

[1] Std. Dev. Used to Set Target	[2] Target (¢/bushel)	[3] Semi-variance Hedge Ratio	[4] (Semi-variance)$^{1/2}$	[5] Standard Deviation of Payoff	[6] Minimum Payoff	[7] Maximum Payoff
1.0	447.19	1.078	77.62	23.78	286.99	476.51
0.9	439.81	1.077	70.62	23.78	287.17	476.77
0.8	432.42	1.075	63.69	23.78	287.40	477.13
0.7	425.05	1.073	56.86	23.78	287.71	477.62
0.6	417.64	1.021	50.15	23.79	288.05	478.14
0.5	410.25	1.068	43.61	23.79	288.44	478.74
0.4	402.86	1.064	37.27	23.80	289.05	479.68
0.3	395.47	1.058	31.17	23.82	289.94	481.05
0.2	388.08	1.051	25.42	23.86	291.01	482.70
0.1	380.69	1.042	20.11	23.91	292.33	484.74
0.0	373.30	1.028	15.40	24.02	294.37	487.90
-0.1	365.91	1.009	11.34	24.22	297.11	492.13
-0.2	385.52	0.988	7.94	24.52	300.20	496.90
-0.3	351.13	0.968	5.23	24.86	303.08	501.35
-0.4	343.71	0.950	3.17	25.23	305.72	505.42
-0.5	336.35	0.928	1.73	35.75	309.03	510.54
-0.6	328.96	0.903	0.75	36.41	312.66	516.14
-0.7	321.57	0.905	0.21	26.36	312.40	515.74
-0.8	314.18	0.894	0.006	26.68	313.79	518.24
-0.9	306.79	0.943	0.00	25.38	306.78	507.06
-1.0	299.40	0.994	0.00	24.44	299.39	495.65
Min.-Var. Hedge Ratio		1.0805	15.60	23.77	286.66	475.99
Unhedged Cash		—	—	73.19	235.50	750.00

Results for the Texas steer hedge are found in table 3. The hedge ratios ranged from a high of 1.146 for a target of 69.15 and 0.904 for the lowest target.[6] The minimum-variance hedge ratio was 1.073. The minimum payoff was highest ($61.04/cwt) for a hedge ratio of 0.904, and the maximum payoff was $89.17/cwt for a hedge ratio of 1.132. The payoff range for the minimum-variance ratio was $60.67/cwt to $88.19/cwt, and the range of the unhedged cash position was $54.50/cwt to $84.75/cwt.

The general result that the semivariance-minimizing hedge ratio is different from the variance-minimizing hedge ratio is consistent with the findings of DeJong, de Roons, and Veld; Lien and Tse (1998, 2000); and Chen, Lee, and Shrestha. How downsize risk is reduced depends on the target and the distribution of risk. As observed in tables 2 and 3 and figure 2, a low target does not necessarily mean a low hedge ratio. For example, in the case of wheat, the hedge ratio decreases from 1.078 to 0.894 as the target falls from 447.19¢/bushel to 314.18¢/bushel. For targets lower than 314.18¢/bushel, the semivariance hedge ratio actually increases. The pattern is quite different for Texas steers. As the target decreases from $77.85/cwt to $69.15/cwt, the semivariance hedge ratio is increasing and then decreases thereafter.

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[6] A hedge ratio greater than 1.0 suggests there is an opportunity to speculate on commodity beyond the cash position. A relatively strong correlation and the fact that the standard deviation of cash prices is greater than the respective futures prices (table 1) can explain the result.

Table 3. Minimum Semivariance Hedge Ratios for Texas Steers ($/cwt)

[1] Std. Dev. Used to Set Target	[2] Target ($/cwt)	[3] Semi-variance Hedge Ratio	[4] (Semi-variance)$^{1/2}$	[5] Standard Deviation of Payoff	[6] Minimum Payoff	[7] Maximum Payoff
1.0	77.85	1.076	6.57	2.116	60.66	88.22
0.9	71.23	1.076	5.98	2.116	60.66	88.24
0.8	76.61	1.077	5.40	2.116	60.66	88.25
0.7	75.90	1.078	4.83	2.116	60.66	88.28
0.6	75.37	1.080	4.27	2.116	60.65	88.31
0.5	74.74	1.082	3.72	2.117	60.64	88.34
0.4	74.12	1.085	3.19	2.117	60.64	88.37
0.3	73.50	1.087	2.68	2.117	60.64	88.40
0.2	72.88	1.089	2.19	2.118	60.63	88.45
0.1	72.26	1.093	1.74	2.119	60.63	88.52
0.0	71.63	1.097	1.35	2.120	60.63	88.62
-0.1	71.01	1.105	1.01	2.123	60.60	88.77
-0.2	70.39	1.116	0.75	2.129	60.58	88.98
-0.3	69.77	1.132	0.55	2.140	60.54	89.17
-0.4	69.15	1.146	0.41	2.153	60.51	88.90
-0.5	68.52	1.126	0.30	2.135	60.55	88.42
-0.6	67.90	1.090	0.23	2.118	60.63	87.84
-0.7	67.28	1.048	0.17	2.120	60.72	87.21
-0.8	66.66	1.001	0.14	2.152	60.82	86.87
-0.9	66.04	0.976	0.11	2.181	60.87	85.89
-1.0	65.41	0.904	0.099	2.309	61.04	84.82
Min.-Var. Hedge Ratio		1.073	1.35	2.116	60.67	88.19
Unhedged Cash		—	—	6.22	54.50	84.75

In terms of risk measurement, the semivariance, measured by its square root in tables 2 and 3, falls as the target falls. This is expected. Likewise, the standard deviation of the semivariance-minimizing hedge is always higher than the variance-minimizing hedge (unless they are equal). The fact that a semivariance hedge can reduce the downside probabilities relative to a minimum-variance hedge is reason enough to pursue a lower partial moment model. For example, from table 2, the lowest outcome for the minimum-variance hedge was 286.66¢/bushel. Hedging 89.4% of wheat (at -0.8 target) rather than 108.05% increases the downside to 313.79¢/bushel, representing a 9.4% (313.79/286.66 - 1.0) increase in the lower bound of profits. Likewise, the maximum value calculated was 518.24¢/bushel, which represents an 8% (518.24/475.99 - 1) increase in the upper payoff level over the minimum-variance hedge.

Of course, such results are not as pronounced for the cattle hedge in table 3. But referring again to table 1, the wheat prices tended to be positively skewed (a greater chance of actual prices falling below the mean), and therefore had more downside risk relative to the target associated with them. Cattle prices were negatively skewed (a greater chance of an outcome above the mean), and therefore they had less downside risk relative to the target. Similar results were found by Lien and Tse (2000) where the minimum semivariance hedge ratio was either less than or greater than the minimum-variance hedge ratio, with the trend signaling a lower semivariance hedge ratio for lower targets.

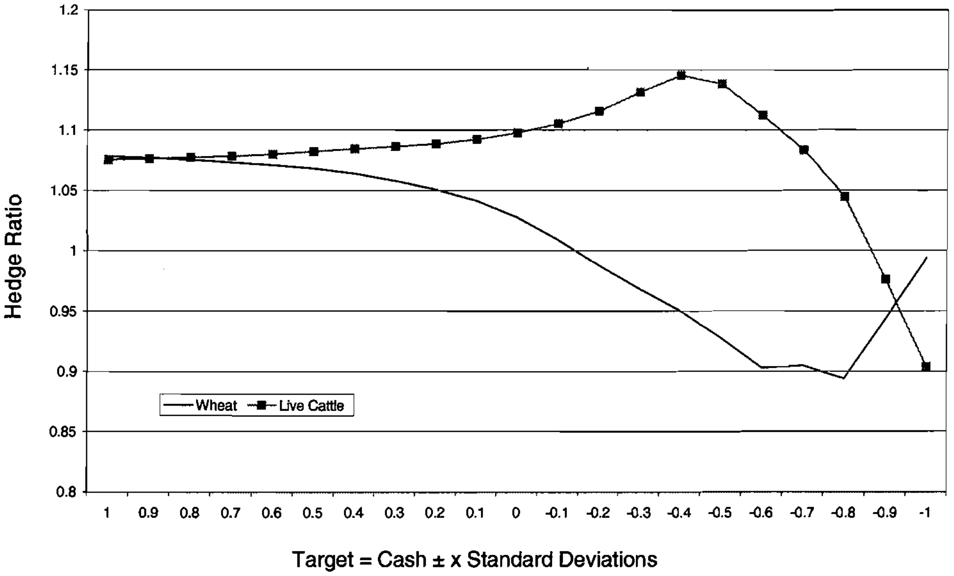

Figure 2. Semivariance-minimizing hedge ratios for various targets

Consistent with our results, Lien and Tse (2000) did not find a uniform decrease in the semivariance hedge ratio; however, their results did not show the concave or convex shapes found in figure 2.

The main conclusion drawn from the wheat and cattle results is that the minimum semivariance hedge will protect the downside more than the minimum-variance hedge. Yet, no general statement can be made regarding the magnitude because the semivariance hedge ratio is sensitive to the nature of the probability distributions of the underlying cash and forward positions as well as the chosen target. However, relative to profits, the cash position will be more variable than the semivariance hedge payoff, which in turn will be more variable than the minimum-variance hedge. This result is consistent with theory because the semivariance-minimizing payoff distribution cannot have a variance lower than the variance-minimizing payoff distribution.

Conclusions

The variance-minimizing hedge implicitly assumes hedgers are willing to forego upside opportunities to reduce total variance. This study proposed a method to derive the downside risk-reducing (semivariance-minimizing) hedge ratio. The expected value of outcomes below a fixed target was used as the basis for deriving first a mathematical expression for semivariance, and then the mathematical expression for the semivariance-minimizing hedge. Because of simultaneity, an iterative rather than parametric approach to solving the problem was presented.

Findings show the semivariance hedge ratio can differ from the minimum-variance hedge ratio. For both Kansas wheat and Texas steers, the hedge position was lower using the semivariance criteria than with the minimum-variance criteria. The amount hedged is contingent on the target selected and the distribution of probabilities below

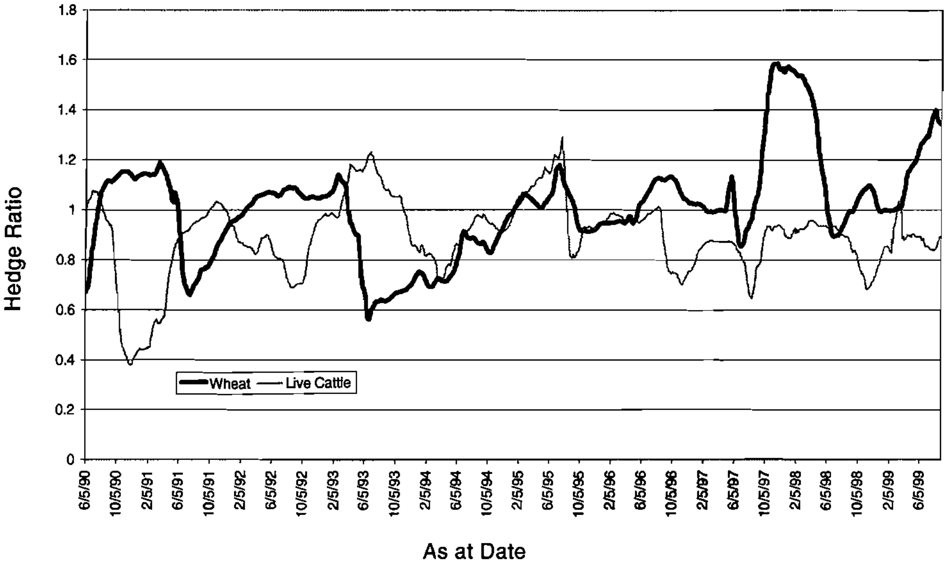

Figure 3. Rolling 250-day variance-minimizing hedge ratios for Kansas wheat and Texas steers

the target. Based on the empirical results, wheat had a greater potential to decrease downside risk through semivariance hedging than Texas steers. These results are generally consistent with the findings of Yang, Bessler, and Leatham that storability does not affect the cointegrated relationship between cash and futures markets. The gains in hedging may not appear to significantly reduce the variance reduction, or the minimum and maximum of hedged payoffs, but when considered in the context of a single or multiple contracts, the downside risk reduction can be quite substantial.

However, a number of related issues, including the sample time frame, basis between cash and forward markets, and the shape of the underlying probability distribution functions, can affect the magnitudes of the gains in hedging from the semivariance approach. While the shape of the distributions and relationship between cash and futures were discussed in the text, in figure 3 we calculate rolling 250-day sample minimum-variance hedge ratios. The variability of these hedge ratios suggests (as do Lien and Tse 1998) the semivariance hedge ratio will also be affected by the data sample frame. Nonetheless, the semivariance hedge is always more effective than the minimum-variance hedge at reducing downside risk.

For both the wheat and steer hedges, the post-hedge probability distribution function had a minimum higher than the minimum-variance hedge and, in the case of wheat, it had a higher maximum. Furthermore, in both cases, the semivariance hedge ratio was less than the minimum-variance hedge ratio for most targets, including those set in-the-money. In addition, an important and distinguishing characteristic of the minimum semivariance hedge ratio is that it is distribution free.

The ideas presented in this study provide a new approach to solving the problem of efficient hedging. The potential gain in efficiency achieved by focusing on downside risk rather than variance when making hedging decisions may be significant. From a practitioner's point of view, applying a semivariance hedge can reduce the number of futures contracts. The extent of reduction depends on the chosen target. An important contribution of this analysis is that it allows the hedger the opportunity to fix a target, above or below expected prices, and to hedge optimally on this target. The minimum-variance hedge ratio does not permit such flexibility. In addition to this practical side of semivariance hedging, a further contribution of this study is the theoretical development of the semivariance hedging model and a numerical solution that can be used by producers, traders, and other hedgers in the futures markets.

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